

# DOWNLINK CHANNEL DECORRELATION IN CDMA SYSTEMS WITH LONG CODES

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**Abstract** - In this paper we develop linear detectors suitable for a Code Division Multiple Access (CDMA) mobile receiver using long codes. The special signal structure in the downlink transmission is exploited to obtain a simple detection rule. A least-squares (LS) detector, a best linear unbiased estimator (BLUE) detector, and a linear minimum mean-square error (LMMSE) detector are derived. For the LMMSE detector we consider an adaptive implementation. The results show that improvement can be achieved using the proposed detectors compared with that of the conventional RAKE receiver.

## I. INTRODUCTION

In CDMA reception at the mobile end there are several special requirements. The resources for processing are severely constrained by the physical size of the receiver and the strict limitations for power consumption. Furthermore, the other users' codes are not necessarily known at the mobile receiver and the estimation of other users' channel parameters especially may involve too complex processing for the mobile resources. This implies that detection algorithms based on simultaneous optimal detection of multiple users [1] or its suboptimal approximations [2], [3] may have to be abandoned. Hence, the multiple-access interference (MAI) has to be suppressed by other techniques.

Adaptive multiuser detection has shown to be a promising technique for interference suppression in a DS-SS system. The low complexity of the adaptive implementation compared with that of other techniques is especially attractive if implemented in a mobile terminal. Unfortunately, the adaptive interference suppression scheme in [4] requires *short* user codes in order to obtain a cyclostationary signal. If *long* codes are used or, as in the downlink, a scrambling code extends over several symbols, the cyclostationary property is lost. In the downlink, the transmission is synchronous and the users can be assigned orthogonal codes. However, in a multipath channel, inter-path interference will destroy the orthogonality and cause

multiple-access interference. In a system employing short codes this interference can be suppressed with adaptive techniques described in for example [4]-[6]. If a long scrambling code is used like in the WCDMA proposal [7], the cross-correlation of the user signals are changing from symbol to symbol and, consequently, other techniques must be used. This paper proposes three linear detectors suitable for CDMA downlink with long codes.

The paper is organized as follows. Section II describes the downlink signal model used. Three linear schemes are derived in Section III. In Section IV, an adaptive implementation is considered. The performance of the receiver and its adaptive implementation is demonstrated with a simplified example in Section V, followed by conclusions.

## II. SIGNAL MODEL

In downlink transmission, a mobile receiver will receive the signals associated with a number of simultaneously active users over the same mobile channel. Figure 1 shows a block diagram of the system considered.

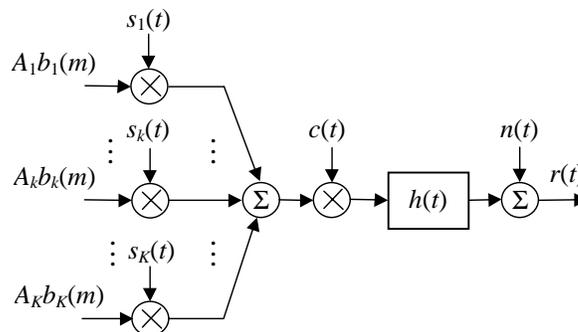


Figure 1: Block diagram of the system.

The system under consideration consists of  $K$  users transmitting information over a channel with  $L$

multipath components. The received signal at the mobile can be written as

$$x(t) = \sum_{m=1}^M \sum_{k=1}^K \sum_{l=1}^L h_l(t) A_k b_k(m) c(t - \tau_l) s_k(t - mT_b - \tau_l) + n(t) \quad (1)$$

where  $h_l$  is the  $l$ th path's channel coefficient, and for the  $k^{\text{th}}$  user,  $A_k$  is the amplitude,  $b_k(m) \in \{-1, 1\}$  is the  $m^{\text{th}}$  bit,  $s_k(t)$  is signature sequence (code) with  $G = T_b/T_c$  number of chips per bit,  $c_n(t) \in \{-1, 1\}$  is the scrambling sequence separating different cells, and  $n(t)$  is additive noise.

The received signal is passed through a chip-matched filter and sampled at chip rate. If the samples from the received sequence are collected in a vector, we can write the received discrete-time signal as

$$\mathbf{r} = \mathbf{HCSd} + \mathbf{n} = \mathbf{Hu} + \mathbf{n} \quad (2)$$

where  $\mathbf{H}$  is a  $(MG+L-1) \times MG$  convolution matrix containing the channel coefficients,  $\mathbf{C}$  is the  $MG \times MG$  scrambling matrix with  $\mathbf{C}^2 = \mathbf{I}_{MG \times MG}$ ,  $\mathbf{S}$  is a  $MG \times MK$  block diagonal matrix whose diagonal consists of spreading matrices,  $\mathbf{d}$  is a  $MK \times 1$  vector consisting of the user amplitudes and transmitted bits. Finally,  $\mathbf{n}$  is a zero-mean vector of noise components with covariance matrix  $\mathbf{Q}$ . Let us also define the vector  $\mathbf{u} = \mathbf{CSd}$  containing the transmitted multiuser signal. The structures of the individual components are given below.

$$\mathbf{S} = \text{diag}[\bar{\mathbf{S}} \bar{\mathbf{S}} \dots \bar{\mathbf{S}}] \quad (3)$$

$$\bar{\mathbf{S}} = [\mathbf{s}_1 \mathbf{s}_2 \dots \mathbf{s}_K] \quad (4)$$

$$\mathbf{C} = \text{diag}[c(1) c(2) \dots c(MG)], c \in \{\pm 1\} \quad (5)$$

$$\mathbf{d} = [\mathbf{d}^T(1) \mathbf{d}^T(2) \dots \mathbf{d}^T(M)] \quad (6)$$

$$\mathbf{d}(m) = [A_1 b_1(m) A_2 b_2(m) \dots A_K b_K(m)] \quad (7)$$

By assigning the users orthogonal codes, we have  $\bar{\mathbf{S}}^H \bar{\mathbf{S}} = G \cdot \mathbf{I}_{KM \times KM}$  and consequently  $\mathbf{S}^H \mathbf{S} = G \cdot \mathbf{I}_{K \times K}$ .

The model in (1) permits the users to have different data-rates like in the ETSI proposal since orthogonality between different users with different rates is preserved with orthogonal channelization codes or multiple codes. Therefore, it is still possible to write the received signal on the form  $\mathbf{r} = \mathbf{Hu} + \mathbf{n}$ , and for a particular user of interest we only need to consider the appropriate symbol interval.

### III. LINEAR DETECTORS

In this section, we look at three different detection schemes suitable for the downlink. The main idea is to

try to estimate the vector  $\mathbf{u}$  containing the orthogonal user signals. Once an estimate of the vector  $\mathbf{u}$  is obtained we can simply apply the conventional matched filter for the demodulation of a single user. The schemes considered are the least-squares (LS) detector, the best linear unbiased estimator (BLUE) detector, and the linear minimum mean-square error (LMMSE) detector.

#### A. LS detector

Our first approach is to obtain the least-squares estimate of the vector  $\mathbf{u} = \mathbf{CSd}$ . After some straightforward calculation we get

$$\hat{\mathbf{u}}_{LS} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{r} = \mathbf{C} \bar{\mathbf{S}} \mathbf{d} + (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{n} \quad (8)$$

We now descramble our estimate obtaining

$$\hat{\mathbf{C}}\mathbf{u}_{LS} = \mathbf{C}(\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{r} = \bar{\mathbf{S}} \mathbf{d} + \mathbf{C}(\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{n} \quad (9)$$

After descrambling the signal, the users are orthogonal to each other and for detecting user one's  $m^{\text{th}}$  bit we need only to apply the matched filter, i.e.,

$$\hat{z}_{LS}(m) = \mathbf{s}_{1m}^H \mathbf{C}(\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{r} \quad (10)$$

where

$$\mathbf{s}_{1m}^H = \begin{bmatrix} \underbrace{0 \dots 0}_{(m-1)G} \mathbf{s}_{1m}^H \underbrace{0 \dots 0}_{(M-m)G} \end{bmatrix} \quad (11)$$

#### B. BLUE detector

The above least-squares detector is not optimal in the case of non-white noise. To include the effect of non-white interference, e.g., adjacent cell interference, we can use the best linear unbiased estimator (BLUE). The derivation is straightforward, see for example [8]. The BLUE estimate of the vector  $\mathbf{u}$  is obtained as

$$\hat{\mathbf{u}}_{BLUE} = (\mathbf{H}^H \mathbf{Q}^{-1} \mathbf{H})^{-1} \mathbf{H}^H \mathbf{Q}^{-1} \mathbf{r} \quad (12)$$

After descrambling the signal we obtain

$$\hat{\mathbf{C}}\mathbf{u}_{BLUE} = \bar{\mathbf{S}} \mathbf{d} + (\mathbf{H}^H \mathbf{Q}^{-1} \mathbf{H})^{-1} \mathbf{H}^H \mathbf{Q}^{-1} \mathbf{n} \quad (13)$$

The users are now orthogonal and we can apply the matched filter in (10) for detection of the desired users symbols. The decision statistic for user one's  $m^{\text{th}}$  bit is

$$\hat{z}_{BLUE}(m) = \mathbf{s}_{1m}^H (\mathbf{H}^H \mathbf{Q}^{-1} \mathbf{H})^{-1} \mathbf{H}^H \mathbf{Q}^{-1} \mathbf{r} \quad (14)$$

If compare the BLUE detector in (14) with the LS detector in (10) we see that the BLUE detector require the knowledge of the noise covariance matrix.

### C. LMMSE detector

It is well known that the above solutions suffer from noise enhancement. An alternative is to use the linear mean-square error (LMMSE) estimate to overcome this problem. We now view  $\mathbf{u}$  as a random vector with the first and second moments given by  $E[\mathbf{u}] = 0$  and  $E[\mathbf{u}\mathbf{u}^H] = \sigma_u^2 \mathbf{I}$  respectively. The LMMSE estimate of  $\mathbf{u}$  is obtained as

$$\hat{\mathbf{u}}_{LMMSE} = \sigma_u^2 \mathbf{H}^H (\sigma_u^2 \mathbf{H}\mathbf{H}^H + \mathbf{Q})^{-1} \mathbf{r} \quad (15)$$

The LMMSE estimate will trade off the noise enhancement by not completely restoring the orthogonality between the users. Applying the matched filter as in (10) we obtain an estimate of the  $m$ th bit as

$$\hat{z}_{LMMSE}(m) = \mathbf{s}_{1m}^H \mathbf{C}\mathbf{H}^H (\sigma_u^2 \mathbf{H}\mathbf{H}^H + \mathbf{Q})^{-1} \mathbf{r} \quad (16)$$

The leftmost  $\sigma_u^2$  in (15) could be neglected in (16) as it only acts as a gain factor.

If we take a closer look at (16) we can identify  $\mathbf{s}_{1m}^H \mathbf{C}\mathbf{H}^H$  as the conventional RAKE receiver for the  $m$ th symbol. The LMMSE estimate is obtained by first applying  $(\sigma_u^2 \mathbf{H}\mathbf{H}^H + \mathbf{Q})^{-1}$  to the received signal and thereafter pass it through a RAKE receiver. In the next section we will look at an algorithm implementation of the LMMSE using this structure.

### IV. ALGORITHM

The detectors above can be used to detect the desired signals in block of several symbols at the time. The LMMSE receiver requires knowledge of both the channel coefficients and the noise covariance matrix  $\mathbf{Q}$ . In this section we look at an adaptive implementation of the LMMSE detector.

For the solution of (16) we only assume the knowledge of the channel coefficients to be able to perform the RAKE combining as discussed in the previous section. To motivate the approach taken here, we first realize that employing adaptive algorithms like the LMS or the RLS for solving (16) would require both the knowledge of a known transmitted sequence and all the user sequences to build a reference signal at the receiver. To overcome this problem, we instead estimate the inverse  $(\sigma_u^2 \mathbf{H}\mathbf{H}^H + \mathbf{Q})^{-1}$  directly. To obtain an estimate we first recognize that

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^H] = (\sigma_u^2 \mathbf{H}\mathbf{H}^H + \mathbf{Q}) \quad (17)$$

For the estimation of  $\mathbf{R}$  we use [9]

$$\mathbf{R}(m) = \alpha \bar{\mathbf{r}}(m) \bar{\mathbf{r}}^H(m) + (1 - \alpha) \mathbf{R}(m-1) \quad (18)$$

where  $\bar{\mathbf{r}}(m)$  is the  $m$ th symbol including its multipath components, and  $\alpha$  is a small factor balancing the past and the present signal information. Applying the matrix inversion lemma to (17) we obtain an estimator for the inverse  $\mathbf{D} = \mathbf{R}^{-1}$  as

$$\mathbf{D}(m) = \frac{1}{1 - \alpha} \left[ \mathbf{D}(m-1) - \frac{\mathbf{t}(m) \mathbf{t}^H(m)}{\frac{\alpha}{1 - \alpha} + \mathbf{r}^H(m) \mathbf{t}(m)} \right] \quad (19)$$

$$\mathbf{t}(m) = \mathbf{D}(m-1) \mathbf{r}(m)$$

An advantage of the LMMSE implementation above is that the channel estimation can be moved to the RAKE receiver and, therefore, the resulting structure does not require any additional knowledge than the RAKE receiver. The estimation of  $\mathbf{R}$  utilizes the power from all the users in the downlink since they propagate over the same channel.

### V. NUMERICAL RESULTS

In this section, we study the performance of the proposed detectors. The performance of the detectors is compared with that of the RAKE receiver.

The system under consideration consists of 8 users transmitting with the same power. We consider a 3-path constant channel where the delays are 0, 1, and 2 chips and the corresponding values of the coefficients are:  $h_1=0.4$ ,  $h_2=0.3e^{j2\pi/6}$ , and  $h_3=0.4e^{j2\pi/3}$  corresponding to those chosen in [2]. The spreading codes are chosen as Walsh-Hadamard codes of length 16. The scrambling code is taken as part of a Gold code sequence as given in [7].

In the simulations we assume the noise to be Gaussian with covariance matrix  $\mathbf{Q} = \sigma^2 \mathbf{I}$ . As a consequence the BLUE and the LS detector will yield the same results and we, therefore, only simulate the block versions of the LS (B-LS) and the LMMSE (B-LMMSE) detectors in (11) and (16) respectively, and the adaptive implementation of the LMMSE detector.

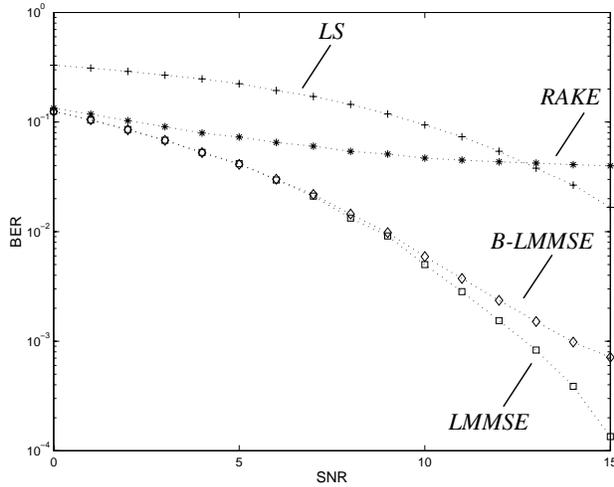
In Fig. 2, the bit-error rate (BER) as a function of the signal-to-noise ratio is shown. The base station transmits with the same power to all the users, here set to unity, i.e.,  $A_k = A_1 = 1$ . For the B-LMMSE and the B-LS detectors a block size of 10 symbols was used. For the adaptive LMMSE  $\alpha$  was chosen to 0.01.

As can be seen from the figure, both the B-LMMSE and the adaptive LMMSE provides better BER as compared with the B-LS and the RAKE. The advantage of using the adaptive LMMSE over its block version is that no knowledge of the noise covariance matrix is needed. The LS detector performs better than

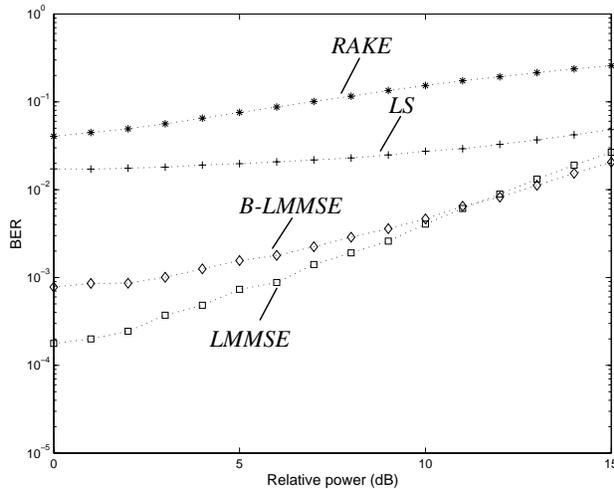
RAKE receiver does only in the high SNR region due to the noise enhancement.

In Fig. 3, three of the interfering users vary their powers relative to the user of interest. The average signal-to-noise ratio for the desired user and the remaining interfering users is fixed at 15 dB.

Fig. 3 clearly shows the advantage of using the LMMSE receiver when the user signals have different powers.



**Figure 2:** BER as a function of SNR in a system with 8 users in a 3-path channel.



**Figure 3:** BER as a function of the relative powers of two interfering users in a system with 8 users and a 3-path channel.

## VI. CONCLUSIONS

In this paper, we have presented detectors suitable for implementation at the mobile receiver in a CDMA system using long scrambling codes. Three detectors were considered: the LS, BLUE, and the LMMSE. An adaptive version of the LMMSE detector was proposed requiring no more knowledge than that of the conventional RAKE receiver. The LMMSE detector performed best of all the schemes. The LS solution suffers from noise enhancement and the performance is only better than the RAKE in the high SNR region. Future work will include channel estimators as a part of the receiver.

## ACKNOWLEDGEMENTS

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