

DOWNLINK TRANSMIT BEAMFORMING IN MULTI-USER MIMO SYSTEMS

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ABSTRACT

The primary issue in downlink beamforming for wireless communications is how to balance the need for high received signal power for each user against the interference produced by the signal at other points in the network. In this paper, we describe several approaches to this problem: channel inversion, regularized channel inversion, vector modulo pre-coding, channel block diagonalization, and coordinated transmit/receive beamforming. While the basic idea behind these algorithms is the same, namely the use of channel information at the transmitter to predict and then counteract the interference produced at each node in the network, each of the algorithms is based on achieving a different performance objective. We compare the various goals of the above algorithms, and detail their respective advantages and disadvantages in terms of computational complexity, required transmit power, network throughput, and assumed receiver capabilities. The results of several simulation studies are presented to quantify these comparisons.

1. INTRODUCTION

Downlink beamforming refers to the problem of using an array of antennas at a particular node (e.g., a base station) in a wireless network to communicate simultaneously with multiple co-channel users. In the communications and information theory literature, this is referred to as the MIMO broadcast channel. The users in the network may have a single antenna, and hence no ability for spatial discrimination, or they may have multiple antennas and the ability to perform some type of interference suppression. In either case, we have a multiple-input, multiple-output (MIMO) communications system. While considerable attention has focused on two-user, point-to-point MIMO wireless links, relatively less work has considered multiple-user scenarios. Much of this work has been on the uplink problem, where a multiple antenna receiver must separate the signals arriving from several different users. Any of the extensive source separation literature is relevant to this problem, although there has been research specifically targeted for MIMO scenarios. The multi-antenna uplink scenario is often referred to as the MIMO multiple access channel (MAC). In this paper, we focus on the multi-user MIMO downlink. Although less frequently addressed in the literature, there is still a considerable body of work on the topic that is too extensive to adequately cover in this paper.

The primary issue for the MIMO downlink is how to balance the need for high received signal power for each user against the interference produced by each signal at other points in the network. Key

to achieving this goal is the availability of channel state information (CSI) at the transmitter. While in principle the receivers themselves could perform interference cancellation via multiuser detection, for example, the desire to keep costs low for the end user in cellular networks usually leads to simpler receiver architectures. The difficulty of obtaining CSI at the transmitter is often not justified in point-to-point MIMO links, since the resulting gain in capacity is only significant at low SNR (where throughput is likely quite low anyway) or when there are more transmit than receive antennas. However, CSI is much more critical in multi-user scenarios, since it is required for interference suppression. In this paper, we will focus on techniques that assume the CSI is in the form of deterministic estimates of the channels themselves, rather than based on statistical information (e.g., channel mean or covariance). This should not be construed as implying that such information cannot be used for the downlink problem, but rather that most work to date has assumed the deterministic CSI case. For an excellent and comprehensive treatment of the issues involved with different types of CSI, see [1].

Perhaps the simplest approach to downlink spatial multiplexing is channel inversion [2, 3], which amounts to using a set of beamformers that "pre-inverts" the channel and ideally removes all inter-user interference. This is the analog on the transmit side of a zero-forcing receive beamformer. As in the receive case, problems arise when the channel is nearly rank deficient, although we will see it is not noise amplification that occurs, but rather signal attenuation. In the spirit of minimum mean-squared error (MMSE) beamforming, a regularization parameter can be added to the channel inverse to dramatically improve performance in such situations [4, 5]. Although the gain of regularized channel inversion is significant, there is still a considerable gap between it and the capacity bound. Algorithms from the class of so-called "dirty paper" coding techniques have recently been shown to more closely approach the sum capacity for the multi-user channel, and in some cases achieve it [6]-[10]. We will describe one such technique, referred to as vector modulo pre-coding [4, 11, 12], that can be framed as an extension of the channel inversion algorithms described earlier.

The primary goal of the algorithms mentioned to this point is maximizing the overall throughput of the network for a fixed transmit power, under the constraint of zero (or nearly zero) interference. An alternative approach that relaxes the zero interference constraint is to minimize the total transmitted power subject to meeting a certain Quality of Service (QoS) criteria for each user, measured for example in terms of bit-error rate (BER) or, more easily, signal-to-interference-plus-noise ratio (SINR). Such techniques have been referred to as power control or interference balancing algorithms. Iterative methods have been found that are guaranteed to find the op-

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timal solution to this problem, assuming a solution exists [13, 14]. The problem can also be posed as a semidefinite optimization problem with convex constraints, and solved using more efficient numerical procedures [15].

Another way of classifying algorithms for the multi-user MIMO downlink is based on assumptions regarding the number of antennas each user possesses. The techniques mentioned above all assume that each user has only one receive antenna. They can trivially be extended to cases involving multiple antenna receivers, provided that the total number of receive antennas for all users is no greater than the number of transmit antennas. Not only is this situation unlikely in practice, there are other reasons why alternative solutions should be sought. If applied when the users have multiple antennas, the solutions mentioned thus far force an association between each individual data stream and an individual receive antenna, whether the receive antennas are shared by the same user or not. While this allows for extremely simple receiver architectures, it ignores the ability of the receivers to perform spatial discrimination of their own, and overly constrains the problem. The result can be either (1) a significant gap between the achievable throughput of these techniques and the capacity of the system in cases where the receivers can obtain CSI, or (2) dramatic increase in required transmit power to achieve a desired QoS, especially in situations where the channels to adjacent receive antennas are not uncorrelated.

With the above in mind, instead of completely diagonalizing the channel as some of the techniques above attempt to do, one could find an optimal *block*-diagonalization. Such an approach removes inter-user interference, but leaves the receiver responsible for separating the multiple data streams sent to it [16]-[21]. The drawback with this approach of course is that the total number of receive antennas among all the users must be less than the number of antennas at the base. As a means of relaxing this constraint, suppose that each user employs a beamformer or beamformers of its own to receive the data stream(s) destined for it. If the transmitter knew what those beamformers were in advance, then it could consider the *effective* channel to each user to be the combination of the propagation channel for that user and the beamformers that user employs. As long as the total number of data streams to all users does not exceed the number of transmit antennas, then any of the algorithms discussed above could be used. The problem of course is that, the optimal receive beamformers depend on the choice of the transmit beamformers, and vice versa. We will discuss iterative techniques in which the transmitter postulates a set of receive beamformers, designs a corresponding set of transmit weights, updates the receive beamformers accordingly, and so on [21][22]-[27].

In the next section, we describe the mathematical model we assume for the multi-user MIMO downlink, and establish a common notation. Section 3 describes algorithms for the case where each user has only a single receive antenna and presents some simulation results illustrating their performance. Section 4 does the same for cases involving multiple antennas per user. In Section 5, we discuss some open problems in the area, reference related work that we did not address in this paper, and offer some conclusions.

2. MATHEMATICAL MODEL AND NOTATION

We will implicitly assume a synchronous communications system in which a basestation simultaneously transmits data to a number of distinct users, whose channels have been determined earlier either through the use of uplink training data (as in a time-division duplex system) or via a feedback channel (as in a frequency-division

duplex system). The basestation is assumed to have n_T antennas, user j has n_{R_j} antennas, and the total number of receive antennas assuming K users is $n_R = \sum_{k=1}^K n_{R_k}$. In a flat-fading propagation environment, the channel separating the base from user j is described by the $n_{R_j} \times n_T$ matrix \mathbf{H}_j , whose rows we denote by $\mathbf{h}_{i_j}^*$ as follows:

$$\mathbf{H}_j^* = [\mathbf{h}_{1j} \cdots \mathbf{h}_{n_{R_j}j}]$$

The symbol $(\cdot)^*$ is used to denote the complex conjugate (Hermitian) transpose. In some of our discussion, we will focus on cases where $n_{R_j} = 1$, in which case we will simply denote the channel as $\mathbf{H}_j = \mathbf{h}_j^*$. We will follow the convention of denoting matrices by capital boldface letters, vectors in lowercase boldface, and scalars as either upper or lowercase letters without boldface.

Assume that at symbol time t , the transmitter desires to send the $m_j \times 1$ vector of symbols $\mathbf{d}_j(t)$ to user j . The choice of appropriate values for m_1, \dots, m_K depends on a number of factors, including the desired user data rates, the available transmit power and achievable SINR, the number of receive antennas (typically $m_j \leq n_{R_j}$ without some type of additional coding or multiplexing), and the number of transmit antennas (usually it is required that $\sum m_k \leq n_T$). We will assume that m_j has been determined beforehand, recognizing the fact that this resource allocation step is critical if optimal system performance is required. The signal destined for user j that is actually broadcast from the transmit antennas at time t is denoted by the $n_T \times 1$ vector $\mathbf{s}_j(t)$. In most cases, the transmitted signal is a linear function of the symbols, *i.e.*, $\mathbf{s}_j(t) = \mathbf{B}_j \mathbf{d}_j(t)$, where the columns of \mathbf{B}_j , denoted $\mathbf{B}_j = [\mathbf{b}_{1j} \cdots \mathbf{b}_{m_j j}]$, correspond to the transmit beamformers for each symbol. In cases where $m_j = 1$, we will simply write $\mathbf{B}_j = \mathbf{b}_j$, $\mathbf{d}_j(t) = d_j(t)$, and $\mathbf{s}_j(t) = \mathbf{b}_j d_j(t) = s_j(t)$. We will also consider algorithms that employ a nonlinear mapping of the symbols to the transmitted data: $\mathbf{s}_j(t) = \mathbf{f}_j(\mathbf{d}_j(t))$.

Accounting for contributions from the signals for all users, the data received by user j can be written as

$$\mathbf{x}_j(t) = \sum_{k=1}^K \mathbf{H}_j \mathbf{s}_k(t) + \mathbf{e}_j(t), \quad (1)$$

where $\mathbf{e}_j(t)$ is assumed to represent spatially white noise and interference with covariance $\mathcal{E}\{\mathbf{e}_j(t)\mathbf{e}_j^*(t)\} = \mathbf{I}$. If linear beamforming is used on the transmit side, then stacking the data together from all of the receivers leads to the following compact expression:

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} \mathbf{x}_1(t) \\ \vdots \\ \mathbf{x}_K(t) \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} [\mathbf{B}_1 \cdots \mathbf{B}_K] \begin{bmatrix} \mathbf{d}_1(t) \\ \vdots \\ \mathbf{d}_K(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} \mathbf{e}_1(t) \\ \vdots \\ \mathbf{e}_K(t) \end{bmatrix} \\ &= \mathbf{H}\mathbf{B}\mathbf{d}(t) + \mathbf{e}(t), \end{aligned} \quad (2)$$

where the definitions of $\mathbf{x}(t)$, \mathbf{H} , \mathbf{B} , $\mathbf{d}(t)$ and $\mathbf{e}(t)$ should be obvious from context. For the sake of simplicity, in what follows we will drop the explicit dependence of the above equations on time.

The transmission rate achieved by (3) for user j is found by application of the standard capacity formula assuming the contributions from other users are treated as noise:

$$R_j = \log_2 \left| \mathbf{I} + \left(\mathbf{I} - \sum_{k \neq j} \mathbf{H}_j \mathbf{B}_k \mathbf{B}_k^* \mathbf{H}_j^* \right)^{-1} \mathbf{H}_j \mathbf{B}_j \mathbf{B}_j^* \mathbf{H}_j \right|.$$

The sum capacity of the system is the sum of the transmission rates for each user maximized over the transmit beamformers:

$$C_S = \max_{\mathbf{B}_1, \dots, \mathbf{B}_K} \sum_{k=1}^K R_k \quad \text{s.t.} \quad \text{Tr} \left(\sum_{k=1}^K \mathbf{B}_k \mathbf{B}_k^* \right) \leq \rho, \quad (4)$$

where ρ represents an upper bound on the total transmit power. Solutions to this problem are difficult to find, but have been formulated using the dirty paper coding (DPC) framework (see, for example, [6]-[10] and [28]-[31]). The capacity region C_R of a given multi-user MIMO system is defined to be the set of all achievable rates $\{R_1, \dots, R_K\}$ given the power constraint. In general, determining C_R is an unsolved problem, but progress has been reported in [9, 32, 33]. While in the sequel we will briefly discuss a DPC technique for the downlink, our focus in this paper will mainly be on suboptimal, but less complex beamforming methods that either attempt to approach C_S , or that attempt to achieve a particular rate point with minimum power.

3. SINGLE ANTENNA RECEIVERS

In this section, we restrict attention to cases where each user has only a single antenna, or $m_j = 1$. This is the most common situation considered in the literature. Since this implies that the receiver is unable to perform any interference suppression of its own (assuming, as mentioned above, that it is too complicated to implement multi-user detection at the receivers), the transmitter is responsible for pre-coding the data in such a way that the interference seen by each user is tolerable. In the discussion that follows, we consider four techniques for solving this problem.

3.1. Channel Inversion

Perhaps the most straightforward solution for the multi-user downlink is *channel inversion* [2, 3], which simply amounts to pre-coding the symbols with the pseudo-inverse of the channel:

$$\mathbf{s} = \frac{1}{\sqrt{\gamma}} \mathbf{H}^* (\mathbf{H} \mathbf{H}^*)^{-1} \mathbf{d}, \quad (5)$$

where it is assumed that $n_T \geq K = n_R$. The scaling factor γ is present to limit the total transmitted power to some predetermined value ρ :

$$\|\mathbf{s}\|^2 = \rho \Rightarrow \gamma = \frac{1}{\rho} \mathbf{d}^* (\mathbf{H} \mathbf{H}^*)^{-1} \mathbf{d}. \quad (6)$$

Ideally, all inter-user interference is cancelled by this approach, and each user sees only the desired symbol in additive noise:

$$x_j = \frac{1}{\sqrt{\gamma}} d_j + e_j. \quad (7)$$

One issue that may be a problem in practice is the fact that the scaling γ is data-dependent, and will in general change from symbol to symbol. To avoid this problem, γ can be chosen so that the *average* transmit power is ρ , which leads to

$$\gamma = \frac{1}{\rho} \text{trace} [(\mathbf{H} \mathbf{H}^*)^{-1}] \quad (8)$$

if the users' symbols are independent and have average unit power.

Obviously, a more serious problem arises if the channel is ill-conditioned. In such cases, at least one of the singular values of $(\mathbf{H} \mathbf{H}^*)^{-1}$ is very large, γ will be large, and the SNR at the receivers

will be low. It is interesting to contrast channel inversion with least-squares or "zero-forcing" (ZF) receive beamforming, which applies a dual of the transformation in (5) to the receive data. Such beamformers are well-known to cause noise amplification when the channel is nearly rank deficient. Here, on the transmit side, ZF produces signal attenuation instead. In fact, as shown in [5], the problem is very serious, even for what one might consider the "ideal" case, *i.e.*, where the elements of \mathbf{H} are independent, identically distributed Rayleigh random variables. If the elements of \mathbf{d} are modeled as independent zero-mean unit-variance Gaussian random variables, it can be shown [5] that the probability density function of γ is given by

$$p(\gamma) = K \frac{\gamma^{K-1}}{(1+\gamma)^{K+1}}, \quad (9)$$

when $n_T = K = n_R$, and γ has an infinite mean! As a consequence, the capacity of channel inversion does not increase linearly with K , unlike the capacity bound.

3.2. Regularized Channel Inversion

Following up on the parallels between channel inversion and ZF receive beamforming, a technique that is often used to reduce the effects of noise amplification is to regularize the inverse in the ZF filter. If the noise is spatially white, this is equivalent to using a minimum mean-squared error (MMSE) criterion to design the beamformer weights. Applying this principle to the transmit side suggests the following solution:

$$\mathbf{s} = \frac{1}{\sqrt{\gamma}} \mathbf{H}^* (\mathbf{H} \mathbf{H}^* + \alpha \mathbf{I})^{-1} \mathbf{d}, \quad (10)$$

where α is the regularization parameter. The presence of a non-zero value for α will mean that the transmit beamformer does not exactly cancel the "mixing" effect of the channel, resulting in some level of inter-user interference. The key is to define a value for α that optimally trades off the numerical condition of the matrix inverse (which impacts the normalization required for the power constraint) against the amount of interference that is produced. In [5], it is shown that choosing $\alpha = K/\rho$ approximately maximizes the SINR at each receiver, and unlike standard channel inversion, leads to linear capacity growth with K .

Figure 1 and 2 compare respectively the symbol error rates and capacity of standard and regularized channel inversion. Figure 1 shows error results for 4×4 and 10×10 channels as a function of SNR (the SNR is defined as ρ since the elements of \mathbf{e} are assumed to have unit power). The elements of the channel matrices were simulated as independent, unit-variance Rayleigh random variables. Note that the performance of standard channel inversion degrades as K increases from 4 to 10, but with regularization it does not. Figure 2 plots capacity as a function of K assuming $n_T = K = n_R$ and $\rho = 10$ dB. The plot also includes the sum capacity of the system, indicating that there is still a considerable gap between the performance of regularized inversion and the bound.

3.3. Vector Modulo Pre-coding

It is clear from the results presented in the preceding section that, even with the improvement offered by regularization, there is still a significant gap between the performance of channel inversion and the sum capacity bound. As mentioned above, dirty paper coding (DPC) techniques more closely approach (and in some cases

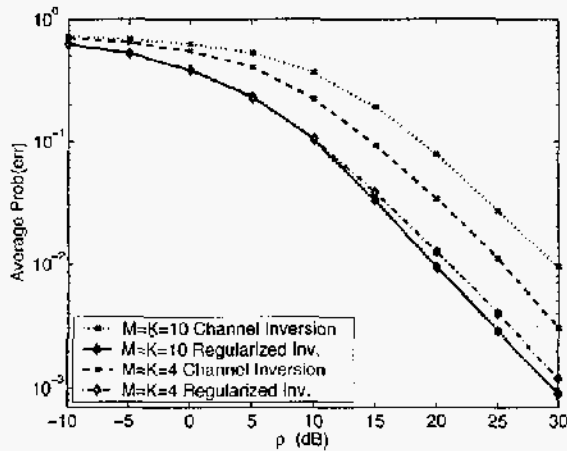


Fig. 1. Comparing uncoded symbol error rates for standard and regularized channel inversion for $K = 4$ and $K = 10$.

achieve) multi-user capacity, and thus may be of interest when capacity is the primary design criterion. DPC is different from other downlink approaches in that the transmitted symbol stream itself, rather than some separate spatial processor (beamformer), is coded so as to reduce inter-user interference. DPC techniques typically employ non-traditional methods such as non-linear coding and high-dimensional lattices, and are often difficult to implement in practice. Although technically DP codes do not constitute beamforming *per se*, they can be used in conjunction with beamforming as illustrated below. In this section we present a simple DPC technique that fits in well with the channel inversion algorithms already discussed.

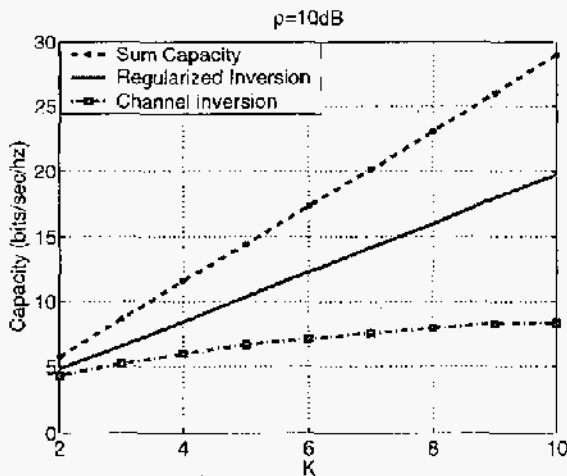


Fig. 2. Comparison of the sum-capacity (dashed line) as a function of K (where $n_T = K$) for $\rho = 10\text{dB}$ with the regularized channel inversion sum-rate (solid line) and the standard channel inversion sum-rate (dash-dotted line).

As discussed above, channel inversion performs poorly because the scaling factor γ in (6) can be large when the channel is ill-conditioned, and the vector \mathbf{d} happens to (nearly) align itself with

a right singular vector of $(\mathbf{H}\mathbf{H}^*)^{-1}$ with large singular value. The idea behind the technique proposed in [4, 11] is to “perturb” the symbol vector \mathbf{d} by some value $\bar{\mathbf{d}}$ such that $\mathbf{d} + \bar{\mathbf{d}}$ is directed towards singular vectors of $(\mathbf{H}\mathbf{H}^*)^{-1}$ with smaller singular values, and in such a way that the receivers can still decode \mathbf{d} without knowledge of $\bar{\mathbf{d}}$. The algorithm in [4, 11] accomplishes this using a vector extension of the modulo pre-coding idea of [34, 35, 36]. In particular, [4, 11] constrains $\bar{\mathbf{d}}$ to lie on a (complex) integer lattice:

$$\bar{\mathbf{d}} = \tau(\mathbf{a} + j\mathbf{b}), \quad (11)$$

where \mathbf{a}, \mathbf{b} are vectors of integers and τ is a real-valued constant, and calculates $\bar{\mathbf{d}}$ based on the following optimization problem:

$$\begin{aligned} \bar{\mathbf{d}} &= \arg \min_{\bar{\mathbf{d}}} (\mathbf{d} + \bar{\mathbf{d}})^* (\mathbf{H}\mathbf{H}^*)^{-1} (\mathbf{d} + \bar{\mathbf{d}}) \\ \text{s.t. } \bar{\mathbf{d}} &= \tau(\mathbf{a} + j\mathbf{b}). \end{aligned} \quad (12)$$

This is an integer-lattice least-squares problem, and can be solved using standard sphere decoding techniques. Since it is used on the transmit side for this application, in [4, 11] it is referred to as sphere *encoding*. Using this method, the vector of data at the receivers is given by

$$\mathbf{x} = \frac{1}{\sqrt{\gamma}} \mathbf{d} + \frac{1}{\sqrt{\gamma}} \tau(\mathbf{a} + j\mathbf{b}) + \mathbf{e}, \quad (13)$$

where, as before, γ is chosen to maintain a constant (average) transmit power ρ .

To eliminate the contribution from the vector perturbation, the receivers employ the modulo function:

$$f_{\tau}(y) = y - \left\lfloor \frac{y + \tau/2}{\tau} \right\rfloor \tau. \quad (14)$$

If γ (or $\mathcal{E}\{\gamma\}$) is known at the receivers, then in the absence of noise,

$$f_{\tau}(\sqrt{\gamma}x_j) = f_{\tau}(d_j + \tau a_j + \tau b_j) = d_j. \quad (15)$$

The modulo parameter τ must also be known at the receiver. Small values of τ are advantageous because they allow for a denser perturbation lattice, and hence more flexibility in maximizing received SINR. However, τ must be chosen large enough to allow for unambiguous decoding. In [4, 11], it is suggested that τ be chosen as

$$\tau = 2(d_{\max} + \Delta/2),$$

where d_{\max} is the distance from the origin to the furthest constellation point, and Δ is the maximum distance between any two constellation points.

Figure 3 shows a plot of the uncoded symbol error probability of the algorithms discussed thus far for a case with $n_T = 10$, $n_R = 10$, a Rayleigh fading channel and 16-QAM signaling. “VBLAST” refers to the use of a successive modulo pre-coding algorithm based on the VBLAST approach [37, 38] (see [11, 39] for details on this technique). “Sphere Encoder” denotes the modulo pre-coding algorithm described above, and “Reg. Sphere Encoder” refers to the use of modulo pre-coding together with regularized channel inversion. Regularization improves performance, but by a smaller margin than in the case of standard channel inversion. It is clear from the plot that, for SNRs high enough to achieve reliable decoding, modulo pre-coding offers a significant improvement in performance. The modulo pre-coding technique presented here represents perhaps the simplest form of DPC for the multi-user MIMO problem, *i.e.*, one involving a simple cubical lattice. As reported in [11], improved performance can be expected if more complicated, higher-dimensional

lattices are employed. Finally, we note that a suboptimal but more computationally efficient version of the modulo pre-coding algorithm has recently been presented in [39].

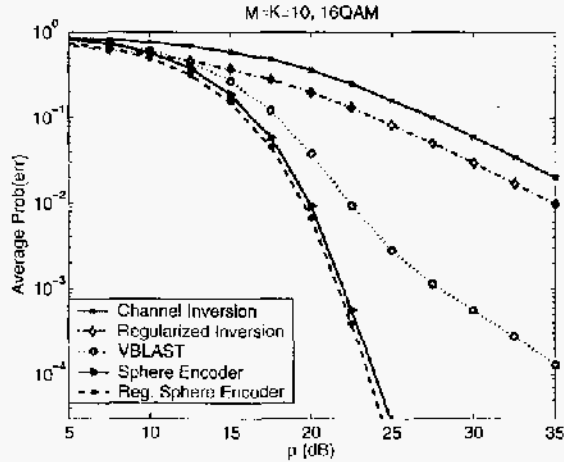


Fig. 3. Uncoded probability of symbol error for various downlink algorithms as a function of transmit power p .

3.4. Power Control

Achieving the sum capacity of a multi-user network may not be the goal of a system designer. In “near-far” scenarios, such an approach may result in one or two strong users taking a dominant share of the available power, potentially leaving weak users with little or no throughput. Consequently, in practice, the dual power control problem is often of more interest, *i.e.*, minimizing power output at the transmitter subject to achieving a desired QoS for each user. QoS can be defined in terms of a given bit error rate or throughput rate, or simply as a certain desired SINR. Assuming the symbols for each user and the additive noise are each unit power, the SINR at receiver j can be expressed as:

$$\text{SINR}_j = \frac{\mathbf{b}_j^* \mathbf{R}_j \mathbf{b}_j}{\sum_{k \neq j} \mathbf{b}_k^* \mathbf{R}_k \mathbf{b}_k + 1}, \quad (16)$$

where either $\mathbf{R}_j = \mathbf{h}_j \mathbf{h}_j^*$ or $\mathbf{R}_j = \mathcal{E}\{\mathbf{h}_j \mathbf{h}_j^*\}$ depending on the type of CSI available at the transmitter.

Given a desired minimum SINR for each user, which we denote by η_j , the power control problem can be formulated as follows:

$$\begin{aligned} & \min_{\mathbf{b}_1, \dots, \mathbf{b}_K} \sum_{k=1}^K \mathbf{b}_k^* \mathbf{b}_k \\ & \text{s.t. } \frac{\mathbf{b}_j^* \mathbf{R}_j \mathbf{b}_j}{\sum_{k \neq j} \mathbf{b}_k^* \mathbf{R}_k \mathbf{b}_k + 1} \geq \eta_j, \quad j = 1, \dots, K. \end{aligned} \quad (17)$$

In [13, 14], iterative algorithms are presented that solve this problem when a feasible solution exists (*i.e.*, if the SINR constraints can be met). An alternative formulation of the problem is presented in [15], where (17) is recast as a minimization over the matrices $\mathbf{W}_j = \mathbf{b}_j \mathbf{b}_j^*$ rather than the beamformers \mathbf{b}_j directly. It is shown that the constraint that \mathbf{W}_j be rank one can be relaxed, and the resulting optimization problem will still have an optimal rank-one

solution. The advantage of this approach is that the problem becomes a semidefinite optimization, for which efficient numerical algorithms exist.

4. MULTIPLE ANTENNA RECEIVERS

The presence of multiple antennas at each receiver node in the network opens up a number of possibilities, including local spatial discrimination (*i.e.*, the transmitter need not remove all interference via pre-coding or beamforming) and the transmission of multiple data streams per user. The corresponding challenges of course are coordinating the signal gain and interference cancellation capabilities of the transmit and receive beamformers, and appropriately allocating resources among all users and among the spatial channels of each individual user. In this paper, we present some basic approaches that address the coordinated beamforming problem, but we leave the resource allocation problem as a topic for future discussion.

4.1. Channel Block-Diagonalization

If $n_R \leq n_T$, *i.e.*, the number of transmit antennas is greater than the number of receive antennas summed over all the users, then the techniques of the previous section could, in principle, be applied without modification. In such cases, each receive antenna is considered a separate “user,” and it receives a single data stream that can be decoded independently of any adjacent antennas. As mentioned above, while this approach results in a very simple receiver, it overly constrains the problem and will lead to suboptimal performance.

Rather than forcing $\mathbf{H}\mathbf{B}$ in (3) to be diagonal (or nearly so), an alternative is to make it block-diagonal [16]–[21]. This removes inter-user interference, but requires that the receiver perform some type of spatial demultiplexing to separate and decode the individual data streams sent to it. To be precise, the goal is to find \mathbf{B} such that

$$\mathbf{H}\mathbf{B} = \begin{bmatrix} \mathbf{M}_1 & & \\ & \ddots & \\ & & \mathbf{M}_K \end{bmatrix}, \quad (18)$$

where \mathbf{M}_j is $n_{R_j} \times n_{R_j}$ assuming that up to n_{R_j} data streams are transmitted to user j (some of the columns of \mathbf{M}_j could be zero so that $m_j \leq n_{R_j}$). There are several criteria that could be used to determine \mathbf{M}_j . Below, we present an algorithm that is sum-capacity-achieving under the block-diagonal constraint [21].

Define $\tilde{\mathbf{H}}_j$ as the following $(n_R - n_{R_j}) \times n_T$ matrix:

$$\tilde{\mathbf{H}}_j = \left[\mathbf{H}_1^T \cdots \mathbf{H}_{j-1}^T \mathbf{H}_{j+1}^T \cdots \mathbf{H}_K^T \right]^T. \quad (19)$$

If we denote the rank of $\tilde{\mathbf{H}}_j$ as \tilde{L}_j , then the nullspace of $\tilde{\mathbf{H}}_j$ has dimension $n_T - \tilde{L}_j > n_{R_j}$. The SVD of $\tilde{\mathbf{H}}_j$ is partitioned as follows:

$$\tilde{\mathbf{H}}_j = \tilde{\mathbf{U}}_j \tilde{\Sigma}_j \left[\tilde{\mathbf{V}}_j^{(1)} \quad \tilde{\mathbf{V}}_j^{(0)} \right], \quad (20)$$

where $\tilde{\mathbf{V}}_j^{(0)}$ holds the $n_T - \tilde{L}_j$ singular vectors in the nullspace of $\tilde{\mathbf{H}}_j$. The columns of $\tilde{\mathbf{V}}_j^{(0)}$ are candidates for user j ’s beamforming matrix \mathbf{B}_j , since they will produce zero interference at the other users. Since $\tilde{\mathbf{V}}_j^{(0)}$ potentially holds more beamformers than the number of data streams that user j can support, an optimal linear combination of these vectors must be found to form \mathbf{B}_j , which

can have at most n_{R_j} columns. To do this, the following SVD is formed:

$$\mathbf{H}_j \tilde{\mathbf{V}}_j^{(n)} = \mathbf{U}_j \begin{bmatrix} \Sigma_j & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_j^{(1)} & \mathbf{V}_j^{(0)} \end{bmatrix}, \quad (21)$$

where Σ_j is $L_j \times L_j$ and $\mathbf{V}_j^{(1)}$ represents the L_j singular vectors with non-zero singular values. The $L_j \leq n_{R_j}$ columns of the product $\tilde{\mathbf{V}}_j^{(n)} \mathbf{V}_j^{(1)}$ represent (to within a power loading factor) the beamformers that maximize the information rate for user j subject to producing zero inter-user interference.

The transmit beamforming matrix will thus have the following form:

$$\mathbf{B} = \begin{bmatrix} \tilde{\mathbf{V}}_1^{(n)} \mathbf{V}_1^{(1)} & \dots & \tilde{\mathbf{V}}_K^{(n)} \mathbf{V}_K^{(1)} \end{bmatrix} \mathbf{\Lambda}^{1/2}, \quad (22)$$

where $\mathbf{\Lambda}$ is a diagonal matrix whose elements scale the power allocated to each "sub-channel." With \mathbf{B} chosen as in (22), the capacity of the block-diagonalization (BD) method becomes

$$C_{BD} = \max_{\mathbf{\Lambda}} \log_2 |\mathbf{I} + \Sigma^2 \mathbf{\Lambda}| \quad \text{s.t.} \quad \text{Tr}(\mathbf{\Lambda}) = \rho, \quad (23)$$

where

$$\Sigma = \begin{bmatrix} \Sigma_1 & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \Sigma_K \end{bmatrix}. \quad (24)$$

The optimal power loading coefficients in $\mathbf{\Lambda}$ are then found using water-filling on the diagonal elements of Σ . Forcing the inter-user interference to zero also allows for a power control formulation of the above approach. This is done by performing water-filling on each Σ_j individually in order to achieve the desired rate for user j , then forming $\mathbf{\Lambda}$ from the diagonal matrices that result for each user.

Figure 4 illustrates the performance of the BD algorithm and several alternatives for a case involving $n_T = 4$ and $n_R = 4$ with $\rho = 10\text{dB}$. The elements of \mathbf{H} were independent Rayleigh random variables with unit variance, and the complementary cumulative distribution function of the capacity achieved by each method is plotted. The BD algorithm is implemented for three different scenarios: four users with one antenna each ($\{1, 1, 1, 1\} \times 4$), two users with two antennas each ($\{2, 2\} \times 4$), and a single user with 4 antennas (referred to as "1 User" in the figure). "Inversion" refers to channel inversion with equal power distributed to each data stream, and "Blind Tx" refers to case where no channel information is available and the users are simply time-multiplexed. Note that the difference between channel inversion and BD in the $\{1, 1, 1, 1\} \times 4$ case is due to the fact that BD employs an optimal power allocation via water-filling. The single-user performance is obviously the best, since it doesn't require the block-diagonal constraint. The improved performance of BD in the $\{2, 2\} \times 4$ case compared with the $\{1, 1, 1, 1\} \times 4$ scenario demonstrates the advantage of relaxing the requirement that the channel be identically diagonalized.

4.2. Coordinated Tx/Rx Beamforming

Strictly speaking, the BD algorithm does not require $n_T \geq n_R$. However, when there are more than just a couple of users, n_R is usually close to the lower bound on the number of transmit antennas¹ In this section, we examine methods that have a less stringent

¹Technically, the BD approach requires

$$n_T > \max \{ \text{rank}(\hat{\mathbf{H}}_1), \dots, \text{rank}(\hat{\mathbf{H}}_K) \}.$$

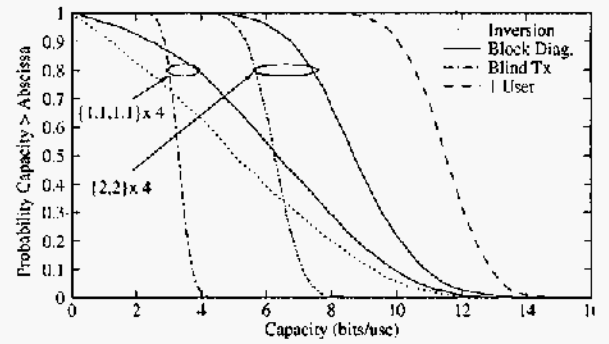


Fig. 4. Complementary cumulative distributions of the sum capacity for $n_T = n_R = 4$ achieved by several transmit beamforming strategies.

constraint on n_T , namely that n_T be no smaller than the total number of data streams to be transmitted. For example, if $m_j = 1$, then $n_T \geq K$ would be required. Obviously, in a real system where the total number of users serviced by a base station is very large, spatial multiplexing must be augmented by other multiple access techniques such as time and frequency multiplexing. A key question is how to best group the K users to be spatially multiplexed together into a given time/frequency slot.

To begin, consider the case where $m_j = 1$, and each receiver uses a beamformer \mathbf{w}_j in decoding the symbol d_j that is sent to it:

$$\tilde{x}_j = \mathbf{w}_j^* \mathbf{x}_j = \sum_{k=1}^K \mathbf{w}_j^* \mathbf{H}_j \mathbf{b}_k d_k + \mathbf{w}_j^* \mathbf{e}_j \quad (25)$$

$$= \sum_{k=1}^K \tilde{\mathbf{h}}_j^* \mathbf{b}_k d_k + \tilde{\mathbf{e}}_j, \quad (26)$$

where $\tilde{\mathbf{h}}_j^* = \mathbf{w}_j^* \mathbf{H}_j$ represents the effective channel from the transmit array to the output of the receive beamformer, and $\tilde{\mathbf{e}}_j = \mathbf{w}_j^* \mathbf{e}_j$ represents the noise at the output of the receive beamformer. If we define $\tilde{\mathbf{H}}^* = [\tilde{\mathbf{h}}_1^* \dots \tilde{\mathbf{h}}_K^*]$, then we obtain an equation identical in form to (3):

$$\tilde{\mathbf{x}} = \tilde{\mathbf{H}} \mathbf{B} \mathbf{d} + \tilde{\mathbf{e}}. \quad (27)$$

Each receiver has a single element of $\tilde{\mathbf{x}}$ associated with it, so (27) has the same dimensions as (3) when $n_{R_j} = 1$. The implication is that, if the transmitter somehow has knowledge of $\mathbf{w}_1, \dots, \mathbf{w}_K$, then it knows $\tilde{\mathbf{H}}$, and hence any of the downlink algorithms in Section 3 for the single-antenna-per-user case could be used.

The composite channel $\tilde{\mathbf{H}}$ could be estimated directly by the transmitter using uplink training data in a reciprocal time-division duplex (TDD) system, assuming that the receiver will use the conjugate of its transmit weights for downlink reception. However, this approach begs the question of how the receiver chose its beamformer, and whether or not any type of optimal solution is possible. An alternative is to assume the base station knows what algorithm each receiver uses in computing its own "optimal" receive beamformer. Since the base generates the interference that each user sees, given CSI it can predict what each user's beamformer will be. For

Since $\text{rank}(\tilde{\mathbf{H}}_j) \leq n_R - n_{R_j}$, it is clear that n_R can be larger than n_T . For example, two users with 3 antennas each could be accommodated by a transmit array with no more than 4 antennas, and possibly fewer depending on the rank of $\tilde{\mathbf{H}}_1$ and $\tilde{\mathbf{H}}_2$.

example, suppose it is known that user j employs MMSE receive beamforming. Then

$$\mathbf{w}_j = [\mathcal{E}\{\mathbf{x}_j \mathbf{x}_j^*\}]^{-1} \mathcal{E}\{\mathbf{x}_j d_j^*\} \quad (28)$$

$$= \left[\sum_{k \neq j} \mathbf{H}_k \mathbf{b}_k \mathbf{b}_k^* \mathbf{H}_j^* + \mathbf{I} \right]^{-1} \mathbf{H}_j \mathbf{b}_j, \quad (29)$$

which can be computed at the transmitter. Alternatively, if the receiver uses maximal ratio combining (MRC), then $\mathbf{w}_j = \mathbf{H}_j \mathbf{b}_j$, which is also known at the transmitter. Whatever the criterion chosen by the receiver, it is likely that the optimal value for \mathbf{w}_j will depend on one or more of the transmit beamformers in \mathbf{B} . On the other hand, the choice of \mathbf{B} in (27) depends on $\bar{\mathbf{H}}$, which in turn depends on the receive beamformers \mathbf{w}_j .

The interdependency of \mathbf{w}_j and \mathbf{B} suggests the following iterative approach:

1. Find an initial value for $\mathbf{w}_1, \dots, \mathbf{w}_K$. For example, they could be chosen as the principle left singular vectors of the respective channel matrices \mathbf{H}_j .
2. Repeat steps 3-4 until convergence.
3. Given $\mathbf{w}_1, \dots, \mathbf{w}_K$, calculate $\bar{\mathbf{H}}$ and find \mathbf{B} using any of the algorithms discussed above.
4. Given \mathbf{B} , recalculate the receive beamformers $\mathbf{w}_1, \dots, \mathbf{w}_K$ according to their respective algorithms.

Convergence can be said to have occurred when no appreciable change in, for example, the achieved SINR or sum rate is observed from one iteration to the next. Algorithms of this general form have been presented in [21][22]-[27]. While analytical results for these approaches are scarce, empirical evidence suggests they have reliable convergence behavior.

In situations where $m_j > 1$, solutions similar to those in Section 4.1 are possible, where in this case it is the effective channel $\bar{\mathbf{H}}$ that is block-diagonalized. Step 3 in the above iterative algorithm is simply replaced by either the capacity or the power control formulation of the BD algorithm. Figure 5 plots the complementary cumulative distribution functions of capacity for the coordinated Tx/Rx beamforming algorithm described above. The SNR for this example is 10dB and the channels were all composed of independent, Rayleigh distributed entries. Several base/user geometries were considered: 4×4 (single-user case), $\{2, 2\} \times 4$ with $m_1 = m_2 = 2$, $\{4, 4\} \times 4$ also with $m_1 = m_2 = 2$, $\{2, 2, 2, 2\} \times 4$ and $\{4, 4, 4, 4\} \times 4$. In the latter two scenarios, one sub-channel is allocated to each user, and channel inversion is used to determine \mathbf{B} . When $m_j = 2$, the BD algorithm is assumed. In all cases, the receive beamformers were calculated as the appropriate left singular vector(s) in equation (21), with \mathbf{H}_j replaced by $\bar{\mathbf{H}}_j$. As expected, the more total receive antennas that are available, the more flexibility there is in finding a good solution, and the higher the capacity. The 4×4 single-user system outperforms the $\{2, 2\} \times 4$ case since it doesn't require the block-diagonal constraint. Similarly, the $\{4, 4\} \times 4$ channel achieves higher capacity than the $\{2, 2, 2, 2\} \times 4$ system since the block-diagonal constraint is less restrictive than full channel diagonalization.

5. DISCUSSION

The goal of this paper has been to present, at a tutorial level, several general approaches that have recently been proposed for the multi-user MIMO downlink. Space does not permit a complete treatment

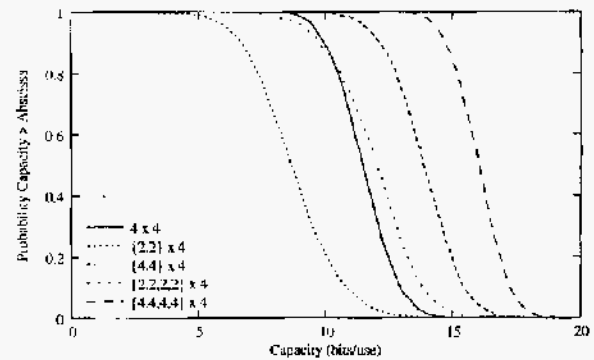


Fig. 5. Complementary cumulative distributions of the sum capacity for coordinated Tx/Rx beamforming in several scenarios.

of this topic; in addition to the references cited above, there have been a number of other important results that have not been mentioned, including methods based on estimating physical channel parameters (e.g., directions of arrival, etc.) [40]-[44], "multi-cell" or multi-basestation MIMO [45, 46], and others [47]-[51]. As we mentioned in the paper, a critical issue that must be addressed prior to applying a downlink beamformer is how to appropriately allocate system resources and schedule users for transmission (i.e., how do we decide which users to spatially multiplex at any given time?). Recent studies in this area include [52]-[66]. In addition, downlink processing is only one narrow aspect of the multi-user MIMO problem. The uplink MIMO multiple access channel (MAC) has received significant attention in recent years (see [1] for a discussion of the MIMO MAC and some of the duality relationships it shares with the MIMO broadcast channel). We have focused solely on the cellular network architectures with a base and users that communicate with the outside world through it. The application of MIMO techniques to *ad hoc* networks remains largely unexplored, although some preliminary studies have been conducted (e.g., see [67]-[70]). Other open problems include extending the techniques we have discussed to cases involving frequency selective fading and partial or incomplete transmit CSI (e.g., where only the statistics of the channel are known). Analyses are also needed to determine the capacity regions for general multi-user scenarios and to quantify the convergence properties of coordinated Tx/Rx beamforming. It appears that the multi-user MIMO problem will continue to be an active and interesting area of research for many years to come.

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