

Dual-channel supply chain operations with working capital constraint: constant vs increasing marginal costs

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Abstract

This paper builds a two-stage game model to capture the operations of a dual-channel supply chain consisting of one supplier and two retailers where one of the retailers is constrained by its working capital and the other is not. With this model, we identify the different effects of the capital constraint on the operations of the supply chain and the corresponding profitability in two different (production) technological settings where the marginal cost is constant and increasing respectively. The results show that (1) in the case of a constant marginal cost, the unconstrained retailer's operations and its profitability are independent of the constrained retailer's working capital, while in the case of an increasing marginal cost, the constrained retailer's working capital have a negative impact on the unconstrained retailer's ordering quantity and profitability; (2) the impact of an change in such working capital on the constrained retailer's operations and profitability is regardless to the feature of the marginal cost; (3) in both setting, the supplier and the constrained retailer benefit from an increase in the constrained retailer's operational capital;(4) in the case of an increasing marginal cost, this produces a conflict between the constrained and the unconstrained channels. Managerial insights are also discussed.

Key words: *supply chain; dual channels; operational capital constraints; marginal cost*

1 Introduction

In recent years, channel conflicts in supply chains are becoming more and more prominent. These conflicts are mainly due to the externalities that one channel makes on the other. For these externalities, a large numbers of studies focus on the market aspect (see, for example, Parka and Keh, 2003; Tsay and Agrawal, 2004; Huang, Yang and Liu, 2013; etc.). However, this literature al most (implicitly) assumes a cost function with the feature of constant marginal costs. Obviously, when the marginal cost is increasing in the quality, more sales of one channel will lead to a higher marginal cost, then tend to increase the wholesale price in the other channel and thus have a negative impact on the latter. Note further that downstream sales usually are constrained by the amount of working capital. Therefore, intuitively, in a setting where working capital is constrained, the order quantity from a constrained

downstream firm would increase in the amount of working capital. When an upstream firm produces with a cost function with the feature of increasing marginal costs, the relaxation of working capital constraints is likely to have a negative externality to the unstrained channel via the effect on the supplier's marginal cost. We address this issue with two-stage game model that describe the operations of a dual-channel supply chain consisting of an upstream supplier and two downstream retailers where one of the retailers is constrained by its working capital and the other is not. We analyze the supply chain in two different setting in terms of constant or increasing marginal cost function. By comparing the equilibriums, we identify the different effects of the capital constraint on the operations of the supply chain and the corresponding profitability in two different (production) technological settings where the marginal cost is constant and increasing respectively.

Our paper is related to the literature which focuses on multi-channel supply chain operations with channel conflicts and their coordination. Among these literatures, Parka and Keh (2003) compare the equilibrium under the indirect and vertically integrated channels with the equilibrium under the hybrid channel with respect to the marketing decision variables, particularly pricing and profit distribution¹. Tsay and Agrawal (2004) develop a model that captures key attributes leading to supply chain channel conflict, and examine ways to adjust the manufacturer–reseller relationship that have been observed in industry². Zhao and Xu (2014) discuss the equilibrium results of dual-channel supply chain and indicate that when manufacturer opened the e-direct channel and operated dual channels distribution system which makes the retail's profit decrease and lead to the channel conflict³. Cai (2010) investigates the influence of channel structures and channel coordination on the supplier, the retailer, and the entire supply chain in the context of two single-channel and two dual-channel supply chains⁴. Huang, Yang and Liu (2013) study a pricing and production problem in a dual-channel supply chain when production costs are disrupted⁵. Batarfi, Jaber and Zanoni (2016) investigate the effect of adopting a dual-channel comprised of a traditional retail channel and a direct online channel on the performance of a two-level supply chain⁶. Giri and Roy (2015) find out the optimal pricing strategies and effort levels of both the direct channel and retail channel using sequential optimization and the Stackelberg game⁷. Xie and Hong (2015) find that there is some certain robustness both in the manufacturer's production quantity and in the offline retail price. When the supply chain is decentralized, the supply chain can be coordinate by changing the wholesale price according to different disruption levels⁸. Dumrong Siri, Fan, Jain and Moinzadeh (2008) develop the conditions under which the manufacturer and the retailer share the market in equilibrium, and show that the difference in marginal costs of the two channels plays an important role in determining the existence of dual channels in equilibrium⁹. Dan, Xu and Liu (2012) examine the optimal decisions on retail services and prices in a centralized and a decentralized dual-channel supply chain and evaluate the impacts of retail services and the degree of customer loyalty to the retail channel on the manufacturer and retailer's pricing behaviors¹⁰. Liu, Cao and Salifou (2015)

investigated the effect of risk aversion on the optimal policies of a dual-channel supply chain under complete information and asymmetric information cases¹¹. Xiao and Jim (2016) study channel priority strategy of a dual-channel supply chain facing potential supply shortage and examine the effects of coordination/decision sequence of channel priority on priority strategy¹². Different from the literature discussed above, this paper studies supply chain operations with different cost characteristics in terms of marginal cost and working capital constraints.

The contribution of this paper is two-fold. First, from the modeling perspective, working capital constraints are incorporated into both supply chain game models with the supplier's constant and increasing marginal cost functions. This complements to the literature that studies these problems in independent manner. Second, we identify the different effects of the capital constraint on the operations of the supply chain and the corresponding profitability in two different (production) technological settings where the marginal cost is constant and increasing respectively.

3 Supply chain operations in a constant marginal cost setting

Consider a supply chain consisting of an upstream supplier and two downstream retailers (1 and 2), where the retailer 2 faces the problem of working capital constraint. The supplier produces and sells a product to both retailers through wholesale price contracts. The retailers sell the product to final customers. The final demand for retailer $i(=1,2)$ is $p_i = a - bq_i$, where $a > 0$, $b > 0$, and p_i and q_i represent the retail price and the quantity demanded respectively. To make our analysis meaningful, we assume that $c < w < p < a$. The supplier produces with a constant marginal cost is $c(> 0)$ and both retailers' retail costs are normalized to be 0. The wholesale prices for retailer $i(=1,2)$ is denoted by $w_i(> 0)$. Finally, Retailer 2' s working capital constraint is captured by an amount $K_2 \leq (a^2 - c^2) / 8b$.

The supplier's profit can be written as

$$\pi_s = (w_1 - c)q_1 + (w_2 - c)q_2 \quad (1)$$

The profit functions of retailer 1 and 2 are

$$\pi_{r1} = (p_1 - w_1)q_1 = (a - w_1 - bq_1)q_1 \quad (2)$$

$$\pi_{r2} = (p_2 - w_2)q_2 = (a - w_2 - bq_2)q_2 \quad (3)$$

The decision sequence is as follow. In stage 1, the supplier decides the wholesale price w_1 and w_2 for retailer 1 and 2. In stage 2, retailer 1 and 2 choose their order quantities q_1 and q_2 . We below solve the model by the backward induction. For convenience, we denote

$$w_2^1 \square (a - \sqrt{a^2 - 8bK_2})/2 \text{ and } w_2^2 \square (a + \sqrt{a^2 - 8bK_2})/2.$$

Lemma 1: In stage 2, for given w_1 and w_2 , retailer 1's and 2's reaction functions are respectively $q_1(w_1) = (a - w_1)/(2b)$ and

$$q_2(w_2) = \begin{cases} (a - w_2)/(2b), & \text{if } w_2 \geq w_2^2 \text{ or } w_2 \leq w_2^1 \\ K_2 / w_2, & \text{otherwise} \end{cases}$$

Proof: In stage 2, for a given w_1 , retailer 1's decision can be captured by

$$\max_{q_1} \pi_{r1} = (p_1 - w_1)q_1 = (a - w_1 - bq_1)q_1$$

The first-order condition with regard to q_1 implies

$$\partial \pi_{r1} / \partial q_1 = a - w_1 - 2bq_1 = 0 \Leftrightarrow q_1(w_1) = (a - w_1)/2b \quad (4)$$

For a given w_2 , retailer 2's decision can be written as

$$\max_{q_2} \pi_{r2} = (p_2 - w_2)q_2 = (a - w_2 - bq_2)q_2 \quad \text{s.t. } w_2q_2 \leq K_2 \leq (a^2 - c^2)/8b$$

Notice that if $w_2q_2 \leq K_2$ is not binding, retailer 2's optimal order quantity is $q_2(w_2) = (a - w_2)/(2b)$. And then we have $w_2q_2(w_2) = w_2(a - w_2)/(2b)$. Thus, if $w_2q_2(w_2) = w_2(a - w_2)/(2b) < K_2 \Leftrightarrow w_2 \geq w_2^2$ or $w_2 \leq w_2^1$, then $q_2(w_2) = (a - w_2)/(2b)$ is optimal. Otherwise the constraint must be binding, the optimal quantity is $q_2(w_2) = K_2 / w_2$.

In stage 1, anticipating the reactions in Lemma 1, the supplier's profit is calculated as

$$\pi_s(w_1, w_2) = \begin{cases} \frac{(w_1 - c)(a - w_1)}{2b} + \frac{(w_2 - c)(a - w_2)}{2b} & \text{if } w_2 \geq w_2^2 \text{ or } w_2 \leq w_2^1 \\ \frac{(w_1 - c)(a - w_1)}{2b} + \frac{(w_2 - c)K_2}{w_2} & \text{otherwise} \end{cases}$$

Lemma 2: In stage 1, the supplier's optimal wholesale price for retailer 1 and 2 are $w_1^* = (a + c)/2$ and $w_2^* = (a + \sqrt{a^2 - 8bK_2})/2$, respectively.

Proof: we first prove that $K_2 \leq (a^2 - c^2)/(8b)$ implies that every w_2 satisfying $w_2q_2(w_2) = w_2(a - w_2)/(2b) < K_2$ is not optimal for the supplier. Consider any such w_2 , the order quantity of retailer 2 is $q_2(w_2) = (a - w_2)/(2b)$, implying that the profit of supplier is

$\pi_s(w_1, w_2) = (w_1 - c)(a - w_1) / 2b + (w_2 - c)(a - w_2) / 2b$, Notice that $K_2 \leq (a^2 - c^2) / (8b)$ and $w_2(a - w_2) / (2b) < K_2$, this means $w_2 < w_2^1 \leq (a + c) / 2$ and $w_2 > w_2^2 \geq (a + c) / 2$. Notice also that for any given w_1 , $\pi_s(w_1, w_2)$ is a quadratic concave function with regard to w_2 , where its symmetry axis is $w_2 = (a + c) / 2$. It means that for all $w_2 < w_2^1 \leq (a + c) / 2$, $\pi_s(w_1, w_2)$ is increasing in w_2 and for all $w_2 < w_2^1 \leq (a + c) / 2$, $\pi_s(w_1, w_2)$ is decreasing in w_2 . Therefore, for any $K_2 \leq (a^2 - c^2) / (8b)$, the supplier do not choose w_2 satisfying $w_2(a - w_2) / (2b) > K_2$. Second, we prove that for any given w_1 , the supplier's optimal wholesale price is $w_2^* = (a + \sqrt{a^2 - 8bK_2}) / 2$. Notice that for any w_2 satisfying $w_2(a - w_2) / (2b) \geq K_2$, the order quantity of retailer 2 is $q_2(w_2) = K_2 / w_2$, thus the profit of supplier is $\pi_s(w_1, w_2) = [(w_1 - c)(a - w_1)] / 2b + [(w_2 - c)K_2] / w_2$. It follows that $\pi_s(w_1, w_2)$ is a increasing strictly in w_2 : $\partial \pi_s(w_1, w_2) / \partial w_2 = cK_2 / w_2^2 > 0$.

Further, since $w_2(a - w_2) / (2b) \geq K_2 \Leftrightarrow w_2^1 \leq w_2 \leq w_2^2$, thus for any w_1 , the supplier's optimal wholesale price is

$$w_2^* = w_2^2 = (a + \sqrt{a^2 - 8bK_2}) / 2 \quad (5)$$

Finally, the first-order condition with regard to w_1 implies

$$\partial \pi_s(w_1, w_2) / \partial w_1 = (a + c - 2w_1) / 2b = 0 \Rightarrow w_1^* = (a + c) / 2 \quad (6)$$

By substituting w_1^* and w_2^2 in the order decisions of retailer 1 and 2 respectively, the optimal order quantities of retailer 1 and 2 are $q_1^* = (a - c) / 4b$ and $q_2^* = K_2 / w_2^2$ (7)

The equilibrium profits of retailer 1 and 2 can be obtained by substituting equation (6) and (7) in equation (2) and by substituting equation (5) and (7) in equation (3) respectively. The supplier's equilibrium profit can be obtained by substituting equation (5), (6) and (7) in equation (1). These are summarized in Proposition 1.

Proposition 1: In the setting of a constant marginal cost, the equilibrium wholesale price (w_1^* and w_2^*), the equilibrium ordering quantities (q_1^* and q_2^*) and the supply chain members' equilibrium profit (π_s^* , π_{r1}^* and π_{r2}^*) can be obtained respectively as

$$w_1^* = \frac{a+c}{2}, \quad w_2^* = \frac{a+\sqrt{a^2-8bK_2}}{2}, \quad q_1^* = \frac{a-c}{4b}, \quad q_2^* = \frac{K_2}{w_2^*}, \quad \pi_s^* = \frac{2a^2+2c^2-6ac+8bK_2+2c\sqrt{a^2-8bK_2}}{8b}$$

$$\pi_{r1}^* = \frac{(a-c)^2}{16b}, \quad \pi_{r2}^* = \frac{a^2-a\sqrt{a^2-8bK_2}-4bK_2}{8b}$$

We now investigate the impact of K_2 on the operations and the corresponding profitability of the supply chain.

Proposition 2: In the setting of a constant marginal cost, for any $K_2 \leq (a^2 - c^2)/(8b)$, an increase in retailer 2's working capital (K_2) leads to a higher equilibrium order quantity for retailer 2 (q_2^*), higher equilibrium profits for the supplier and retailer 2 (π_s^* and π_{r2}^*), but a lower equilibrium wholesale price of retailer 2 (w_2^*). The retailer 1's equilibrium wholesale price (w_1^*), equilibrium order quantity (q_1^*) and equilibrium profit (π_{r1}^*) are independent of retailer 1's working capital (K_2).

Proof: All the partial derivatives with respect to K_2 can be directly calculated as

$$\frac{\partial w_1^*}{\partial K_2} = 0, \quad \frac{\partial w_2^*}{\partial K_2} = \frac{-2b}{\sqrt{a^2-8bK_2}} < 0, \quad \frac{\partial q_1^*}{\partial K_2} = 0, \quad \frac{\partial q_2^*}{\partial K_2} = \frac{1}{\sqrt{a^2-8bK_2}} > 0, \quad \frac{\partial \pi_{r1}^*}{\partial K_2} = 0$$

$$\frac{\partial \pi_{r2}^*}{\partial K_2} = \frac{a-\sqrt{a^2-8bK_2}}{2\sqrt{a^2-8bK_2}} > 0, \quad \frac{\partial \pi_s^*}{\partial K_2} = 1 - \frac{c}{\sqrt{a^2-8bK_2}} = \frac{\sqrt{a^2-8bK_2}-c}{\sqrt{a^2-8bK_2}} > 0 \Leftrightarrow K_2 < \frac{a^2-c^2}{8b}$$

4 Supply chain operations in incremental marginal cost setting

In this section, we replace the assumption of a constant marginal cost by an increasing one. That is, we assume that the supplier's marginal cost is $C = hq^2/2$, where $q = q_1 + q_2$. Other assumptions are the same as those in Section 3. This implies that the supplier's profit can be re-written as

$$\pi_s = w_1q_1 + w_2q_2 - h(q_1 + q_2)^2/2 \quad (8)$$

Since both retailers' decision-making problems do not change at all, their optimal reactions are the same as those in Lemma 1. In stage 1, anticipating these reactions, the supplier's profit can be re-calculated as

$$\pi_s(w_1, w_2) = \begin{cases} \frac{w_1(a-w_1)}{2b} + \frac{w_2(a-w_2)}{2b} - \frac{h}{2} \left(\frac{a-w_1}{2b} + \frac{a-w_2}{2b} \right)^2 \square f(w_1, w_2) & \text{if } w_2 \notin (w_2^1, w_2^2) \\ \frac{w_1(a-w_1)}{2b} + K_2 - \frac{h}{2} \left(\frac{a-w_1}{2b} + \frac{K_2}{w_2} \right)^2 \square g(w_1, w_2) & \text{otherwise} \end{cases}$$

Lemma 3: In stage 1, when $K_2 < [a^2(b+h)]/[2(2b+h)^2]$, the supplier's optimal wholesale prices for retail 1 and 2 are $w_1^\# = [(4ab+3ac) - c\sqrt{a^2-8bK_2}]/(8b+2c)$ and $w_2^\# = (a + \sqrt{a^2-8bK_2})/2$.

Proof: We first give the lower and the upper bounds of the supplier's optimal wholesale prices under the condition of $K_2 < [a^2(b+h)]/[2(2b+h)^2]$. In fact, given this condition, it follows from a few calculations that $w_2^1 < w_1^{**} = [a(b+h)]/(2b+h) = w_2^{**} < w_2^2$, where $w_1^{**} = w_2^{**} = [a(b+h)]/(2b+h)$ are the wholesale prices for retailers 1 and 2 respectively without any capital constraint. They are the solution to the following Problem:

$$\max_{w_1, w_2} \pi_s(w_1, w_2) = [w_1(a-w_1)]/2b + w_2(a-w_2)/2b - h((a-w_1)/2b + (a-w_2)/2b)^2 / 2 \quad (9)$$

and can be obtained by solving the corresponding first-order conditions:

$$\frac{\partial \pi_s(w_1, w_2)}{\partial w_1} = \frac{a-2w_1}{2b} + \frac{h}{2b} \left(\frac{a-w_1}{2b} + \frac{a-w_2}{2b} \right) = 0 \quad \text{and} \quad \frac{\partial \pi_s(w_1, w_2)}{\partial w_2} = \frac{a-2w_2}{2b} + \frac{h}{2b} \left(\frac{a-w_1}{2b} + \frac{a-w_2}{2b} \right) = 0$$

The rest of the proof is divided into three steps. Step 1: we show that for any w_1 , every $w_2 < w_2^1$ is not optimal wholesale price that the supplier offers to the retailer.

Let $r_2(w_1)$ be the solution to $\partial \pi_s(w_1, w_2)/\partial w_2 = 0$. Note that for all w_1 such that $r_2(w_1) \geq w_2^1$, $\partial \pi_s(w_1, w_2)/\partial w_2 > 0$. Thus every vector (w_1, w_2) (where $w_2 < w_2^1$) is not the supplier's optimal wholesale price. For all w_1 such that $r_2(w_1) < w_2^1$, the supplier's optimal wholesale price for retailer 2 is $r_2(w_1)$. However, $K_2 < [a^2(b+h)]/[2(2b+h)^2]$ implies that for all $w_2 < w_2^1$, $r_1(w_2) < r_2^{-1}(w_2)$ where $r_1(w_2)$ solves $\partial \pi_s(w_1, w_2)/\partial w_1 = 0$. Therefore, for a fixed $w_2 = r_2(w_1)$, the supplier is willing to decrease the wholesale price for retailer 1 from w_1 to $r_1(r_2(w_1)) < w_1$, since $\pi_s(w_1, r_2(w_1)) = f(w_1, r_2(w_1)) < f(r_1(r_2(w_1)), r_2(w_1)) = \pi_s(r_1(r_2(w_1)), r_2(w_1))$.

Step 2: we prove that for any w_1 , every $w_2 > w_2^2$ is not the optimal wholesale price

for retailer 2. For all w_1 satisfying $r_2(w_1) \leq w_2^2$, since $\pi_s(w_1, w_2)$ increases in w_2 for all $w_2 \geq w_2^{**}$, all vectors (w_1, w_2) with $w_2 > w_2^2$ are not optimal for the supplier. For all w_1 satisfying $r_2(w_1) > w_2^2$, the supplier's optimal wholesale price for retailer 2 is $r_2(w_1)$. However, $K_2 < [a^2(b+h)]/[2(2b+h)^2]$ implies that for all $w_2 > w_2^2$, $r_1(w_2) > r_2^{-1}(w_2)$ where $r_1(w_2)$ is the solution to $\partial \pi_s(w_1, w_2) / \partial w_1 = 0$. Therefore, for any given $w_2 = r_2(w_1)$, the supplier is willing to increase its wholesale price for retailer 1 from w_1 to $r_1(r_2(w_1)) > w_1$, since $\pi_s(w_1, r_2(w_1)) = f(w_1, r_2(w_1)) > f(r_1(r_2(w_1)), r_2(w_1)) = \pi_s(r_1(r_2(w_1)), r_2(w_1))$.

Step 3: we show that the supplier's optimal wholesale prices are $w_1^\#$ and $w_2^\#$ for retailer 1 and 2 respectively. Based the results in steps 1 and 2 above, we need to only consider w_2 's such that $w_2^1 \leq w_2 \leq w_2^2$. In the case, for any given w_1 , the supplier's profit is

$$\pi_s(w_1, w_2) = [w_1(a - w_1)] / 2b + K_2 - h((a - w_1) / 2b + K_2 / w_2)^2 / 2$$

Notice that for any $K_2 < [a^2(b+h)]/[2(2b+h)^2]$, we have

$$\partial \pi_s(w_1, w_2) / \partial w_2 = [h((a - w_1) / 2b + K_2 / w_2) K_2] / (w_2)^2 > 0$$

That is, $\pi_s(w_1, w_2)$ increases in w_2 . We thus have $\pi_s(w_1, w_2) < \pi_s(w_1, w_2^2)$ for all w_2 satisfying $w_2^1 \leq w_2 \leq w_2^2$, implying that for any given w_1 , the supplier's optimal wholesale price for retailer 2 is $w_2^\# = w_2^2 = (a + \sqrt{a^2 - 8bK_2}) / 2$ (10)

This optimality implies that if w_1 is chosen, the supplier can obtain a profit of $\pi_s(w_1, w_2^\#)$. The first-order condition with regard to w_1 is

$$\partial \pi_s(w_1, w_2^\#) / \partial w_1 = (a - 2w_1) / 2b + h((a - w_1) / 2b + K_2 / w_2^\#) / 2b = 0 \quad (11)$$

With (10), solving this equation gives $w_1^\# = [(4ab + 3ah) - h\sqrt{a^2 - 8bK_2}] / [2(4b + h)]$. By substituting $w_1^\#$ and $w_2^\#$ in the order decisions of retailer 1 and 2 respectively, the optimal order quantities of retailer 1 and 2 are $q_1^\# = [4ab - ah + h\sqrt{a^2 - 8bK_2}] / [4b(4b + h)]$ and

$q_2^\# = 2K_2 / a + \sqrt{a^2 - 8bK_2}$. Finally, By substituting $w_1^\#, w_2^\#, q_1^\#$ and $q_2^\#$ into supply chain members' profit functions, we can obtain the corresponding equilibrium profits. We summarize the equilibrium results in Proposition 3.

Proposition 3: In the supply chain game model with an increasing marginal cost, the equilibrium wholesale price ($w_1^\#$ and $w_2^\#$), the equilibrium ordering quantities ($q_1^\#$ and $q_2^\#$) and the supply chain members' equilibrium profit ($\pi_s^\#, \pi_{r1}^\#$ and $\pi_{r2}^\#$) can be obtained respectively by

$$w_1^\# = \frac{(4ab + 3ah) - h\sqrt{a^2 - 8bK_2}}{2(4b + h)}, \quad w_2^\# = \frac{a + \sqrt{a^2 - 8bK_2}}{2}, \quad q_1^\# = \frac{(4ab - ah) + h\sqrt{a^2 - 8bK_2}}{4b(4b + h)}$$

$$q_2^\# = \frac{2K_2}{a + \sqrt{a^2 - 8bK_2}}, \quad \pi_{r1}^\# = \frac{(4ab - ah + h\sqrt{a^2 - 8bK_2})^2}{16b(4b + h)^2}, \quad \pi_{r2}^\# = \frac{a^2 - a\sqrt{a^2 - 8bK_2} - 4bK_2}{8b}$$

$$\pi_s^\# = \frac{[(4ab + 3ah) - h\sqrt{a^2 - 8bK_2}][(4ab - ah) + h\sqrt{a^2 - 8bK_2}]}{8b(4b + h)^2} + K_2 - \frac{h(2a - \sqrt{a^2 - 8bK_2})^2}{2(4b + h)^2}$$

We below investigate the impact of K_2 on the operations and the corresponding profitability of the supply chain in the increasing-marginal-cost setting.

Proposition 4: In the supply chain game model with an increasing marginal cost, for any $K_2 < [a^2(b+h)]/[2(2b+h)^2]$, an increase in retailer 2's working capital (K_2) leads to a higher equilibrium wholesale price of the retailer 1 ($w_1^\#$), equilibrium order quantity of the retailer 2 ($q_2^\#$), equilibrium profits of the supplier and retailer 2 ($\pi_s^\#$ and $\pi_{r2}^\#$), but a lower equilibrium wholesale price of the retailer 2 ($w_2^\#$), equilibrium order quantity ($q_1^\#$) and equilibrium profit ($\pi_{r1}^\#$) of the retailer 1.

Proof: We differentiate all the equilibrium variables with respect to K_2 and thus have

$$\frac{\partial w_1^\#}{\partial K_2} = \frac{2bh}{(4b+h)\sqrt{a^2-8bK_2}} > 0, \quad \frac{\partial w_2^\#}{\partial K_2} = \frac{-2b}{\sqrt{a^2-8bK_2}} < 0, \quad \frac{\partial q_1^\#}{\partial K_2} = \frac{-h}{(4b+h)\sqrt{a^2-8bK_2}} < 0$$

$$\frac{\partial q_2^\#}{\partial K_2} = \frac{1}{\sqrt{a^2-8bK_2}} > 0, \quad \frac{\partial \pi_{r1}^\#}{\partial K_2} = \frac{-h(4ab-ah+h\sqrt{a^2-8bK_2})}{2(4b+h)^2\sqrt{a^2-8bK_2}} < 0, \quad \frac{\partial \pi_{r2}^\#}{\partial K_2} = \frac{a-\sqrt{a^2-8bK_2}}{2\sqrt{a^2-8bK_2}} > 0$$

$$\frac{\partial \pi_s^\#}{\partial K_2} = \frac{(4b+2h)\sqrt{a^2-8bK_2}-2ah}{(4b+h)\sqrt{a^2-8bK_2}} > 0 \Leftrightarrow (4b+2h)\sqrt{a^2-8bK_2}-2ah > 0 \Leftrightarrow K_2 < \frac{a^2(b+h)}{2(2b+h)^2}$$

By comparing Proposition 2 and 4, we obtain the following differences. In the case of a

constant marginal cost, although an increased amount of the constrained retailer's working capital has a positive impact on the contracted quantity, the wholesale price, the profitability of the supplier and the constrained retailer itself, it does not have any positive or negative impact on the unconstrained channel's operations and its profitability for the retailer. In the case of an increasing marginal cost, the supplier and the constrained retailer benefit from an increased amount of constrained retailer's working capital at the cost of the unconstrained retailer in the sense that the unconstrained retailer buys (sells to final consumers) a smaller quantity at a higher wholesale price and thus gets a lower profit. This implies a conflict between the constrained and the unconstrained channels. The reason for this conflict is that the relaxation of the working capital constraint allows for a higher contracted quantity in the constrained channel and thus leads to a higher marginal cost for the supplier who in turn offers the wholesale price to the unconstrained retailer. In a word, a relaxation of the working capital constraint in the constrained channel produces a negative externality to the unconstrained channel via an increase in the supplier marginal cost.

5 Conclusions

The paper builds a two-stage game model to capture the operations of a dual-channel supply chain and derives the equilibriums in two different settings where the supplier's marginal cost is respectively constant and increasing in its output. By comparing the equilibriums and the corresponding comparative statics, we mainly show that whether the supplier's marginal cost is constant or increasing is an alternative key to understand the conflicts (in the case of the existence of downstream working capital constraints) between two channels. This result complements the market-focusing literature on channel conflicts from a cost aspect: the supplier's increasing marginal cost makes a relaxation of the working capital constraints in one channel provide a negative externality to the channel without any working capital constraint. Finally, this paper assumes that the supply chain is operated in a complete information setting. However, this assumption may not coincide with the reality perfectly. Thus an extension of this research to an incomplete information setting is a valuable work in the future.

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