Double Up on Heaven

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Having read (Arntzenius, Elga, and Hawthorne 2004), Donald Trump Jr. knows about the situation that faced his father when he died and went to the afterlife: by default, he was supposed to spend eternity in Purgatory (0 utils per day), but before going there he was offered the possibility of going to Hell (-1 util per day) for one day in exchange for being allowed to spend the following two days in Heaven (1 util per day) before returning to Purgatory. He was also informed that if he rejected the offer, he would never get a similar chance; and that if he accepted it, he would be asked again after having spent one day in Hell whether he would accept staying in Hell for a second day in order to increase his number of subsequent days in Heaven to four, and so on. That placed Donald Trump Sr. in the difficult position of having to make a series of choices, each of which seemed advantageous, but which in combination would lead to the horrible result of him staying in Hell forever.

When Trump Jr. himself arrives at the Pearly Gates, St. Peter has something slightly different in store for him. He only has to make a single binary choice. The first option is to go to Purgatory for all eternity. The second again involves going to Hell for a number of days and then, when he leaves Hell, going to Heaven for twice as many days. In that scenario, it will be decided by chance how many days he will spend in Hell: he will start in Hell and every time he has endured a number of days there that is a power of three, a fair, regular die will be thrown, and if it shows six, he will move to Heaven the next day. According to St. Peter, this means that Trump Jr. gets to enjoy Heaven without the risk of languishing in Hell for all eternity. That is, with probability 1 he will spend some finite number of days in Hell, twice as many days in Heaven, and then the rest of his afterlife in Purgatory.

However, Trump Jr. notices a problem. If he takes the second option, the function that maps a day's number to the probability of him being in Hell on that day is $p : \mathbb{N} \to [0, 1]$, defined recursively by $p(1) = 1$ and $p(n) = \frac{5}{6} \cdot p(\lceil \frac{n}{3} \rceil)$, $n > 1$, where $\lceil x \rceil$ is the smallest integer not less than x.[1]

[1] Here is why: Let $n \in \mathbb{N} \setminus \{1\}$ be given and let m be the largest number in $\mathbb{N}_0$ such that $3^m < n$. Then $3^m < n \leq 3^{m+1}$ and $3^{m-1} < \lceil \frac{n}{3} \rceil \leq 3^m$. Therefore, the recursive definition ensures that p of any natural number in the interval $(3^m, 3^{m+1}]$ is $\frac{5}{6}$ times p of any natural number in the interval $(3^{m-1}, 3^m]$, as it should. This table may be helpful:

n	1	2	3	4 ··· 9	10 ··· 27	28 ··· 81	···
$\lceil \frac{n}{3} \rceil$		— 1 —		2 ··· 3	4 ··· 9	10 ··· 27	···
$p(\lceil \frac{n}{3} \rceil)$		— 1 —		— $\frac{5}{6}$ —	— $(\frac{5}{6})^2$ —	— $(\frac{5}{6})^3$ —	···
$p(n)$	1	— $\frac{5}{6}$ —		— $(\frac{5}{6})^2$ —	— $(\frac{5}{6})^3$ —	— $(\frac{5}{6})^4$ —	···

Since being in Heaven on day n is equivalent to having spent at least $\lceil \frac{n}{3} \rceil$ and less than n days in Hell, the probability of him being in Heaven on day n is $p(\lceil \frac{n}{3} \rceil) - p(n)$. It follows that the expected utility of day n is

$$\begin{aligned} 1 \cdot (p(\lceil \tfrac{n}{3} \rceil) - p(n)) + (-1) \cdot p(n) &= p(\lceil \tfrac{n}{3} \rceil) - 2 \cdot p(n) \\ &= p(\lceil \tfrac{n}{3} \rceil) - 2 \cdot \tfrac{5}{6} \cdot p(\lceil \tfrac{n}{3} \rceil) = -\tfrac{2}{3} \cdot p(\lceil \tfrac{n}{3} \rceil) \end{aligned}$$

for all $n > 1$. Since p is positive for all arguments and the expected utility for day 1 is -1, this means that every day has a negative expected utility.

Trump Jr. is as baffled as his father was. Via the second option, St. Peter promises him twice as many days in Heaven as in Hell, but when Trump Jr. considers any given day in his great beyond, the first option – purgatory from the beginning – is expected to be better for his well-being on that day. He can't tell which of the two choices is the better one. He is aware that there is no well-defined expected utility for the totality of his afterlife if he takes the second option, but that seems to him to be the absence of what could otherwise have been used to settle the matter, rather than any reason to be less baffled by the conflict. Is the zero probability possibility of him staying in Hell forever making the second option a bad choice by, so to speak, contributing $0 \cdot -\infty$ to the expected utility?

What should Trump Jr. do?

Acknowledgments

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References

Arntzenius, F., A. Elga, and J. Hawthorne (2004). Bayesianism, infinite decisions, and binding. *Mind 113*, 251–283.