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# Driving the Gravity-Mode Pulsations in $\gamma$ Doradus Variables

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## ABSTRACT

The  $\gamma$  Doradus stars are a newly-discovered class of gravity-mode pulsators which lie just at or beyond the red edge of the  $\delta$  Scuti instability strip. We present the results of calculations which predict pulsation instability of high-order  $g$ -modes with periods between 0.4 and 3 days, as observed in these stars. The pulsations are driven by the modulation of radiative flux by convection at the base of a deep envelope convection zone. Pulsation instability is predicted only for models with temperatures at the convection zone base between  $\sim 200,000$  and  $\sim 480,000$  K. The estimated shear dissipation due to turbulent viscosity within the convection zone, or in an overshoot region below the convection zone, can be comparable to or even exceed the predicted driving, and is likely to reduce the number of unstable modes, or possibly to quench the instability. Additional refinements in the pulsation modeling are required to determine the outcome. A few  $\gamma$  Doradus stars have been observed that also pulsate in  $\delta$  Scuti-type  $p$ -modes, and at least two others have been identified as chemically peculiar. Since our calculated driving region is relatively deep,  $\gamma$  Doradus pulsations are not necessarily incompatible with surface abundance peculiarities or with  $\delta$  Scuti  $p$ -mode pulsations driven by the H and He-ionization  $\kappa$  effect. Such stars will provide useful observational constraints on the proposed  $\gamma$  Doradus pulsation mechanism.

*Subject headings:* stars: oscillations; stars: variables: other

## 1. Introduction

The  $\gamma$  Doradus stars are a newly-discovered class of pulsating variables which pulsate in low-degree ( $\ell$ ) nonradial gravity modes (see Kaye et al. 1999 and references therein). These oscillations are characterized by periods between 0.4 and 2.9 days seen in both broad-band photometry and in line-profile variations. Recent results largely based on HIPPARCOS photometry indicate a preliminary  $\gamma$  Doradus instability strip with a domain bounded by 7200–7700 K on the zero-age main sequence and 6900–7500 K near the terminal-age main sequence (Handler 1999).

Early modeling efforts were unable to confirm pulsational instability in this portion of the Hertzsprung-Russell diagram (e.g., Gautschy & Löffler 1996). Papers at recent conferences have examined aspects of the physics of these stars that may be important for pulsation driving, e.g.,

diffusion (Turcotte 2000), metallicity (Guzik et al. 2000), convection (Guzik et al. 2000; Wu & Goldreich 2000), or surface boundary conditions (Löffler 2000). In this *Letter*, we present the first theoretical models that predict pulsation instability of gravity modes corresponding to those observed in  $\gamma$  Dor variables. These models also offer a natural explanation for the localized position of these stars in the H-R diagram.

## 2. Evolution and Pulsation Modeling

We use an updated version of the Iben (1963, 1965a,b) evolution code, including the latest OPAL (Iglesias & Rogers 1996) opacities; convection is treated in the standard mixing-length theory (Böhm-Vitense 1958). The composition profile, luminosity, mass, and effective temperature of models on the evolution sequence are used to generate 2000-zone models in hydrostatic equi-

librium. The zones are distributed to resolve the interior just outside the convective core (where a large number of  $g$ -type nodes are present), and the envelope (where pulsation driving occurs). We use the Pesnell (1990) linear nonadiabatic pulsation code to calculate the frequencies and test the stability of  $\ell=1$  and 2 modes.

We present results for a zero-age main sequence (ZAMS) and two evolved main-sequence models of  $1.62M_{\odot}$  with  $Z = 0.03$  (Table 1). The models have relatively deep envelope convection zones due to their high metallicity and cool effective temperatures. We find that the first two models are unstable to many high-order  $g$ -modes with frequencies between 4.4 and 24  $\mu\text{Hz}$  ( $P \sim 0.4$  to 2.6 days), coinciding with the observed range of  $\gamma$  Dor periods. The growth rates (fractional change in mode kinetic energy per period) for the unstable modes range from  $10^{-4}$  to  $10^{-8}$  per period (Fig. 1). The number of excited modes and the maximum growth rate *decrease* with *increasing* convection zone (CZ) depth along the evolution sequence (compare Models 1, 2, and 3), with only one  $\ell=1$   $g$ -mode predicted for Model 3 with the deepest CZ. The mode kinetic energy varies by three orders of magnitude over the observed frequency range (Fig. 1), and reaches a minimum around 11  $\mu\text{Hz}$  ( $P \sim 1$  day). The minimum mode kinetic energy also increases with increasing CZ depth. The models are stable to pulsations with frequencies between  $\sim 25$  and  $\sim 150$   $\mu\text{Hz}$  ( $P \sim 0.08$  to 0.46 days), but are unstable to  $\delta$  Scuti-like  $p$ -modes at frequencies higher than the radial fundamental mode ( $P \lesssim 0.08$  d).

### 3. Proposed Driving Mechanism

For these models, the pulsation driving occurs at the base of the envelope convection zone, where the opacity is increasing and the transition from fully radiative to fully convective transport is abrupt (Fig. 2). Also barely discernable in Fig. 2 is a very small amount of driving due to the  $\kappa/\gamma$  effect near the top of the envelope CZ around 12,000 K, where hydrogen is ionizing. The Pesnell code adopts the “frozen-in convection” approximation, in which fluctuations in the convective luminosity are set to zero during the pulsation cycle. Because convection does not adapt to transport the additional luminosity during the pulsation cycle in

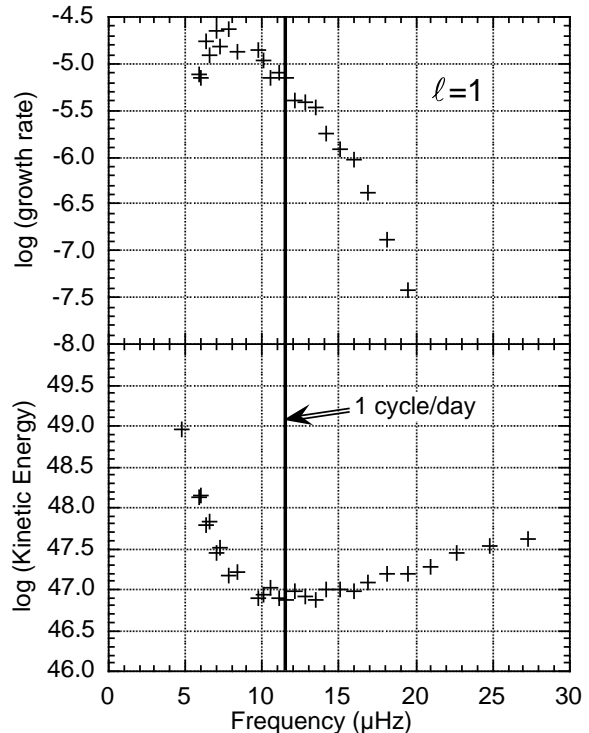


Fig. 1.— *Top Panel:* Log growth rate ( $\Delta\text{KE}/\text{KE}$  per period) versus frequency for  $1.62 M_{\odot}$   $Z = 0.03$  Model 2. Note the peak near a period of 1 day ( $\nu = 11.6 \mu\text{Hz}$ ). *Bottom Panel:* log KE vs. frequency for the same model. Note the minimum near a period of one day.

TABLE 1  
 PROPERTIES OF  $1.62 M_{\odot}$   $Z = 0.03$  MODELS

Model Property	ZAMS Model #1	Evolved Model #2	Evolved Model #3
$T_{\text{eff}}$ (K)	7160	6932	6659
Luminosity ( $L/L_{\odot}$ )	5.61	6.24	6.64
$\log g$ ( $GM/R^2$ )	4.27	4.17	4.07
Age (Gyr)	0.000	0.765	1.31
Core H Abundance	0.682	0.487	0.300
CZ* Base Radius ( $R/R_{\star}$ )	0.974	0.947	0.907
CZ Base Temperature (K)	177,445	297,273	480,170
Convective Timescale <sup>†</sup> at CZ Base (days)	0.34	1.4	4.6
Unstable $\ell=1$ $g$ -mode Period Range (days)	0.48 to 2.6	0.6 to 2	1.16
Peak Growth Rate ( $\Delta KE/KE$ per period)	$6.7 \times 10^{-5}$	$2.3 \times 10^{-5}$	$4.0 \times 10^{-9}$
Minimum log Kinetic Energy (ergs)	46.24	46.87	47.56
Radial Fundamental Period (days)	0.0503	0.0616	0.0771

\*CZ  $\equiv$  envelope convection zone

<sup>†</sup> $\equiv$  local pressure scale height/local convective velocity

this approximation, the luminosity is periodically blocked at the CZ base, resulting in pulsation driving.

This mechanism, introduced as “convective blocking” by Pesnell (1987) and further developed by Li (1992), is independent of the classical  $\kappa/\gamma$  mechanism. This mechanism was first suggested to explain white dwarf pulsations by Cox et al. (1987) and Cox (1993), but is not viable for white dwarfs because the convective timescale of their thin envelope CZ is much shorter than the pulsation period; thus, the frozen-in convection approximation is not valid.

In the case of  $\gamma$  Dor stars, however, this mechanism may operate if these stars have sufficiently deep envelope convection zones so that the *local* convective timescale ( $\equiv$  local pressure scale height/local convective velocity) at the CZ base is comparable to or longer than the pulsation period. In this case, convection does not have time to completely adapt to the changing conditions at the CZ base during the pulsation cycle, and the frozen-in convection approximation is reasonable. This lo-

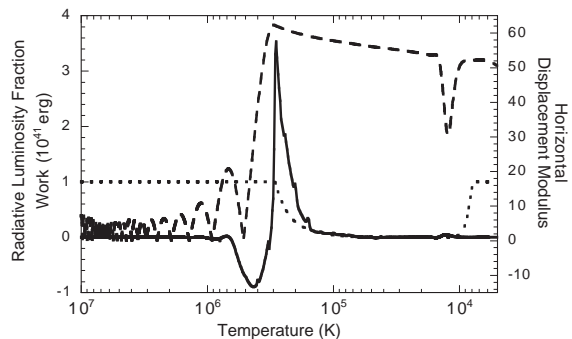


Fig. 2.— Luminosity fraction transported by radiation (short dashes); work (solid line); and modulus of nonadiabatic horizontal displacement (long dashes, with usual linear theory normalization) vs. temperature for 1.5 day mode of Model 2 with largest growth rate ( $2.3 \cdot 10^{-5}$  per period). Note that driving (positive work) occurs at the transition between radiative and convective luminosity transport at the envelope CZ base.

cal convective timescale criterion is satisfied, or nearly satisfied, for our models with convective timescales at the CZ base of 0.34 days (Model 1), 1.4 days (Model 2), and 4.6 days (Model 3).

On the other hand, if the convection zone extends too deep (i.e., comparable to that of Model 3), the driving becomes weaker. In deep and more nearly adiabatic stellar layers, all variations usually are smaller than nearer the surface; and the amplitude of a periodic radiative luminosity wave impinging on the CZ base grows smaller as the CZ becomes deeper. Since the frozen-in convection is blocking a smaller luminosity variation, there is less driving. Then the radiative damping can dominate and stabilize pulsations. We find that there is a locus of convective envelope depths for which  $\gamma$  Doradus-like pulsations can occur: The temperature at the CZ base must be between  $\sim 200,000$  and  $\sim 480,000$  K.

While Table 1 presents results for  $Z = 0.03$  models, we emphasize that this driving mechanism does *not* require models with higher-than-solar metallicity. We also tested the pulsation stability of a  $1.45 M_{\odot}$ ,  $Z = 0.02$  model sequence. These models have lower luminosities (4.3–4.7  $L_{\odot}$ ), but about the same effective temperature (6900–7100 K), CZ base temperature (260,000–310,000 K), and local CZ base convective timescale ( $\sim 1$  d) as our  $1.62 M_{\odot}$  models. We find a similar range of excited  $g$ -mode frequencies, with the growth rates diminishing, and frequency range narrowing with increasing convection zone depth.

These models neglect diffusive element settling and radiative levitation. We find that when diffusive settling of helium and heavier elements is included, but radiative levitation is neglected, both the envelope convection zone and the helium ionization zone rapidly disappear, and the models become pulsationally stable. However, Turcotte (2000) finds that radiative levitation supports and concentrates Fe-peak elements in a layer around 200,000 K to produce a localized convection zone. A convection zone at this location is exactly what we require to modulate the emergent flux and drive  $g$ -modes.

#### 4. Effect of Shear Dissipation

For these nonradial  $g$ -modes with periods of  $\sim 1$  day, the modulus (square root of the sum of

squares of the real and imaginary part) of the nonadiabatic horizontal eigenfunction at the CZ base is about 60 times the radial displacement, which is normalized to one at the photosphere in the linear theory (Fig. 2). The nonadiabatic horizontal displacement is nearly identical to the adiabatic one, except for a discontinuity near the top of the CZ caused by entropy variations in this region where hydrogen is ionizing. This nonadiabatic effect is smaller for lower-order modes, which depend less on the structure of the model near the surface.

We estimate the dissipation due turbulent viscosity for horizontal shearing motion within the convection zone, using typical parameters from Model 2 and the 1.5-day mode. The work (force  $\times$  distance) due to the shear force is given by

$$\frac{2}{\pi} \int \frac{dr}{2l} \frac{1}{3} \rho v l |\nabla u| A d \simeq 2.5 \cdot 10^{43} \text{ erg/cycle.} \quad (1)$$

Here,  $\eta = \frac{1}{3} \rho v l$  is the dynamic turbulent viscosity;  $\rho$ ,  $v$ , and  $l$  are the local density, convective velocity, and mixing length;  $\nabla u$  is the local radial gradient of the horizontal velocity during the pulsation period, estimated as the difference in horizontal displacement between adjacent zones during the pulsation period, divided by the period and the zone thickness;  $A$  is the local spherical stellar area;  $d$  is the distance over which the horizontal motion occurs, which is  $4 \times$  the horizontal displacement  $\times R_{\star}$ . Note that the  $R_{\star}$  factor is due to the conventional linear theory normalization of the radial eigenfunction to  $1 R_{\star}$  at the photosphere. We will be comparing the dissipation rates to our calculated growth rates, which use this normalization. The integral is weighted by  $dr/2l$  to account for damping occurring over length scales of the colliding convective eddies. We reduce the estimate by  $2/\pi$  to roughly account for the spatial variation in amplitude over the areal surface for modes of different spherical harmonic degree  $\ell$  and azimuthal order  $m$ .

Dividing this dissipation rate by the typical mode kinetic energy of  $\sim 10^{47}$  ergs for modes with period  $\sim 1$  day (see Fig. 1), we find that only modes with growth rates larger than  $2.5 \cdot 10^{-4}$ , about a factor of ten larger than predicted by our models, can overcome this shear dissipation. How-

ever, we have probably considerably overestimated the dissipation, for several reasons. First, such large dissipation, if it existed, would reduce or even eliminate horizontal velocity gradients within the CZ (Brickhill 1990). Second, the effect of dissipation on the work is not as straightforward as this simple estimate indicates. Each mass shell reaches its equilibrium position at a different phase, and the variations of the shear forces with phase must be included self-consistently in the solution for the work integral. The nonadiabatic horizontal eigenfunction solution, especially near the top of the CZ, may also be affected by the assumptions of mixing-length theory, frozen-in convection, and the diffusion approximation, which are not as accurate near the stellar surface. For lower-order modes, the horizontal displacement, and the discontinuity near the CZ top become smaller, but the predicted growth rates for these modes also are smaller.

The horizontal displacement amplitude drops rapidly below the CZ base, and the first node of the eigenfunction is several pressure scale heights below the base. Thus, there could also exist damping due to horizontal shear in a possible overshooting region at the CZ base. If we assume that the turbulent viscosity at the CZ base extends one pressure scale height below the base, we estimate a dissipation rate of  $2.6 \cdot 10^{43}$  erg/cycle, comparable to the estimate above for the shear dissipation within the convection zone. This estimate is also probably too large, since it assumes a considerable overshooting distance. However, these shear dissipation estimates, both in and below the convection zone, show that such dissipation should not be neglected, and that possibly only a few modes with the larger predicted growth rates will be able to overcome such dissipation.

## 5. Observational Considerations and Future Modeling

To date, two  $\gamma$  Doradus variables have been observed that also pulsate in  $\delta$  Scuti-like  $p$ -modes (Handler et al. 2000). A few stars with surface abundance peculiarities also have been observed that exhibit  $\gamma$  Doradus pulsations, e.g., the  $\lambda$  Boötis star HR 8799 (Gray & Kaye 1999) and the Am star HD 221866 (Kaye & Gray 2000). These stars offer interesting observational tests for pro-

posed  $\gamma$  Doradus pulsation driving mechanisms.  $\delta$  Scuti pulsations are driven by the H and He ionization  $\kappa$  effect, higher in the stellar envelope than our proposed  $\gamma$  Doradus driving region, so in our scenario could co-exist with longer-period  $\gamma$  Dor pulsations. Surface abundance peculiarities such as those found in  $\lambda$  Boötis and Am stars develop due to accretion, diffusion, or radiative levitation processes. It is possible that a large envelope convection zone would arrest development of surface abundance peculiarities. In the case of  $\lambda$  Boötis stars, the envelope convection zone might mix accreted low- $Z$  material throughout the convection zone, making it more shallow and turning off the  $\gamma$  Dor pulsations. On the other hand, it is also possible that the surface abundance is decoupled from conditions in deeper layers, and that diffusion and levitation produce abundance gradients that generate a localized convection zone deeper in the envelope to modulate the emergent flux. Additional modeling as well as observations are needed to sort out these possibilities.

Our proposed pulsation driving mechanism depends critically on the modulation of convective flux by the pulsation, which has not yet been incorporated in the models presented here. However, we believe that the frozen-in convection approximation reasonably represents the physical processes given the long convective timescale at the CZ base. We are implementing a time-dependent convection (TDC) treatment that explicitly accounts for fluctuations in convective luminosity during the pulsation cycle; preliminary results show that TDC reduces, but does not completely quench, the pulsation driving. TDC actually may improve agreement with observations since the models with frozen-in convection predict up to 20 unstable  $\ell=1$   $g$ -modes, and even more  $\ell=2$   $g$ -modes, whereas many  $\gamma$  Dor stars appear to be monop periodic, and no more than a few modes are observed at a given time for any  $\gamma$  Dor star. We are also in the process of incorporating a self-consistent treatment for the effects of shear dissipation into the nonadiabatic pulsation solution, which may also affect the number of predicted unstable modes.

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## REFERENCES

- Böhm-Vitense, E. 1958, *Z. Astrophys.*, 54, 114
- Brickhill, A.J. 1990, *MNRAS*, 246, 510
- Cox, A.N., Starrfield, S.G., Kidman, R.B. & Pesnell, W.D. 1987, *ApJ* 317, 303
- Cox, A.N. 1993, in *New Perspectives in Stellar Pulsation and Pulsating Variable Stars*, eds. J. Nemeč & J. Matthews (Cambridge: Cambridge U. Press), 107
- Gautschy, A. & Löffler, W. 1996, *DSSN*, 10, 6
- Gray, R.O. & Kaye, A.B. 1999, *AJ*, 118, 2993
- Guzik, J.A., Kaye, A.B., Bradley, P.A., Cox, A.N. & Neuforge, C. 2000, in *Delta Scuti and Related Stars*, eds. M. Breger & M. Montgomery (ASP Conf. Ser.), in press
- Handler, G. 1999, *MNRAS*, 309, L19
- Handler, G. et al. 2000, in preparation
- Iben, I. 1963, *ApJ*, 138, 452
- Iben, I. 1965a, *ApJ*, 141, 993
- Iben, I. 1965b, *ApJ*, 142, 1447
- Iglesias, C.A. & Rogers, F.J. 1996, *ApJ*, 464, 943
- Kaye, A.B. & Gray, R.O. 2000, in preparation
- Kaye, A.B., Handler, G., Krisciunas, K., Poretti, E., & Zerbi, F.M. 1999, *PASP*, 111, 840
- Löffler, W. 2000, in *The Impact of Large-Scale Surveys on Pulsating Star Research*, eds. L. Szabados & D. Kurtz (ASP Conf. Ser.), 447
- Li, Y. 1992, *A&A*, 257, 145
- Pesnell, W.D. 1987, *ApJ*, 314, 598
- Pesnell, W.D. 1990, *ApJ*, 363, 227
- Turcotte, S. 2000, in *Delta Scuti and Related Stars*, eds. M. Breger & M. Montgomery (ASP Conf. Ser.), in press

Wu, Y. & Goldreich, P. 2000, in *The Impact of Large-Scale Surveys on Pulsating Star Research*, eds. L. Szabados & D. Kurtz (ASP Conf. Ser.), 508