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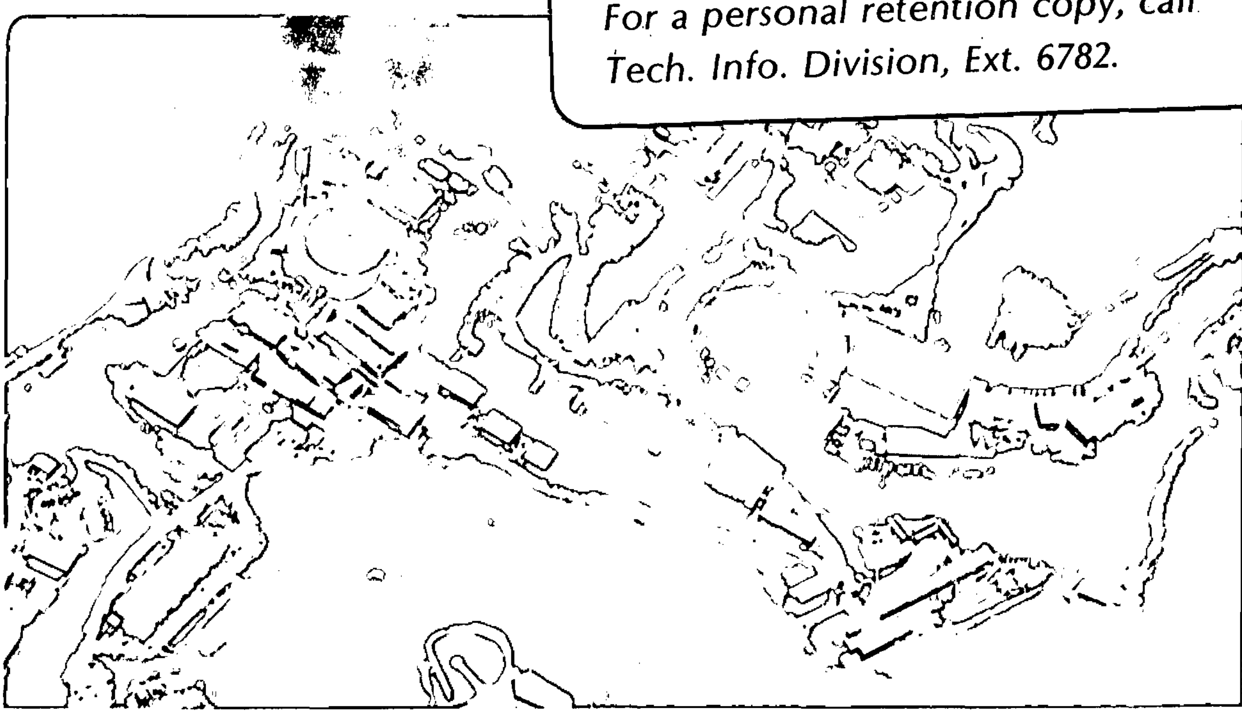

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ABSTRACT

Feynman-graph rules are formulated for the strong-interaction components of the topological expansion--defined as those graphs all of whose vertices are zero-entropy connected parts. These rules imply a "topological asymptotic freedom" and admit a corresponding perturbative evaluation where the zeroth order exhibits topological supersymmetry.

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I. INTRODUCTION

A topological basis for strong interactions has been constructed[1,2,3] and recognized as likely encompassing electroweak interactions.[4,5,6] Rules associating amplitudes with electroweak-boson-lepton topologies are similar to those of Lagrangian field theory inasmuch as these topologies do not admit duality transformations (contractions); the elementary vertex functions are correspondingly structureless. In contrast the duality aspect of strong-interaction topology requires unorthodox Feynman rules--with structured vertices--and this paper presents a detailed proposal. Our rules turn out to embody a qualitative feature that we call "topological asymptotic freedom" (TAF) because of its similarity to the asymptotic freedom of QCD. TAF makes it possible to approach hadron dynamics perturbatively once zero-entropy connected parts are known. Even though the (nonlinear) zero-entropy problem cannot itself be treated perturbatively, plausible general assumptions about zero-entropy amplitudes allow immediate qualitative inferences concerning physical hadrons.

We shall state our dualized rules through the shorthand of "thickened" Feynman graphs where each elementary-hadron momentum line is accompanied by 2, 3 or 4 quark lines. This shorthand exhibits "color" while concealing the topological meaning of quark isospin and generation. For strong interactions the latter deficiency is unimportant; flavor may be represented as an index on each quark line. The chiral degree of freedom, together with spin, we handle in a standard manner through a 4-valued Dirac spinor index at the <u>end</u> of each quark line. Our prescription should be understandable to readers familiar with Lagrangian Feynman rules, even if they are unacquainted with topological particle theory.

II. THICKENED FEYNMAN GRAPHS WITH "COLORED" QUARK LINES

It is shown in Ref. (1) that the full content of strong-interaction topology--which involves a pair of intersecting surfaces--can be transferred to a thickened momentum graph with associated quark lines. This idea--a slight generalization of the original Harari-Rosner scheme[7,8]--we now recapitulate.

By a thickened momentum graph we mean a Feynman graph with uniformly (cyclically) ordered vertices such as in the 2-vertex example of Fig. 1. Such a graph is implicitly embedded in an oriented 2-dimensional bounded surface,* a notion important when we add quark lines.

Quark lines running parallel to momentum lines distinguish different types of elementary hadron. With the convention that graph vertices are always oriented "clockwise", the three categories of elementary hadron are shown in Fig. 2, where the relative locations of the different quark lines can be interpreted as a "color" label. (Attaching the "color"label is no more than a convenience.) A quark line by itself on one side of the momentum line carries color #1; our convention is that color #1 flows in agreement with surface orientation. Quarks that appear in "diquark" pairs are colored #2 and #3, #2 being adjacent to the momentum line; colors #2 and #3 flow in opposition to surface orientation. Figure 3 presents an example of the Fig. 1 momentum graph embellished with quark lines. Notice here the color

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* In the example of Fig. 1 the surface is a torus minus a disk.[1]

permutations or "switches" along intermediate elementary-baryon lines, but notice also that "color" is conserved. Any of 5 "color switches" can occur along an internal baryon line and any of 3 "color switches" along an internal baryonium line; all switches conserve color. Switches always occur <u>between</u> vertices, and there is no more than one color switch between any pair of adjacent vertices.[1] Trivial vertices must, however, be included--as we discuss later.

Each quark line is to be understood as carrying a definite 2-valued isospin and a definite 4-valued generation index, so these quantities are even more obviously conserved than is "color". A given quark line may change color but maintains isospin and generation (i.e., flavor). In the following section we describe how Dirac indices attach to quark lines.

Every external color permutation is to be included in the topological expansion, with all possible switches along internal momentum lines. Thus, in "color" space an external baryon "wave function" is totally symmetric and normalized to 6, while an external baryonium wave function analogously is normalized to 4.[9] We shall develop a formalism that performs economically the sum over internal color switches. The external and internal summation suppresses the "color" degree of freedom for <u>physical</u> hadrons, but graph counting depends on "color", so dynamical calculations must attend to this "inaccessible" attribute of elementary baryons and baryoniums. There happen, for example, to be no closed quark loops in Fig. 3, even though there are 2 momentum loops, so the "weight" of this topology is 1. We show by contrast in Fig. 4 a single momentum loop where there may be 0, 1, or 2 closed quark loops. Reference (10) has explained that the "weights" of these topologies are, respectively, 1, 32 and $(32)^2$, the number 32 being the total number of different quarks: 2 spins x 2 chiralities* x 2 isospins x 4 generations. "Color" does not contribute to quark multiplicity in this counting, but color switching affects the number of closed quark loops.

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* The formalism of Sec. III with closed quark loops and spin sums leads naturally to traces of products of Dirac matrices, where the factor 4 = 2 spins x 2 chiralities is automatic. Only the flavor factor of 8 need be added.

III. CHIRALITY AND SPIN

We now come to an aspect of topological theory that may be puzzling to those conversant with Lagrangian field theory. One source of puzzlement is that each elementary hadron has only one momentum line but more than one spin-1/2 "quark" line, even though momentum and spin are tied together by the Poincaré group. Feynman rules for spin-momentum propagation nevertheless have the same general structure as for the familiar case of a lepton--where there is one "quark" line for each momentum line. A second source of puzzlement is the need to recognize the Dirac propagator of each "quark" as composed of two separate pieces, with different chiral properties. For physical hadrons these two pieces are always added but, as with "color", the quark's inaccessible chiral degree of freedom cannot be ignored in topological dynamics.

The 4-component Dirac space for each quark separates in the Weyl basis into "ortho" and "para" 2-component subspaces. Each quark belongs to an elementary hadron and when a hadron line from one vertex is "plugged" into a hadron line from another vertex the associated quark lines also are plugged. In the rest system of the hadron, on mass shell, the quark plug in the ortho-para-decomposed Dirac space has the form of a 2 x 2 matrix

$$
\begin{array}{c|cc|c}
 & \text{ortho} & \text{para} & \\
\end{array}
$$

$$
\begin{array}{cc}
\text{ortho} & \text{para}
\end{array}
\qquad
\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}
\begin{array}{l} \text{ortho} \\ \\ \text{para} \end{array}
$$

which becomes in general the Feynman-propagator matrix

$$1 + \frac{\gamma \cdot p}{m_0} \ , \tag{III.1}$$

in view of the fact that (Weyl basis)

$$\gamma \cdot p = \begin{pmatrix} 0 & \sigma \cdot p \\ \tilde{\sigma} \cdot p & 0 \end{pmatrix} \tag{III.2}$$

where $\sigma = (1, \vec{\sigma})$, $\tilde{\sigma} = (1, -\vec{\sigma})$ in the notation of Stapp.[11] Here p is the energy-momentum of the elementary hadron to which the quark belongs--understood in the Feynman sense as directed in agreement with the quark-line direction. Remember that topological quarks do not individually carry momentum. However, any quark plug is part of a hadron plug. The real positive nonzero parameter m_0 in Formula (III.1) is the unique mass shared by all elementary hadrons.

Each quark line carries its own 4-component Dirac space which we label with an index i. (The index i may be thought of as carrying flavor.) Thus the chiral-spin plug matrix for an elementary meson, with momentum directed as in Fig. 5, is

$$(1 + \chi)_{meson} = \left(1 + \frac{\gamma_i \cdot P}{m_0}\right) \left(1 - \frac{\gamma_j \cdot P}{m_0}\right) , \tag{III.3}$$

acting in a Dirac-product space of 4^2 dimensions.

The reason we have written $1 + \chi$ in Formula (III.3) is to isolate the zero-entropy part of the meson plug matrix. Formula (III.2) shows that the $\gamma \cdot p/m_0$ part of the quark-plug matrix only contributes when the quark undergoes an ortho-para transition. Now a zero-entropy quark line is purely ortho (0) or purely para (1),[3,12] so when the meson-plug matrix (III.3) is seen as a sum of 4 matrices, one is the zero-entropy unit matrix while the other three are nonzero-entropy--corresponding respectively to ortho-para transition for quark i, for quark j or for both i and j. These four terms are representable through crosses on quark lines, as in Fig. 6. Such a cross-an O-P "switch"--associates with a matrix $\gamma \cdot p/m_0$.

The baryon-plug or baryonium -plug matrix involves a product of 3 or 4 matrices like (III.1). It is convenient to add a "color" plug-matrix and thus to define for baryon and baryonium plugs,

$$(1 + \chi)_{baryon} \equiv \left(1 + \gamma_i \cdot \frac{p}{m_0}\right)\left(1 + \gamma_j \cdot \frac{p}{m_0}\right)\left(1 + \gamma_k \cdot \frac{p}{m_0}\right)(1 + \pi_1 \cdots \pi_5) \tag{III.4}$$

$$(1 + \chi)_{baryonium} \equiv \left(1 - \gamma_i \cdot \frac{p}{m_0}\right)\left(1 - \gamma_j \cdot \frac{p}{m_0}\right)\left(1 + \gamma_k \cdot \frac{p}{m_0}\right)\left(1 + \gamma_\ell \cdot \frac{p}{m_0}\right)(1 + \pi_6 + \ldots \pi_8) \tag{III.5}$$

corresponding to the quark labels of Fig. 7. In Formula (III. 4) the symbols $\pi_1 \ldots \pi_5$ designate the 5 possible "color" switches of the baryon's 3 quarks (3 odd permutations and 2 even). The elementary-baryon plug matrix thus acts in a space of $4^3 \times 6$ dimensions. Similarly in Formula (III. 5) π_6, π_7, π_8 represent the 3 possible "quark color" switches within baryonium; the elementary-baryonium plug matrix acts in a space of $4^4 \times 4$ dimensions. In Ref. (12) Stapp shows from unitarity arguments that the zero-entropy part of any elementary-hadron plug matrix is the unit matrix, so normalizing factors are <u>not</u> to be added to the forms (III. 3), (III. 4) and (III. 5).

Stapp's S-matrix analysis[12] has justified the foregoing plug-matrices when $p^2 = m_0^2$--"on shell". We assume that these matrices may be used off-shell in Feynman formulas. A word should now be added about the external spin-chirality wave functions attached to the ends of the Feynman graph when computing an S-matrix element.

We use for each quark the standard u or v component spinor. In the Weyl (ortho-para) basis

$$u(p,\ s) = \begin{pmatrix} \sqrt{\sigma \cdot p/m_0}\ \eta(s) \\ \sqrt{\bar{\sigma} \cdot p/m_0}\ \eta(s) \end{pmatrix} \qquad \text{(III.6)}$$

$$v(p,\ s) = \begin{pmatrix} \sqrt{\sigma \cdot p/m_0}\ \eta(s) \\ -\sqrt{\bar{\sigma} \cdot p/m_0}\ \eta(s) \end{pmatrix} \qquad \text{(III.7)}$$

where η is a 2-component Pauli spin vector, with s denoting the spin. The normalization*

* Previous papers--e.g. Ref. (9)--have introduced the factor 2 by a different but equivalent prescription.

$$\bar{u}(p,\ s)\ u\ (p,\ s) = 2$$

$$\bar{v}(p,\ s)\ v\ (p,\ s) = -\ 2 \qquad \text{(III.8)}$$

reflects the fact that ortho and para zero-entropy quark lines are always added together in building the S matrix. That is, as emphasized by Ref. (9), the ortho-para degree of freedom brings a factor 2 for <u>each</u> quark line. Comparing Formulas (III.2), (III.6) and (III.7) we see that, as usual, u (p) is the eigenvector of $\gamma \cdot p/m_0$ with eigenvalue $+\sqrt{p^2/m_0^2}$ while v(p) belongs to the eigenvalue $-\sqrt{p^2/m_0^2}$.

The overall phase of an S-matrix element will be given by a Feynman-like rule that includes the prescription of consistently inserting $-\bar{v}_i$* while u_i, $\bar{u}_i$ and v_i appear with + signs. We now relate this rule to the quark-plug matrix (III.1). Intermediate Feynman lines arise from elementary-hadron plugs where the accompanying quark plug is of one of the two types shown in Fig. 8. According to the usual Feynman prescription, in case (a) we have a factor $u_\alpha(p,\ s)\ \bar{u}_\beta(p,\ s)$ while in case (b) we have a factor $-\ v_\alpha(-p,\ s)\bar{v}_\beta(-p,\ s)$, where α and β label the 4 components of the Dirac space. Summing over spin we find from <u>both</u> (a) and (b) the result (III.1). Thus, as with usual Feynman rules, intermediate quark lines can be treated by a uniform prescription.

* This rule is equivalent in Q.E.D. to specifying the opposite electric charges of particle and antiparticle.

Stapp[12] has emphasized that, as in Feynman's original rules, a closed quark loop leads to a factor (-1), but the definition of a closed loop is obscured by the distinction between momentum lines and quark lines. Stapp finds the correct counting of closed-loop (-1) factors to be achieved by eliminating all color switches from the embellished graph [13] and then counting closed quark loops. Thus, for example, for each of the 3 graphs of Fig. 4 there are two closed-loop factors of (-1)*

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* The notion of "quark loop" vs. "diquark loop" may be helpful here. One may say that diquark loops have factors (+1) while quark loops have factors (-1)--regardless of switching.

IV. ZERO ENTROPY

Zero-entropy graphs contain no switches and are all contractible to single-vertex (planar) trees.[1] Corrections to zero entropy include ortho-para or color switches on internal lines as well as closed loops that may be noncontractible because of nonplanarity. Thus Fig. 1, even without any quark switches, is not zero entropy.*

Zero entropy is characterized by "topological supersymmetry,"[14,10] as illustrated by Fig. 9 which depicts the 3-particle vertices. In each case the M function is the same function of momentum variables,

$$M_3^0(p^2, p'^2, p''^2), \tag{IV.1}$$

but a suppressed unit matrix acts in spin-chirality-"color" spaces of different dimensionality. Consider, for example, the purely-mesonic Fig. 9(a). Here there are 3 quark lines, all "colored" #1, each carrying a pair of 4-valued Dirac indices. The value of the S-matrix element belonging to Fig. 9(a) is obtained by multiplying M_3^0, which depends only on momentum, with the Dirac factors

$$[-\bar{v}_k(p'', s_k'')v_k(p', s_k')][\bar{u}_j(p', s_j')v_j(p, s_j)][\bar{u}_i(p, s_i)u_i(p'', s_i'')] \tag{IV.2}$$

Corresponding S-matrix factors associate with the other vertices of

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* Without any quark switches Fig. 1 could be contracted to a single vertex with 2 "naked" tadpoles. As discussed in Ref. (15) such a nonplanar vertex is "implicitly weak" and does not contribute to our definition of "strong interaction".

Fig. 9; here "color" must be considered.[9] In a sense these factors are "kinematical"; zero-entropy dynamics resides in the supersymmetric M functions whose dependence on spin and chirality, as well as on "color", is trivial.

Zero-entropy dynamical equations, whether on shell or off shell, ascribe to each closed momentum loop a uniform weight[10]

$$f = (-32)^2 - 32 \tag{IV.3}$$

because there may invariably attach either a diquark loop or a quark loop. As explained in Ref. (10), it is possible to renormalize M_N^0 functions by the rule

$$M_N^{OR} \equiv f^{\frac{N-2}{2}} M_N^0 \tag{IV.4}$$

so as to achieve dynamical equations for M_N^{OR} corresponding to loop-weight 1. One then deals with a planar spinless, colorless, flavorless dynamics; all quark lines may be ignored.

For the special case N = 3 depicted in Fig. 9, it is convenient to define a dimensionless* parameter g_0 by

$$g_0 \equiv \frac{1}{m_0} M_3^0(m_0^2, m_0^2, m_0^2). \tag{IV.5}$$

Correspondingly

---

* The general dimensional rule is

$$\dim M^N = (\text{energy})^{4-N}$$

$$g_{OR}^2 = f\, g_0^2 \tag{IV.6}$$

It has been argued[10,16] that $g_{OR}^2/16\pi^2$ has order of magnitude unity, which means in view of (IV.3) that g_0 is small--similar in order of magnitude to e. This fact is important to our definition of "strong interactions"[15] and can be exploited in a perturbative strategy to evaluate the Feynman expansion. Deduction of zero-entropy M functions, however, cannot be approached perturbatively. We are faced here with nonlinear bootstrap dynamics.

As discussed in Ref. (1) it is necessary to associate a zero-entropy 2-line connected part

$$M_2^0(p^2) = M_2^{OR}(p^2)$$

with a trivial vertex. What is the connection between $M_2^0(p^2)$ and the familiar notion of a "mass operator" $\Sigma_0(p^2)$? Comparison of the discontinuity formula satisfied by $M_2^0(p^2)$ with that satisfied by $\Sigma_0(p^2)$ leads to the association*

$$M_2^0(p^2) = -\Sigma_0(p^2). \tag{IV.7}$$

$\Sigma_0(p^2)$ is a real analytic function with branch points at $p^2 = (2m_0)^2$, $(3m_0)^2$....

$$\Sigma_0(p^2) = \frac{(p^2 - m_0^2)^2}{\pi} \int_{4m_0^2}^{\infty} dx \frac{\operatorname{Im} \Sigma_0(x)}{(x-p^2)(x-m_0^2)^2}, \tag{IV.8}$$

---

* Recall that positive scattering amplitudes correspond to negative mass shifts.

with Im $\Sigma_0(p^2) < 0$* given by sums of absolute-value squares of zero-entropy vertex functions, integrated over the mass shell as schematically indicated in Fig. 10. We further anticipate that the analytic dependence of any zero-entropy vertex function on the p^2 carried by any one of its attached lines will include all $\ell = 0$ poles <u>except</u> the elementary particle pole at $p^2 = m_0^2$. Formula (IV.8) with Fig. 11 then implies this same property for $\Sigma_0(p^2)$. Consistency considerations, to be discussed elsewhere, have led us further to assume through (IV.8) that

$$\Sigma_0(m_0^2) = 0$$

and

$$\left[\frac{d}{dp^2} \Sigma_0(p^2) \right]_{p^2 = m_0^2} = 0. \qquad \text{(IV.9)}$$

The possibility of continuing zero-entropy functions M_N^0 off mass shell and of defining a zero-entropy mass operator is essential to the Feynman rules that will be developed in the following section.

In connection with off-shell continuation it is difficult to avoid thinking of zero entropy in a ϕ^3 Lagrangian spirit as being defined by the (renormalized) parameters m_0 and g_{OR}. Although some concepts from such a model--such as Fig. 10--appear relevant, we point out three major points of departure:[15] (1) Only planar graphs contribute to the building of zero entropy. (2) There is no reason to suppose meaning for a perturbative expansion in powers of

* See previous footnote.

g_{OR}. (3) Although the value of m_0 simply sets the mass scale, the value of g_{OR} is preseumed to be determined by duality.*

* Think, for example, of a planar Bethe-Salpeter equation based on cubic vertices of strength g_{OR}; g_{OR} must have that value which makes the mass of the lowest bound state equal to m_0.

V. STRONG FEYNMAN VERTICES AS ZERO-ENTROPY CONNECTED PARTS

The contraction rules of Ref. (1) imply that most internal elementary-hadron lines in Feynman graphs carry a switch. Following Ref. (15) let us describe a line without either ortho-para or "color" switch as "naked". Internal naked hadron lines almost all disappear through contraction; in fact, by the terminology of Ref. (15) there are no naked internal hadron lines in a purely-strong-interaction topology. If any naked internal hadron survives after all contractions have been made, then one (or both) of its vertices is "weak"--either "explicitly" or "implicitly".

A vertex touching only "clothed" (internal) hadron lines--carrying a switch--and single ends of naked hadron lines (no naked tadpoles) is a zero-entropy connected part and is defined in Ref. (15) to be a "strong" vertex. External hadron lines are always naked, but the only internal naked lines that survive contraction are "tadpoles"--with both ends at the same vertex--or lines impinging on a vertex that receives an electroweak boson. The latter vertices are "explicitly weak" and the former are "implicitly weak", as explained in Ref. (15). Neither is a zero-entropy connected part.

It follows that any strong-interaction component of the topological expansion is built from zero-entropy vertices, naked external hadron lines and clothed internal hadron lines. In the following section we show how a "clothed" propagator is calculated from the zero-entropy mass matrix in conjunction with the plug factors of Sec. III. Thus, assuming the zero entropy problem has been solved to provide the vertices, a complete prescription for calculating any purely hadronic interaction will have been furnished.

VI. THE ELEMENTARY-HADRON PROPAGATOR WITH SWITCHES AND TRIVIAL VERTICES

The adjective "clothed" is potentially misleading; we are talking about a propagator with switches and trivial vertices but no closed loops. The topological expansion includes an indefinitely-large number of switches, but one can in closed form sum over all numbers of switches and of trivial vertices. Because all strong-interaction components of the topological expansion are built by connected sum of zero-entropy single-vertex components, it follows that exactly one switch intervenes between any pair of adjacent strong-interaction vertices. The sum over all numbers of switches between two nontrivial vertices is indicated in Fig. 11. Each switch corresponds to a factor

$$\frac{i\chi}{p^2 - m_0^2} \qquad \text{(VI.1)}$$

and each trivial vertex to a factor $- i\ \Sigma_0(p^2)$. If the sum of the series is denoted by $i\ D_x(p)$, then

$$D_{x}(p) = \frac{1}{\dfrac{p^2 - m_0^2}{\chi} - \Sigma_0(p^2)} \qquad \text{(VI.2)}$$

It is essential that the unit part of the plug matrix has been omitted; each plug must contribute complexity to the topology. The operator χ has been constructed as the sum over all plugs where some mismatch--either in color or in chirality or in both--has occured.

The required Feynman rules have now been given. If all zero-entropy connected parts M_N^0 are known, including M_2^0, any strong-interaction component of the topological expansion is computable. It may be remarked that the on-shell value of $M_2^0(p^2)$ and its first derivative vanish; an approximation at the zero-entropy level never requires the 2-line connected part. But to calculate corrections to zero entropy the trivial vertex is essential. A related remark is that the external lines of our Feynman graphs are always naked and vertices receiving external lines are always nontrivial.

VII. TOPOLOGICAL ASYMPTOTIC FREEDOM (TAF)

Our Feynman rules attribute all breaking of supersymmetry to the switching matrix χ and thus to the propagator D_x; strong vertices are supersymmetric. We now argue that even $D_x(p)$ is asymptotically supersymmetric--in the limit $p \to \infty$.

The formulas (III.3), (III.4) and (III.5) for χ may be written shorthand as

$$\chi = N\Gamma P - 1 \qquad \text{(VII.1)}$$

here P is the "color-singlet" projection operator ($P^2 = P$) in an N-dimensional quark-permutation space, and Γ acts only on spin-chirality. For mesons N = 1, for baryons N = 6 and for baryoniums N = 4. Each P has one eigenvalue 1 and N - 1 eigenvalues 0. It is then possible to rewrite Formula (VI.2) as

$$D_x = \frac{P}{\frac{p^2 - m_0^2}{\chi} - \Sigma_0} + \frac{1 - P}{\frac{p^2 - m_0^2}{\chi} - \Sigma_0}$$

$$= \frac{P}{\frac{p^2 - m_0^2}{\chi^P} - \Sigma_0} + \frac{1 - P}{-(p^2 - m_0^2) - \Sigma_0}, \qquad \text{(VII.2)}$$

where

$$\chi^P = N\Gamma - 1. \qquad \text{(VII.3)}$$

We may regroup the terms of (VII.2) to write

$$D_x = D_x^0 + D_x^1 \qquad \text{(VII.4)}$$

where

$$D_x^0 \equiv \frac{1}{-(p^2 - m_0^2) - \Sigma_0} \qquad \text{(VII.5)}$$

$$D_x^1 = P \left\{ \frac{1}{\frac{p^2 - m_0^2}{\chi^P} - \Sigma_0} - \frac{1}{-(p^2 - m_0^2) - \Sigma_0} \right\}. \qquad \text{(VII.6)}$$

It is apparent that D_x^0 is supersymmetric while D_x^1 is not. But let us examine D_x^1 as $p \to \infty$.

From Formula (IV.8) and the condition that Im $\Sigma_0(x)$ is negative definite, we know that $\Sigma_0(p^2)$ behaves roughly linearly as $p^2 \to \infty$. Because $\Gamma_{meson} \sim p^2$, $\Gamma_{baryon} \sim p^3$, $\Gamma_{baryonium} \sim p^4$, the first term within the bracket of (VII.6) then asymptotically approaches $[-\Sigma_0(p^2)]^{-1}$. What about the second term?

It is reasonable from experience both with dispersion relations and with Feynman itegrals to expect*

$$\Sigma_0(p^2) \underset{p^2 \to \infty}{\gtrsim} p^2 \ln p^2 \qquad \text{(VII.7)}$$

If we so assume, then the second term of the bracket (VII.6) also approaches $[-\Sigma_0(p^2)]^{-1}$ and the two terms asymptotically cancel, yielding

$$D_x(p) \underset{p \to \infty}{\sim} D_x^0(p^2). \qquad \text{(VII.8)}$$

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* We expect nontrivial zero-entropy vertex functions to decrease strongly as $p^2 \to \infty$.

There is striking similarity here to the QCD feature of large - p^2 gluon decoupling--the feature that has been called "asymptotic freedom". The parallel becomes even more compelling if we remember that the complete (not shorthand) topology associates "color" and chirality switching with lines on a "classical surface"* that have been called "topological gluons"[3]--for reasons recognized before development of the Feynman rules described here.

Even if p^2 is not extremely large the term D_x^1 appears to be sufficiently small so as to be treated as a perturbation. In this connection we note that the projection operator P brings a factor 1/N which depresses D_x^1 for baryons and baryoniums. There is no such depression for mesons but closed loops of maximum weight (32^2) cannot contain meson lines. We therefore expect supersymmetrical features to survive in the properties of physical hadrons. Detailed study is needed to identify those physical quantities with the best chance for exhibiting approximate supersymmetry.**

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* The classical surface houses the thickened Feynman graph but is not identical thereto because the classical surface contains junction lines where sheets join in threes. The Feynman graph resides on a single sheet.

** The results of Ref. (9) encourage early attention to dimensionless coupling constants.

Smallness of D_x^1 does not mean that zero-entropy is a good approximation. In particular the supersymmetric (gound state) hadron mass m_0^1--which emerges from the zeroth order of the D_x^1 expansion--is not expected to be close to m_0. The ratio m_0^1/m_0 is an interesting characteristic of the theory, but note that our Feynman rules are given in terms of m_0. Mass renormalization has not been attempted here, although future development in such a direction may be anticipated.

VIII. SUMMARY

We have presented complete Feynman rules for strong-interaction components of the topological expansion in terms of zero-entropy connected parts. The new feature is Formula (VI.2) for the elementary-hadron propagator $D_x(p)$ with "color" and chirality switching, together with the auxiliary formulas (III.3), (III.4) and (III.5) for the switching matrix χ. These formulas have been integrated with a previously-developed thickened Feynman graph--that carries quark lines as well as momentum lines--and with phase and normalization rules given by Stapp.

It appears that, for certain physical questions, color and chirality switching may be treated perturbatively through a power series in D_x^1 as defined by Formula (VII.6). The zeroth-order terms of such an expansion exhibit full topological supersymmetry even though they do not correspond to zero entropy.

We identify the idea that $p \to \infty$ produces simplicity by the term "topological asymptotic freedom" (TAF). Crudely speaking, a very large number of switches restores the supersymmetry characteristic of zero entropy (where there are no switches). On the reverse side of the coin it is noteworthy that $\chi_{meson}(p) \to 0$ as $p \to 0$, suggesting that for some questions involving low-momentum mesons it may be profitable to treat switching as "weak" rather than "strong".

We comment in closing that one anticipates large supersymmetry breaking from infrared electroweak-boson contributions--topological theory's analogue of the Higgs mechanism. Techniques for handling zero-mass scalar and vector internal lines in Feynman graphs remain to be developed, and at this point we do not know which physical questions can be approached without having solved the infrared problem. We suppose nevertheless that application of common sense will allow certain interesting issues to be dealt with through the machinery here described. The old idea--that for certain purposes electroweak interactions are negligible--cannot be completely wrong.

ACKNOWLEDGMENT

Extended discussion with J. Finkelstein, R. Espinosa and especially H. P. Stapp has assisted development of the rules described here. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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FIGURE CAPTIONS

1. Example of a Feynman graph with ordered vertices.
2. Colored quark lines accompanying Feynman (momentum) lines of elementary hadrons.
3. Example of quark lines attached to the Feynman graph of Fig. 1.
4. Examples of weights for Feynman graphs.
   - a) 0 closed quark loops.
   - b) 1 closed quark loop.
   - c) 2 closed quark loops.
5. A meson line with quark indices.
6. The four possible meson plugs.
7. (a) Baryon line with quark indices.
   (b) Baryonium line with quark indices.
8. Quark plugs.
   - a) Quark-line direction agrees with particle flow.
   - b) Quark-line direction opposes particle flow.
9. Zero-entropy 3-hadron connected parts.
   - a) 3 mesons
   - b) 1 meson, 2 baryons
   - c) 1 baryonium, 2 baryons
   - d) 3 baryoniums
10. Discontinuity of the zero-entropy mass operator.
11. Infinite series for elementary-hadron "clothed" propagation between nontrivial vertices.

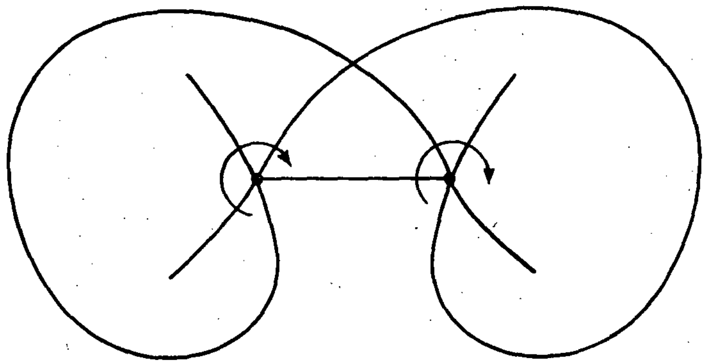

FIGURE 1

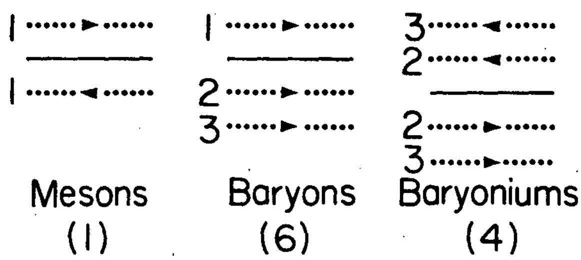

FIGURE 2

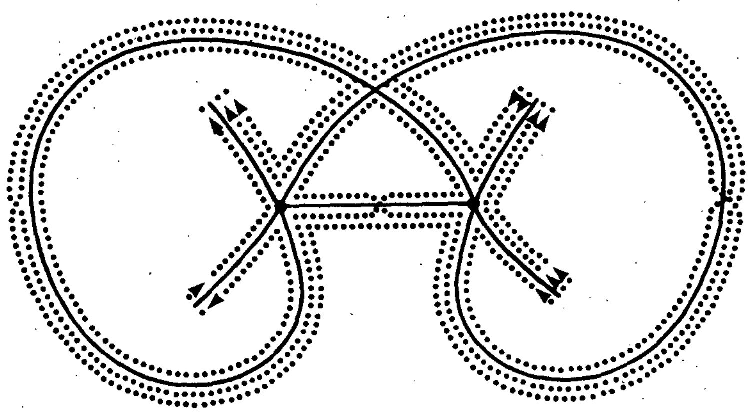

FIGURE 3

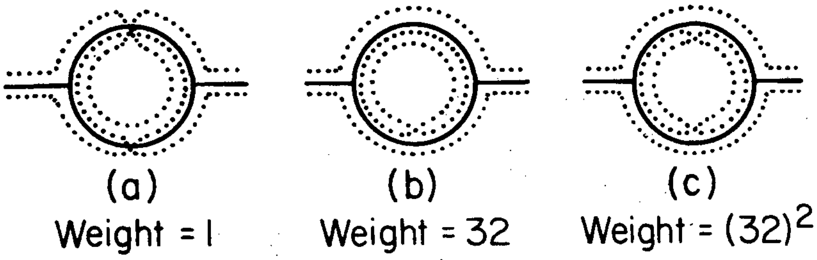

FIGURE 4

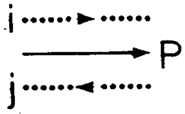

FIGURE 5

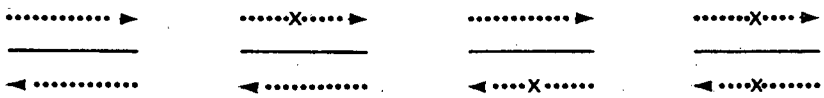

FIGURE 6

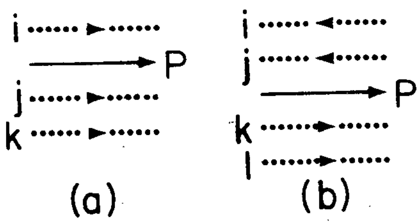

FIGURE 7

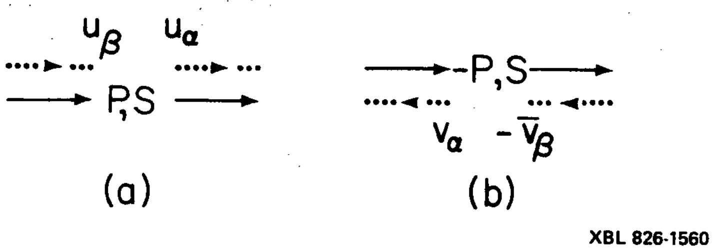

FIGURE 8

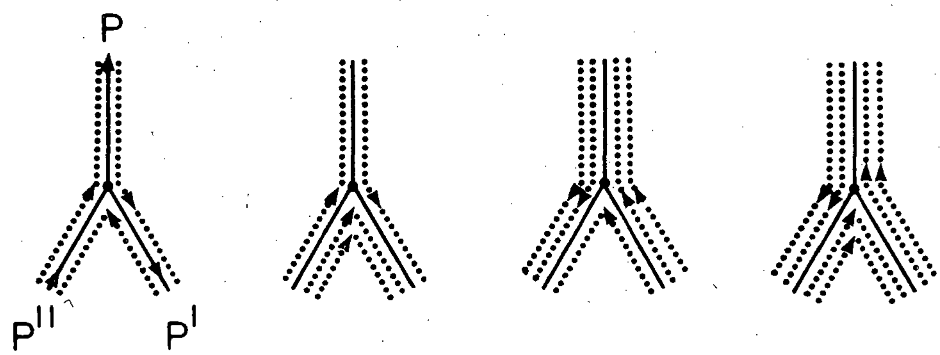

FIGURE 9

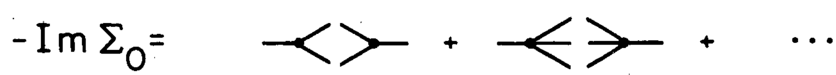

FIGURE 10

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FIGURE 11

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