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DUALITY AND UNNATURAL PARITY TRAJECTORIES

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A B S T R A C T

Global duality arguments can have but a limited value when applied to the low intercept unnatural parity trajectories. Difficulties which then arise are analyzed. Information on the isospin 0 $\rho\pi$ channel would help choosing between different schemes.

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1. DUALITY AND MESON SPECTRA

Duality [1] and the absence of direct channel resonances with exotic quantum numbers readily leads to exchange degeneracy [2),3] between exchanged trajectories. This is particularly simple in the case of meson-meson scattering where the high intercept vector and tensor trajectories are exchanged with strong couplings. As is well known [4], the absence of exotic states imposes degeneracy between the four ρ, f^o, ω and A_2 trajectories and also degeneracy between the vector and tensor K^* trajectories and the φ and f' isoscalar trajectories. This is remarkably well met by the facts [5),6]. It is further compatible with approximate SU_3 symmetry and leads to the observed nonet structure with standard mixing angle : $tg^2\theta = \frac{1}{2}$ [2]. Conversely, the observed mass pattern for the vector and tensor mesons, with the equality of slopes it shows, or at least strongly suggests, is a conspicuous support for the duality scheme. It is therefore most interesting to see how far the duality constraints are obeyed and if they also lead for the trajectories which are next to the vector and tensor ones in intercepts and are associated with the pseudoscalar and pseudovector mesons, to properties which are just as well met by the facts. It is the purpose of this note to stress that this is not the case but that the scheme appears to be but gradually broken as one includes trajectories with lower and lower intercepts. As a result, several interesting questions about the meson mass spectra come up.

It has been argued that the unnatural parity trajectories here considered can only be exchanged but in reactions such as pseudoscalar vector, vector-vector or baryon-baryon scattering, the thresholds of which are high as compared to the lowest meson masses. As a result duality equations cannot be followed with the confidence attached to them in the case of pseudoscalar-pseudoscalar scattering [7]. Only a broken duality remains valid, when satisfying all duality equations would have imposed degeneracies in mass which are not acceptable. We prefer to follow here the attitude that duality, as used in the framework of finite energy sum rules [1], actually leads to stringent constraints but for the high partial waves in one channel and the leading Regge trajectories in the crossed channel. They are powerful because it so happens that the contributions from Regge cuts are but a correction to those from the high intercept vector and tensor meson trajectories. However they are only of highly questionable value when low partial waves or low intercept trajectories are considered [8),9)]. Contributions from Regge cuts could well be of such importance as to overshadow those from trajectories with negative intercepts. For the same reasons, the neat success of duality arguments in the timelike region, where trajectories are drawn through the observed resonance points, should not be expected to be as good in the scattering region. This is in particular the case for line reversal symmetry [10)], when only the relations between trajectory values, notwithstanding further relations among residue functions, are actually tested. The soundness of duality arguments, so far used with leading trajectories, is therefore not obvious when extended to lower intercept trajectories. It is, however, very interesting to see how far one may actually go. It should be stressed furthermore that the equality of slopes for the leading meson and baryon trajectories is one of the strongest supports of the duality scheme [5)]. However, if the ρ and f^{o} trajectories must have the same slope, a low lying trajectory with a somewhat different slope could be tolerated. As a matter of fact, the only candidate for the needed ρ' state [4)], seems at present to be the bump observed in the two and four pion distributions at 1550 MeV [11)]. The slope thus obtained, when associating it with the ε on the same exchange degenerate daughter trajectory, is then smaller than the parent's one. One could therefore also expect smaller slope for the low intercept trajectories associated with unnatural parity particles.

Even though dual models [12)] which provide us with reaction amplitudes for a model world of zero width resonances and straight line trajectories impose a unique slope, one should be ready for more flexibility when dealing phenomenologically with the actual particle masses and couplings. Detailed information on unnatural parity mesons is in fact most precious at assessing how this dual model world differs from the actual one.

2. DUALITY AND UNNATURAL PARITY TRAJECTORIES

Conclusions drawn from the absence of exotic states affect only those states which are strongly coupled to the crossed channel [3]. We therefore consider pseudoscalar-vector and vector-vector scattering rather than pseudoscalar-scalar, which has been independently analyzed [13]. As a first example, we consider the reaction :

$$\rho^- \pi^+ \rightarrow \rho^- \pi^+$$

disregarding, as is usually done the diffractive part of the amplitude [14]. The absence of u channel exotic states imposes exchange degeneracy between the ρ and f^0 trajectories and the ω and A_2 trajectories, respectively exchanged in the t and s channels. The π trajectory is also exchanged in the s channel. Even though a line joining the π to the A_1 points has the "canonical" slope [15] of 0.9 GeV^{-2}, exchange degeneracy limited to the π and A_1 is not suitable. Trajectories with opposite charge conjugation have to be degenerate and since they have the same G parity they must have different isospin [7),5]. In the particular charge case just considered this simply follows from the absence of isospin two states. It is therefore not the A_1 which one expects on the pion trajectory (if there is anything like it) but a pseudovector meson with isospin zero. This meson could be the H which has been observed in the neighbourhood of 1 GeV, if it stabilizes there [16]. Continuing the π (and separately) the A_1 trajectories as straight lines, we should then find two 2^- particles with isospins 1 and 0 and negative G parity in the neighbourhood of 1.5 GeV [17], one of them being the recurrence of the pion. According to present evidence such particles could be expected but at a somewhat higher mass. We are led to a quartet of trajectories with both signatures and allowed isospin, see Fig. 1.

The same arguments can be applied to the reaction :

$$\rho^+ \rho^- \rightarrow \rho^+ \rho^-$$

where the η is strongly coupled. The absence of exotic state in the u channel will then again impose exchange degeneracy between the η

and the B trajectories, provided duality arguments hold at such a low intercept level. Indeed, the line joining the η to the B has the canonical slope. The D trajectory which would tentatively be degenerate with it has also to be exchange degenerate with an isovector trajectory the first member of which would be a 2^- state. Again two 2^- particles with isospin 0, 1 and positive G parity are expected in the neighbourhood of 1.6 GeV, one of them being the recurrence of the η. Here again one needs four degenerate trajectories and the doubling of states is in effect what is expected from the quark model [18]. Indeed the existence of a possible η-B exchange degenerate trajectory with slope 0.9 GeV^{-2} is a hint that duality arguments can still at least partly apply for intercept as low as -0.25. The η intercept could in fact be determined from duality arguments used in a different framework [19].

Difficulties, however, occur. If we now include KK^* scattering in the analysis, absence of exotic states requires exchange degeneracy between trajectories of same isospin and opposite signatures and charge conjugations. As a result the π and B trajectories would have now to be exchange degenerate when the same should hold for the η and H ! Indeed, as remarked by Goldhaber, Fox and Quigg [20], a line drawn through the π and B points hits an isospin 1 state at 1640 MeV for which there is evidence for 2^- spin parity quantum numbers. If the π and the η on the one hand, and the H and the B (a still open possibility), were respectively degenerate in mass, everything might work. However, the π-η mass difference, as an experimental fact, is at odds with duality as it readily is in the Veneziano model [21].

Further difficulties arise. If the H and B states are associated through duality with the π and η the SU_3 pattern must be different. If, for instance, two channels such as $\pi\rho \rightarrow \pi\rho$ and $\pi\rho \rightarrow KK^*$ are considered, the couplings of the π and the H to the same helicity states should obey :

$$\frac{g_{\pi\rho\pi}}{g_{\pi K^* K}} = \frac{g_{H\rho\pi}}{g_{HK^*K}} \qquad (1)$$

This follows from duality and factorization but excludes that the H could be an octet member as the π is. The F-type coupling of the π would lead otherwise to a ratio in (1) which is the opposite of what is obtained from the D-type coupling of the H. It is then necessary to introduce mixing between the H and a new state H' and tempting to take the H as mainly singlet to save the mass formula. The H' should however decouple from $\pi\rho$ since no extra isospin 1 state is otherwise there to forbid exotics. This last property, combined with (1), will again yield a standard nonet structure which in turn imposes H-B degeneracy. It is therefore necessary either to abandon the duality constraints for the H' on the ground that it is a low intercept trajectory or, keeping duality at this level, to predict the H in the neighbourhood of 1.2 - 1.3 GeV rather than around 1 GeV. In this case one should have a nonet grouping the H, the B, the upper component of the Q bump at 1320 GeV [22], and a H' in the 1.4 GeV region with main decay in 3π (no $\rho\pi$) or K^*K. The small mass of the pion would be at odds with the duality scheme. We would have no exchange degenerate trajectories with both isospins supporting the pion with a slope of 0.9 GeV^{-2}. The particular mass of the pion would be a result of approximate chiral symmetry which would win over any constraint we might impose from duality.

On the other hand, the A_1, the D and the lower component of the Q bump at 1230 MeV would make up an almost perfect octet. The "canonical" slope found for the line joining the η to the D would, as in the πA_1 case, not result from simple duality constraints. The octet mass formula ensures the same slope for the whole multiplet. It could be tempting to also apply duality arguments for another possible isospin zero negative signature trajectory which would then be degenerate with the η' trajectory. The expected mass of the 1^+ state would be identical with the E mass. This D' state should, however, now decouple from $\rho\rho$ as the η', if exotic states are to be ruled out. A standard nonet structure would again prevail. This decoupling appears, however, as very embarrassing. The large width of η decay into $\gamma\gamma$, as compared to what one deduces from the U spin scalar character of the electromagnetic decay interaction, calls for $\eta - \eta'$ mixing with a significant $\eta'\gamma\gamma$ coupling [23]. Forbidding the $\eta'\rho\rho$ coupling would void this interpretation. Furthermore the η' decays 20%

of the time into $\rho\gamma$ when this is but an electromagnetic mode as compared to the dominant $\eta\pi\pi$ strong mode. Again forbidding the $\eta'\rho\rho$ coupling is not tenable. As a result we have to admit that the exclusion of possible exotic states through duality constraints has but little meaning when trajectories of intercept -1, such as the $\eta'D'$ trajectory, are considered. We therefore have to leave duality, as simply used through finite energy sum rules at this level of intercept and leave the D as basically an octet member [22]. All the questions dealt with in connection with the duality relations between the $\pi - \eta$ and the H-B multiplets will again come up for the A_1-D multiplet and the 2^- multiplets. Here again one could observe exchange degeneracy at the A_1 level, with an isospin 0 2^- state already at 1.5 GeV or on the other hand a nonet structure at the 2^- level. The mass of the A_1 would then, as the mass of the pion, not fit with duality arguments. Approximate chiral symmetry would again win over duality at determining it.

3. CONCLUSION

The constraints obtained from duality provide many relations among the meson masses but are compatible but with a nonet structure which seemed to be excluded for at least two of the unnatural parity multiplets. It is, however, to be expected that duality arguments, as developed in the framework of finite energy sum rules, will be less and less reliable as lower and lower intercepts are taken into account. This is obvious from the procedure according to which exchanged trajectories are "built up" out of direct channel resonances which saturate the left-hand side of relations, the prototype of which is [1] :

$$\int_0^N \mathrm{Im}\, A^{(-)}(\nu,t)\,d\nu = \sum_i \beta_i(t)\,\frac{N^{\alpha_i(t)+1}}{\alpha_i(t)+1} \qquad (2)$$

Determining trajectories require a further extrapolation in t which only favours the high partial waves over lower ones where exotic states could be present. As a result successes met with leading trajectories cannot be readily expected at the next lower level where phenomenological analysis has to be carried out in detail. As we have stressed, the clarification of the H question is extremely important to know whether exchange degeneracy arguments still apply for a pion trajectory. One then predicts the H in the neighbourhood of the A_1. It is further possible to incorporate the needed $H-A_1$ degeneracy into the duality scheme with an interesting model using a triplet of degenerate trajectories [24]. This may be tempting if constraints obtained from analyzing KK^* amplitudes are disregarded [7].

However duality might not apply to pion exchange with approximate chiral symmetry winning over duality. The H would then be expected in the neighbourhood of the B (or the A_2 in the $\rho\pi$ channel), keeping the slope of 0.9 GeV^{-2} obtained when joining the η and B points as a hint that duality constraints are yet not fully dimmed at this level. Of course, information on the 2^- pattern would also greatly help. The apparent absence of state below the $\pi_A(1640)$ favours the standard nonet choice for the H, the π and the A_1 escaping duality constraints still obeyed at the η-B level.

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R E F E R E N C E S

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6) Two remarks are in order : the two A_2 trajectories are lumped together with an effective global coupling to the $K\bar{K}$ and $\eta\pi$ channels. The f' is the particular combination of octet and singlet states which decouples from $\pi\pi$, whether they are degenerate in mass or not. Duality is compatible with a mass splitting. See further Ref. 24).

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22) There is some rationale at taking but a small mixing, if some at all, between the two strange states. The lowest component of the Q lump is in a mass ratio of $\sqrt{2}$ with the $K^*(890)$. This corresponds to the same SU_3 breaking parameter for the non-strange and strange part of a phenomenological Lagrangian with chiral symmetry involving vector mesons and pseudoscalar and axial mesons (with positive charge conjugation).

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FIGURE CAPTION

Chew-Frautschi plot for unnatural parity mesons. The ρ trajectory is drawn to give the "canonical" slope. Black and white dots correspond to non-strange and strange mesons, respectively.

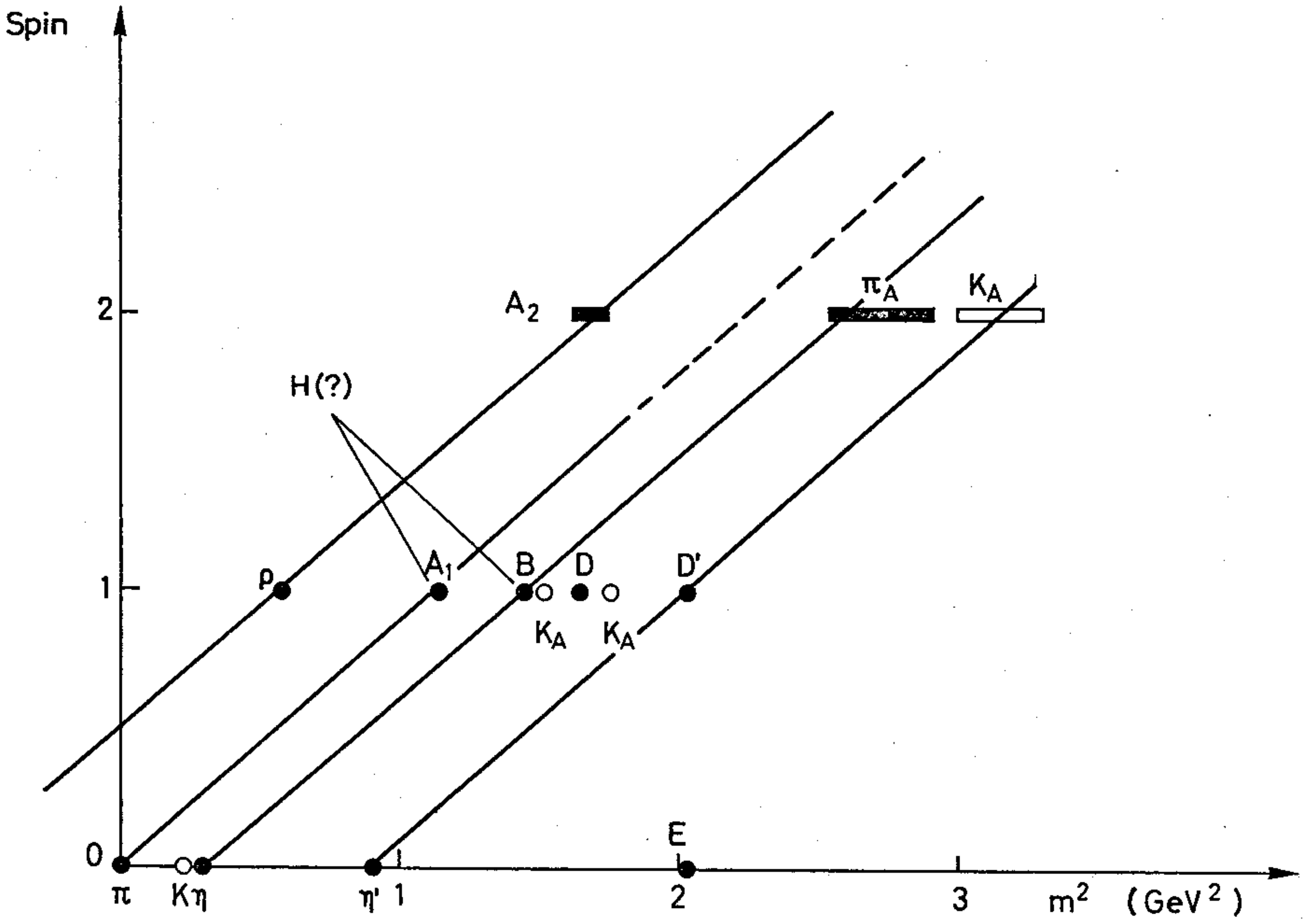

FIG.1