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Research note

Driving-stress waveform and the determination of rock internal friction by the stress–strain curve method

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Summary. Harmonic distortion in the stress–time function applied to rock specimens affects the measurement of rock internal friction in the seismic wave periods by the stress–strain hysteresis loop method. If neglected, the harmonic distortion can cause measurements of rock internal friction to be in error by 30 per cent in the linear range. The stress–time function therefore must be recorded and Fourier analysed for correct interpretation of the experimental data. Such a procedure would also yield a value for internal friction at the higher harmonic frequencies.

Direct measurement of rock internal friction by stress–strain hysteresis loops is the most important method in the seismic frequency range. A periodic stress variation is applied to a rock specimen, and the resulting stress–strain hysteresis loop in the steady state gives a measure of rock internal friction. Harmonic distortion in the stress–time function, however, can affect the determination of rock internal friction by as much as 30 per cent in the linear range, so that the stress–time function applied to the rock specimen must be recorded for correct interpretation of the experimental data.

The various measures of linear anelasticity are related by (e.g. O'Connell & Budiansky 1978)

$$\phi = Q^{-1} = \Delta/\pi = \delta W/2\pi W = \alpha\lambda/\pi \tag{1}$$

for small dissipations ($\phi \lesssim 10^{-2}$), where ϕ is the phase lag between the applied sinusoidal stress and the resulting strain, Q^{-1} is determined by the half-width of a mechanical resonance curve, Δ is the logarithmic decrement for free vibration, δW is the amount of strain energy W dissipated in a cycle and α is the amplitude attenuation per unit length. One necessary condition for equation (1) to hold is that the various measures are defined with respect to sinusoidal time variations of stress and strain. When harmonic components are present in the stress–time variation, additional analysis is needed to determine the internal friction from the measure of $\delta W/2\pi W$.

Consider a linear anelastic solid with an internal friction of 0.01 constant to ± 1.25 per cent over a frequency range from 3×10^{-4} to 10 Hz. One model (model A5) of such a linear anelastic solid, constructed from a superposition of 12 relaxation mechanisms, is given by

Liu *et al.* (1976). The strain $\epsilon(t)$ is related to the stress $\sigma(t)$ in the steady-state sinusoidal loading by

$$\epsilon(t) = \mathrm{Re}\left\{\frac{\sigma(t)}{M_r}[A(\omega) - iB(\omega)]\right\} \tag{2}$$

where

$$\sigma(t) = \mathrm{Re}\left\{\sigma_0 + \sigma_1 \exp(i\omega t)\right\},$$

M_r is the relaxed modulus, ω is the angular frequency, σ_0 and σ_1 are constant stress parameters, and

$$\begin{aligned} A(\omega) &= 1 - \sum_{k=1}^{12}\left\{\omega^2\tau_{\epsilon k}(\tau_{\epsilon k} - \tau_{\sigma k})/(1 + \omega^2\tau_{\epsilon k}^2)\right\} \\ B(\omega) &= \sum_{k=1}^{12}\left\{\omega(\tau_{\epsilon k} - \tau_{\sigma k})/(1 + \omega^2\tau_{\epsilon k}^2)\right\}, \end{aligned} \tag{3}$$

where the relaxation times $\tau_{\epsilon k}$, $\tau_{\sigma k}$ are given by Liu *et al.* (1976). Rewriting equation (2),

$$\sigma(t) = \sigma_0 + \sigma_1 \cos \omega t$$

and

$$\epsilon(t) = \frac{\sigma_0}{M_r} + \frac{\sigma_1}{M_r}[A(\omega)\cos \omega t + B(\omega)\sin \omega t].$$

The amount of strain energy dissipated in one period T is given by

$$\delta W_S = \int_{t=0}^{T} \sigma \, d\epsilon = \frac{\sigma_1^2}{M_r}\pi B(\omega). \tag{4}$$

where the subscript S means single-frequency component. When the steady-state periodic stress time function has several harmonic components, e.g.

$$\sigma(t) = \sum_{n=0}^{N} \sigma_n \cos n\omega t,$$

the strain is given by

$$\epsilon(t) = \frac{1}{M_r}\sum_{n=0}^{N} \sigma_n[A(n\omega)\cos n\omega t + B(n\omega)\sin n\omega t],$$

and the amount of strain energy dissipated in one period T is given by

$$\delta W_M = \int_{t=0}^{T} \sigma \, d\epsilon = \int_{t=0}^{T} \frac{1}{M_r}\left(\sum_{n=0}^{N} \sigma_n \cos n\omega t\right)$$
$$\times \sum_{n=1}^{N}[-\sigma_n A(n\omega)n\omega \sin(n\omega t) + \sigma_n B(n\omega)n\omega\cos(n\omega t)]\, dt = \frac{1}{M_r}\sum_{n=1}^{N}\sigma_n^2 \pi n B(n\omega), \tag{5}$$

where the subscript M means multiple-frequency components. The two measures $\delta W_S/2\pi W_S \equiv \phi$ and $\delta W_M/2\pi W_M$ can differ by as much as 30 per cent depending on the harmonic components.

Consider first the case of triangular loading function. Gordon & Davis (1968) have measured internal friction ϕ in a variety of crystalline rocks. Their procedure was to measure the internal friction at high frequency (90 kHz) over the strain amplitude range $10^{-10} < \epsilon_0 < 10^{-5}$ by the driven-resonance method, and at low frequency (14 mHz) in the range $10^{-5} < \epsilon_0 < 10^{-3}$ by direct determination of the stress–strain curve. The loading sequence in the determination of the stress–strain curve is described by them (Gordon & Davis 1968, p. 3925) as follows: 'The platens are advanced at a constant speed, reversed, and returned to the starting position at the same speed to complete the strain cycle. To hold alignment, a small compressive load is always held on the sample,.... Measurements of ϕ are made only after a steady state is attained'. Such a loading sequence can be approximated by a triangular time function. The truncated Fourier series

$$\sigma(t) = 1.5 + (8/\pi^2)\cos(2.8\times10^{-2}\pi t) + (8/9\pi^2)\cos(8.4\times10^{-2}\pi t) + (8/25\pi^2) \times \cos(1.4\times10^{-1}\pi t) + (8/49\pi^2)\cos(1.96\times10^{-1}\pi t) + (8/81\pi^2)\cos(2.52\times10^{-1}\pi t) + (8/121\pi^2)\cos(3.08\times10^{-1}\pi t) \quad (6)$$

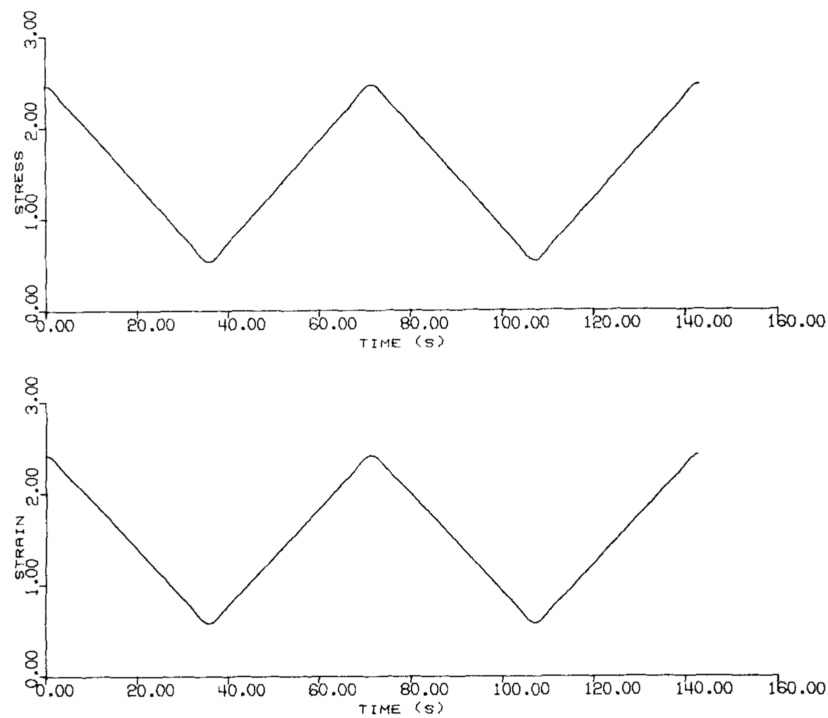

Figure 1. A steady-state stress–time function that approximates a triangular time function and its strain response. The linear anelastic rheology is described by equation (3).

approximates a steady-state triangular stress time function with an amplitude of 0.9663 (arbitrary stress units) and a period of 71.429 s; the steady-state strain response is shown in Fig. 1. The frequency content of equation (5) consists of the fundamental frequency $f_0 =$ 14 mHz and its odd harmonics up to $11f_0 = 154$ mHz. The internal friction ϕ at these

Table 1. Internal friction ϕ calculated by the linear anelastic model A5 at the component frequencies of the steady-state stress–time function shown in Figs 1 and 3.

Frequency (mHz)	Internal friction
14	1.0065×10^{-2}
28	1.0089×10^{-2}
42	1.0193×10^{-2}
70	1.0237×10^{-2}
98	1.0156×10^{-2}
126	1.0099×10^{-2}
154	1.0077×10^{-2}

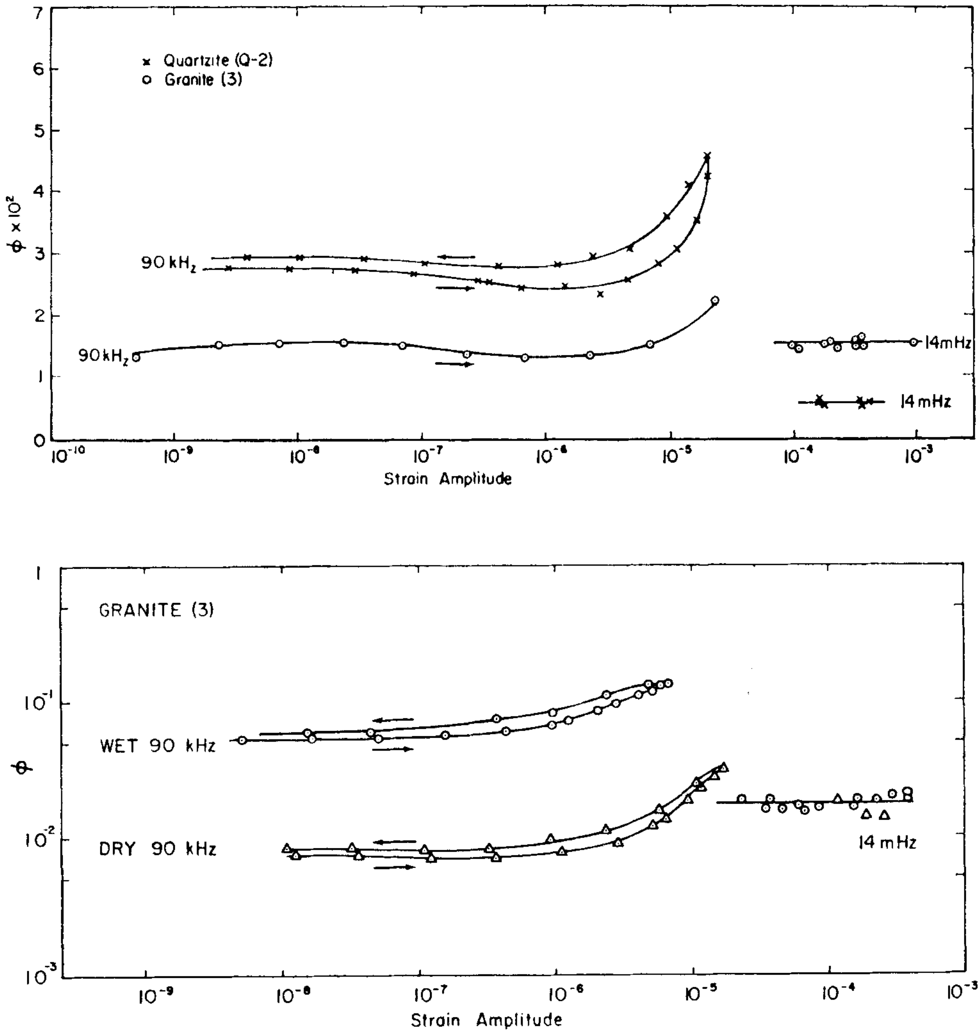

Figure 2. Discontinuity in the internal friction values between the driven resonance method (at 90 kHz) and the stress–strain curve method (at 14 mHz) (after Gordon & Davis 1968, Figs 5 and 8).

frequencies, calculated by the linear superposition model A5, are listed in Table 1. The strain energy dissipation per cycle when

$$\sigma(t) = 1.5 + 0.9663 \cos(2.8 \times 10^{-2} \pi t), \tag{7}$$

calculated by model A5 and equation (4), is $\delta W_S = 2.8168 \times 10^{-2} (M_r = 1)$. The strain energy dissipation per cycle when $\sigma(t)$ is given by equation (6) and calculated by model A5 and equation (5) is $\delta W_M^{(i)} = 2.0817 \times 10^{-2}$. Since the $\sigma(t)$ given by equations (6) and (7) have the same amplitude, the corresponding strain energies agree to within ~ 1 per cent when $\phi = 0.01$. The value of $\delta W_M^{(i)}/W$ calculated by equations (5) and (6) is seen to be less by 30 per cent than $\delta W_S/W$ calculated by equations (4) and (7). This result and the results in Table 1 demonstrate that determination of internal friction according to equation (5) can underestimate ϕ by as much as 30 per cent, a consequence of using a stress–time function with harmonic-frequency content, even though the internal friction is the same at these frequencies as at the fundamental frequency.

Examination of the experimental results presented by Gordon & Davis (1968) shows a discontinuity between the internal friction values determined by the driven-resonance method at 90 kHz and by the stress–strain curve method at 14 mHz (Fig. 2). The frequency dependence of internal friction and the large strains (10^{-3}–10^{-4}) in the stress–strain determination could contribute to this discontinuity. However, correction of the effect on ϕ due to the triangular time function would decrease the discontinuous jump in values of ϕ between the driven-resonance method and the stress–strain curve method.

Consider next the case when the force output at seismic frequencies of an electromagnetic force transducer without a permanent magnet is proportional to the square of the

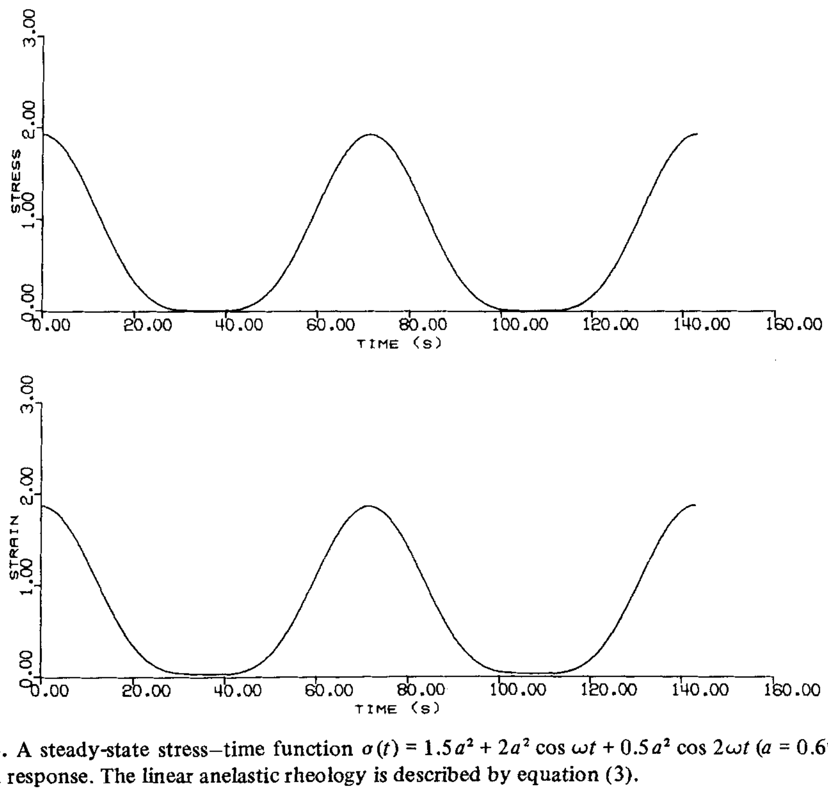

Figure 3. A steady-state stress–time function $\sigma(t) = 1.5a^2 + 2a^2 \cos \omega t + 0.5a^2 \cos 2\omega t$ ($a = 0.6951$) and its strain response. The linear anelastic rheology is described by equation (3).

input current. Such a force transducer was employed by Peselnick *et al.* (1979) in their determination of rock internal friction. The steady-state stress–time function is given by

$$\sigma(t) = (a + b\cos\omega t)^2 = a^2 + \frac{b^2}{2} + 2ab\cos\omega t + \frac{b^2}{2}\cos 2\omega t, \qquad a \geqslant b > 0. \tag{8}$$

The maximum harmonic distortion occurs when $a = b$. The time function ($a = b = 0.6951$)

$$\sigma(t) = 0.7247 + 0.9663\cos(2.8\times10^{-2}\pi t) + 0.2416\cos(5.6\times10^{-2}\pi t) \tag{9}$$

also has an amplitude of 0.9663 (arbitrary stress units) and a period of 71.429 s (Fig. 3). The steady-state strain response is shown in Fig. 3. The values of internal friction ϕ at the fundamental frequency $f_0 = 14$ mHz and at $2f_0 = 28$ mHz are listed in Table 1. The strain energy dissipation per cycle calculated by model A5 and equations (5) and (9), is $\delta W_M^{(ii)} = 3.1682\times10^{-2}$. This value is higher than $\delta W_S = 2.8168\times10^{-2}$ by 12 per cent, in contrast with the previous example, where $\delta W_M^{(i)}$ is less than δW_S by 30 per cent. Here, the internal friction ϕ can be overestimated by as much as 12 per cent if calculated according to equation (5).

In summary, the stress–strain function applied to the rock specimen in the determination of rock internal friction by the stress–strain curve method must be recorded and Fourier analysed for correct interpretation of the experimental data. Such a procedure would also yield the internal friction at the higher harmonic frequencies. The present research note points out the correct procedure for the determination of rock internal friction in the linear range. However, the reinterpretation of experimental data of the rocks tested by Gordon & Davis (1968) and by Peselnick *et al.* (1979) does not prove or disprove the linearity of internal friction of these rock specimens.

Acknowledgments

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