Drone Routing Optimizer for Aerial Inspections of Energy and Railway Infrastructures

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Long range drones (either fixed wing or VTOL) represent a novel approach for monitoring and inspection of extensive longitudinal infrastructures such as power lines, pipelines and railway tracks including overhead lines. This paper proposes a method to find efficient system of systems configurations regarding drone range requirements and base locations to operate from. Using an heuristic optimizer, sensitivity analyses are presented for inspection flights of the German Railway system of *Deutsche Bahn* as well as the US natural gas pipeline system. It is concluded that longer drone ranges facilitate either more efficient mission designs or alternatively enable fewer bases of operations and thus less capital expenditure while limiting mission efficiency through increased average target approach distances.

I. Nomenclature

$2-opt$	=	2 Edge Exchange Optimization - an optimization algorithm
$3-opt$	=	3 Edge Exchange Optimization - an optimization algorithm
TSP	=	Traveling Salesman Problem
UAV	=	Unmanned Aerial Vehicle
VRP	=	Vehicle Routing Problem
VTOL	=	Vertical take-off and landing drone

II. Introduction

INSPECTIONS of vast infrastructures like energy transportation assets or railroads traditionally come at significant cost, either requiring considerable manual labor or expensive aerial assets like helicopters. Natural gas pipelines for one are required to be checked for leaking methane as well as external threats through unauthorized construction work on a regular basis [1]. Internationally helicopter inspection flights are usually carried out in bi-weekly intervals, sometimes voluntarily exceeding mandatory intervals to increase safety further [2]. Similarly, power lines as well as railroads including their overhead lines are monitored for defects, albeit usually still carried out by ground based means [3].

Hence it is proposed to use automated drones to conduct inspections flights at a possibly lower cost level [4] [3] [5]. Flight missions can be performed by either of two drone concepts:

- **Fixed Wing Systems** provide benefits in range capability while on the other hand require extensive operations infrastructure, are generally higher invest and less mission efficient when operating from sparse bases because of approach distances.
- **VTOL or Rotary Systems** tend to be more affordable individually while requiring larger numbers of individual systems to compensate for limitations in range capability. This could potentially incur higher labor costs.

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In order to analyse the appropriateness of certain airborne system of systems available for covering vast infrastructures such as energy and rail networks, a method for establishing efficient vehicle navigation and routing had to be developed in order to compare many possible systems. Requirements for the method encompassed the efficient generation of routing solutions across a range of parameters such as the number and location of bases to operate from as well as the available range a vehicle can offer. The number of operating bases forms a trade-off decision between being as close as possible to the areas of operation (on average) and thus operating efficiently and operating as few bases as possible in order to reduce the overall investment and system complexity. The algorithm shall set a minimum required range for each investigated base configuration as to cover all requisite targets. The influence of UAV range on routing and mission efficiency as well as the share of approach and departure distances is analyzed also in respect to overall mission lengths.

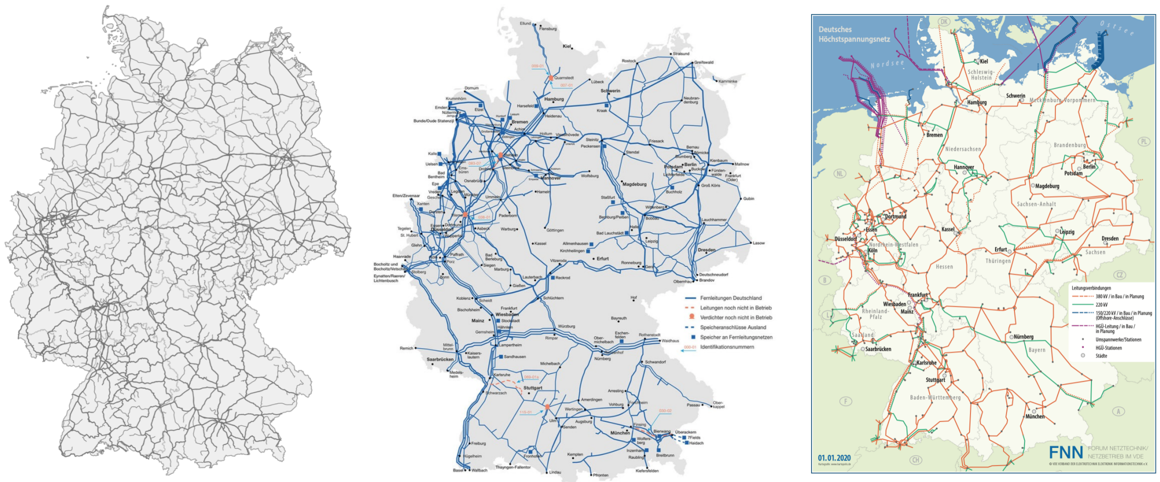

Fig. 1 Railway System [6], Long Distance Gas Pipelines [7], Power Transmission Grid [8] - Germany

III. Algorithms for Drone Navigation

Aerial Navigation in a broader sense describes the overall process of pre-flight planning as well as in-flight measuring and adjusting of flight parameters in order to follow the pre-defined flight plan. In a more differentiated view, the former part of pre-flight planning is often described as *route determination* or *routing* and specifies a desired sequence of waypoints whereas the latter part of measuring and adapting heading, azimuth, position and velocity on a tactical level is described by *navigation* in a more narrow sense [9].

The narrower distinction proves useful when identifying suitable algorithmic approaches for the task at hand. Tactical in-flight drone navigation often utilizes optical image recognition and fusion with RADAR, LIDAR and sometimes ultrasonic perception data or communication data like Global Navigation Satellite Systems for orientation. For this and increasingly also for the subsequent trajectory planning deep learning algorithms, commonly referred to as AI are the means of choice [10] [11]. Furthermore trajectory planning algorithms that follow a waypoint sequence under real time flight performance constraints play an important role.

Pre-flight planning, hereinafter also referred to as vehicle routing, usually employs combinatorial optimization algorithms for solving shortest path problems under certain airspace restrictions in order to fulfill a certain transportation task. This paper however diverts from the classical origin-to-destination setting with known start/end locations in that in presumes different sets of possible operating base positions within the mission area. These operating bases constitute start and end points for individual flights along the target infrastructure. This in itself is very similar to the well known Traveling Salesman Problem (TSP) within which a "salesman" has to find the shortest route to visit all target "cities" within his route before returning to his home base [12]. Range limitations furthermore complicate the problem as the target "cities" (= infrastructure waypoints) have to be visited with several flights (on possibly multiple vehicles). These have to be planned which leads to a TSP-generalization known as the *Vehicle Routing Problem (VRP, see Figure 2)*. The VRP problem structure was applied in the context of combined truck and drone parcel delivery for instance [13].

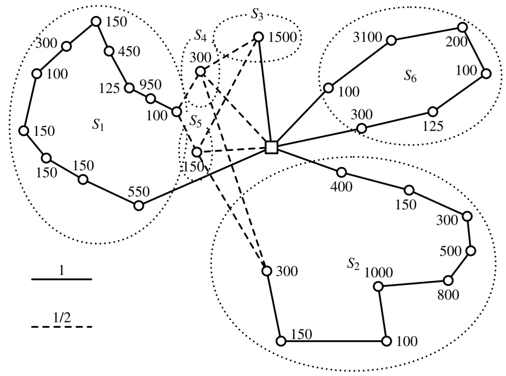

Fig. 2 Instance of a vehicle routing problem [14]

The vehicle routing problem is known to be NP-hard [15]*. From a practitioners perspective, this means that it is hard to find solutions which are provably optimal, meaning that the running time of any algorithm which computes optimal solutions grows quickly with the size of the input setting unless $P = NP$. However, there are algorithms which work well in practice, meaning that they scale well enough for being used on real-world instances with a reasonable running time. Furthermore, they often produce solutions which are very close to optimal solutions.

Hence, NP-hard combinatorial optimization problems like the Vehicle Routing Problem usually employ approximation algorithms to find not the optimal but a near optimal solution [17, 18] . Besides *branch-and-bound*, *branch-and-cut* and *set-covering* based algorithms, classical *heuristics* and *meta-heuristics* are among the tool set suitable for solving generic capacitated vehicle routing problems. Classical heuristics were mostly developed between 1960 and 1990 and typically provide "good quality solutions with modest computing times" while only exploring a limited part of the search space. At the expense of longer compute times meta-heuristics can provide even better results by applying sophisticated neighbourhood search rules or recombination of common part solutions [19].

In between heuristics for the vehicle routing problem, Toth and Vigo [19] differentiate between algorithms for *route construction*, *route improvement* and *two-phase heuristics*: Route construction algorithms create feasible solutions for routes until the capacity limit (here: the drone's range) is reached while trying to keep to total mission length minimal. Route improvement heuristics take a pre-defined route (intra-route) or set of routes (inter-route) and iterate on those in order to shorten the total route length while keeping the solution valid (i.e. not loose any vertices). Two-phase heuristics decompose the problem into allocating the vertices to routes and constructing actual route sequences while these steps can be performed in either order.

A. Formal Problem Definition

For this application, each individual Vehicle Routing Problem (given a number of Bases $|B|$ and a drone range u) is modelled on a *graph* whose *arcs* represent the geodesic distance between target waypoints of the infrastructure which represent the graph's *vertices*. Although this paper does not incorporate any flight restriction zones yet, instead of the geodesic, the distance between any two vertices could in theory be substituted through route distances (found by path search algorithms) which avoid any defined flight restriction zones. The vertices are modelled *undirected*, i.e. the "cost" or distance toward a certain waypoint is the same as back. Theoretically however, directed vertices could make sense if one wanted to model in head or tailwind conditions, possibly even time variant. The targets are visited by a *fleet* of *vehicles* (drones) which are assigned to a *home depot* (also: *base*). As the present flight mission is not to transport goods but to perform sensor tasks the only *capacity* constraint of the vehicle is defined by its range u it can fly before returning to the base.

The optimization problem is constituted as follows:

Given: We are given a set of targets T, hypothetical drone ranges $u \in U$ and a set of bases of operation B.

*This means that an efficient algorithm for solving the VRP could be used to solve all problems in NP efficiently. The existence of such an efficient algorithm is considered unlikely. Its existence would imply $P = NP$. The question whether this equality holds is one of the seven millennium problems [16].

Solve the VRP for each tuple of (B, u_v): The Graph $G = (V, V \times V)$ has nodes that are given by both targets and bases, i.e. $V = B \cup T$ and a distance function $d : V \times V \mapsto \mathbb{Q}_{\geq 0}$. We are looking for m routes $R_i = (r_{i,1}, r_{i,2}, \ldots, r_{i,|R_i|})$ on nodes of G, i.e. $r_{i,j} \in V \quad \forall \quad 1 \leq i \leq m, 1 \leq j \leq |R_i|$ such that the first and last node is a base, i.e. $r_{i,1} = r_{i,|R_i|}$ and $r_{i,1} \in B$. The length of each route is upper-bounded by the range of the drones, c.f. Equation 2. Equation 3 ensures that all targets are covered. Our aim is to minimize the overall mission length of distance ℓ.

$$\min \ell \qquad \text{s.t.} \tag{1}$$

$$\sum_{j=1}^{|R_i|} d(r_{i,j}, r_{i,j+1}) \leq u \tag{2}$$

$$\{r_{i,j} | 1 \leq i \leq m, 2 \leq j < |R_i|\} = T \tag{3}$$

$$\sum_{1 \leq i \leq m} \sum_{1 \leq j < |R_i|} d(r_{i,j}, r_{i,j+1}) \leq \ell \tag{4}$$

For different base locations B and drone ranges u, we obtain optimized mission lengths $\ell_{B,u}$ which can be analyzed in order to surface strengths and weaknesses of many/few bases and the value of the available drone range in each context.

B. Algorithmic Approach

As an exemplary data set on which to show trade offs between different drone system configurations, this paper utilises railway track data by Deutsche Bahn *GEO-Kilometer* [6]. In total it contains ca. 34.000 track waypoints with 1 km separation distance for sufficient aerial resolution over a total area of approximately 350.000 km^2.

Having a five-digit amount of nodes in the overall graph, the algorithm classes of branch-and-bound, branch-and-cut and set-covering were deemed as to compute intensive as to be used practically to solve multiple VRPs. Hence this paper applies heuristics to minimize total mission length $\ell_{B,u}$ as a trade-off between solution quality and compute run time. As the heuristic approach produced good results a more compute-intensive metaheuristic did not seem necessary.

1. Base Positioning

While in reality geographical locations of operating bases might be constrained to available real estate, in this paper, for purposes of exemplifying the impact of different numbers of bases $|B|$ in a scalable way, B is provided by a k-means clustering algorithm with restarts in respect to the set of targets T. For a detailed description of the k-means algorithm in general, see [20].

In order to further speed up the average calculation of the VRPs (especially for larger numbers of bases), the graph is split into a subgraph for each base with the respective subset of targets T_{base} . This reduces the size of the distance matrix ($O(n^2)$) as well as the run time of *route construction* and *intermediate optimization* (both $O(n^2)$). While subdivision into several smaller problems results in itself into shorter run times because of the superlinear time complexity of the solver, further performance gains can be realized by computing individual sub-solutions in parallel.

2. Route Construction Heuristic

To construct routes for the VRP solution, this paper uses the *Clarke and Wright algorithm* [21]. Also known as the *savings method*, the Clarke-Wright algorithm is initialized by creating elementary routes from the base b to each of the assigned targets. These routes $(b, \ldots, i, b)$ and $(b, j, \ldots, b)$ are then iteratively merged into larger routes $(b, \ldots i, j, \ldots, b)$ as long as the resulting route does not violate constraints (like the maximum drone range or a share of it). The sequence within each sub-route is not altered at this step.

The routes which are merged next are chosen according to the largest saving of total route length through any possible merge. In preparation of these merge selections, savings between all targets, i, j are precomputed and stored in a savings priority queue with highest priority for largest savings according to $s_{ij} = d(i, b) + d(j, b) - d(i, j)$. Out of the priority queue, only pairs which represent a first or last target within a route are considered as a route concatenation of in-between targets cannot be executed.

As the approach with a central savings priority queue constantly considers all current routes with respect to the largest saving before each iterative merging step, this variant is considered as the *parallel version* of the Clarke and Wright algorithm. The *sequential version* where an individual route is consecutively merged with more and more elementary routes is viewed as inferior in terms of solution quality by Thoth and Vigo [14].

3. Intra-Route Improvement

Concatenations between the start and/or end targets of routes however, can lead to suboptimal sequences within the merged route. This requires an additional route improvement algorithm to optimize the sequence of targets within each route.

To do that the *λ-opt algorithm* as developed by Lin [22] was used. Here in each iteration, λ edges of the route are removed and the remaining sub-sequences are checked for re-connection possibilities that shorten the overall route distance. There are again different variants of the λ-opt algorithm [23]: Within each iteration, either the best edge swap of all possible edge swaps in the current state of the route can be performed or the first swap that results in an improvement. Furthermore the edge tuples for potential swapping are either searched globally across the entire route or just locally where beneficial swaps are most likely.

Because a global search is very compute intense with only limited improvements in solution quality, in this paper, we propose a *2-opt* local search as a trade-off where we define a neighbourhood size within which two edges have to be in relation to each other. The best possible swap is implemented in each iteration. Furthermore we limit the maximum number of iterations as there are diminishing returns towards the end of finding to local optimum.

More sophisticated derivatives for future refinements of the presented research increase λ which improves solution quality at the expense of larger time complexity $O(n^{\lambda})$. Lin and Kernighan [24] proposed a *variable-opt* algorithm that adapts λ dynamically. This enables high solution quality while maintaining computational performance.

4. Alternating Route Construction and Intermediate Improvement

With sequential route construction and intra-route improvement it quickly became apparent that this results in the available drone ranges not being utilized to the full extent as the route merging is stopped with non optimized routes. Hence, the approach was implemented to interlink both steps. Every time a route construction iteration produces a route which is longer than 0.5 times the available range, a full intra-route improvement is performed. Additionally, intra-route improvement is applied again after route construction has finished in order to improve also smaller routes that have not yet been optimized.

With this variant of intermediate improvement, however, it may occur that targets from a inner route sequence are swapped to be the first or last target of the improved route. As the Clarke Wright route construction algorithm skips inner route targets during the selection process of the next merge, it is possible that the new end or start target of the route represents a now valid merge combination after being already skipped before intermediate improvement. Hence, the savings entry has to be reinserted into the priority queue.

IV. Railroad Inspection Germany

The drone routing optimizer is employed in two case studies, the first being the German railroad network operated by *DB Netz AG*. *DB Netz AG* provides a dataset containing geopositions of the network on their open data portal [6]. Of interest for inspection are possible corrosion of electrical conductors, visible damage to insulators and structural equipment as well as the visual condition of the track bed itself (e.g. gravel abrasion rock flour indicating movement in track foundation, loose rail fasteners or broken ties). While this paper does not discuss sensors and sensor requirements in detail, it may be noted that usually optical inspection methods either in the visual or infrared spectrum can be employed which is suitable to be carried out by airborne sensors as long as the angular resolution and shutter speed are sufficient.

As can be seen in Figure 3 the Drone Routing Optimizer qualitatively produces good results, both placing bases in plausible locations and covering all target waypoints with the calculated mission routing. Each colour represents an individual *route* (e.g. sortie) while combined they form a *mission* which visits each waypoint once. Figure 4 exemplifies how differences in UAV range affect the routing, especially noticeable with approach segments from the base to target infrastructure.

As expected, smaller ranges necessitate many more individual sorties, each with significant approach and departure flight time to actual mission targets. This explains the almost vertically asymptotic behavior of mission length when approaching the limit for a specific base configuration's minimum required UAV range (see Figure 5). In between base configurations it is noticeable that dense configurations (many bases) approach an infrastructure-specific minimum mission length already at relatively small UAV ranges. Sparse configurations (few bases) profit from marginal UAV range much longer until they experience diminishing returns from efficiently adding additional route targets to a flight. This is also reflected in that in Figure 7 the average share of actual range usage in sparse configurations remains high

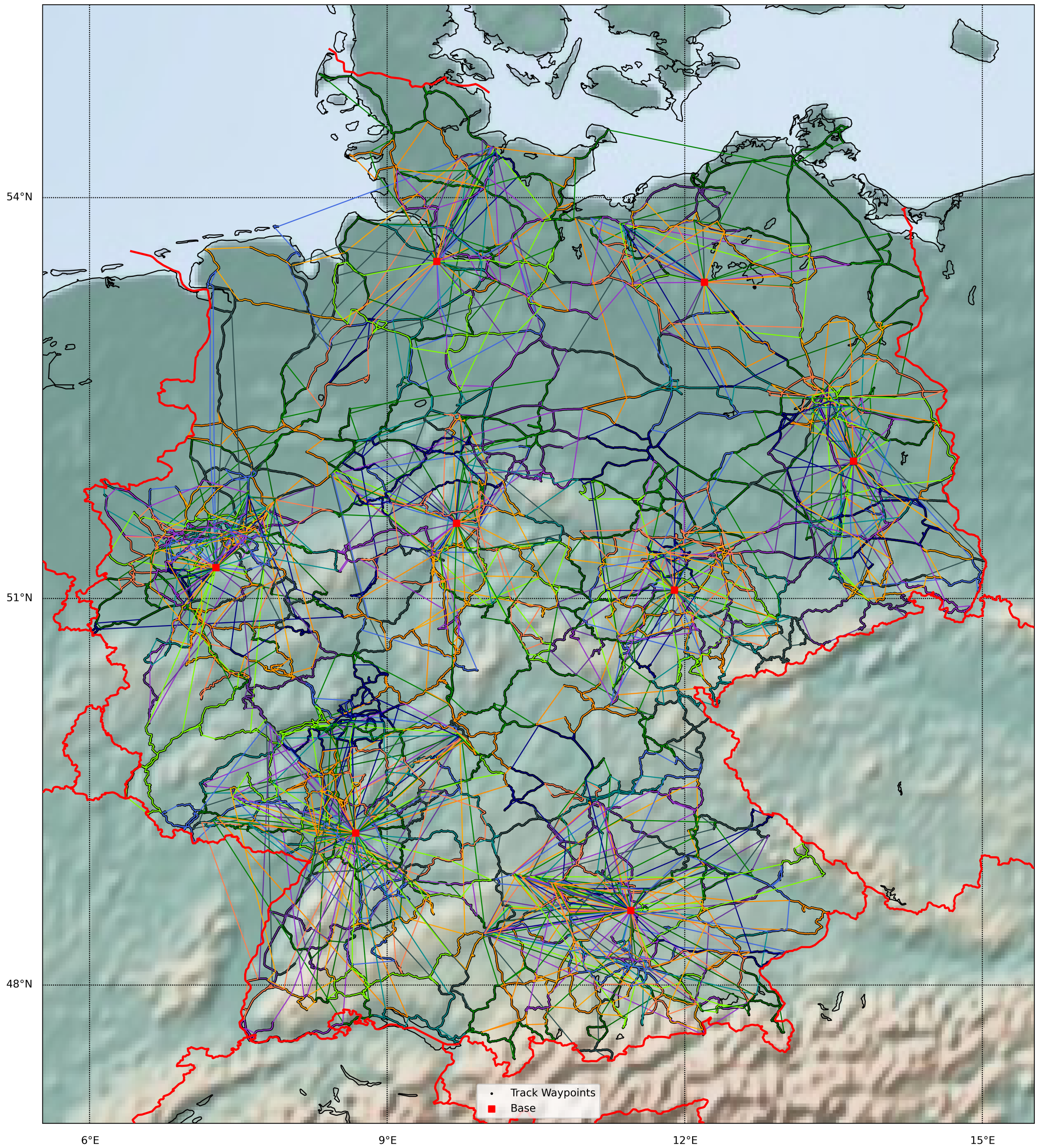

Fig. 3 Routing Solution for 8 Bases of Operation and 500km drone range

for longer UAV ranges as opposed to the drop in range usage for dense configurations, meaning that marginal range capability is actually used beneficially.

Inversely, this is reflected in the share of mission flight time that is spent over target infrastructure versus mission flight time spent on approach and return to operating base (see Figure 6). The monotonically increasing character of the mission share on target infrastructure along increasing available UAV range is also a strong indicator for the quality of solution results provided by the applied algorithm. Any intermediate decrease of this share along with increased range would point to globally suboptimal solutions being stuck in local optima at specific available ranges.

Noticeable is that mission efficiency for extremely dense configurations (50 and 100 bases) maxes out at a lower value of just below 60% while medium density configurations achieve up to 63%. This effect leads to higher values

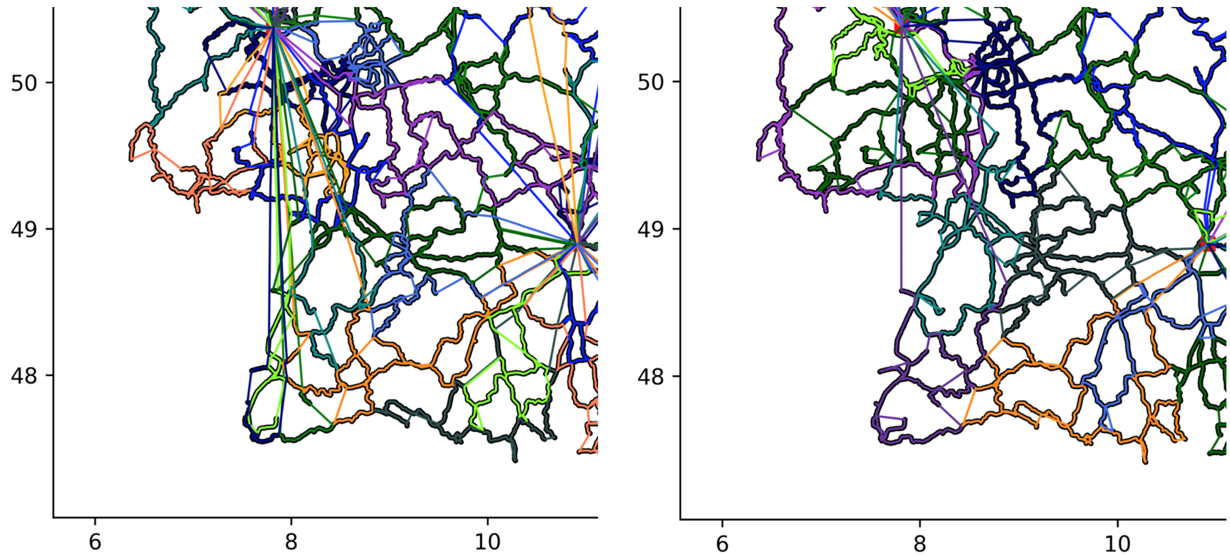

Fig. 4 Comparison of routings with a drone range of 674km (l) and 1100km (r); 4 Bases

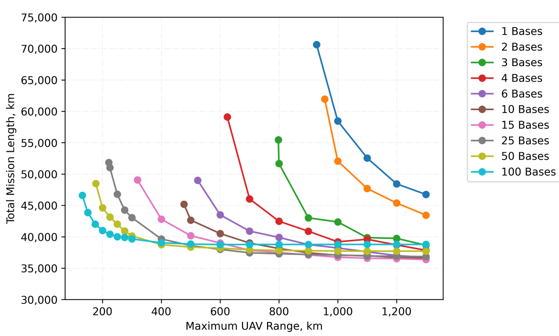

Fig. 5 Overall Mission Lengths

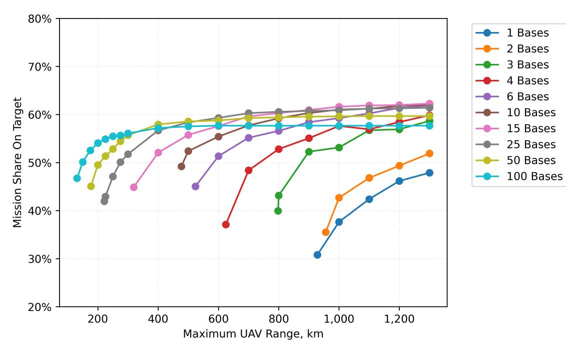

Fig. 6 Mission Share on Target Infrastructure

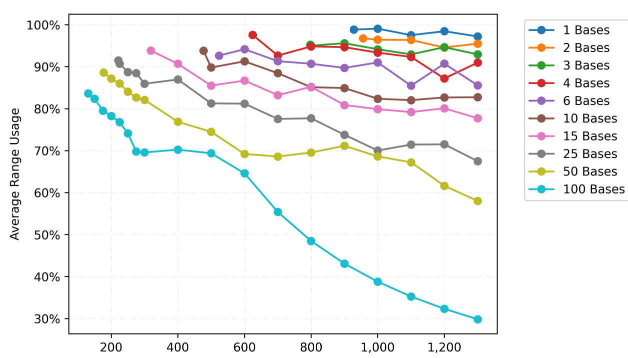

Fig. 7 Average Range Usage

of mission length for especially long UAV ranges compared to medium density configurations. A probable reason behind this effect is, that there is an infrastructure specific optimum of bases above which there is only additional, unnecessary approach and departure distances to the targets to be covered. Another reason could lie in the algorithm for placing the bases which puts the base into the gravimetric center of a target cluster but not necessarily right onto a target infrastructure. This however would be representative for real world base locations as suitable property might not be directly next to target infrastructure, either. With long enough ranges making the benefit of additional bases redundant, the drawbacks predominate.

Agnostic of how the assumed ranges would be realised in a specific aerial vehicle design, it becomes clear that capital efficient sparse base configurations require a minimum range represented as a vertical asymptote of mission range in Figure 5. Depending on the cost intensity per base of operation the cost optimal solution has to be determined with concrete cost models of actual UAVs.

V. Pipeline Inspection United States

The second case study deals with inspection flights of the US natural gas transmission pipeline system as provided by the United States Energy Information Administration [25]. Pipelines in general are checked for issues like unauthorised construction work, corrosion, leakage and vegetation interference among other things [1] [26]. While in general inspection tasks suitable for aerial inspection are similar internationally, detailed regulation and inspections intervals vary from country to country [27]. Similarly to railway power line inspections, detailed sensor requirements are not discussed in this paper. However, the interest in aerial inspection either by helicopter or by drone is rapidly rising in recent year [28] [29] [30].

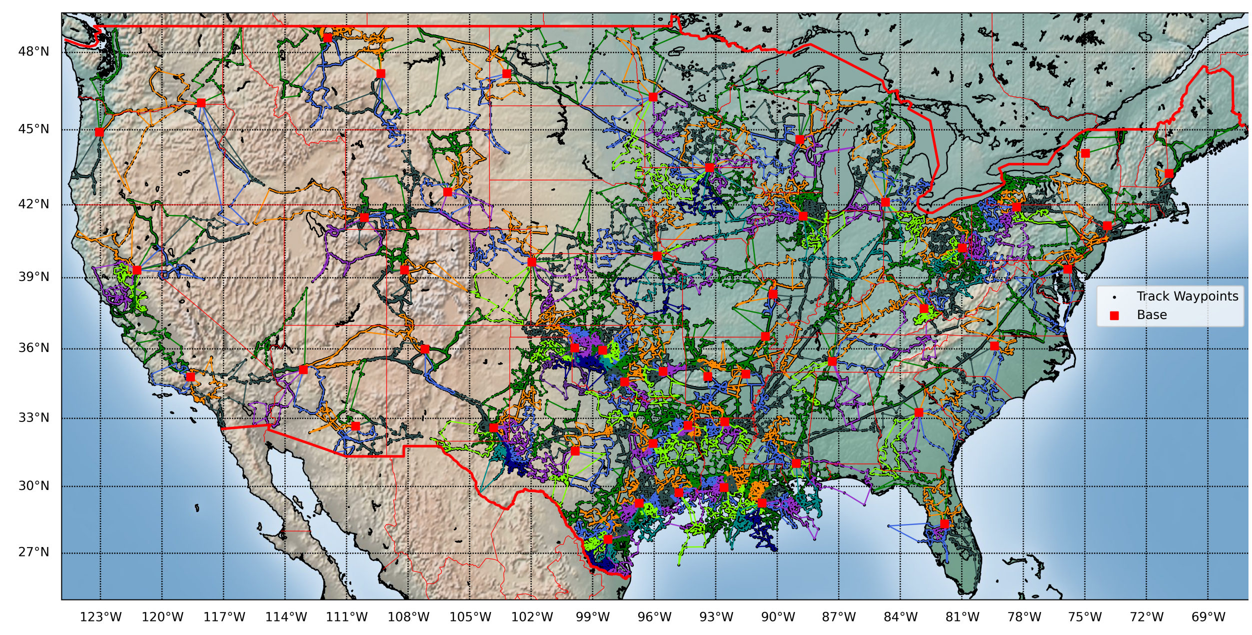

Fig. 8 Routing Solution for the US Natural Gas Pipeline Network with 50 bases and 1300 km range

As the United States mainland is a significantly larger area of operation and gas pipeline infrastructure also has different geometric characteristics as compared to railway infrastructure, the results for mission length and average range usage vary regarding UAV range and base configurations in contrast to the previous case. Obviously there is a tendency that the most efficient configurations do have more bases as compared to the railway case (see Fig. 9). Configurations from 30 up to 2000 bases were investigated as compared to 1 to 100 bases for the German Railway. Also, longer range UAVs tend to have a significant advantage over shorter range versions as additional range within a specific configuration can indeed be efficiently applied so as the average range usage does not drop too much (see Fig. 10). Note also that the densest configurations which correspond the the lower range UAVs of $150 km$ to $400 km$ do drop in average

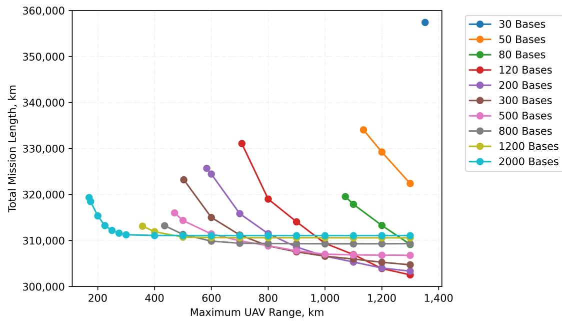

Fig. 9 Pipeline - mission lengths

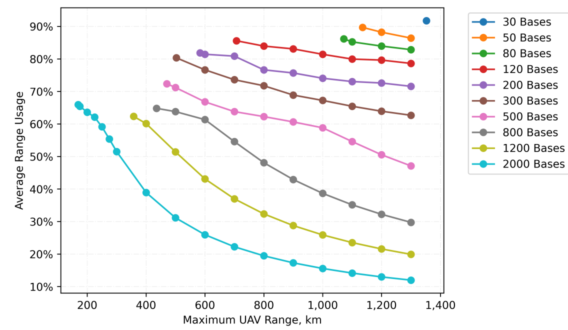

Fig. 10 Pipeline - average range usage

range usage much earlier with respect to UAV range. Accordingly, they reach their mission length optimum already at much less UAV range without any further improvements in mission length. Configurations with medium density clearly dominate the lowest mission lengths at high UAV ranges. Even sparser configurations would certainly become even more dominant, would the practicable UAV range not be limited by flight time during daylight for the camera sensors.

VI. Conclusion

An algorithm for efficiently routing drones along large linear infrastructures for inspection purposes was designed and applied to two use cases: The German Railway Network and the US natural gas transmission pipeline system. Using both case studies, different specifications of drone system-of-systems could be bench-marked against each other in order to find operationally efficient configurations of drone ranges and operation base densities.

It could be shown that operational efficiency of certain system-of-systems specifications is hugely dependent on the type of infrastructure that has to be inspected. In the case of the German Railway Network the most efficient configurations were of 10 to 50 bases. The operational efficiency within a configuration varied comparatively little across available UAV ranges once a configuration specific (approximate) "range saturation" value was reached. However, the sparser a configuration, the higher the value of range saturation until the mission efficiency only increases marginally anymore. Overall, if a suitable UAV was chosen, total mission lengths between configurations of 3 to 100 bases, varied only about 10% of total mission length.

The US pipeline case however has shown a larger sensitivity regarding the choice of suitable UAV range for a certain base configuration in that the densest configurations reach a minimum mission length with limited mission efficiency already at low UAV ranges, while sparser configurations have the potential to utilize higher UAV ranges efficiently and thus reduce mission length further. As the available daylight hours per day are limited to 11h in this study, the UAV cruise speed limited the maximum practical range to no more than 1300km. An increase in UAV cruise speed, enabled by sensors with a higher frame rate and shutter speed to maintain data quality could open up even more efficient base configurations capable of exploiting even higher UAV ranges. Most likely because of this daylight limitation the configurations between 80 and 200 bases each with optimal UAV ranges differ only about 4% of total mission length.

This paper indicates that there is significant impact of the right choice of UAV performance specification as well as base configuration in respect to geographic infrastructure properties. Especially, finding the right match of UAV and configuration is crucial for mission efficiency. However, future work should establish a cost model to quantify system-of-systems options against each other from an economical perspective. Different UAV performance requirements (VTOL vs. fixed wing) and especially base configurations may very well have considerably different capital and operational expenditures and costs.

Finally it may be noted that autonomous aerial inspections carry huge promise of inspection cost reduction on one hand and improve operational safety of energy infrastructure through more frequent inspections enabled by a lower cost basis.

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