Downlink Fluid Model of CDMA Networks

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***Abstract*—Classical CDMA models mostly consider hexagonal networks. In spite of their simplicity, the hexagonal networks need complex calculations. We develop an analytical fluid model of the network, simpler than the hexagonal one. It can provide explicit formulas of capacity. As an application of this model, we indicate the densification problem of a UMTS network which means the increase of the number of base stations in a limited zone to satisfy a traffic need.**

Keywords: fluid model, CDMA, interferences

I. INTRODUCTION

Considering a mobile and its server base station (BS), the amount of interference, due to the other BSs of the network for the downlink, depends on many parameters such as the powers, the positions and the number of the transmitters and receivers: BSs or mobiles. An analytical study of the capacity needs a model to calculate these interferences. Classical models mostly consider hexagonal networks [1-2] (sometimes random ones, see [5]). In spite of their simplicity, the hexagonal networks need complex computations. The purpose of this study is to develop an analytical network model, simpler than the classical hexagonal one. It can allow obtaining explicit expressions of characteristics of the network (capacity, throughput), and to investigate other kind of problems such as the densification

The paper is organized as follows. First we recall the downlink analysis of a WCDMA network. We particularly insist on the fundamental influence of the interferences due to the other stations and show that it can be interpreted as a characterization of the CDMA network. Afterwards, we introduce the fluid model and establish the expressions of the interference parameter factor. We validate the model, comparing it to a hexagonal one. Finally we propose a possible application: the densification of a UMTS network.

II. ANALYSIS OF WCDMA NETWORKS

Considering the downlink of a multi-service WCDMA system [6], the required minimum power received at a mobile from its BS carrying a call is determined by a condition concerning the Signal to Interferences ratio (SIR): it has to be at least equal to a minimum threshold target value [4] [5]. Using this condition in the mono service case, the cell manages one kind of mobiles, we establish the influence of the network on the SIR target. Afterwards, we focus the analysis on mobiles positioned at different distances from the BS: Our goal is to obtain the explicit dependence of the interference factor. The equation, established for one traffic channel, is generalized to the whole power used by the base station of the cell: all traffic channels and common channels powers. We will assume that each mobile only uses one kind of service.

II.1 Impact of the network

Let us consider a network of N_{BS} base stations. A mobile *i* is connected to the base station *b* of this network. Using the equation of the transmission traffic channel power [4], we can write the following relation:

$$\frac{P_{received\ ,i}}{\alpha_{ortho}\, I_{own} + I_{other} + Noise} \geq \left(\frac{C}{I}\right)_{t\arg et\,,i} \qquad \text{(II.1)},$$

$P_{received\ ,i}$ received power by the mobile *i*, I_{own} is the interferences due to the common channels and the traffic channels of the other mobiles in the same cell, and I_{other} is the interferences due to the other BSs. $\left(\frac{C}{I}\right)_{t\arg et,i}$, noted γ_i, represents the level of the signal to interference target ratio for the service used by the mobile *i.* Introducing the transmission power P_{TCH_i} of the traffic channel between the base station *b* and the mobile *i. Noise* stands for the level of noise floor at the mobile receiver, $g_{j,i}$ the total pathloss between the mobile *i* and the base station *j;* this term thus particularly contains the pathloss depending on the distance and the shadowing and α_{ortho} the factor of orthogonality, the relation can be rewritten as

$$\frac{P_{TCH,i}\, g_{b,i}}{\alpha_{ortho}(P_b - P_{TCH,i_i})g_{b,i} + \sum_{j=1, j\neq b}^{N_{BS}} P_j g_{j,i} + Noise} \geq \gamma_i \qquad \text{(II.2)}$$

P_b is the total transmission power of the base station *b*, *including the common channels assumed orthogonal.* We express that the SIR received has to be at least equal to γ_i as

$$\frac{1}{\alpha_{ortho}\left(\frac{P_b}{P_{TCH_i}} - 1\right) + \frac{1}{P_{TCH_i} g_{b,i}} \sum_{j=1, j\neq b}^{N_{BS}} P_j g_{j,i} + \frac{Noise}{P_{TCH_i} g_{b,i}}} = \gamma_i \qquad \text{(II.3)}$$

<u>Remark:</u> If the cell is isolated and the *Noise* very low compared to the received signal, the second and the third term of the

(*) Eitan Altman has been collaborating with France Telecom R&D within a research contract denominator disappear. The SIR target only depends on the fraction of the power of the BS b dedicated to the traffic channel. To minimize the other base stations influences allows maximizing the received SIR. The interferences and the *Noise* could be considered as perturbations on received SIR.

This equation shows the influence of the network, especially the positions of the base stations, on the cell. The second term of the denominator appears as a fundamental characteristic parameter of the system.

II.2 Role of the intercellular interference parameter

For each mobile i managed by a BS b, we can define the parameter f_i , as the ratio between the total power received by the mobile i coming from the other base stations to the total power received by its server base station b:

$$f_i = \frac{I_{other}}{I_{own} + P_{TCH_i} g_{b,i}} = \frac{1}{P_b g_{b,i}} \sum_{j=1, j\neq b}^{N_{BS}} P_j g_{j,i} \quad \text{(II.4)}$$

This ratio strongly depends on the position of the mobile in the cell. Let's assume that all the base stations have the same power $P_i = P_b$, which is reasonable if the traffic is uniform. The expression of f_i becomes, in the case of an UMTS system,

$$f_i = \sum_{j=1, j\neq b}^{N_{BS}} \frac{g_{j,i}}{g_{b,i}} \qquad \text{(II.5)},$$

Introducing $\beta_i = \dfrac{\gamma_i}{1+\alpha_{ortho}\gamma_i}$, we can express from (II.2)

$$P_{TCH_i} = \beta_i(\alpha_{ortho} P_b + f_i P_b + \frac{1}{g_{b,i}} Noise) \quad \text{(II.6)}$$

II.3 Model

The key modelling step of the model we propose consists in replacing a given fixed finite number of base stations by an equivalent continuum of base stations which are distributed according to some distribution function. When a uniform traffic and a uniform BS density are assumed, and using a model where the pathloss $g_{b,i}$ is only a function of the distance r (between the base station b and the mobile i), f_i only depends on this distance. So this interference parameter which was written as a function of i (with an index i) can be written now as a function of r: The mobiles positioned at the same distance from the BS have the same interference parameter f_r , because the density of BS is considered as uniform. All the mobiles positioned on circles s whose centre is the server BS b have thus the same pathloss, which can be written $g_{b,s}$, and the same interference factor. Classical network models consider either hexagonal or random positions of the base stations. They are also considered as uniform but locally there are not uniform because the power received at a point of the cell depends not only on the distance r between the server BS and the mobile, but also on an angle θ characterizing the relative positions of the base stations. If we consider a unique service and mobiles positioned on circles around the BS b, the "type" s could represent the mobiles that are positioned on the circle s.

III. CALCULATION OF f_r : FLUID MODEL

If the network is homogeneous with a density of stations ρ_{BS} per unit of surface, and a density of mobiles ρ_{MS} , at any point of the network, the power received by a mobile is calculated as follow. When we consider a mobile i at a distance r from his server BS, there are $\rho_{BS}.r.dr.d\theta$ base stations which contribute to I_{other} . And their influence on the mobile i is $\rho_{BS}.r.dr.d\theta.Kr^{\eta}$ with the pathloss model $L(r) = Kr^{\eta}$, where η is the pathloss factor. So if we consider the whole network, we can write $I_{own} = P_b g_{b,i} = Kr^{\eta}$ and

$$I_{other} = \int_{2R_c - x}^{R-x} \rho_{BS}.r.dr.d\theta.Kr^{\eta} = -\frac{\rho_{BS}.2\pi}{\eta+2} K\left[(2R_c - x)^{\eta+2} - (R-x)^{\eta+2}\right]$$

with R_c : cell radius, R: network "radius", x: distance between the base station b and the mobile, in the considered cell. The parameter f_i only depends on the distance r. We denote it f_r :

$$f_r = \frac{I_{other}}{I_{own}} = \frac{\int_{2R_c - r}^{R-r} \rho_{BS}.u.du.d\theta.Ku^{\eta}}{Kr^{\eta}} = -\frac{\frac{\rho_{BS}.2\pi}{\eta+2} K\left[(2R_c - r)^{\eta+2} - (R-r)^{\eta+2}\right]}{Kr^{\eta}}$$

We obtain

$$f_r = -\frac{\rho_{BS}.2\pi}{(\eta+2)r^{\eta}}.\left[(2R_c - r)^{\eta+2} - (R-r)^{\eta+2}\right] \quad \text{(II.7)}$$

We replaced a discrete network of N_{BS} base stations by a continuous one, whose density is ρ_{BS} . We assumed that the network is uniform. If it is not the case, for example ρ_{BS} becomes $\rho_{BS}(r,\theta)$ the approach remains the same, only (II.7) differs. Here, the expression of the interference parameter f_r appears very simple (II.7). It depends on the density of base stations, the radius of a cell and the radius of the considered network. This expression opens a large number of possibilities of analysis for CDMA networks because it gives precisely the influence of such a network on every point of a given cell.

IV. VALIDATION OF THE MODEL

To validate the model, we choose to compare it to a hexagonal classical one. The validation of the model will consist on comparing the analytical values of the interference parameter to the ones obtained with the hexagonal network.. We build a numerical hexagonal network and calculate numerically at each point of a given cell, the interference parameter. The base stations are omni-directional. We calculate f_i for a mobile *i*, positioned in a cell of such a network, with a numerical method, using its definition given by the expression (II.7): We choose the pathloss parameter $\eta = -3$, a radius cell $R_c = 1$ km. To simplify notations, we drop the index *i* of the interference parameter: $f = f_r$. To validate the model, we need to verify the expression (II.7). In particular, to validate the fluid model we need to check whether indeed *f* depends on the distance only. We further check the values of *f* obtained with the two methods to analyze the influence of the radius of the network *R* and the one of the cell radius R_c, the influence of the BS density and the influence of the pathloss parameter η

Fig 1 shows the hexagonal network used for the validation. Each point represents an omni directional station. Each point corresponds to the location of a BS. Fig. 2 shows the points in the analyzed hexagonal cell of iso-interference factor *f*. Four cases are given (0.084, 0.31, 0.83, 1.48).The locations at which these values are obtained are very close to circles (with some deviations that depend on the *angle* θ). The approximation that ignores the dependence on the angle is what the fluid model predicts. The corresponding radii are 400m, 600m, 800m 900m (measured from the server BS). The corresponding relative deviations from exact circles are less than 10^{-3} for distances up to 600m, and up to 3% for distances up to 900m.

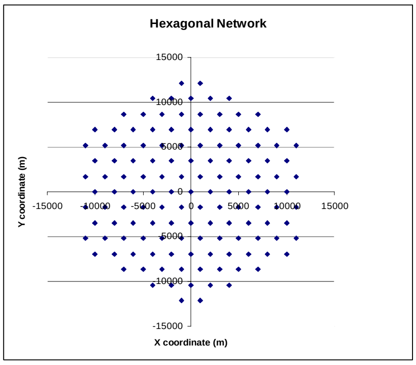

Figure 1: Hexagonal network

The relative deviation reaches 10% at the edge of the cell (1000 m). Next, we compare the real values obtained in the two approaches. Fig. 3 compares *f* obtained with the hexagonal network to the fluid one using the equation (II.7). We observe that the values are identical. For the largest values of the distance *r*, over 800m, we can see that *f* "hexagonal" has variations at a given distance. These variations correspond to all the values obtained at that distance. The values and the curves obtained confirm that the analytical approach we followed is justified: we can modelize a hexagonal network by a continuous density BS network. This model allows calculating the parameter *f* at each point of a cell, with a very simple way, as we made. However, it seems interesting to precise the influence of the network size. Indeed, the values of *f*, and the dependence of these values only on the distance as we observed, may vary if the network is different. We thus considered four networks sizes, and compared the two approaches.

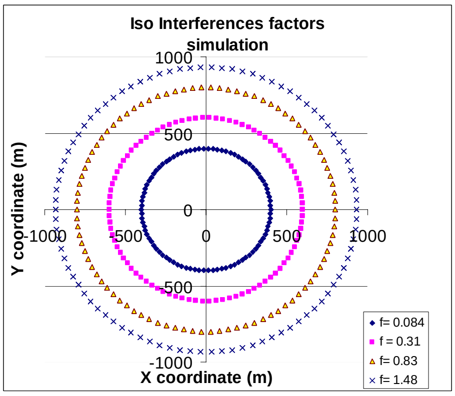

Figure 2: Hexagonal network: Interference factor as a function of the location

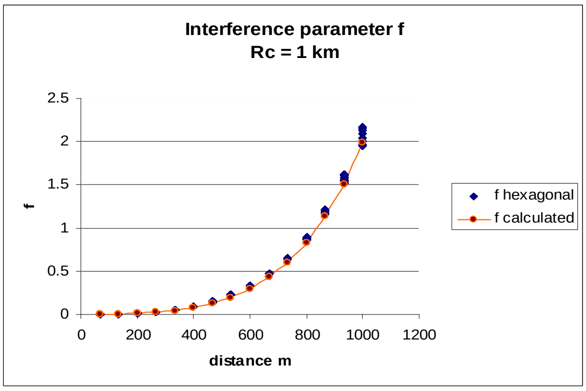

Figure 3: Comparison of the interference parameter, hexagonal and analytically calculated, vs distance from the BS

Remark

The presented curve of the figure 3 is drawn for a network radius of $R = 40\, R_c$. For $R = 2R_c$ $R = 4R_c$ and $R = 12R_c$ the same behaviour of the curves are obtained.

Fig. 4 shows the values of *f* in the hexagonal case (obtained numerically) for different network sizes: 2 R_c, 4 R_c, 12 R_c, 40 R_c. We observe that the size of the network has an influence on *f*: it becomes larger when the network is larger. The differences observed on the examples can reach up to 0.5. However, this influence decreases when the size of the network increases: The network whose size is 12 R_c gives almost the same values of *f* than the one whose radius is 40 R_c. We moreover observe that the variations of f, at a given distance, seem the same whatever the network size is. Consequently the relative variations of f decrease when R increases, because f increases with R. One could think that the fluid model would be better with a large network, i.e. the variations of f (at a given distance) would be smaller when a larger network radius is considered. The numerical calculations show that it is not the case. It means that the validity of the fluid model does not depend on the size of the network: for any given network size, the fluid model gives values very close to the hexagonal's ones.

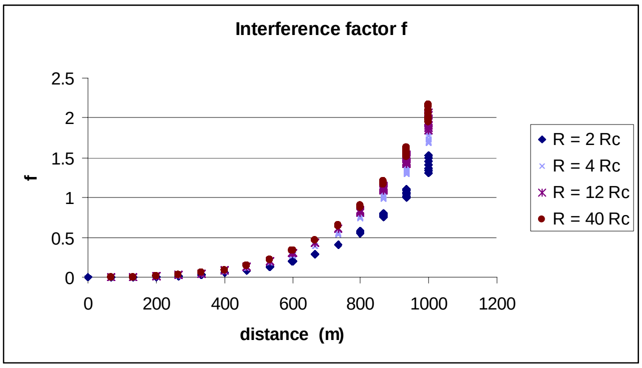

Figure 4: Interferences factor numerical values for different network size.

We can understand this result as follows. The first circle of stations around the analysed one gives the principal influence on the parameter f. The second one has a lower influence, and the following ones have lower and lower influences. At a given distance from the server BS, the numerical calculated values of f vary because there is not a "continuum" of stations (hexagonal network). A second circle added to the network (Fig 5) increases the total power received at each point of the cell of the server BS. However, this increase is very low compared to the power due to the first circle of stations because of the great distances of the considered stations, and also because of the value of pathloss factor η (-3). Considering mobiles at the same distance from the server base station (i.e. circles), the variations of the power they receive, due to their position along the circle from the network are thus not very important: For a given distance, the power received can be considered *as almost* the same.

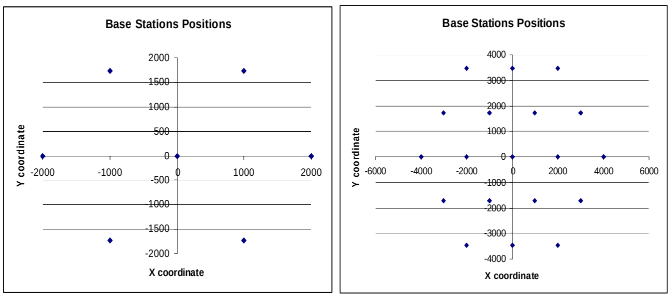

Figure 5: hexagonal network: 1 circle and 2 circles of stations around the server BS

Eq (II.7) shows that f also depends on the BS density. To validate this dependency, we compare the results obtained with three BS densities, corresponding to different cell radius: 0.5 km, 1 km and 2 km. The curves (fig. 6a and 6b) show f obtained using fluid model for cell radius of 0.5 km and 2 km are close to the ones obtained with the hexagonal one.

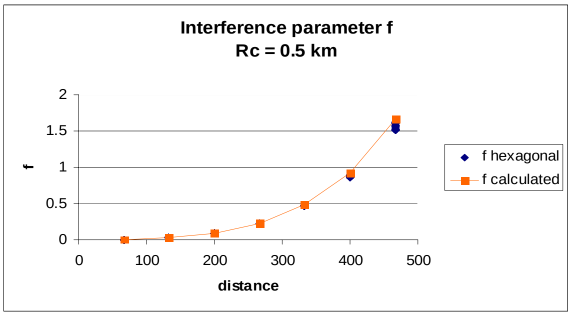

Figure 6a: Interference parameter with $R_c = 0.5$ km

Next we check the influence of the factor η. Fig.7 shows the results with $\eta = -4$. We can observe that the model is still accurate, but for the larger values of the distance, the results are more dispersed than before.

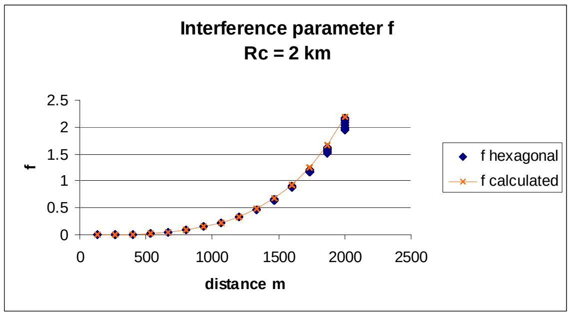

Figure 6b: Interference parameter with $R_c = 2$ km

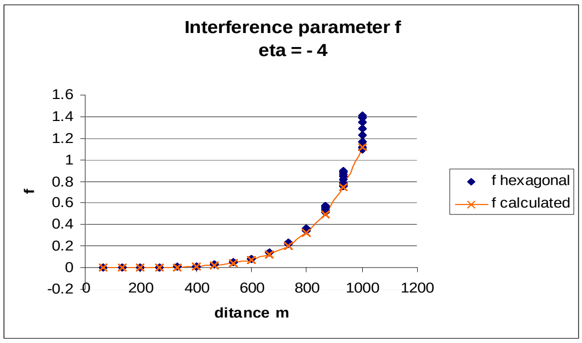

Figure 7: Interference parameter with $\eta = -4$

V. APPLICATIONS DENSIFICATION

To satisfy the traffic demand, we need engineering rules for choosing the density of BSs. If the BSs are already deployed with an insufficient density in a zone (e.g. a hot spot), we may need to add BSs: this is the "densification". Our fluid approach provides such rules. Considering a cell of a UMTS network, we established the following condition in order to guarantee the SIR target, for the downlink [3]

$$\sum_i N_i \beta_i < \frac{(1-\varphi)}{\alpha_{ortho} + F_b} \quad \text{(V.1)}$$

φ represents the fraction of the total power of the base station used by the common channels. This relation means that the number N_i of mobiles of type i is *limited* by a factor depending on the mean interference factor. That one was defined as $F_b = \frac{1}{N_{MS_b}} \sum_{m=1}^{N_{MS_b}} f_m$ (V.2)

Here, N_{MS_b} represents the mobiles of the cell considered and f_m the interference factor on the mobile *m*, whatever its type. Equation (V.2) considers all the mobiles of the cell. Equations (V.1) and (V.2) show that the limited capacity is related to the interference parameter f_m related to a mobile *m*: This last one is proportional to the density of base station (see equation II.7). We define a zone as a geographical area covered by several cells whose number is to be determined. The capacity of a zone is the sum of capacities of the cells covering the zone. And the capacity of each cell is related to the set of N_i satisfying (V.1). We can deduce that the capacity of a zone is directly connected to the density of base stations. To modify this capacity, one solution can consist on the densification. Using the equation (V.1), we can determine the accurate density of base stations to satisfy an increasing traffic.

VI. Conclusion

We established that for a CDMA system, the whole network has a fundamental influence on each given cell. We considered the interference parameter *f* which represents the weight of the other base stations on a mobile in a given cell. We showed that this parameter characterizes the impact of the network. We developed a *fluid model* to establish an analytical expression of it, replacing the N_{BS} discrete BS by a density ρ_{BS} of BS. Comparing this model to a hexagonal one to analyse its validity, this model seems accurate. The model is especially accurate until 80% of the cell radius, because the relative error is less than 1 %. For distances over 80% of the radius, the error can reach 10%. Its validity does not depend on the size of the network. It allows some analytical calculations which are not possible, or which are difficult, with a hexagonal model. Compared to a hexagonal one, this model is simpler and easier to manipulate. Its precision is analogue: the parameters of the network are *a priori* neither best nor worse. And moreover due to the fact this approach is analytic it allows making a more powerful analysis of CDMA systems. As an application of our model we presented the densification of a UMTS network. It becomes possible to calculate the density of BS to satisfy certain traffic.

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