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DSP complexity of mode-division multiplexed receivers

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Abstract: The complexities of common equalizer schemes are analytically analyzed in this paper in terms of complex multiplications per bit. Based on this approach we compare the complexity of mode-division multiplexed digital signal processing algorithms with different numbers of multiplexed modes in terms of modal dispersion and distance. It is found that training symbol based equalizers have significantly lower complexity compared to blind approaches for long-haul transmission. Among the training symbol based schemes, OFDM requires the lowest complexity for crosstalk compensation in a mode-division multiplexed receiver. The main challenge for training symbol based schemes is the additional overhead required to compensate modal crosstalk, which increases the data rate. In order to achieve 2000 km transmission, the effective modal dispersion must therefore be below 6 ps/km when the OFDM specific overhead is limited to 10%. It is concluded that for few mode transmission systems the reduction of modal delay is crucial to enable long-haul performance.

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1. Introduction

Single mode fiber (SMF) is the standard for all long haul transmission systems so far but may not be able to meet the explosively increasing capacity demand of optical communication in the near future. One of the possible solutions is to use space division multiplexing (SDM) as this is a dimension which has not been explored yet. There are different ways to achieve SDM transmission: weakly coupled multicore fibers (MCF), strongly coupled MCF, and multimode fibers (MMF). Types of MMF in which only a few modes are supported, i.e., few mode fibers (FMF), have attracted a lot of attention in the recent years [1–3]. In [4], the transmission over 1200 km of differential group delay (DGD) compensated FMF with three-fold mode-multiplexing has been demonstrated. The FMF transmission research is looking into optical amplification as well to achieve longer transmission distances [5]. Recently [6], used inline amplifiers to transmit over 50 km FMF and [7] demonstrated transmission with the aid of Raman amplification.

The optimization of few mode fiber design is still an active area of research. One of the problems with the realization of transmission over FMF is that these fibers have a high modal dispersion. This results in a high number of equalization taps which might make long-haul transmission using FMFs impractical due to the required DSP complexity. Randel et al. [8] for instance used 36 finite impulse response filters with 120 taps for a 6 x 6 MIMO equalizer to demonstrate 33-km transmission using a three-fold FMF with 60 ps/km modal dispersion. This concept clearly does not scale to long haul transmission due to issues with complexity. Over the last few years the modal dispersion of FMF significantly reduced and it is expected that this trend will continue. In [7], the modal dispersion parameter between LP_{01} and LP_{11} was 27 ps/km, after a few months the effective DGD decreased to less than 50 ps for 30 km with the help of cascading two fibers with both 1.5 ns DGD but in one of them LP_{01} travels faster, in the other one LP_{11} [4].

Although most of the papers on FMF transmission until now [4, 6, 7] use three fold multiplexing, considering fibers with two linearly polarized modes (LP_{01}, LP_{11}), it is desired to increase the number of modes to further increase fiber capacity [9]. Recently, we reported the complexity comparison of various equalization schemes of coherent receivers for both blindly adapted and training symbol (TS) based approaches with respect to the equalization of modal dispersion and crosstalk for a 6 x 6 mode multiplexed receiver [10]. In this paper, we

extend this work by showing the details of analysis and present how the complexity scales with increasing number of modes. The complexity is calculated in terms of complex multiplications per transmitted bit. We conclude that there is a significantly larger difference between TS and blindly adapted schemes compared to what was observed for SMF [11]. For FMF, among TS based algorithms, orthogonal frequency division multiplexing (OFDM) provided the lowest complexity. However, the reach of algorithm that use TS channel estimation is limited by the allowable overhead for cyclic prefix (CP). For a 10% CP, it is observed that the maximum modal dispersion that can be tolerated between LP_{21} and LP_{01} is 6 ps/km for 2000 km of 10 x 10 MIMO transmission.

2. Equalizer complexity for few mode fiber

In this paper, we focus on long-haul transmission with 100 Gb/s transponders. We concentrate here on coherent detection in which impairments are equalized in the electrical domain since it is the standard for 100 Gb/s long haul [11]. The complexity analysis focuses on the complexity required for chromatic dispersion (CD), differential modal delay (DMD) and polarization mode dispersion (PMD) equalization. The channel is assumed to be linear; the nonlinear effects are not taken into account in this work. The nonlinear investigation is less interesting for FMF as the nonlinear limit is significantly higher than that of SMF [12]. The laser phase noise is also excluded. Equalizer schemes can be divided into two groups: blindly adapted and TS based. In this work the most common equalizer schemes are compared. The time domain equalizer (TDE) and hybrid frequency domain time domain equalizer (FDE/TDE) are investigated for a blindly adapted equalizer approach. FDE, OFDM and hybrid FDE/OFDM are studied for a TS based equalizer approach. The complexity is assessed in terms of complex multiplications per transmitted bit which includes forward error correction overhead, but excludes TS based equalization specific overhead bits [11]. Polarization-multiplexing is assumed for all configurations. The channel impulse response is determined by CD, DMD and PMD. The lengths of the fiber impulse length due to CD (τ_{CD}), PMD (τ_{PMD}) and DMD (τ_{DMD}) can be defined as

$$\tau_{CD} = \frac{c}{f_c^2} |D \cdot d| \Delta f, \tag{1}$$

where c is the speed of light, D is the chromatic dispersion parameter, d is the fiber length, f_c is the optical carrier frequency, Δf is the signal's spectral bandwidth;

$$\tau_{PMD} = p \cdot \sqrt{d}, \tag{2}$$

where p is the PMD parameter;

$$\tau_{DMD} = \chi \cdot d, \tag{3}$$

where χ is the DMD per km, i.e. modal dispersion. For the rest of this section, the block diagrams are depicted for a scenario with five-fold multiplexing each with two polarizations is used, over a three mode fiber (LP_{01}, LP_{11}, LP_{21}). Consequently, the number of tributaries, denoted as ζ, is 10.

2.1. Blind equalizers

Figure 1 illustrates two common blind equalizers These schemes adaptively update the coefficients of the finite impulse response (FIR) filters in equalizer using feedback. The coefficients are assumed to be updated once per symbol duration [11]. Some of the branches in the butterfly structure are not shown for simplification of the figure.

The minimum numbers of equalizer taps to compensate the CD and DGD are

$$N_{CD} = \left\lceil \frac{\tau_{CD}}{T} \right\rceil n_{SC}, \tag{4}$$

$$N_{\mathrm{DGD}} = \left\lceil \frac{\tau_{\mathrm{DGD}}}{T} \right\rceil n_{\mathrm{SC}}, \tag{5}$$

where $\lceil x \rceil$ is the ceiling function, gives the smallest integer greater than or equal to x, n_{SC} is the oversampling ration for single carrier (SC), T is inverse of the symbol rate. For FMF, both DMD and PMD delay contribute to DGD.

2.1.1. Time domain equalizer

TDE uses a finite impulse response (FIR) filter to compensate for channel effects. Figure 1(a) depicts a 10 x 10 MIMO TDE. The coefficient update doubles the complexity for the common algorithms like least mean squares, constant modulus or Godard's algorithm [11]. There are ζ^2 FIR filters and each FIR filter requires N_{CD} plus N_{DGD} taps, resulting in ζ^2 times ($N_{\mathrm{CD}} + N_{\mathrm{DGD}}$) complex multiplications. The equalizer gives ζ times $\log_2(M)$ output bits where M is the number of points in constellation. As a result a TDE with ζ tributaries have

$$C_{\mathrm{TDE}} = \frac{2\zeta^2 \cdot (N_{\mathrm{CD}} + N_{\mathrm{DGD}})}{\zeta \cdot \log_2(M)} = \frac{2\zeta \cdot (N_{\mathrm{CD}} + N_{\mathrm{DGD}})}{\log_2(M)} \tag{6}$$

complex multiplications per bit.

2.1.2. Hybrid frequency domain/time domain equalizer

It is proven that when the equalizer length exceeds a certain length, FDE is less complex compared to TDE [13]. The equalizer length can by far be more than hundreds of samples at high bit rates for CD equalization, especially for the links without in-line CD compensation. In such cases it is beneficial to compensate for CD in the frequency domain. The single carrier FDE/TDE in Fig. 1(b) converts the signal into the frequency domain with a fast Fourier transform (FFT), compensates for the bulk CD, and transforms the signal back into the time domain with another FFT where subsequently a butterfly structure (TDE block) is used to compensate for dynamic time-varying effects such as DGD as well as residual CD. The coefficients of the FDE block, CD compensation, are estimated only once at the beginning of transmission. The FDE part uses 2ζ FFTs with size N in addition to the ζN complex multiplications with coefficients ($C'11, ..., C'_{1010}$). The FFT complexity is $N/2\log_2(N)$. In order to reduce the complexity of the FDE part an overlap and save method can be used, with which the number of output bits is decreased by N_{CD}-1 [11]. The decimation from n_{sc} to 1 takes places in the TDE part, also reducing the number of useful bits from the FDE part by a factor of n_{sc}. Thus, the number of useful bits per block on all tributaries is $\zeta \cdot \log_2 M / n_{\mathrm{SC}} (N - N_{CD} + 1)$. To sum up, the complexity of the FDE/TDE method is

$$C_{\mathrm{FDE/TDE}} = \frac{\zeta \cdot N + 2\zeta \left(\dfrac{N}{2} \log_2 N \right)}{\zeta \dfrac{\log_2 M (N - N_{CD} + 1)}{n_{SC}}} + \frac{2\zeta \cdot N_{DGD}}{\log_2 M} = \frac{N + 2 \left(\dfrac{N}{2} \log_2 N \right)}{\dfrac{\log_2 M (N - N_{CD} + 1)}{n_{SC}}} + \frac{2\zeta \cdot N_{DGD}}{\log_2 M}. \tag{7}$$

The Eq. (7) shows that the CD compensation part complexity is independent of number of tributaries.
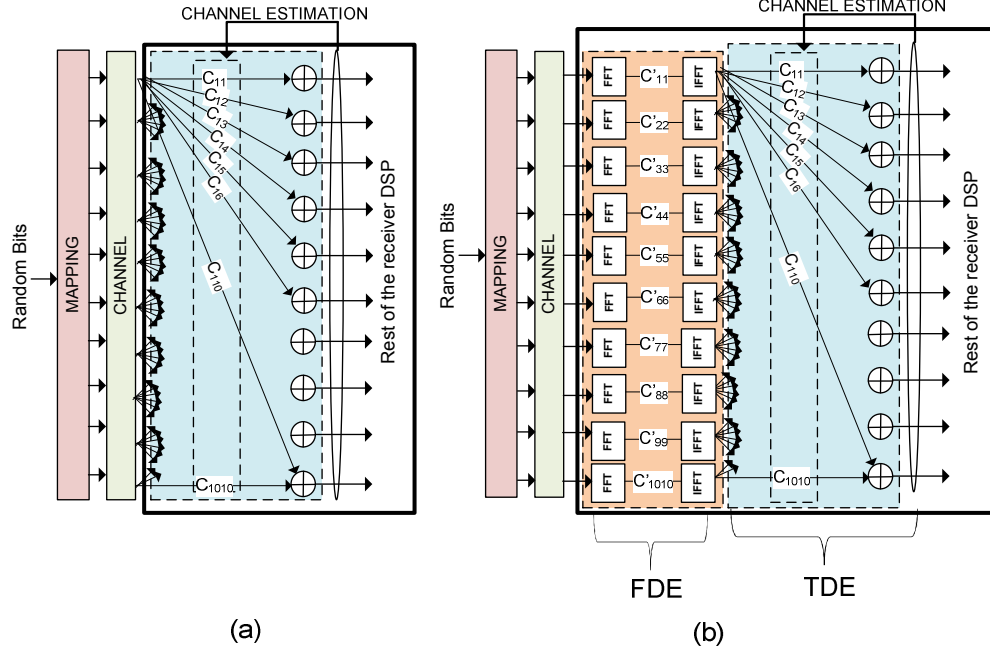
Fig. 1. Block diagram of (a) SC system with TDE (b) SC system with FDE/TDE for 10 x 10 MIMO.

2.2. Training symbol based equalizers

The key of TS based transmission is blockwise transmission. The data are sent in blocks, which can be referred to as frames as well. Each subsequent block is separated with a guardband, often referred to as cyclic prefix (CP). The TSs are used to calculate equalizer coefficients at the receiver. The CP and TS are inserted at transmitter for the TS based equalizers as shown in Fig. 2 and both add overhead to the total data rate. With each TS the the coefficients are updated at the receiver. The repetition rate R_{TS} of the TS needs to be as low as possible to limit the TS-overhead, but sufficiently fast in order to react fast enough to channel dynamics. The CP must be long enough to accommodate the impact of CD and DGD in order to avoid intersymbol interference of neighboring symbols at the receiver.

2.2.1. Frequency domain equalizer

The block diagram of FDE is shown in Fig. 2. The complexity calculation for one frame of FDE includes the 2ζ FFT complexities, calculation of channel matrices and their inverse at every channel estimation update and the multiplication of the received frame with the inverse channel matrix [11]. The calculation of channel matrices result in $\zeta^2 N$ complex multiplications, their inverse approximately $\zeta^3 N$ complex multiplications [14] and multiplication of the received frame with the inverse channel matrix requires $\zeta^2 N$ complex multiplications. The complexity of the FDE equalizer can be expressed as

$$C_{FDE} = \frac{2\zeta\left(\dfrac{N}{2}\log_2 N\right) + \zeta^2 N + \dfrac{\zeta^2 N + \zeta^3 N}{1/R_{TS}}}{\zeta N \dfrac{\log_2 M}{n_{SC}}} \cdot T_f = \frac{2\left(\dfrac{N}{2}\log_2 N\right) + \zeta N + \dfrac{\zeta N + \zeta^2 N}{1/R_{TS}}}{N \dfrac{\log_2 M}{n_{SC}}} \cdot T_f . (8)$$
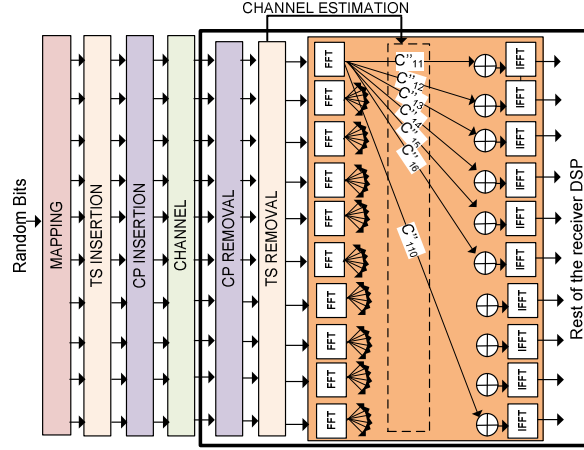
where T_f is the time for one frame.

Fig. 2. Block diagram of single carrier system with FDE for 10 x 10 MIMO.

2.2.2. Orthogonal frequency division multiplex

Figure 3 depicts OFDM. For OFDM, the IFFT is done at transmitter side rather than at the receiver, but other than that it has many common properties with the single carrier FDE. The complexity of OFDM is calculated similarly as FDE,

$$C_{\text{OFDM}} = \frac{2\left(\dfrac{N}{2}\log_2 N\right) + \zeta N_u + \dfrac{\zeta N_u + \zeta^2 N_u}{\dfrac{1/R_{\text{TS}}}{T_f}}}{N_u \log_2 M}. \tag{9}$$

The oversampling in OFDM is the ratio of modulated subcarriers (N_u) to all subcarriers (N).
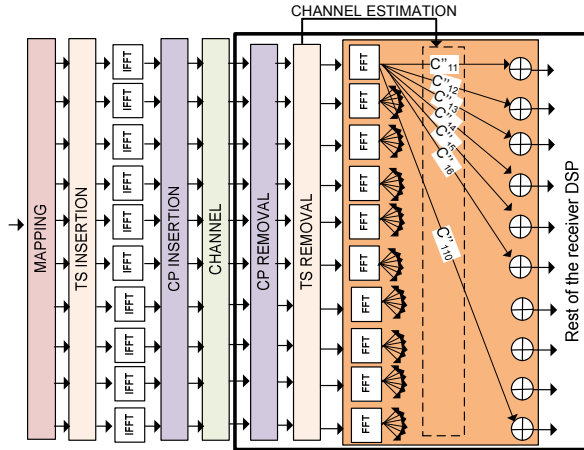

Fig. 3. OFDM block diagram for 10 x 10 MIMO.

2.2.3. Hybrid frequency division multiplex/orthogonal frequency division multiplex

The CD is statically compensated for FDE/OFDM architecture, similar to the FDE/TDE architecture shown in Fig. 4. As a result the CP compensates only for DMD and PMD, and becomes shorter. The complexity of FDE/OFDM is simply a sum of the separate complexity of FDE (for CD compensation) as in Eq. (7) and OFDM (for DMD and PMD compensation),

$$C_{FDE/OFDM} = \frac{N_1 + 2\left(\dfrac{N_1}{2}\log_2 N_1\right)}{\dfrac{\log_2 M (N_1 - N_{CD} + 1)}{n_{OFDM'}}} + \frac{2\left(\dfrac{N}{2}\log_2 N\right) + \zeta N_u + \dfrac{\dfrac{\zeta N_u + \zeta^2 N_u}{1/R_{TS}}}{T_f}}{N_u \log_2 M}, \quad (10)$$

where N_1 and N are the FFT sizes of the FDE and OFDM parts respectively, $n_{OFDM'}$ is the oversampling of OFDM including the effects of CP and TS overheads derived as

$$n_{OFDM} = \frac{N_u}{N} \quad (11)$$

$$n_{OFDM'} = n_{OFDM} \cdot (1 + \varepsilon_{TS}) \cdot (1 + \varepsilon_{CP}), \quad (12)$$

where ε_{TS} and ε_{CP} are the TS and CP overheads respectively. The overheads ε_{TS} and ε_{CP} are

$$\varepsilon_{TS} = \frac{N_{Training}}{N_{TrSpacing}}, \quad (13)$$

$$\varepsilon_{CP} = \frac{T_{CP}}{T_f}. \quad (14)$$

The number of TS used in one training block spacing $N_{TrSpacing}$ is $N_{Training}$ and the time for CP is denoted as T_{CP}.
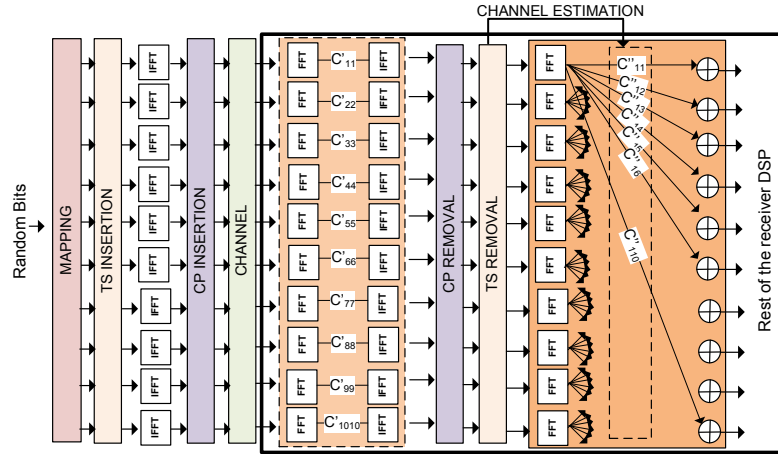

Fig. 4. FDE/OFDM block diagram for 10 x 10 MIMO.

3. Analytical results

The parameters chosen for the analytical analysis are as follows. The net bit rate per mode is 100 Gb/s using polarization multiplexed QPSK. The FEC overhead is chosen as 11% and the combined maximum overhead for TS and CP is 10% for TS based approach. The oversampling factors for both single and multi-carrier schemes are 1.5. The PMD parameter, p, is 0.02 ps/√km. The chromatic dispersion parameter, D, is chosen as 20 ps/nm/km to be compatible with recent publications such as [6]. The TS based equalization technique uses one TS for FMF, which is valid under the assumption that with subcarrier multiplexing one can find orthogonal symbols [15]. Consequently, one TS is sufficient for the MIMO scenario and the TS overhead is kept as low as possible. The channel estimation is updated every 10 μs so

that TSs can dynamically react to changes in optical channel [16], however, the optimum repetition rate for FMF should still be investigated. The complexity is calculated in a two-dimensional matrix, one variable is the sampling rate and the other is the size of the FFT. The maximum sampling rate is limited to 60 GS/s, compatible with the sampling rates of commercially available current analog-to-digital and digital-to-analog converters. For each configuration, we find the optimum sampling rate resulting in minimum complexity satisfying the overhead constraint.
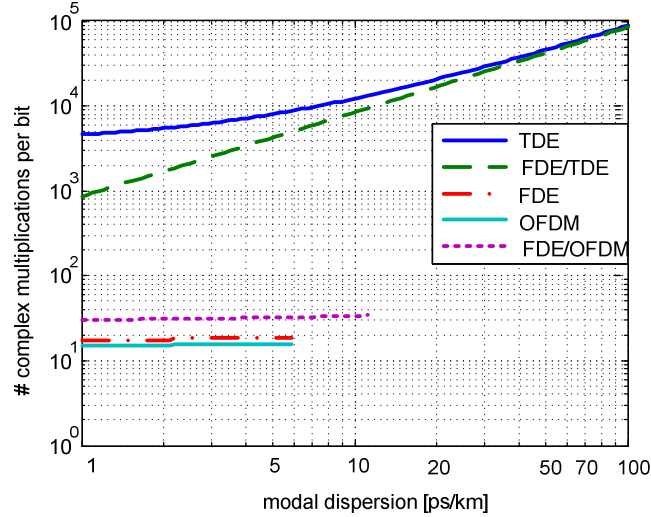

Fig. 5. Complex multiplications per bit in terms of modal dispersion for 2000 km and 10 x 10 MIMO. Total overhead for cyclic prefix and training symbols is maximum 10%.

Figure 5 shows the complex multiplications per bit for the equalizer types described above as a function of modal dispersion of FMF for 2000 km transmission distance. The number of tributaries, ζ, is 10 in Fig. 5. The blind equalizers, TDE and FDE/TDE, use more than 1000 multiplications even for a modal dispersion of 5 ps/km, which is significantly higher compared to TS based equalizers. Most of the multiplications required for FDE/TDE are caused by time domain operations. The efficiency of FDE/TDE is greatly reduced because the modal dispersion is, unlike CD, time variant and therefore cannot be compensated for with the FDE part. The TS based equalizer complexity is significantly lower; however the reach is limited by the maximum allowed overhead. Since the modal dispersion causes the need for a larger CP, the required overhead increases. Among the TS based equalizers, OFDM has the lowest complexity. However, it cannot tolerate more than a modal dispersion of 5.9 ps/km modal dispersion due to the 10% overhead constraint, where the number of complex multiplications per bit for OFDM is only 17.6. The reach of OFDM can be extended by adding the FDE block before it. The FDE block that reduces the required CP would allow the DSP to cope with higher modal dispersion, 11.2 ps/km with higher number of complex multiplications, i.e. 37.5.
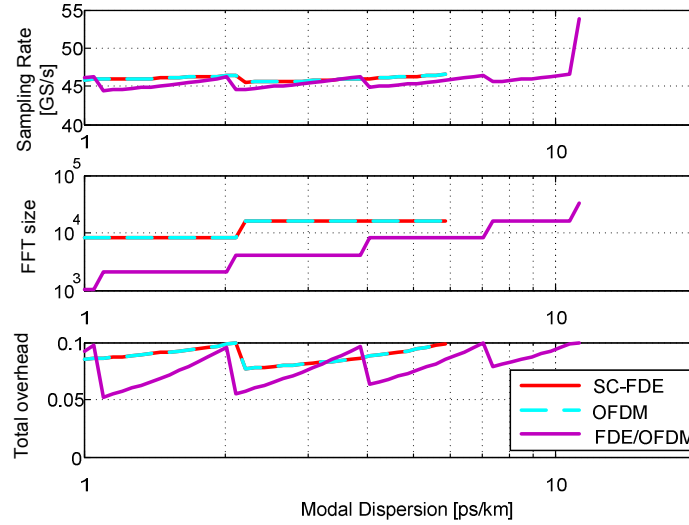
Fig. 6. The sampling rate, FFT size and total OFDM overhead vs. modal dispersion for 10 x 10 MIMO and 2000 km transmission.

The TS based equalizers have a lower complexity but require a higher data rate due to extra overheads. The sampling rate, the FFT size N and the total of TS and CP overheads are shown in terms of modal dispersion in Fig. 6 for a transmission distance of 2000 km. As the CP increases due to the growing modal dispersion, the sampling rate increases in order to keep the overhead under the maximum value. When the total overhead rises to the maximum value, the FFT size is doubled to lower the overhead. The FFT size becomes 16k for OFDM at the maximum modal dispersion it can tolerate and sampling rate at this point is 47 GS/s. It should be noted that, the FFT size has to be upper limited. For example, for FDE/OFDM and a 2 ps/km modal dispersion, the optimum solution is a 2048 FFT and almost 10% overhead. However, an FFT of higher than 10000 and a overhead of less than 5% would be less optimum due to the fact that not only higher FFT size is less tolerant to phase noise but also due to the increase of the TS overhead.
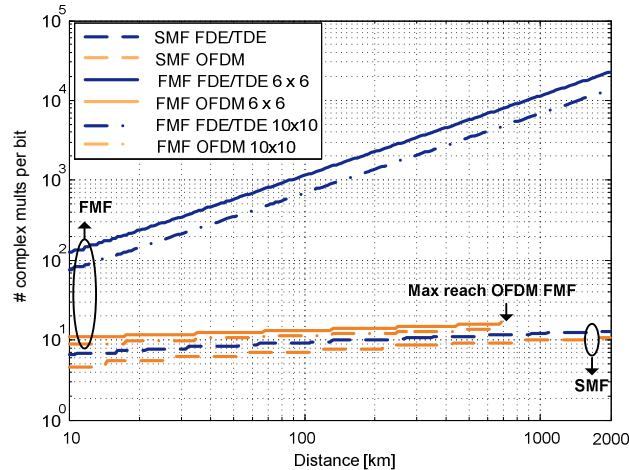

Fig. 7. Complex multiplications per bit in terms of distance for few mode and single mode fiber.

In Fig. 7 the complexity of FMF is compared with SMF. In this figure, the best TS based (OFDM) and blind equalization (FDE/TDE) techniques are chosen. The equalizer complexity

is shown as a function of transmission distance, and we directly compare the required complexity for distortion equalization of both SMF and FMF. Both 6 x 6 and 10 x 10 MIMO scenarios are plotted. The modal dispersion between highest and lowest order modes is chosen as 27 ps/km for both 6 x 6 and 10 x 10 cases. The modal dispersion of 27 ps/km has been shown for 2 mode FMF in [7] and is possible to achieve it in the near future for 3 mode case with DMD compensated fibers. The sum of N_{CD} and N_{DGD} for FMF increases up to 2276 taps at 2000 km for FMF whereas it does not exceed 1 for SMF where only PMD has to be compensated. The FDE/TDE equalizer complexity is highly dependent on the time domain operations which are determined by τ_{DMD} and τ_{PMD}. Since, τ_{DMD} is 0 for SMF (2 x 2), the FMF results in the much higher FDE/TDE complexity. On the other hand, the complexity of FDE/TDE for SMF is only 12.7 multiplications per bit at 2000 km.
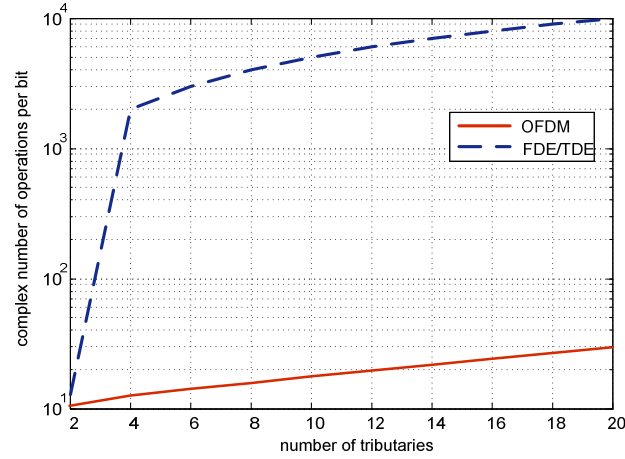

Fig. 8. Complex multiplications per bit in terms of number of tributaries for few mode fiber.

As explained in Section 2.2, the time shift between tributaries caused by CD and DMD is compensated with the CP for the TS based equalizers. From Eq. (8) and Eq. (9), it is deduced that the FFT size is the dominating factor on complexity and the number of modes has less impact. As a result, the complexity of a TS based equalizer for FMF is not significantly different than that for SMF. On the other hand, OFDM cannot exceed a transmission distance of 689 km due to the 10% overhead constraint. The number of complex multiplications per bit at the maximum reach is and 15.6 for 6 x 6 and 19.7 for 10 x 10 case.

Figure 8 illustrates how the complexity scales as the number of tributaries, ζ, increase. One more time the best TS based (OFDM) and blind equalization (FDE/TDE) techniques are chosen. The transmission distance is 2000 km. The modal dispersion is 5.9 ps/km between the highest and lowest modes since this is the maximum modal dispersion that can be tolerated in order to transmit over 2000 km with OFDM for 10 x 10 case as shown in Fig. 5. In both cases the complexity increases linearly with the number of processed bits, but for FDE/TDE the complex multiplications per bit increases to 10000 as ζ increases to 20 whereas for OFDM it only increases to 28. It should be noted that as ζ increases, it would be more difficult to keep the modal dispersion at 5.9 ps/km.

The TS-based equalization reach is limited with CP overhead but it is not significantly more complex than SMF equalization schemes, even though the modal dispersion of the FMF with 10 x 10 MIMO needs to come down to less than 5.9 ps/km to transmit over 2000 km.

4. Conclusion

In this paper several equalization methods were compared in terms of complexity for space division multiplexed optical systems. We showed that training symbol based equalization provides several orders of lower complexity compared to blind equalization for long distance transmission systems. The complexity of blind algorithms scales with the size of the MIMO

matrix whereas that of TS-based algorithms is virtually unaffected. As a result, a blind approach is not feasible for a practical FMF system. The main challenge of the TS-based algorithm is that it requires additional overhead for channel equalization, which again increases the data rate. If the OFDM specific overhead is limited to 10%, a fiber with a modal dispersion as low as 6 ps/km between LP_{01} and LP_{21} is required for 2000 km of 10 x 10 MIMO transmission. The reach of OFDM cannot exceed 689 km again due to the overhead constraint for the modal dispersion of 27 ps/km between LP_{01} and LP_{11}. This shows that for few mode transmission systems the reduction of modal delay is essential to enable long-haul performance.

Acknowledgments