Downlink Power Allocation for Multi-class Wireless Systems

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Abstract

In this paper we consider a power allocation problem in multi-class wireless systems. We focus on the downlink of the system. Each mobile has a utility function that characterizes its degree of satisfaction for the received service. The objective is to obtain a power allocation that maximizes the total system utility. Typically, natural utility functions for each mobile are non-concave. Hence, we cannot use existing convex optimization techniques to derive a global optimal solution. We develop a simple (distributed) algorithm to obtain a power allocation that is asymptotically optimal in the number of mobiles. The algorithm is based on dynamic pricing and consists of two stages. At the *mobile selection* stage, the base-station selects mobiles to which power is allocated. At the *power allocation* stage, the base-station allocates power to the selected mobiles. We provide numerical results that illustrate the performance of our scheme. In particular, we show that our algorithm results in system performance that is close to the performance of a global optimal solution in most cases.

Index Terms

Power allocation, downlink, wireless networks, and non-convex optimization.

I. INTRODUCTION

In recent years, the area of power control in wireless networks has received significant interest from both academic and industrial researchers. Power control plays an important role in the efficient management of code division multiple access (CDMA) networks. Since voice has been the main service provided by

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wireless networks thus far, most research efforts have been devoted to voice systems. In voice systems, typically all users have the same quality of service (QoS) requirements and it is important that the signal to interference and noise ratio (SINR) exceeds some minimum threshold. Hence, the main purpose of power control in such systems is to eliminate the near-far effect by equalizing the SINR of each user by setting it at the minimum SINR threshold [1], [2].

In the next generation of wireless networks, it is expected that services will have significantly differing characteristics from the current voice-dominated systems. Already, the demand for various services with different QoS requirements such as video and data is increasing. The required bandwidth for these services is much higher than that for voice services, further compounding the scarcity of resources in wireless systems. Therefore, to more efficiently accommodate services with different characteristics, we need a new approach for power control in next generation wireless networks. A distinguishing feature of many of these new services is their elasticity, i.e., they can adjust transmission rates (to some degree) based on the channel conditions and the congestion level of the system. Hence, by appropriately exploiting the elasticity of such services, we can maintain high network efficiency and prevent network congestion. Moreover, such services are highly asymmetric, requiring more bandwidth in the downlink than the uplink. This implies that, in the next generation of wireless networks, efficient resource allocation for the downlink becomes an important issue [3], [4], [5].

Recently, the concept of utility (and pricing) from economics has been used to develop network control algorithms by exploiting the elasticity of the services. The utility represents the degree of a user's (service's) satisfaction when it acquires a certain amount of the resource, and the price is the cost per unit resource that the user needs to pay. Hence, services with heterogeneous QoS requirements (elasticity) can be modeled with different utility functions. The basic idea of these algorithms is to control the users' behavior by pricing resources appropriately to obtain the desired results, (e.g., high utilization for the overall system and fairness among users).

In wireline networks, utility and pricing based algorithms have been studied for distributed flow control of best effort services [6], [7], [8]. In these works, the utility function is assumed to be a concave function of the allocated rate, which results in a convex programming problem. Hence, the Karush-Kuhn-Tucker (KKT) conditions or the duality theorem can be used to obtain the optimal solution.

Utility (and pricing) based control algorithms can also be applied to the power control problem in wireless networks. However, the main difficulty in solving the problem is that, in general, it cannot be formulated as a convex programming problem, since the utility function may not be concave [9], [10], [11], [12]. Thus, neither the KKT conditions nor the duality theorem provides a sufficient condition for the optimal solution. In most works on utility and pricing for power control, only Nash equilibria, which are inefficient from the point of overall system utility [13], have been obtained.

In [9], [10], [11], a power control problem is formulated as a non-cooperative N-person game in which each mobile transmits a power level that maximizes its (net) utility without considering the behavior of the other mobiles. They show that their algorithms converge to Nash equilibria. Further, in [10], [11], the authors show that, by introducing pricing, system efficiency can be improved. In these works, the base-station informs each mobile of a fixed price per unit power and each mobile transmits at a power level that maximizes its net utility (utility minus cost for power allocation). They show that the system utilization significantly depends on the choice of price. However, they do not provide a systematic algorithm to find an optimal price. In [12], [14], a downlink resource allocation problem is considered with restricted types of utility functions. In [12], only voice services are considered and utility functions are modeled as step functions and in [14], utility functions are modeled as concave functions. In these works, the authors obtain the optimal prices for maximizing the total system utility and the total revenue. In [15], [16], capacity regions and optimal power and rate allocation schemes are studied from an information theoretic point of view.

In this paper, we study the downlink power allocation problem for multi-class wireless networks. We use a utility based framework as in other works. However, the situation considered here differs from previous works in many aspects. Primarily, we consider general types of utility functions that are suitable for multi-class systems and may be non-concave. This generalization requires a significantly different analysis than the works of [12], [14]. We also study the problem of maximizing total system utility for heterogeneous users that provides a higher system utility than those considered in [9], [10], [11].

We put an emphasis on the efficiency of the system. However, due to the non-convexity of the problem, obtaining a global optimal power allocation is difficult and, if feasible, would require a very complex algorithm. Therefore, we develop a simple (distributed) algorithm that provides an asymptotically (in the number of mobiles) optimal power allocation. This algorithm can be implemented in either a distributed or centralized way. If implemented in a centralized way, the base-station must know certain information about the mobiles, such as path gain from the base-station to the mobile, the interference level at the mobile, the utility function of the mobile, and so on. In addition the computational burden is imposed all on the base-station. If implemented in a distributed way, the base-station need not know the detailed information about the mobiles, and the computational burden can be distributed among the base-station and mobiles. This is suitable for the case when the base-station does not know the utility function of the mobile [17], [18]. However this requires iterative communication between the base-station and mobiles for the algorithm to converge. In this case, our problem can be expressed as a utility and dynamic pricing problem. The dynamic pricing attribute is also another distinguishing feature of this work compared with other works.

The rest of the paper is organized as follows. In Section II, we describe the system model considered in this paper and formulate the basic problem. In Section III, we present the power allocation algorithm, which consists of the mobile selection stage and the power allocation stage. In Section IV, we study the asymptotic optimality (in the number of mobiles) and the lower bound on the performance of our power allocation. In Section V, we study a special case when all mobiles are homogeneous. Numerical results are provided in Section VI. Finally, we conclude in Section VII.

II. System Model and Problem Description

Our objective is to determine the appropriate power levels at which the base-station should communicate to the different mobiles (the downlink power allocation problem in a multi-class wireless network). We focus on a single cell consisting of a single base-station and M mobiles. The system is assumed to be time-slotted. At each time-slot, the power allocation algorithm is executed. A time-slot in our system is an arbitrary interval of time and could consist of one packet or several packets. We focus on a time-slot assuming that the path gain, background noise, and intercell interference for each mobile do not change during this time-slot. Each mobile communicates with the base-station. For downlink communication, the base-station has a maximum power limit, P_T. It allocates power to each mobile within the power limit (i.e., the sum of the power allocated to each mobile cannot exceed the power limit). Each mobile i, $i = 1, 2, \cdots M$, has its own utility function, U_i that represents the degree of mobile i's satisfaction of the received QoS and is a function of the "generic" signal quality for mobile i. We first define γ_i, the "generic" signal quality for mobile i as follows:

$$\begin{aligned}\gamma_i(\bar{P}) &= \frac{N_i G_i P_i}{G_i \theta(\sum_{m=1}^{M} P_m - P_i) + I_i} \\ &= \frac{N_i P_i}{\theta(\sum_{m=1}^{M} P_m - P_i) + A_i},\end{aligned} \tag{1}$$

where

P_i: Allocated power for mobile i.

$\bar{P}$: Power allocation vector, $(P_1, P_2, \cdots, P_M)$.

N_i: Constant for mobile i.

G_i: Path gain from the base-station to mobile i.

I_i: Background noise and intercell interference to mobile i.

A_i: "Goodness" of the transmission environment of mobile i, which is defined by I_i/G_i.

M: Number of mobiles in the cell.

θ: Orthogonality factor ($0 \leq \theta \leq 1$).

Note that if $\theta \neq 0$, γ_i and the utility function U_i depend not only on mobile i's own power allocation but also on the power allocations of all the other mobiles. In the above equation, if $N_i = 1$, then the signal quality metric γ_i represents the SINR for mobile i. If N_i is the processing gain for mobile i, which is defined by W/R_i, where W is the chip rate for the CDMA network and R_i is the data rate at which the base-station transmits to mobile i, then γ_i represents the bit energy to interference density ratio of mobile i, $(E_b/I_0)_i$, in the CDMA system. If $N_i = W$, then $\gamma_i = (E_b/I_0)_i R_i$ of mobile i in the CDMA system. In this case, for a given power allocation, i.e., for a given $\gamma_i(\bar{P})$, R_i and $(E_b/I_0)_i$ have an inversely proportional relationship. Hence, there exist appropriate R_i^* and $(E_b/I_0)_i^*$ for a given $\gamma_i(\bar{P})$, and they may have different values for different $\gamma_i(\bar{P})$. Hence, in this case, each mobile may receive variable data rates (i.e., variable processing gains) and they can be adjusted appropriately based on the power allocation. Thus, the utility value can depends on R_i^* and $(E_b/I_0)_i^*$.

We further assume that U_i has the following properties.

Assumptions:

(a) U_i is an increasing function of γ_i.

(b) U_i is twice continuously differentiable.

(c) $U_i(0) = 0$.

(d) U_i is bounded above.

(e) If $\sum_{i=1}^{M} P_i = P_T$[1], then $U_i(\gamma_i(\bar{P}))$ is one of three types: a sigmoidal-like[2], a strictly concave, or a strictly convex function of P_i, its own power allocation.

[1] We will show this in Lemma 1.

[2] A function $f(x)$ is said to be a sigmoidal-like function if it has one inflection point, x^o and $\frac{d^2 f(x)}{dx^2} > 0$ for $x < x^o$ and $\frac{d^2 f(x)}{dx^2} < 0$ for $x > x^o$.

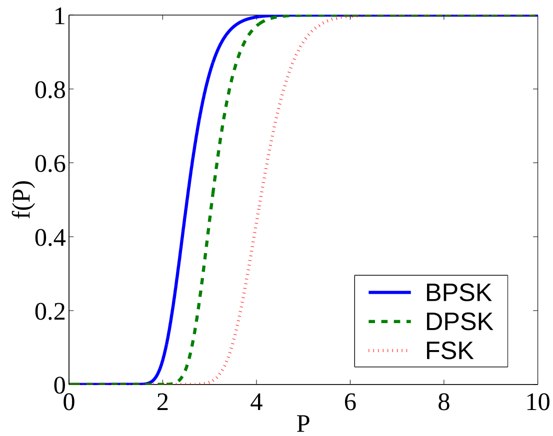

Fig. 1. Probabilities of packet transmission success for BPSK, DPSK, and FSK modulation schemes.

Note that typically, most utility functions used in wireline or wireless networks can be represented by three types of functions in assumption (e) [11], [19].

For instance, we can define the utility function of each mobile i as its expected throughput, which is defined as $U_i(\gamma_i(P)) = R_i f_i(\gamma_i(P))$, where R_i is the data rate received at mobile i and $f_i(\gamma_i(P))$ is the probability of packet transmission success of mobile i. In Fig. 1, we provide the probability of packet transmission success for various modulation schemes such as Binary Phase-Shift Keying (BPSK), Differential Phase-Shift Keying (DPSK), and Frequency-Shift Keying (FSK) [20]. We assume that a packet consists of 800 bits without channel coding and set $P_T = 10$, $\theta = 1$, $N_i = 16$, and $A_i = 0.7407$. As shown in this figure, the probability of packet transmission success is represented by a sigmoidal-like function of its power allocation. Hence, in this case, we have sigmoidal-like utility functions.

The goal of this paper is to obtain the power allocation for each mobile that maximizes the total system utility (i.e., the sum of the utilities of all mobiles). The basic formulation of this problem is given by the following optimization problem:

$$\begin{aligned} \text{(A)} \qquad & \max_{\bar{P}} \sum_{i=1}^{M} U_i(\gamma_i(\bar{P})) \\ & \text{subject to} \quad \sum_{i=1}^{M} P_i \le P_T, \\ & \qquad\qquad\quad 0 \le P_i \le P_T, \quad i = 1, 2, \cdots, M. \end{aligned}$$

In problem (A), if we define the utility function of the mobile as its expected throughput, the objective of this problem will be to maximize the total expected throughput of the system. Further, if each i corresponds to each sub-carrier in an Orthogonal Frequency Division Multiplexing (OFDM) system, problem (A) can be applied to power allocation for sub-carriers in the OFDM system.

III. Power Allocation

We consider only the distributed solution, i.e., a user based approach. However, the algorithm can be easily executed in a centralized way at the base-station, if each mobile i informs the base-station of the "goodness" of its transmission environment, A_i and its utility function, U_i. Our power allocation algorithm consists of two stages. In the first stage, mobiles to which power is allocated are selected, and, then, power is allocated to the selected mobiles in the second stage. Before we describe the details of our power allocation algorithm, we decompose problem (A) as a mobile problem and a base-station problem. To do this, we need certain results outlined next.

The following lemma will show that to maximize the total system utility, the base-station must transmit at its maximum power limit, P_T.

Lemma 1: If $\bar{P} = (P_1, P_2, \cdots, P_M)$ is a power allocation and $\sum_{i=1}^{M} P_i < P_T$, then we can find another power allocation $\bar{P}^* = (P_1^*, P_2^*, \cdots, P_M^*)$ such that $\sum_{m=1}^{M} P_m^* = P_T$ and $\sum_{i=1}^{M} U_i(\gamma_i(\bar{P}^*)) > \sum_{i=1}^{M} U_i(\gamma_i(\bar{P}))$.

Proof: See Appendix A. ■

Hence, the base-station always transmits at the maximum power level, P_T and $\sum_{i=1}^{M} P_i = P_T$. So, we can rewrite $\gamma_i(\bar{P})$ in (1) as

$$\begin{aligned}\gamma_i(\bar{P}) &= \frac{N_i P_i}{\theta(\sum_{m=1}^{M} P_m - P_i) + A_i} \\ &= \frac{N_i P_i}{\theta(P_T - P_i) + A_i} \\ &\triangleq \gamma_i(P_i),\ i = 1, 2, \cdots, M.\end{aligned}$$

Note that $\gamma_i(P_i)$ does not depend on the power allocation for the other mobiles and so problem (A) is equivalent to the following problem.

$$\begin{aligned}\text{(B)} \qquad & \max_{\bar{P}} \sum_{i=1}^{M} U_i(\gamma_i(P_i)) \\ \text{subject to} \quad & \sum_{i=1}^{M} P_i \le P_T, \\ & 0 \le P_i \le P_T, \quad i = 1, 2, \cdots, M.\end{aligned}$$

Since $\sum_{i=1}^{M} P_i = P_T$, from assumption (e) on the utility function, $U_i(\gamma_i(P_i))$ is one of three types: a sigmoidal-like, a strictly concave, or a strictly convex function of P_i. We now define P_i^o as

$$P_i^o = \begin{cases} \text{the inflection point of } U_i(\gamma_i(P_i)), & \text{if } U_i(\gamma_i(P_i)) \text{ is sigmoidal-like} \\ 0, & \text{if } U_i(\gamma_i(P_i)) \text{ is concave} \\ P_T, & \text{if } U_i(\gamma_i(P_i)) \text{ is convex} \end{cases}.$$

Note that since we allow non-concave utility functions, in general, (B) is a non-convex optimization problem. We will develop a simple (distributed) algorithm that attempts to approximate the performance of the global optimal solution, and show that the performance of this algorithm asymptotically (in the number of mobiles) converges to the global optimum. To that end, we will use the following result.

Lemma 2: Let us define a Lagrangean function associated with problem (B) as

$$L(\bar{P}, \lambda) = \sum_{i=1}^{M} U_i(\gamma_i(P_i)) + \lambda(P_T - \sum_{i=1}^{M} P_i),$$

$$S = \{\bar{P} | \bar{0} \le \bar{P} \le \bar{P}_T\},$$

and

$$Y(\lambda) = \{\bar{x} \in S \mid L(\bar{x}, \lambda) = \max_{\bar{P} \in S}\{L(\bar{P}, \lambda)\}\},$$

where $\bar{0} = (0, 0, \cdots, 0)$ *and* $\bar{P}_T = (P_T, P_T, \cdots, P_T)$*. Then, for any* $\lambda \ge 0$*,* $\bar{P}(\lambda) \in Y(\lambda)$ *is a global optimal solution of the following problem.*

$$\begin{aligned} &\max_{\bar{P}} \sum_{i=1}^{M} U_i(\gamma_i(P_i)) \\ &\textit{subject to} \quad \sum_{i=1}^{M} P_i \le \sum_{i=1}^{M} P_i(\lambda) \\ &\qquad\qquad 0 \le P_i \le P_T, \qquad i = 1, 2, \cdots, M, \end{aligned} \tag{2}$$

where $\bar{P}(\lambda) = (P_1(\lambda), P_2(\lambda), \cdots, P_M(\lambda))$.

Proof: This immediately follows from Property 6.6 in [21]. ■

Lemma 2 implies that if we find a λ^* above such that $\sum_{i=1}^{M} P_i(\lambda^*) = P_T$ (when P_T is the threshold in problem (A)), the global optimal solution of problem (A) can be obtained. However, when we cannot find such a λ^* (this case is described later in this section), $\bar{P}(\lambda)$ is a global optimal solution of the perturbed problem that differs from problem (A) by $|P_T - \sum_{i=1}^{M} P_i(\lambda)|$ on the constraint. From Theorem 5.4 in [21], we can show that $\sum_{i=1}^{M} U_i(\gamma_i(P_i^o)) - \sum_{i=1}^{M} U_i(\gamma_i(P_i(\lambda))) \le \lambda(P_T - \sum_{i=1}^{M} P_i(\lambda))$, where $\bar{P}^o = (P_1^o, P_2^o, \cdots, P_M^o)$ is a global optimal solution of problem (A). Hence, if $P_T \approx \sum_{i=1}^{M} P_i(\lambda)$, we expect that $\sum_{i=1}^{M} U_i(\gamma_i(P_i^o)) \approx$

$\sum_{i=1}^{M} U_i(\gamma_i(P_i(\lambda)))$. Therefore, in this paper, we will attempt to minimize this quantity by considering the following problem, and later on, we will also show that it provides an asymptotically (in the number of mobiles) optimal power allocation.

$$\text{(C)} \qquad \min_{\lambda}\{P_T - \sum_{i=1}^{M} P_i(\lambda)\}$$
$$\text{subject to} \quad \bar{P}(\lambda) = \arg\max_{\bar{0}\le\bar{P}\le\bar{P}_T}\{L(\bar{P},\lambda)\}$$
$$\sum_{i=1}^{M} P_i(\lambda) \le P_T.$$

We first consider the equation, $\bar{P}(\lambda) = \arg\max_{\bar{0}\le\bar{P}\le\bar{P}_T}\{L(\bar{P},\lambda)\}$. Since $L(\bar{P},\lambda)$ is separable in $\bar{P}$, $\bar{P}(\lambda)$ solves the equation if and only if it solves the following problem.

$$(\mathrm{D_i}) \qquad P_i(\lambda) \in \{0 \le q \le P_T \mid L_i(q,\lambda) = \max_{0\le P\le P_T} L_i(P,\lambda)\},\ i = 1,2,\cdots,M,$$

where $L_i(x,\lambda) = U_i(\gamma_i(x)) - \lambda x$. Note that the parameters in problem $(\mathrm{D_i})$ correspond only to mobile i. By this property, we can decompose problem (C) as the mobile problem $(\mathrm{D_i})$ for each mobile i and the following base-station problem.

$$\text{(E)} \qquad \min_{\lambda}\{P_T - \sum_{i=1}^{M} P_i(\lambda)\}$$
$$\text{subject to} \sum_{i=1}^{M} P_i(\lambda) \le P_T.$$

We can interpret the decomposed problems as follows. Based on λ, the price per unit power, each mobile i tries to maximize its net utility (i.e., the utility minus the cost) by solving problem ($\mathrm{D_i}$). This is a greedy procedure and is typically known as a non-cooperative property. In our formulation, by solving problem (E) based on the power request of each mobile, the base-station adjusts the price λ dynamically to reduce the performance difference between the global optimal power allocation and its power allocation by minimizing $\{P_T - \sum_{i=1}^{M} P_i(\lambda)\}$. Therefore, this problem can be interpreted as a utility and dynamic pricing problem. Using this interpretation, we can implement the power allocation algorithm in a distributed way. However, a solution to problem (C) (or equivalently problems $(\mathrm{D_i})$ and (E)) may result in an inefficient power allocation, i.e., $\sum_{i=1}^{M} P_i < P_T$. Further, due to the discontinuity and non-uniqueness of $P_i(\lambda)$ (we will show this later), if we implement the distributed solution using standard gradient descent techniques, the resultant power allocations could oscillate (i.e., there would be no equilibrium solution). Hence, we will devise a strategy to ensure that our solution will in fact have an efficient power allocations ($\sum_{i=1}^{M} P_i = P_T$) as well as have a stable solution. To that end, we divide the algorithm in two stages. The first stage is the mobile selection stage. In this part, mobiles that can be allocated positive power are selected. The second stage is the power allocation stage. Here, only selected mobiles participate at the power allocation stage and power is optimally allocated to the selected mobiles.

A. Mobile Selection

Before, we develop an algorithm for mobile selection, we first study the properties of $P_i(\lambda)$ in problem ($\mathrm{D_i}$). We define λ_i^{max} for mobile i as:

$$\lambda_i^{max} = \min\{\lambda \geq 0 \mid \max_{0 \leq P \leq P_T} \{U_i(\gamma_i(P)) - \lambda P\} = 0\}.$$

The parameter λ_i^{max} will play an important role in mobile selection. From Appendix B, it can be calculated by

$$\lambda_i^{max} = \begin{cases} \frac{dU_i(\gamma_i(P))}{dP}|_{P=0}, & \text{if } P_i^o = 0 \\ \frac{dU_i(\gamma_i(P))}{dP}|_{P=P_i'}, & \text{if } 0 < P_i^o < P_T \text{ and } P_i' \text{ exists} \\ \frac{U_i(\gamma_i(P_T))}{P_T}, & \text{otherwise} \end{cases}, \tag{3}$$

where P_i' is a solution of the following equation.

$$U_i(\gamma_i(P)) - P\frac{dU_i(\gamma_i(P))}{dP} = 0, \; P_i^o \leq P \leq P_T.$$

Further, we define λ_i^{min} as

$$\lambda_i^{min} = \max\{\lambda \geq 0 \mid P_i(\lambda) = P_T\}.$$

We now summarize the properties of $P_i(\lambda)$. Details are provided in Appendix C.

(P1) $P_i(\lambda)$ is discontinuous and has two values (zero and positive) at $\lambda = \lambda_i^{max}$, if U_i is a convex or a sigmoidal-like function. In this case, the positive value is greater than or equal to P_i^o.

(P2) $P_i(\lambda)$ is continuous function of λ, if U_i is a concave function.

(P3) $P_i(\lambda)$ is a positive, continuous, and decreasing function of λ for $\lambda_i^{min} \leq \lambda < \lambda_i^{max}$.

(P4) $P_i(\lambda) = 0$ for $\lambda > \lambda_i^{max}$.

(P5) $P_i(\lambda) = P_T$ for $\lambda \leq \lambda_i^{min}$.

When the price is λ_i^{max}, $P_i(\lambda_i^{max})$ can have two values. One is zero and the other is positive. In the sequel, unless explicitly mentioned, $P_i(\lambda_i^{max})$ will denote the positive value. Hence, with a slight abuse of the notation, we redefine $P_i(\lambda)$ in problem $(\mathrm{D_i})$ as

$$P_i(\lambda) = \arg \max_{0 \le P \le P_T} \{U_i(\gamma_i(P)) - \lambda P\}.$$

Note that there exists a λ_i^{max} such that $P_i(\lambda) = 0$ for $\lambda > \lambda_i^{max}$ and $P_i(\lambda) > 0$ for $\lambda < \lambda_i^{max}$. Hence, we call λ_i^{max} the *maximum willingness to pay* of mobile i.

Using these properties of $P_i(\lambda)$, we can characterize the optimal mobile selection for problem (C), where the optimal mobile selection is defined as follows.

Definition 1: We call a subset of mobiles S an optimal mobile selection for an optimization problem, if there exists a λ^* that makes $\bar{P}^* = (P_1^*, P_2^*, \cdots, P_M^*)$ a global optimal solution of the problem, where

$$P_i^* = \begin{cases} P_i(\lambda^*), & \textit{if } i \in S \\ 0, & \textit{otherwise} \end{cases} .$$

In the following, without loss of generality, we assume that $\lambda_1^{max} > \lambda_2^{max} > \cdots > \lambda_M^{max}$ [3].

Proposition 1: Selecting mobiles 1 from K for power allocation is an optimal mobile selection for problem (C), where

$$K = \max\{1 \le j \le M \mid \sum_{i=1}^{j} P_i(\lambda_j^{max}) \le P_T\}.$$

Further, if

$$\sum_{i=1}^{K} P_i(\lambda_{K+1}^{max}) \ge P_T, \ K < M \tag{4}$$

or

$$K = M, \tag{5}$$

it is an optimal mobile selection for problem (A).

Proof: See Appendix D. ■

Proposition 1 implies that the mobiles are selected in a *decreasing order of* λ_i^{max}.

By using Proposition 1, we can develop a distributed algorithm for mobile selection.

Mobile Selection Algorithm (MSA)

(i) The base-station broadcasts its maximum power limit, P_T, to all mobiles.

(ii) Each mobile i reports its λ_i^{max} to the base-station.

[3] Since λ_i^{max} of each mobile i depends on its channel condition, in general, each mobile i has different λ_i^{max}. If some mobiles have the same λ_i^{max}, they can be ordered randomly.

(iii) Let $K = 1$.

(iv) If $K = M$, select mobiles from 1 to K and stop.

(v) The base-station broadcasts price, λ_{K+1}^{max}.

(vi) Each mobile i reports its power request $P_i(\lambda_{K+1}^{max})$ to the base-station.

(vii) If $\sum_{i=1}^{K+1} P_i(\lambda_{K+1}^{max}) > P_T$, select mobiles from 1 to K and stop.

Otherwise, let $K = K + 1$ and go to (iv).

The MSA needs $O(M)$ iterations for selecting mobiles.

B. Power Allocation for the Selected Mobiles

After the base-station selects mobiles using the MSA in the previous subsection, it allocates its power to the selected mobiles. In this subsection, we assume that mobiles i, $i = 1, 2, \cdots, K$ are selected and $\lambda_1^{max} > \lambda_2^{max} > \cdots > \lambda_K^{max}$. In the proof of Proposition 1 in Appendix D, we have shown that if the condition in (4) or (5) is satisfied, to solve problem (C), we have to find a λ^* such that $\sum_{i=1}^{K} P_i(\lambda^*) = P_T$, and it is also a global optimal power allocation for problem (A). Further, we have shown that otherwise, i.e., if

$$\sum_{i=1}^{K} P_i(\lambda_{K+1}^{max}) < P_T \text{ and } \sum_{i=1}^{K+1} P_i(\lambda_{K+1}^{max}) > P_T,\ K < M, \tag{6}$$

the optimal solution of problem (C) is λ_{K+1}^{max} and $\sum_{i=1}^{K} P_i(\lambda_{K+1}^{max}) < P_T$. Hence, in this case, the amount of power that is allocated to the selected mobiles is less than P_T at the optimal solution of problem (C). However, from Lemma 1, we can increase the total system utility by allocating residual power to the mobiles. Hence, the purpose of this stage is to find a λ^* that satisfies $\sum_{i=1}^{K} P_i(\lambda^*) = P_T$. If there exists such a power allocation, from Lemma 2, it is a global optimal power allocation for the selected mobiles.

To that end, the base-station problem (E) for the selected mobiles can be rewritten as

$$\begin{aligned} \text{(F)} \qquad & \min_{\lambda} |P_T - \sum_{i=1}^{K} P_i(\lambda)| \\ & \text{subject to } \sum_{i=1}^{K} P_i(\lambda) \le P_T, \\ & \qquad\qquad 0 \le \lambda \le \lambda_K^{max}. \end{aligned}$$

Hence, in the power allocation stage, the base-station solves problem (F) and each selected mobile i, $i = 1, 2, \cdots, K$ solves its problem (D_i). The next proposition will show that the solution of problem (F) and problem (D_i) is a global optimal solution for the set of selected mobiles.

Proposition 2: There exists a power allocation, $\bar{P}^K(\lambda^*) = (P_1(\lambda^*), P_2(\lambda^*), \cdots, P_K(\lambda^*))$, which is a solution of problem (F) and problem $(\mathrm{D_i})$. *Further, it satisfies $\sum_{i=1}^{K} P_i(\lambda^*) = P_T$, i.e., it is a global optimal solution of the following optimization problem:*

$$
\begin{aligned}
\text{(G)} \qquad & \max_{\bar{P}} \sum_{i=1}^{K} U_i(\gamma_i(P_i)) \\
& \textit{subject to} \quad \sum_{i=1}^{K} P_i \le P_T, \\
& \qquad\qquad\qquad 0 \le P_i \le P_T, \quad i = 1, 2, \cdots, K.
\end{aligned}
$$

Proof: See Appendix E. ■

As we have discussed before, Proposition 2 also implies that if the condition in (4) or (5) is satisfied, it is also a global optimal power allocation for all mobiles. But, when the condition in (6) is satisfied, it may not be a global optimal power allocation for all mobiles. However, we will show that our power allocation asymptotically optimal in the number of mobiles.

The power allocation algorithm can be implemented in several ways. First, if we consider problem (F), we can use line search algorithms such as a golden section algorithm [21], since $|P_T - \sum_{i=1}^{K} P_i(\lambda)|$ is a unimodal function. Secondly, since we know that $P_T - \sum_{i=1}^{K} P_i(\lambda)$ has a unique root, λ^* for $0 \le \lambda \le \lambda_K^{max}$ and it is an optimal solution of problem (F), we can use root finding algorithms such as a bisection algorithm. Finally, if we consider problem (G), we can use a gradient based algorithm [8] or a penalty based algorithm [6], since problem (G) is equivalent to the following convex programming problem.

$$
\begin{aligned}
\text{(H)} \qquad & \max_{\bar{P}} \sum_{i=1}^{K} U_i(\gamma_i(P_i)) \\
& \text{subject to} \quad \textstyle\sum_{i=1}^{K} P_i \le P_T, \\
& \qquad\qquad\quad P_i(\lambda_K^{max}) \le P_i \le P_T, \quad i = 1, 2, \cdots, K.
\end{aligned}
$$

Since $P_i(\lambda_K^{max}) \ge P_i(\lambda_i^{max}) \ge P_i^o$, $i = 1, 2, \cdots, K$, $U_i(\gamma_i(P_i))$ is a concave function for $P_i(\lambda_K^{max}) \le P_i \le P_T$, $i = 1, 2, \cdots, K$, which makes problem (H) a convex programming problem.

In this subsection, we implement the power allocation algorithm using a simple bisection algorithm.

Power Allocation Algorithm

Let ϵ be a small positive constant.

(i) Set $a^{(1)} = 0$, $b^{(1)} = \lambda_K^{max}$ and $n = 1$.

(ii) The base-station broadcasts the price $\lambda^{(n)} = \frac{a^{(n)}+b^{(n)}}{2}$ to all selected mobiles.

(iii) Each mobile i reports its power requests $P_i(\lambda^{(n)})$ to the base-station.

(iv) If $|b^{(n)} - a^{(n)}| \leq 2\epsilon$ or $P_T = \sum_{i=1}^{K} P_i(\lambda^{(n)})$, allocate power to the selected mobiles as $\bar{P}^K(\lambda^{(n)}) = (P_1(\lambda^{(n)}), P_2(\lambda^{(n)}), \cdots, P_K(\lambda^{(n)}))$ and stop.

Otherwise, go to (v).

(v) If $P_T < \sum_{i=1}^{K} P_i(\lambda^{(n)})$, set $a^{(n+1)} = \lambda^{(n)}$ and $b^{(n+1)} = b^{(n)}$.

Otherwise, set $a^{(n+1)} = a^{(n)}$ and $b^{(n+1)} = \lambda^{(n)}$.

(vi) $n = n + 1$ and go to (ii).

If the power allocation algorithm stops at iteration n^*, we have $|\lambda^{n^*} - \lambda^*| \leq \epsilon$, where λ^* is an optimal solution of problem (F). Hence, a smaller value of ϵ can provide a more accurate solution. Further, we can easily show that $\frac{1}{2^{n^*}}\lambda_K^{max} \leq 2\epsilon$. Hence,

$$n^* = \min\{n \geq \frac{\log \lambda_K^{max} - \log 2\epsilon}{\log 2},\ n = 1, 2, \cdots\}.$$

IV. Asymptotic Optimality and a Lower Bound on the Performance

In this section, we first study the asymptotic optimality in the number of mobiles of our power allocation and, then, also study the lower bound on the worst case performance.

Before we show the asymptotic optimality of our power allocation, we first study the upper bound on the global optimal power allocation of problem (A). Let us define $U_i^u(P)$ as

$$U_i^u(P) = \begin{cases} \lambda_i^{max} P, & \text{if } 0 \leq P \leq P_i(\lambda_i^{max}) \\ U_i(\gamma_i(P)), & \text{if } P_i(\lambda_i^{max}) \leq P \leq P_T \end{cases}. \tag{7}$$

We now consider the following optimization problem.

$$\begin{aligned} \text{(U)} \qquad & \max \sum_{i=1}^{M} U_i^u(P_i) \\ & \text{subject to} \quad \sum_{i=1}^{M} P_i \leq P_T, \\ & \qquad\qquad\quad 0 \leq P_i \leq P_T, \quad i = 1, 2, \cdots, M. \end{aligned}$$

Since we can easily show that for each mobile i, $U_i^u(P_i) \geq U_i(\gamma_i(P_i))$, $\forall P_i$, problem (U) gives us an upper bound on the total achievable system utility, i.e.,

$$\sum_{i=1}^{M} U_i(\gamma_i(P_i^*)) \leq \sum_{i=1}^{M} U_i^u(P_i^u),$$

where $\bar{P}^* = (P_1^*, P_2^*, \cdots, P_M^*)$ and $\bar{P}^u = (P_1^u, P_2^u, \cdots, P_M^u)$ be optimal solutions of problems (A) and (U), respectively.

We now obtain a bound on the difference between our power allocation and the upper bound on the global optimal power allocation.

Proposition 3: Let $\bar{P}^p = (P_1^p, P_2^p, \cdots, P_M^p)$ *be our power allocation and* $\bar{P}^u = (P_1^u, P_2^u, \cdots, P_M^u)$ *be an optimal solution of problem (U). Then,*

$$\sum_{i=1}^{M} U_i^u(P_i^u) - \sum_{i=1}^{M} U_i(\gamma_i(P_i^p)) \leq u_{max},$$

where $u_{max} = \max_{1\leq i\leq M}\{U_i(\gamma_i(P_T))\}$.

Proof: See Appendix F. ■

Proposition 3 shows that the maximum difference between the system utility obtained by our power allocation and the upper bound on the system utility is at most the utility of one mobile. Further, since we assume that the utility function of each mobile is bounded, from Proposition 3, our power allocation can be shown to be asymptotically optimal in the following sense.

Corollary 1: Let $\bar{P}^p = (P_1^p, P_2^p, \cdots, P_M^p)$ *be our power allocation and* $\bar{P}^* = (P_1^*, P_2^*, \cdots, P_M^*)$ *be an optimal solution of problem (A). If* $\sum_{i=1}^{M} U_i(\gamma_i(P_i^*)) \to \infty$ *as* $M \to \infty$*,*

$$\frac{\sum_{i=1}^{M} U_i(\gamma_i(P_i^p))}{\sum_{i=1}^{M} U_i(\gamma_i(P_i^*))} \to 1, \text{ as } M \to \infty.$$

Corollary 1 implies that if there are many mobiles requiring a small amount of power in the system (i.e., if the orthogonality factor of the system is small, or if each mobile has a large processing gain or a good transmission environment), our power allocation scheme will yield a solution close to the global optimal solution.

We now study the worst case performance of our algorithm. The next proposition provides us a lower bound on the performance of our algorithm.

Proposition 4: Let $\bar{P}^p = (P_1^p, P_2^p, \cdots, P_M^p)$ *be our power allocation and* $\bar{P}^* = (P_1^*, P_2^*, \cdots, P_M^*)$ *be an optimal solution of problem (A). Then,*

$$\frac{\sum_{i=1}^{M} U_i(\gamma_i(P_i^P))}{\sum_{i=1}^{M} U_i(\gamma_i(P_i^*))} \geq \frac{u_{min}}{u_{max} + u_{min}},$$

where $u_{min} = \min_{1\leq i\leq M}\{U_i(\gamma_i(P_T))\}$ *and* $u_{max} = \max_{1\leq i\leq M}\{U_i(\gamma_i(P_T))\}$.

Proof: See Appendix G. ■

The implication is that the worst case performance can be poor when there are only few mobiles in the system with utility functions that are quite different from each other and maximum willingness to pays that are inversely proportional to utility values. However, in general, utility functions are comparable with each other and a mobile with a higher utility value has a higher maximum willingness to pay than a mobile with a lower utility value. Such a situation is unlikely to occur, especially when network providers will likely require that users pick utility functions from a pre-defined set.

V. Special Case: Single Class of Mobiles

In this section, we study, for illustration, a special case of our method in which all mobiles are homogeneous, i.e., each mobile i has the same $U_i = U$ and the same $N_i = N$. We present this case because it provides some insight.

In the homogeneous case, we can show the following properties. Details are provided in Appendix H. Let $\bar{P}^* = (P_1^*, \cdots, P_M^*)$ be a global optimal power allocation.

(S1) If $A_i < A_j$, then $\lambda_i^{max} > \lambda_j^{max}$.

(S2) If $A_i < A_j$, then $\gamma_i(P_i^*) \geq \gamma_j(P_j^*)$.

(S3) If $P_k^* = 0$, then $P_j^* = 0$ for all j such that $A_j > A_k$.

Property (S1) shows the relationship between A_i and λ_i^{max}. This implies that in the homogeneous case, mobiles are selected in an increasing order of A_i by the MSA since mobiles are selected in a decreasing order of λ_i^{max} by the MSA. This also implies that the mobile in a better transmission environment has a greater chance to be selected by the MSA than the mobile in a worse transmission environment. Furthermore, from property (S2), the former achieves a higher utility than the latter. Property (S3) implies that, at the global optimal solution, mobiles are selected in an ascending order of A_i. By properties (S1) and (S3), the order of mobile selection in our power allocation is the same as that of the global optimal solution. Hence, the set of mobiles selected by the MSA is a subset of the set of mobiles selected by the global optimal solution and the relationship between mobiles in each set is as follows:

$$A_j \leq A_i, \text{ for } i, j \in V, j \in Z \text{ and } i \notin Z,$$

where V is the set of mobiles selected at the global optimal solution and Z is the set of mobiles selected at our power allocation. This implies that the MSA excludes only those mobiles that obtain relatively low utility in the global optimal mobile selection and, thus, the difference between their achieved performance should be small.

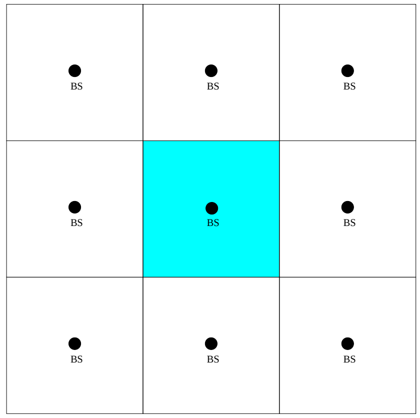

Fig. 2. Cellular network model.

VI. Numerical Results

In this section, we provide numerical results of our power allocation scheme for the CDMA network. Hence, the parameters N_i and γ_i in (1) correspond to the processing gain and E_b/I_0 for mobile i, respectively. For simplicity, we model the cellular system with nine square cells, as shown in Fig. 2. We assume that the base-station is located at the center of each cell and that each base-station has the same maximum power limit, P_T. We focus on the cell at the center of the system assuming that the base-stations in the other cells transmit at the maximum power level, P_T. We model the path gain from a base-station i to a mobile j, $G_{i,j}$ as follows:

$$G_{i,j} = \frac{K_{i,j}}{d_{i,j}^{\alpha}},$$

where $d_{i,j}$ is the distance from the base-station i to mobile j, α is a distance loss exponent, and $K_{i,j}$ is the log-normally distributed random variable with mean 0 and variance σ^2 (dB) that represents shadowing [22]. The parameters for the system are summarized in Table I. For the simulation, we use a sigmoid utility

TABLE I

Parameters for the System

Maximum power (P_T)	10
Orthogonality factor (θ)	1
Distance loss exponent (α)	4
Variance of log-normal distribution (σ^2)	8
Length of the side of the cell	1000

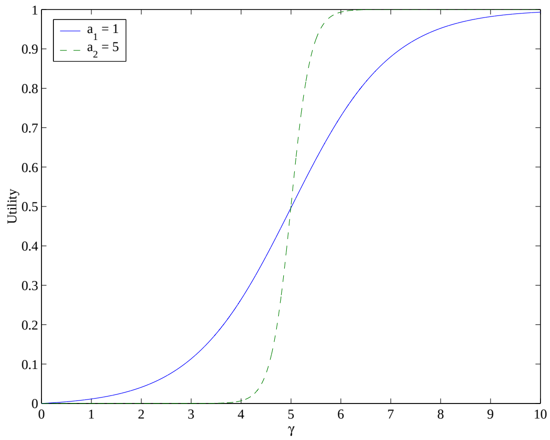

Fig. 3. Sigmoid functions with different a ($b = 5$).

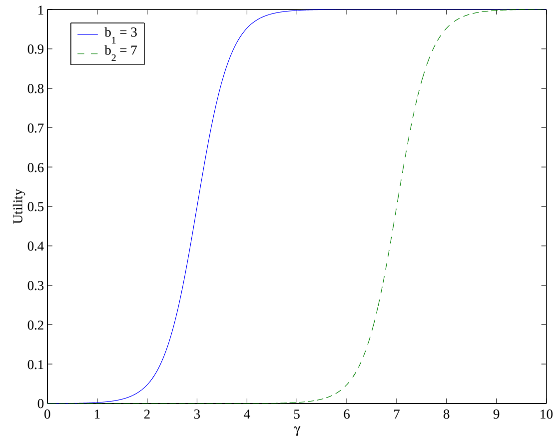

Fig. 4. Sigmoid functions with different b ($a = 3$).

TABLE II

COMPARISON OF UTILITY FOR THE HOMOGENEOUS CASE ($b = 7(dB)$, $N = 64$, $M = 10$, 95% CONFIDENCE)

a	0.5	1	2	4	8
Our	5.967 ± 0.009	7.004 ± 0.011	7.756 ± 0.013	8.256 ± 0.014	8.539 ± 0.015
Global	6.012 ± 0.009	7.093 ± 0.011	7.885 ± 0.012	8.392 ± 0.014	8.661 ± 0.014
Upper	6.236 ± 0.009	7.336 ± 0.011	8.131 ± 0.012	8.657 ± 0.013	8.956 ± 0.013
Our/Global	0.992	0.987	0.984	0.984	0.986
Our/Upper	0.957	0.955	0.954	0.954	0.953

function. The sigmoid utility function is expressed as

$$U(\gamma) \;=\; c\{\frac{1}{1+e^{-a(\gamma-b)}} - d\}. \tag{8}$$

We normalize the sigmoid utility function such that $U(0) = 0$ and $U(\infty) = 1$ by setting $c = \frac{1+e^{ab}}{e^{ab}}$ and $d = \frac{1}{1+e^{ab}}$. The sigmoid utility functions with different values for a and b are provided in Figs. 3 and 4, respectively. For each experiment, we run the simulation program 10^4 times and tabulate the average values (e.g., the total system utility and the selection ratio of mobiles in each class, which is defined as the ratio of the number of selected mobiles to the number of mobiles in each class). At each time epoch of the simulation, each mobile is generated at a new location (with new path gain) in the cell via an independent uniform distribution.

We first provide simulation results for the single class case. We compare our power allocation, the global optimal power allocation, and the upper bound on the global power optimal allocation. For the global

TABLE III

COMPARISON OF UTILITY FOR THE HOMOGENEOUS CASE($a = 3$, $N = 64$, $M = 10$, 95% CONFIDENCE)

$b(dB)$	3	5	7	9	11
Our	9.887 ± 0.006	9.391 ± 0.012	8.072 ± 0.014	6.302 ± 0.011	4.697 ± 0.009
Global	9.907 ± 0.004	9.475 ± 0.011	8.213 ± 0.013	6.459 ± 0.01	4.803 ± 0.007
Upper	9.923 ± 0.004	9.596 ± 0.01	8.472 ± 0.013	6.749 ± 0.009	5.09 ± 0.006
Our/Global	0.998	0.991	0.983	0.976	0.978
Our/Upper	0.996	0.979	0.955	0.934	0.923

TABLE IV

COMPARISON OF UTILITY FOR THE HOMOGENEOUS CASE ($a = 3$, $b = 7(dB)$, $M = 10$, 95% CONFIDENCE)

N	8	16	32	64	128
Our	1.987 ± 0.002	2.982 ± 0.002	5.040 ± 0.008	8.065 ± 0.014	9.884 ± 0.006
Global	1.995 ± 0.001	2.991 ± 0.001	5.253 ± 0.008	8.210 ± 0.013	9.913 ± 0.005
Upper	2.227 ± 0.001	3.433 ± 0.001	5.544 ± 0.007	8.466 ± 0.013	9.944 ± 0.004
Our/Global	0.996	0.997	0.959	0.982	0.997
Our/Upper	0.892	0.868	0.909	0.953	0.994

optimal power allocation, we use an exhaustive search method. However by properties in Section V, the search region can be reduced significantly for the single class case. In Tables II – IV, we provide the total system utilities for each power allocation, varying the values of a, b, and N. Table II indicates that as the value of a increases, the total system utility increases. As shown in Fig. 3, as the value of a increases, less power is required to achieve the same utility for the concave region and more power for the convex region. In general, in our power allocation, mobiles that are allocated positive power are in the concave region, as shown in Lemma 3 in Appendix B. Hence, generally, if other conditions are same, a mobile with a larger value of a in its utility function requires less power than a mobile with a smaller value of a to achieve the same utility. We say that, in this case, the former is more *efficient* than the latter. Hence, the results indicate that as the mobiles in the system get more efficient, the total system utility increases. Similar results are provided in Tables III and IV. If other conditions are same, a mobile with a smaller value of b requires less power than a mobile with a larger value of b to achieve the same utility, as shown in Fig. 4, and the mobile

TABLE V

COMPARISON OF PERFORMANCES OF TWO CLASSES ($a_1 = 2$, $b_1 = b_2 = 7(dB)$, $N_1 = N_2 = 64$, $M = 10$, 95% CONFIDENCE)

a_2	0.5	1	2	4	8
Selection ratio of class 1	0.815 $\pm$ 0.003	0.795 $\pm$ 0.003	0.789 $\pm$ 0.003	0.789 $\pm$ 0.003	0.791$\pm$ 0.002
Selection ratio of class 2	0.645 $\pm$ 0.004	0.728 $\pm$ 0.003	0.787 $\pm$ 0.003	0.83 $\pm$ 0.003	0.854$\pm$ 0.002
Our	6.887 $\pm$ 0.012	7.388 $\pm$ 0.012	7.756 $\pm$ 0.013	8.013 $\pm$ 0.014	8.168$\pm$ 0.025
Upper	7.177 $\pm$ 0.012	7.737 $\pm$ 0.012	8.137 $\pm$ 0.012	8.396 $\pm$ 0.013	8.559$\pm$ 0.026
Our/Upper	0.96	0.955	0.953	0.954	0.954

TABLE VI

COMPARISON OF PERFORMANCES OF TWO CLASSES ($a_1 = a_2 = 1$, $b_1 = 9(dB)$, $N_1 = N_2 = 64$, $M = 10$, 95% CONFIDENCE)

b_2	5	7	9	11	13
Selection ratio of class 1	0.569 $\pm$ 0.003	0.549 $\pm$ 0.003	0.572 $\pm$ 0.004	0.658 $\pm$ 0.003	0.755$\pm$ 0.004
Selection ratio of class 2	0.873 $\pm$ 0.003	0.755 $\pm$ 0.004	0.571 $\pm$ 0.004	0.339 $\pm$ 0.005	0.149$\pm$ 0.003
Our	6.896 $\pm$ 0.013	6.290 $\pm$ 0.011	5.575 $\pm$ 0.008	4.9 $\pm$ 0.012	4.45$\pm$ 0.013
Upper	7.23 $\pm$ 0.013	6.663 $\pm$ 0.011	5.99 $\pm$ 0.008	5.338 $\pm$ 0.013	4.901$\pm$ 0.011
Our/Upper	0.954	0.944	0.931	0.918	0.908

with a smaller value of b is more efficient than the mobile with a larger value of b. Therefore, as the value of b decreases, the total system utility increases, as shown in Table III. Also, if other conditions are same, a mobile with a larger value of N requires less power than a mobile with a smaller value of N to achieve the same γ_i (and thus, the same utility) from (1) and the mobile with a larger value of N is more efficient than the mobile with a smaller value of N. Therefore, as the value of N increases, the total system utility increases, as shown in Table IV.

We also provide the ratio of the system utilities of our power allocation to that of other allocations in Tables II- IV. As shown in these tables, the ratios are quite close to 1 in most cases, which implies that the system utility achieved by our power allocation is quite close to that achieved by the global optimal power allocation.

In Tables V– VII, simulation results for a system with two classes of mobiles are provided. Each class is generated with probability 0.5. For the utility function of mobiles in class i, we set $a = a_i$, $b = b_i$ and

TABLE VII

COMPARISON OF PERFORMANCES OF TWO CLASSES ($a_1 = a_2 = 1$, $b_1 = b_2 = 7(dB)$, $N_1 = 64$, $M = 10$, 95% CONFIDENCE)

N_2	16	32	64	128	256
Selection ratio of class 1	0.901 ± 0.003	0.814 ± 0.003	0.736 ± 0.004	0.778 ± 0.003	0.85 ± 0.003
Selection ratio of class 2	0.107 ± 0.002	0.366 ± 0.003	0.731 ± 0.004	0.943 ± 0.002	0.995 ± 0.001
Our	4.9473 ± 0.018	5.664 ± 0.012	6.999 ± 0.011	8.338 ± 0.015	9.065 ± 0.015
Upper	5.337 ± 0.017	6.039 ± 0.012	7.332 ± 0.011	8.611 ± 0.014	9.275 ± 0.013
Our/Upper	0.927	0.938	0.955	0.968	0.977

$N = N_i$. In this case, we do not provide the system utility achieved by the global optimal power allocation, since it is not easy to obtain. However, as shown in the tables, the ratios of the system utility achieved by our power allocation to that achieved by the upper bound on the global optimal power allocation are quite close to 1 in most cases. This implies that, even in multi-class cases, the system utility achieved by our power allocation is close to that achieved by the global optimal power allocation. In fact, as in the single class cases, the ratio between these two allocations is much closer to 1 than the ratio between our power allocation and the upper bound on the global optimal power allocation.

We also provide the mobile selection ratio for each class in Tables V– VII. The results show that the class of mobiles with a larger value of a (a smaller value of b, or a larger value of N) has a higher selection ratio than one with a smaller value of a (a larger value of b, or a smaller value of N). This implies that in our power allocation, the mobile that is more efficient has a higher priority to be selected than the mobile that is less efficient. This efficient utilization of power results in our power allocation achieving high system utility. On the other hand, from the results, our power allocation in which only the efficiency of the system is considered could be unfair to some mobiles (that are less efficient).

VII. CONCLUSION

In this paper, we have developed a downlink power allocation algorithm for multi-class wireless networks by using a utility based framework allowing general types of utility functions. The algorithm can be implemented in a distributed way using a utility and dynamic pricing framework. We have shown that it provides an asymptotically (in the number of mobiles) optimal power allocation. Further, numerical results show that its performance is close to that of the global optimal power allocation.

Power is a fundamental resource in wireless networks and other resource allocation problems in wireless networks, such as data rate and time must be studied based on the power allocation scheme. Therefore, even though in this paper, we consider only the power allocation problem in wireless networks, our framework can be extended to other resource allocation problems as well [23].

In this paper, we have focused only on the efficiency of the system without considering fairness among the mobiles, which could result in unfair power allocation for some mobiles. However, in resource allocation, considering fairness as well as efficiency such as the opportunistic scheduling schemes in [24], [25], [26] is an important issue and is a topic for future research.

Appendix

A. *Proof of Lemma 1*

If $\sum_{i=1}^{M} P_i < P_T$, there exists an $\alpha > 1$ such that

$$\sum_{i=1}^{M} P_i < \alpha \sum_{i=1}^{M} P_i = P_T.$$

We define $P_i^* = \alpha P_i$ for $i = 1, 2, \cdots, M$, then

$$\begin{aligned}\gamma_i(\bar{P}^*) &= \frac{N_i P_i^*}{\theta(\sum_{j=1}^{M} P_j^* - P_i^*) + A_i} \\ &= \frac{\alpha N_i P_i}{\theta(\sum_{j=1}^{M} \alpha P_j - \alpha P_i) + A_i} \\ &> \frac{\alpha N_i P_i}{\theta(\sum_{j=1}^{M} \alpha P_j - \alpha P_i) + \alpha A_i} \\ &= \gamma_i(\bar{P}),\ i = 1, 2, \cdots, M.\end{aligned}$$

Therefore, $U_i(\gamma_i(\bar{P}^*)) > U_i(\gamma_i(\bar{P}))$ for all i, since U_i is an increasing function of γ_i.

B. *Proof of (3)*

We first prove the following lemma, from which if mobile i requests positive power $P_i(\lambda)$ at price λ, then $P_i(\lambda) = P_T$ or $U_i(\gamma_i(P_i(\lambda)))$ is in the concave region.

Lemma 3: $P_i(\lambda) = 0$ or $P_i^o \leq P_i(\lambda) \leq P_T$.

Proof: If $0 < P(\lambda) < P_T$, it must satisfy the first and the second order necessary conditions for optimality [21], i.e., $\frac{dU_i(\gamma_i(P))}{dP}|_{P=P(\lambda)} = \lambda$, $\frac{d^2U_i(\gamma_i(P))}{dP^2}|_{P=P(\lambda)} \leq 0$, since $P(\lambda)$ is an interior point. This implies that $P_i(\lambda) = 0$ or $P_i^o \leq P_i(\lambda) \leq P_T$. ■

From Lemma 3, if the utility function U_i, of mobile i, is convex, then mobile i will always request a power level of 0 or P_T.

We now prove (3). We first define $w_i(\lambda)$ as

$$w_i(\lambda) = \max_{P_i^o \leq P \leq P_T} \{U_i(\gamma_i(P)) - \lambda P\},$$

which is a non-increasing function of λ. Then, by Lemma 3,

$$\max_{0 \leq P \leq P_T} \{U_i(\gamma_i(P)) - \lambda P\} = \max\{0, w_i(\lambda)\}$$

and

$$\lambda_i^{max} = \min\{\lambda \geq 0 \mid w_i(\lambda) \leq 0\}. \tag{9}$$

We now define $q_i(\lambda) = \arg\max_{P_i^o \leq P \leq P_T} \{U_i(\gamma_i(P)) - \lambda P\}$, $\lambda_i^T = \frac{dU_i(\gamma_i(P))}{dP}|_{P=P_T}$, and $\lambda_i^o = \frac{dU_i(\gamma_i(P))}{dP}|_{P=P_i^o}$. Then, since $U_i(\gamma_i(P))$ is a concave function for $P_i^o \leq P \leq P_T$, $\frac{dU_i(\gamma_i(P))}{dP}$ is a decreasing function for $P_i^o \leq P \leq P_T$. Hence, $\lambda_i^T \leq \lambda_i^o$ and

$$q_i(\lambda) = \begin{cases} P_T, & \text{if } \lambda < \lambda_i^T \\ q_i^*(\lambda), & \text{if } \lambda_i^T \leq \lambda \leq \lambda_i^o \\ P_i^o, & \text{if } \lambda > \lambda_i^o \end{cases}, \tag{10}$$

where $q_i^*(\lambda)$ is a unique solution of

$$\frac{dU_i(\gamma_i(P))}{dP} = \lambda, \ P_i^o \leq P \leq P_T. \tag{11}$$

Therefore,

$$w_i(\lambda) = \begin{cases} U_i(\gamma_i(P_T)) - \lambda P_T, & \text{if } \lambda < \lambda_i^T \\ U_i(\gamma_i(q_i^*(\lambda))) - \lambda q_i^*(\lambda), & \text{if } \lambda_i^T \leq \lambda \leq \lambda_i^o \\ U_i(\gamma_i(P_i^o)) - \lambda P_i^o, & \text{if } \lambda > \lambda_i^o \end{cases}. \tag{12}$$

Since, by the assumptions on the utility functions, $\frac{dU_i(\gamma_i(P))}{dP}$ is a continuous function, $q_i(\lambda)$ is a continuous function and, thus, $w_i(\lambda)$ is a continuous function. Further, we can easily show that $w_i(\lambda)$ is a decreasing function for $\lambda \leq \lambda_i^o$, $w_i(\lambda_i^o) \leq 0$, and $w_i(0) > 0$. This implies that $w_i(\lambda) = 0$ has a unique solution for $0 \leq \lambda \leq \lambda_i^o$ and, by (9), λ_i^{max} is its solution. Hence, there exists a unique λ_i^{max} for mobile i.

We now consider the following equation:

$$U_i(\gamma_i(P)) - \frac{dU_i(\gamma_i(P))}{dP}P = 0, \; P_i^o \leq P \leq P_T. \tag{13}$$

Then, by (10) - (12), there exists a solution P_i' of the equation in (13) if and only if there exists a solution λ_i' of $w(\lambda) = 0$, $\lambda_i^T \leq \lambda \leq \lambda_i^o$. We first assume that there exists a solution P_i' of the equation in (13) and let $\lambda_i' = \frac{dU_i(\gamma_i(P))}{dP}|_{P=P_i'}$. Hence, $q_i(\lambda_i') = P_i'$ and $w_i(\lambda_i') = 0$. This implies that $\lambda_i^{max} = \lambda_i' = \frac{dU_i(\gamma_i(P))}{dP}|_{P=P_i'}$. We now assume that there is no solution of the equation in (13). This implies that $w_i(\lambda) \neq 0$, $\lambda_i^T \leq \lambda \leq \lambda_i^o$. However, since we have shown that $w_i(\lambda) = 0$ has a solution for $0 \leq \lambda \leq \lambda_i^o$, it has a solution $\lambda_i^{max} < \lambda_i^T$. Hence, by (12), $\lambda_i^{max} = \frac{U_i(\gamma_i(P_T))}{P_T}$.

If $P_i^o = 0$, i.e., $U_i(\gamma_i(P))$ is a concave function, the equation in (13) always has a solution at $P_i' = P_i^o = 0$. Hence, in this case, $\lambda_i^{max} = \lambda_i^o = \frac{dU_i(\gamma_i(P))}{dP}|_{P=0}$. If $P_i^o = P_T$, i.e., $U_i(\gamma_i(P))$ is a convex function, we can easily show that $U_i(\gamma_i(P_T)) < \frac{dU_i(\gamma_i(P))}{dP}|_{P=P_T}P_T$. Hence, the equation in (13) has no solution and, thus, $\lambda_i^{max} = \frac{U_i(\gamma_i(P_T))}{P_T}$.

C. *Properties of* $P_i(\lambda)$

In this subsection, we study the properties of $P_i(\lambda)$. Throughout this subsection, λ_i^T, λ_i^o, $q_i(\lambda)$, and $w_i(\lambda)$ are defined as in Appendix B and we will use their properties that have been shown there. For convenience, we summarize some useful properties as follows:

(B1) $\lambda_i^T \leq \lambda_i^o$.

(B2) $w_i(\lambda_i^{max}) = 0$.

(B3) If there is a solution P_i' of the equation in (13), then $q_i(\lambda_i^{max}) = P_i'$ and $\lambda_i^T \leq \lambda_i^{max} \leq \lambda_i^o$.

(B4) If there is no solution of the equation in (13), then $\lambda_i^{max} < \lambda_i^T$.

(B5) If $P_i^o = 0$, then $\lambda_i^{max} = \lambda_i^o$ and there exists a solution $P_i' = 0$ of the equation in (13).

(B6) If $P_i^o = P_T$, then there is no solution of the equation in (13).

(B7) $w_i(\lambda)$ is a decreasing function for $\lambda \leq \lambda_i^o$.

Further, by using the definitions of $w_i(\lambda)$ and $q_i(\lambda)$, we can represent $P_i(\lambda)$ as

$$P_i(\lambda) \;\in\; \begin{cases} \{0\}, & \text{if } w_i(\lambda) < 0 \\ \{0, q_i(\lambda)\}, & \text{if } w_i(\lambda) = 0 \\ \{q_i(\lambda)\}, & \text{if } w_i(\lambda) > 0 \end{cases}. \tag{14}$$

Property 1:

$$P_i(\lambda_i^{max}) \in \begin{cases} \{0\}, & \textit{if } P_i^o = 0 \\ \{0, P_i'\}, & \textit{if } 0 < P_i^o < P_T \textit{ and } P_i' \textit{ exists} \\ \{0, P_T\}, & \textit{otherwise} \end{cases},$$

where P_i' is a solution of the equation in (13).

Proof: From (B2) and (14), $P_i(\lambda_i^{max}) \in \{0, q_i(\lambda_i^{max})\}$. If there exists a solution P_i' of the equation in (13), then $q_i(\lambda_i^{max}) = P_i'$ by (B3). Otherwise, $\lambda_i^{max} < \lambda_i^T$ by (B4) and, thus, $q_i(\lambda_i^{max}) = P_T$ by (10). Hence, if $P_i^o = 0$, then, $P_i(\lambda_i^{max}) \in \{0\}$ by (B5). If $P_i^o = P_T$, then $P_i(\lambda_i^{max}) \in \{0, P_T\}$ by (B6). ■

Property 2: $P_i(\lambda) \in \{0\}$ for $\lambda > \lambda_i^{max}$.

Proof: If $P_i^o = 0$, then $q_i(\lambda) = P_i^o = 0$ for $\lambda > \lambda_i^{max}$ by (B5) and (10). Hence, $P_i(\lambda) \in \{0\}$ for $\lambda > \lambda_i^{max}$ by (14). If $P_i^o \neq 0$, then we can easily show that $w_i(\lambda)$ is a decreasing function. This implies that $w(\lambda) < 0$ for $\lambda > \lambda_i^{max}$ by (B2). Hence, $P_i(\lambda) \in \{0\}$ for $\lambda > \lambda_i^{max}$ by (14). ■

Property 3: $P_i(\lambda)$ is non-increasing in λ. Moreover, $P_i(\lambda)$ is a decreasing and continuous function of λ for $\lambda_i^{min} \leq \lambda < \lambda_i^{max}$, if $\lambda_i^{min} \neq \lambda_i^{max}$, where $\lambda_i^{min} = \max\{\lambda \geq 0 | P_i(\lambda) = P_T\}$.

Proof: We will prove this by considering two different cases.

We first suppose that there is no solution of the equation in (13) or that these exists P_i', a solution of the equation in (13) and $P_i' = P_T$. By Property 2, $P_i(\lambda) \in \{0\}$ for $\lambda > \lambda_i^{max}$ and by Property 1, $P_i(\lambda_i^{max}) \in \{0, P_T\}$. In this case, we can show that $\lambda_i^{max} \leq \lambda_i^T$ by (B4) and (3). Hence, $w_i(\lambda) > 0$ for $\lambda < \lambda_i^{max}$ by (B1), (B2), and (B7) and $q_i(\lambda) = P_T$ for $\lambda < \lambda_i^{max}$ by (10). This implies that $P(\lambda) \in \{P_T\}$ for $\lambda < \lambda_i^{max}$ by (14). Hence, $P(\lambda)$ is non-increasing in λ. Further, by the definition of λ_i^{min}, in this case, $\lambda_i^{max} = \lambda_i^{min}$.

We now suppose that there exists P_i', a solution of the equation in (13) and $P_i' \neq P_T$. In this case, $\lambda_i^T \leq \lambda_i^{max} \leq \lambda_i^o$ by (B3). However, since $P_i' \neq P_T$, $\lambda_i^T < \lambda_i^{max} \leq \lambda_i^o$. By Property 2, $P_i(\lambda) \in \{0\}$ for $\lambda > \lambda_i^{max}$. By Property 1 and (B3), $P_i(\lambda_i^{max}) = \{0, P_i'\}$ and $P_i' = q_i(\lambda_i^{max})$. In a similar way to the above case, we can show that $w_i(\lambda) > 0$ for $0 \leq \lambda < \lambda_i^{max}$. This implies that $P_i(\lambda) \in \{q_i(\lambda)\}$ for $0 \leq \lambda < \lambda_i^{max}$ by (14). Hence, by (10), $P_i(\lambda) \in \{P_T\}$ for $\lambda < \lambda_i^T$. Again, by (10), $q_i(\lambda)$ is a solution of

$$\frac{dU_i(\gamma_i(P))}{dP} = \lambda, \; P_i^o \leq P \leq P_T$$

for $\lambda_i^T \leq \lambda \leq \lambda_i^{max}$. Since, by the assumptions on the utility function, $\frac{dU_i(\gamma_i(P))}{dP}$ is a continuous and decreasing function for $P_i^o \leq P \leq P_T$, $q_i(\lambda)$ is a continuous and decreasing function for $\lambda_i^T \leq \lambda \leq \lambda_i^{max}$. Hence, $P_i(\lambda)$ is a continuous and decreasing function for $\lambda_i^T \leq \lambda < \lambda_i^{max}$ and $P_i(\lambda) > q_i(\lambda_i^{max})$ for $\lambda_i^T \leq \lambda < \lambda_i^{max}$. Further, by the definition of λ_i^T, $P_i(\lambda_i^T) \in \{P_T\}$. This implies that $\lambda_i^{min} = \lambda_i^T$. Hence, $P(\lambda)$ is a non-increasing in λ and it is a continuous and decreasing function for $\lambda_i^{min} \leq \lambda < \lambda_i^{max}$. ■

D. *Proof of Proposition 1*

When $K = \max\{1 \leq j \leq M \mid \sum_{i=1}^{j} P_i(\lambda_j^{max}) \leq P_T\}$, we can have the following three cases.

We first consider the case when $K = M$. Then, $\sum_{i=1}^{M} P_i(\lambda_M^{max}) \leq P_T$. Since each $P_i(\lambda)$, $i = 1, 2, \cdots, M$ is a continuous and non-increasing for $\lambda \leq \lambda_M^{max}$, we can find a λ^* such that $\sum_{i=1}^{M} P_i(\lambda^*) = P_T$ and $\lambda^* \leq \lambda_M^{max}$. This implies that λ^* is an optimal solution of problem (C). Hence, selecting mobiles from 1 to M is an optimal mobile selection for problem (C). Further, since $\sum_{i=1}^{M} P_i(\lambda^*) = P_T$, by Lemma 2, it is an optimal mobile selection for problem (A).

We now consider the case when $K < M$ and $\sum_{i=1}^{K} P_i(\lambda_{K+1}^{max}) \geq P_T$. In this case, in a similar way to the above case, we can find a λ^* such that $\sum_{i=1}^{M} P_i(\lambda^*) = \sum_{i=1}^{K} P_i(\lambda^*) = P_T$ and $\lambda_{K+1} \leq \lambda^* \leq \lambda_K$ (if $\lambda^* = \lambda_{K+1}^{max}$, $P_{K+1}(\lambda^*)$ implies zero). Hence, selecting mobiles from 1 to K is an optimal mobile selection for problems (A) and (C).

Finally, we consider the case when $K < M$ and $\sum_{i=1}^{K} P_i(\lambda_{K+1}^{max}) < P_T$. Then, by the definition of K, $\sum_{i=1}^{K+1} P_i(\lambda_{K+1}^{max}) > P_T$. In this case, due to the non-increasing property of $P_i(\lambda)$, $\lambda^* = \lambda_{K+1}^{max}$ is an optimal solution of problem (C) and selecting mobiles from 1 to K is an optimal mobile selection for problem (C). However, since $\sum_{i=1}^{M} P_i(\lambda^*) = \sum_{i=1}^{K} P_i(\lambda^*) < P_T$ (where $P_{K+1}(\lambda^*)$ implies zero) and $\sum_{i=1}^{M} P_i(\lambda^*) = \sum_{i=1}^{K+1} P_i(\lambda^*) > P_T$ (where $P_{K+1}(\lambda^*)$ implies positive), in this case, there is no λ^o such that $\sum_{i=1}^{M} P_i(\lambda^o) = P_T$ and this mobile selection may not be an optimal mobile selection for problem (A).

E. *Proof of Proposition 2*

By Proposition 1, $\sum_{i=1}^{K} P_i(\lambda_K) \leq P_T$ and $P_i(\lambda)$, $i = 1, 2, \cdots, K$ is a non-increasing and continuous function for $0 \leq \lambda \leq \lambda_K$. Hence, there always exists $\lambda^* \leq \lambda_K^{max}$ that satisfies $\sum_{i=1}^{K} P(\lambda^*) = P_T$. Therefore, by Lemma 2, $\bar{P}^K(\lambda^*) = (P_1(\lambda^*), P_2(\lambda^*), \cdots, P_K(\lambda^*))$ is a global optimal solution for problem (G).

F. Proof of Proposition 3

We assume that mobiles from 1 to K are selected by the MSA and $\lambda_1^{max} > \lambda_2^{max} > \cdots > \lambda_M^{max}$. We define $P_{i,u}(\lambda)$ and $\lambda_{i,u}^{max}$ as

$$P_{i,u}(\lambda) = \arg\max_{0\le P\le P_T}\{U_i^u(P) - \lambda P\}$$

and

$$\lambda_{i,u}^{max} = \min\{\lambda \ge 0 \mid \max_{0\le P\le P_T}\{U^u(P) - \lambda P\} = 0\}.$$

Then, we can easily show that

$$P_{i,u}(\lambda) = \begin{cases} P_i(\lambda)\ (>0), & \text{if } \lambda < \lambda_i^{max} \\ \{P \mid 0 \le P \le P_i(\lambda_i^{max})\}\ (\ni P_i(\lambda)), & \text{if } \lambda = \lambda_i^{max} \\ P_i(\lambda)\ (=0), & \text{if } \lambda > \lambda_i^{max} \end{cases}. \tag{15}$$

This implies that $\max_{0\le P\le P_T}\{U^u(P) - \lambda P\} > 0$ for $\lambda < \lambda_i^{max}$ and $\max_{0\le P\le P_T}\{U^u(P) - \lambda P\} = 0$ for $\lambda \ge \lambda_i^{max}$. Hence, $\lambda_{i,u}^{max} = \lambda_i^{max}$.

We first suppose that the condition in (4) or (5) is satisfied. In this case, from the proof of Proposition 1 in Appendix D, there exists a λ^* such that $P_i(\lambda^*) = P_i^p$, $i = 1, 2, \cdots, M$ and $\sum_{i=1}^M P_i(\lambda^*) = P_T$. Since, $P_i^p \in P_{i,u}(\lambda^*)$ by (15) and $\sum_{i=1}^M P_i^p = P_T$, by Lemma 2, $\bar{P}^p$ is a global optimal solution of problem (U). Hence, we can take $\bar{P}^u = \bar{P}^p$. Further, if $P_i^p > 0$, then $P_i^p \ge P_i(\lambda_i^{max})$. This implies that, by the definition of U_i^u in (7), $U_i^u(P_i^p) = U_i(\gamma_i(P_i^p))$ for $i = 1, 2, \cdots, M$. Hence,

$$\sum_{i=1}^M U_i^u(P_i^u) = \sum_{i=1}^M U_i(\gamma_i(P_i^p)).$$

We now suppose that neither of the condition in (4) nor (5) is satisfied, i.e., from (6), $\sum_{i=1}^K P_i(\lambda_{K+1}^{max}) < P_T$ and $\sum_{i=1}^{K+1} P_i(\lambda_{K+1}^{max}) > P_T$, $K < M$. In this case, from Propositions 1 and 2, $P_i^p = P_i(\lambda^*)$ for $i = 1, 2, \cdots, K$ and $P_i^p = 0$ for $i = K+1, K+2, \cdots, M$, where λ^* satisfies $\sum_{i=1}^K P_i(\lambda^*) = P_T$. This implies that $\lambda^* \le \lambda_{K+1}^{max}$ and, thus, $P_i^p \ge P_i(\lambda_{K+1}^{max})$ for $i = 1, 2, \cdots, K$. Hence,

$$\sum_{i=1}^M U_i(\gamma_i(P_i^p)) = \sum_{i=1}^K U_i(\gamma_i(P_i^p)) \ge \sum_{i=1}^K U_i(\gamma_i(P_i(\lambda_{K+1}^{max}))). \tag{16}$$

We now define

$$P_i^u = \begin{cases} P_i(\lambda_{K+1}^{max}), & \text{if } i = 1, 2, \cdots, K \\ P_T - \sum_{i=1}^K P_i(\lambda_{K+1}^{max}), & \text{if } i = K+1 \\ 0, & \text{if } i = K+2, K+3, \cdots, M \end{cases}.$$

Then, $\sum_{i=1}^{M} P_i^u = P_T$ and $P_i^u \in P_i(\lambda_{K+1}^{max})$ for $i = 1, 2, \cdots, M$ by (15). Hence, by Lemma 2, it is a global optimal solution of problem (U). Further, since $P_i(\lambda_{K+1}^{max}) \geq P_i(\lambda_i^{max})$ for $i = 1, 2, \cdots, K$, by the definition of U_i^u in (7),

$$U_i^u(P_i^u) \;=\; U_i(\gamma_i(P_i(\lambda_{K+1}^{max}))),\ i = 1, 2, \cdots, K. \tag{17}$$

Therefore, by (16) and (17),

$$\begin{aligned}
\sum_{i=1}^{M} U_i^u(P_i^u) - \sum_{i=1}^{M} U_i(\gamma_i(P_i^p)) &\leq \sum_{i=1}^{K} U_i^u(P_i(\lambda_{K+1}^{max})) + U_{K+1}^u(P_T - \sum_{i=1}^{K} P_i(\lambda_{K+1}^{max})) - \sum_{i=1}^{K} U_i(\gamma_i(P_i(\lambda_{K+1}^{max}))) \\
&= U_{K+1}^u(P_T - \sum_{i=1}^{K} P_i(\lambda_{K+1}^{max})) \\
&\leq U_{K+1}^u(P_T) \\
&= U_{K+1}(\gamma_{K+1}(P_T)) \\
&\leq \max_{1 \leq i \leq M} \{U_i(\gamma_i(P_T))\}.
\end{aligned}$$

G. Proof of Proposition 4

By Proposition 3,

$$\frac{\sum_{i=1}^{M} U_i(\gamma_i(P_i^p))}{\sum_{i=1}^{M} U_i(\gamma_i(P_i^*))} \geq \frac{\sum_{i=1}^{M} U_i(\gamma_i(P_i^p))}{u_{max} + \sum_{i=1}^{M} U_i(\gamma_i(P_i^p))}.$$

Since by Proposition 2, our power allocation is a global optimal power allocation for the selected mobiles,

$$\sum_{i=1}^{M} U_i(\gamma_i(P_i^p)) \geq U_j(\gamma_j(P_T)) \geq u_{min},$$

where mobile j is one of the selected mobiles by the MSA. This implies that

$$\frac{\sum_{i=1}^{M} U_i(\gamma_i(P_i^p))}{\sum_{i=1}^{M} U_i(\gamma_i(P_i^*))} \geq \frac{u_{min}}{u_{max} + u_{min}}.$$

H. Proof of Properties in the Homogeneous Case

1) Proof of property (S1): Since $U_i(\gamma) = U_j(\gamma)$ and $N_i = N_j$, $A_i < A_j$ implies that $U(\gamma_i(P)) > U(\gamma_j(P))$ for $0 \leq P \leq P_T$. By the definition of λ_i^{max},

$$U(\gamma_i(P)) - \lambda_i^{max} P \leq 0,\ 0 \leq P \leq P_T.$$

Hence,

$$U(\gamma_j(P)) - \lambda_i^{max} P < 0,\ 0 \leq P \leq P_T.$$

This implies that

$$\lambda_j^{max} < \lambda_i^{max},$$

since $\max_{0 \le P \le P_T}\{U(\gamma_j(P)) - \lambda_j^{max} P\} = 0$.

2) Proof of property (S2): We will prove this by using contradiction. Suppose that $\bar{P}^* = (P_1^*, \cdots, P_M^*)$ is an optimal power allocation and $\gamma_i(P_i^*) < \gamma_j(P_j^*)$. Then, $\frac{NP_i^*}{\theta(P_T - P_i^*) + A_i} < \frac{NP_j^*}{\theta(P_T - P_j^*) + A_j}$ and, thus, $P_i^* < \frac{\theta P_T + A_i}{\theta P_T + A_j} P_j^*$. Let $P_j' = \frac{\theta P_T + A_j}{\theta P_T + A_i} P_i^*$ and $P_i' = P_i^* + P_j^* - P_j' = P_j^* - \frac{A_j - A_i}{\theta P_T + A_i} P_i^*$. Then, $P_i' + P_j' = P_i^* + P_j^*$,

$$\begin{aligned}
\gamma_j(P_j') &= \frac{N \frac{\theta P_T + A_j}{\theta P_T + A_i} P_i^*}{\theta(P_T - \frac{\theta P_T + A_j}{\theta P_T + A_i} P_i^*) + A_j} \\
&= \frac{NP_i^*}{\theta(P_T - P_i^*) + A_i} \\
&= \gamma_i(P_i^*),
\end{aligned}$$

and

$$\begin{aligned}
\gamma_i(P_i') &= \frac{N(P_j^* - \frac{A_j - A_i}{\theta P_T + A_i} P_i^*)}{\theta(P_T - P_j^* + \frac{A_j - A_i}{\theta P_T + A_i} P_i^*) + A_i} \\
&> \frac{N(P_j^* - \frac{A_j - A_i}{\theta P_T + A_i} \frac{\theta P_T + A_i}{\theta P_T + A_j} P_j^*)}{\theta(P_T - P_j^* + \frac{A_j - A_i}{\theta P_T + A_i} \frac{\theta P_T + A_i}{\theta P_T + A_j} P_j^*) + A_i} \\
&= \frac{NP_j^*}{\theta(P_T - P_j^*) + A_j} \\
&= \gamma_j(P_j^*),
\end{aligned}$$

where the inequality comes from the fact that $A_i < A_j$ and $P_i^* < \frac{\theta P_T + A_i}{\theta P_T + A_j} P_j^*$. Therefore, $U(\gamma_j(P_j')) = U(\gamma_i(P_i^*))$ and $U(\gamma_i(P_i')) > U(\gamma_j(P_j^*))$. This implies that

$$U(\gamma_i(P_i')) + U(\gamma_j(P_j')) > U(\gamma_i(P_i^*)) + U(\gamma_j(P_j^*))$$

and utilities for all other mobiles are unchanged, which is the contradiction.

3) Proof of property (S3): This immediately follows from property (S2).

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