

Duality Constraints and Representation Mixings in Light-Like Chiral Algebra

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Within the framework of light-like chiral algebra $SU(3)_i \times SU(3)_l$, the mixing scheme for meson resonances are investigated. The mixing angles of representations are determined from duality constraints with an assumption about the variation of Regge residues. In particular, equal-weight mixing between the pseudoscalar and axial vector mesons is obtained. All the observed pionic couplings are fitted well with one parameter for each helicity. Consistency of our results with mass relations of Weinberg's and others is also studied.

§ 1. Introduction

Recently Ida¹⁾ has investigated possible chiral properties of hadron resonances and emphasized the importance of light-like charges²⁾ in hadron classification.

The success of the PCAC hypothesis in the Goldberger-Treiman and the Adler-Weisberger relations can be most naturally understood by chiral symmetry of fundamental dynamics and its Nambu-Goldstone realization. In this case, for ordinary space-like charges Q_α , $Q_{5\alpha}$ (in the $SU(3)$ symmetry limit) one obtains

$$Q_\alpha|0\rangle=0 \quad \text{but} \quad Q_{5\alpha}|0\rangle\neq 0.$$

So physical hadron states cannot be assigned to simple representations of the space-like chiral charge algebra. On the contrary, light-like charges annihilate the vacuum,²⁾

$$\hat{Q}_\alpha|0\rangle=\hat{Q}_{5\alpha}|0\rangle=0.$$

In other words, the vacuum is a singlet of light-like chiral charge algebra $SU(3)_i \times SU(3)_l$. So physical hadron states may be assigned to simple representations of this algebra. On the other hand, $\hat{Q}_{5\alpha}$ does not commute with P^2 and J^2 even in the chiral limit because of the spontaneous breaking of chiral symmetry.¹⁾ Therefore hadron resonances are not generally irreducible under $SU(3)_i \times SU(3)_l$, that is, representation mixings occur. If we can determine this mixing pattern and mixing angles, which give mass relations and axial couplings (through PCAC hypothesis), then we may have a clue to make the structure of hadron dynamics clear.

Here we note the conceptual difference between the approach of ours and that of Melosh and others.³⁾ Melosh assumes from the observed hadron spectrum

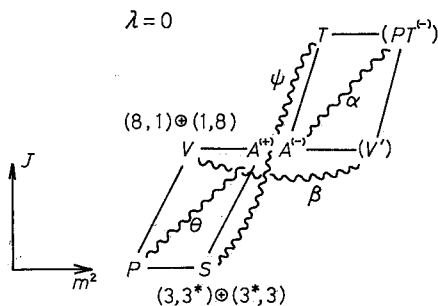
that there exist $SU(6)_W$ generators W_i which approximately commute with the Hamiltonian, that is, hadrons of each $SU(6)_W$ multiplet are approximately degenerate. On the contrary, in our scheme mass splitting between ρ and π , for instance, is unavoidable and mass relations and π -couplings are to be determined simultaneously.

Duality and the absence of exotic resonances require that the imaginary parts of scattering amplitudes in exotic channels be strongly suppressed.⁴⁾ This requirement relates various vertices to each other. If we adopt a specific classification and mixing pattern, axial couplings are expressed in terms of mixing angles. Thus, duality constraints give us information on the mixing angles. In this paper we impose duality constraints on meson-meson scattering and study the mixing of light-like chiral irreducible representations.

In § 2 we briefly review the basic concepts and mixing patterns in chiral classification of mesons. Duality constraints and their consequences are given in § 3. Comparison with experiment and discussions are made in § 4. Appendices I, II and III contain light-like chiral expressions of meson states and equations of duality constraints for meson-meson scattering, respectively.

§ 2. Chiral classification and mixing patterns for mesons

For the chiral assignments and mixing patterns, we adopt the following scheme studied in detail by Ida¹⁾ (Fig. 1).



This scheme is based on the concept of "bloc". Examples for it are given by $(P, S, V, A^{(+)})$, $(A^{(-)}, V', T, PT^{(-)})$ for $J_z = \lambda = 0$ and $(V, A^{(+)}, A^{(-)}, V')$, $(T, PT^{(-)}, PT^{(+)}, T')$ for $\lambda = 1$.

The concept of bloc stems from the following observations.

1) The one-particle saturation of the Adler-Weisberger sum rule indicates:

- i) Only a few particles strongly couple to the nucleon through pion emission. They lie near each other on the J - m^2 plane.

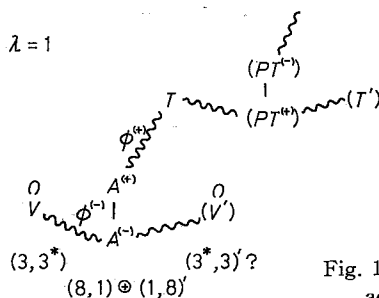


Fig. 1. A schematic diagram of the scheme we adopt for helicity 0 and 1.

- ii) Only a few partial wave (especially $l=0$ and $l=1$) have dominant contributions.
- 2) Higher-lying blocs can be considered as Regge recurrences of the lowest one.

It should be noted that a bloc of physical particles approximately corresponds to a complete set of nonexotic chiral multiplets $(1+8, 1)$, $(1, 1+8)$, $(3, 3^*)$ and $(3^*, 3)$. Further, the bloc consists of two types of partners; one is chiral-type partner such as P and S $((3, 3^*) + (3^*, 3))$, which is connected through $l=0$ π -transition, and the other is " $SU(6)$ "-type partner such as P and V $((8, 1) + (1, 8))$, which is connected through $l=1$ π -transition.

We consider only the lowest two blocs with alternative signatures, neglecting mixings with higher blocs for simplicity.*) Figure 1 shows all the possible mixings among multiplets with the same unitary spin and the same charge parity, and further for the case $\lambda=0$, with the same natural parity.

Physical states are expressed in terms of the mixing angles as given in Appendix I.

§ 3. Duality constraints

Here we consider the duality constraints on the u -channel helicity amplitudes for meson-meson scattering. With an exotic quantum number in the s -channel, we get a relation between u -channel Reggeon trajectories and their residues. (Conditions on the t -channel exchange trajectories are not imposed here because vertices without pseudoscalar meson cannot be evaluated through PCAC.)

The constraints are written as⁴⁾.

$$\sum_i \tau_i \beta_{i,\lambda}^A(u) s^{\alpha_i(u)} = 0, \quad (3.1)$$

where $\alpha_i(u)$ stands for the i -th u -channel Regge trajectory, and τ_i for its signature. $\beta_i(u)$ is the Regge residue which we assume is written as

$$\beta_{i,\lambda}^A(u) = \frac{|X(\lambda)_{n \rightarrow AP_\alpha}|^2}{\tilde{\beta}_{i,\lambda}(u_n)} \cdot \tilde{\beta}_{i,\lambda}(u) \cdot K_\alpha^A(u), \quad (3.2)$$

where

$$|X(\lambda)_{n \rightarrow AP_\alpha}|^2 = \left| \frac{\langle n | \hat{Q}_{\alpha} | A \rangle}{(2\pi)^3 2p^+ \delta^{(2)}(p_{A\perp} - p_{n\perp}) \delta(p_A^+ - p_n^+)} \right|^2,$$

A stands for the external particle, P_α for the pseudoscalar octet, n for the pole $\alpha_i(u_n) = n$ and $K_\alpha^A(u)$ for the kinematical factor***) which depends on u and on external particle mass. Clearly Eq. (3.2) is a simple form of continuation from

*) These higher blocs consist of uncertain resonances and we suppose the mixings with them to be weak.

**) We may take the form of $K_\alpha^A(u)$ as $(u - m_A^2)^2 / F_\alpha^2$ so that $\beta_{i,\lambda}^A(u_n) = ((u_n - m_A^2)^2 / F_\alpha^2) \times |X(\lambda)_{n \rightarrow AP_\alpha}|^2$ is guaranteed at the pole position.

the pole position where $\beta_{i,\lambda}^A(u_n)$ is expressed as

$$\beta_{i,\lambda}^A(u_n) = |X(\lambda)_{n \rightarrow AP_\alpha}|^2 K_\alpha^A(u_n).$$

The essential point of our assumption is that the factor $\tilde{\beta}_{i,\lambda}(u)$ is independent of the external particle.^{*)}

Duality constraint (3.1) combined with Eq. (3.2), leads to the following equation for the exchange degenerate trajectories α_i and α_j :

$$\frac{|X(\lambda)_{n \rightarrow AP_\alpha}|^2}{\tilde{\beta}_{i,\lambda}(u_n)} = \frac{|X(\lambda)_{m \rightarrow AP_\alpha}|^2}{\tilde{\beta}_{j,\lambda}(u_m)}, \quad (3.3)$$

where $\tilde{\beta}_{i,\lambda}(u) = \tilde{\beta}_{j,\lambda}(u)$.

3.1 The case $\lambda = 0$

1) $P_8 P_8 \rightarrow P_8 P_8$

We present the constraints in $\pi\pi \rightarrow \pi\pi$, $K\pi \rightarrow K\pi$ and $KK \rightarrow KK$ in Appendix II. From Eqs. (A.2) and (A.4), we get

$$\begin{aligned} C_0^{K^*-K^{**}} \cos^2 \theta \cos^2 \beta &= \sin^2 \theta \sin^2 \psi, \\ C_0^{\rho-A_2} \cos^2 \theta \cos^2 \beta &= \sin^2 \theta \sin^2 \psi, \end{aligned} \quad (3.4)$$

where $C_{\lambda}^{\rho-A_2}$ and $C_{\lambda}^{K^*-K^{**}}$, etc., are defined by

$$C_{\lambda}^{\rho-A_2} = \frac{\tilde{\beta}_{A_2,\lambda}(u=m_{A_2}^2)}{\tilde{\beta}_{\rho,\lambda}(u=m_{\rho}^2)}$$

and so on. From Eq. (3.4) we get

$$C_0^{K^*-K^{**}} = C_0^{\rho-A_2} = C_0^{V-T}. \quad (3.5)$$

In Eq. (A.1) we require that f' decouples from $\pi\pi$,^{**) then}

$$\tan \theta_T = \frac{1}{\sqrt{2}} \frac{\sin \psi}{\sin \psi'}. \quad (3.6)$$

Substituting Eq. (3.6) into Eq. (A.1), we get from the consistency with Eq. (3.4)

$$\sin^2 \theta_T = \frac{1}{3}. \quad (3.7)$$

From Eqs. (3.6) and (3.7) we get

$$\sin^2 \psi = \sin^2 \psi'. \quad (3.8)$$

^{*)} Although in general the quantity at the pole

$$\frac{|X(\lambda)_{n \rightarrow AP_\alpha}|^2}{|X(\lambda)_{m \rightarrow AP_\alpha}|^2} = \frac{\beta_{i,\lambda}^A(u_n)/K_\alpha^A(u_n)}{\beta_{j,\lambda}^A(u_m)/K_\alpha^A(u_m)}$$

is dependent on A and on the trajectories i and j , duality constraint eliminates the dependence of it on the latter. With our assumption of A -independence we get Eq. (3.3).

^{**) If we accept the fact that f' -trajectory is not degenerate with ρ -trajectory, the very duality constraint (A.1) requires the decoupling of f' from $\pi\pi$.}

Further, if we assume $\phi-f'$ and $\omega-f$ exchange degeneracies (EXD), then

$$\tan^2 \theta_V = \tan^2 \theta_T = \frac{1}{2}. \quad (3.9)$$

2) $A_8^{(+)}P_8 \rightarrow A_8^{(+)}P_8$

Constraints for this case are obtained by replacing $\cos \theta$ and $\sin \theta$ with $-\sin \theta$ and $\cos \theta$, respectively, in the equations for $P_8P_8 \rightarrow P_8P_8$. Thus we get

$$C_0^{V-T} \sin^2 \theta \cos^2 \beta = \cos^2 \theta \sin^2 \psi. \quad (3.10)$$

From Eqs. (3.4) and (3.10), the following results are obtained:

$$\sin^2 \theta = \cos^2 \theta = \frac{1}{2}, \quad (3.11)$$

$$C_0^{V-T} \cos^2 \beta = \sin^2 \psi, \quad (3.12)$$

except for unacceptable ones; $\psi=0$ and $(C_0^{V-T}=0$ or $\beta=\pi/2)$.

3) $V_8P_8 \rightarrow V_8P_8$

In this case we treat the unnatural parity exchange. We present the constraints in Appendix II.

From Eq. (A.7) and assuming $\pi-B$ EXD and $A_1-(PT)_1$ EXD, we obtain

$$\begin{aligned} C_0^{\pi-B} \cos^2 \beta \cos^2 \theta &= \sin^2 \beta \sin^2 \alpha, \\ C_0^{A_1-(PT)_1} \cos^2 \beta \cos^2 \theta &= \sin^2 \beta \cos^2 \alpha. \end{aligned}$$

Putting $\cos^2 \theta = \frac{1}{2}$, we get

$$\tan^2 \beta = \frac{1}{2} (C_0^{\pi-B} + C_0^{A_1-(PT)_1}), \quad (3.13)$$

and

$$\tan^2 \alpha = \frac{C_0^{\pi-B}}{C_0^{A_1-(PT)_1}} \quad (3.14)$$

if $\beta \neq 0$. In the case $\beta=0$, the above equations are still valid independent of the value of α , so long as $C_0^{\pi-B} = C_0^{A_1-(PT)_1} = 0$.

In Eq. (A.5), if we assume that H' and $(PT)_0'$ decouple from $\rho\pi$ (and that $\pi-H$ and $A_1-(PT)_0$ trajectories are exchange-degenerate), we would obtain the ideal mixings for $A^{(-)}$ and PT , but it seems rather doubtful.

3.2 The case $\lambda=1$

We consider scattering of $V_8P_8 \rightarrow V_8P_8$ and $A_8^{(\pm)}P_8 \rightarrow A_8^{(\pm)}P_8$. Constraints on $\rho\pi \rightarrow \rho\pi$, $K^*\pi \rightarrow K^*\pi$, $A_1\pi \rightarrow A_1\pi$, $K_\alpha^*\pi \rightarrow K_\alpha^*\pi$ and $K_\beta^*\pi \rightarrow K_\beta^*\pi$ are presented in Appendix III.

From Eq. (A.9), we get

$$C_1^{K^*-K^{**}} \cos^4 \phi^{(-)} = \sin^2 \phi^{(-)} \sin^2 \phi^{(+)}. \quad (3.15)$$

If we assume in Eq. (A.8) that ϕ decouples from $\rho\pi$, then

$$\tan \theta_V = \frac{1}{\sqrt{2}} \frac{\cos \phi^{(-)}}{\cos \phi^{(-)'}}.$$

Substituting this equation into Eq. (A·8), we have, from the consistency with Eq. (3·15),

$$C_1^{K^*-K^{**}} = C_1^{0-A_2} \equiv C_1^{V-T},$$

$$\sin^2 \theta_V = \frac{1}{3}.$$

Thus

$$\cos^2 \phi^{(-)} = \cos^2 \phi^{(-)'} \quad (3\cdot16)$$

From Eq. (A·12), we get

$$C_1^{K^*-K^{**}} \cos^2 \phi^{(+)} \sin^2 \phi^{(-)} = \sin^2 \phi^{(+)} \cos^2 \phi^{(+)} \quad (3\cdot17)$$

From Eqs. (3·15) and (3·17), the following results are obtained:

$$\cos^2 \phi^{(-)} = \sin^2 \phi^{(-)} = \frac{1}{2}, \quad (3\cdot18)$$

$$\sin^2 \phi^{(+)} = \frac{1}{2} C_1^{K^*-K^{**}}, \quad (3\cdot19)$$

except for trivial ones; $\phi^{(+)} = \pi/2$ and $C^{K^*-K^{**}} \cos^4 \phi^{(-)} = \sin^2 \phi^{(-)}$.

If we require in Eq. (A·10) that f' decouples from $\pi\pi$, then from the consistency with Eq. (3·17), we get

$$\sin^2 \theta_T = \frac{1}{3},$$

$$\cos^2 \phi^{(+)} = \cos^2 \phi^{(+)'} \quad (3\cdot20)$$

§ 4. Comparison with experiment and discussion

We now summarize our results obtained above:

$$\sin^2 \theta = \cos^2 \theta = \frac{1}{2}, \quad (4\cdot1)$$

$$\sin^2 \phi^{(-)} = \cos^2 \phi^{(-)} = \frac{1}{2}, \quad (4\cdot2)$$

$$\sin^2 \phi = C_0^{V-T} \cos^2 \beta, \quad (4\cdot3)$$

$$\tan^2 \beta = \frac{1}{2} (C_0^{P-A^{(-)}} + C_0^{A^{(+)}-PT}), \quad (4\cdot4)$$

$$\tan^2 \alpha = \frac{C_0^{P-A^{(-)}}}{C_0^{A^{(+)}-PT}}, \quad (\text{only when } \beta \neq 0) \quad (4\cdot5)$$

$$\sin^2 \phi^{(+)} = \frac{1}{2} C_1^{V-T}. \quad (4\cdot6)$$

Also we have got nonet type mixings

$$\text{between } T \text{ and } S; \quad \phi = \psi' \quad \text{for } \lambda = 0,$$

$$\text{between } \begin{cases} V \text{ and } A^{(-)}; & \phi^{(-)} = \phi^{(-)'} \\ T \text{ and } A^{(+)}; & \phi^{(+)} = \phi^{(+)'} \end{cases} \quad \text{for } \lambda = 1.$$

Equation (4·1), which is the most important result of our investigation, leads to Weinberg's mass relation⁵⁾ and to correct $\rho \rightarrow 2\pi$ coupling in the case $\beta \sim 0$. In fact, when $\beta = 0$, the equality of the matrix elements of mass operator P^2

$$\langle (8, 1)^+ | P^2 | (8, 1)^+ \rangle = \langle (8, 1)^- | P^2 | (8, 1)^- \rangle^*$$

is written as¹⁾

$$m_V^2 = \cos^2 \theta \cdot m_{\rho^2} + \sin^2 \theta \cdot m_{A^{(+)}}^2. \quad (4.7)$$

Using our result (4.1), we find that Eq. (4.7) is just the well-known mass relation of Weinberg. Further the coupling constant $f_{\rho\pi\pi}$ is given through PCAC as¹⁾

$$\frac{f_{\rho\pi\pi}}{m_{\rho^2}} = \frac{1}{F_{\pi^2}} \cos^2 \theta. \quad (4.8)$$

This equation (4.8) with our result $\cos^2 \theta = \frac{1}{2}$ coincides with the KSFR relation⁶⁾ which is well satisfied experimentally.

Equation (4.2) predicts that

$$g_{\omega\rho\pi} = \frac{2}{F_{\pi}} \cos^2 \phi^{(-)} = \frac{1}{F_{\pi}} = 10.5 \text{ GeV}^{-1}. \quad (4.9)$$

This rather deviates from the semi-experimental value ($15 \pm 2 \text{ GeV}^{-1}$) which is obtained from $\omega \rightarrow \pi^0 + \gamma$ through the assumption of vector meson dominance.⁷⁾

Equations (4.3) ~ (4.6) are dependent on the C 's. We consider the following two special cases.

(I) C is a universal constant independent of helicity and exchanged trajectories: $C \equiv C_{\chi^V-T} = C_{\chi^{P-A^{(-)}}} = C_{\chi^{A^{(+)}}-PT} (\lambda=0, 1)$. Then Eqs. (4.3) ~ (4.6) can be rewritten as

$$C = \tan^2 \beta = \tan^2 \psi = 2 \sin^2 \phi^{(+)}, \quad (4.10)$$

$$\sin^2 \alpha = \cos^2 \alpha = \frac{1}{2}. \quad (4.11)$$

All transitions are expressed by the single parameter C as presented in Table I.

Further, when C is set as

$$C = \frac{|X(0)_{f \rightarrow \pi\pi}|^2}{|X(0)_{\rho \rightarrow \pi\pi}|^2} \sim 0.2, \quad (4.12)**$$

all the pion couplings are evaluated and compared with experiments in Table I. Our results for $\lambda=0$ are fairly well, though they, as will be explained in Case II, rather deviate from experiments for $\lambda=1$. Even for $\lambda=0$, this Case I meets with a difficulty on the mass relation of Weinberg's type, which is written as

) $|(8, 1)^\rangle$ is the eigenstate of reflection operator:

$$|(8, 1)^*\rangle = \frac{1}{\sqrt{2}} \{ |(8, 1)\rangle \pm |(1, 8)\rangle \}.$$

**) $|X(\lambda)_{\alpha \rightarrow \beta\pi}|$ is related to the decay width $\Gamma_{\alpha \rightarrow \beta\pi}$ as follows:

$$\Gamma_{\alpha \rightarrow \beta\pi} = \frac{p^3}{2\pi F_{\pi}^2} \left(\frac{p}{p^*} \right)^{2(l-1)} \frac{1}{2J_{\alpha}+1} \sum_{\lambda=-J_{\alpha}}^{J_{\alpha}} |X(\lambda)_{\alpha \rightarrow \beta\pi}|^2,$$

where $F_{\pi} \doteq 94 \text{ MeV}$ and p^* is the pion momentum in the light-like limit,

$$p^* = \frac{m_{\alpha}^2 - m_{\beta}^2}{2m_{\alpha}}.$$

$$m_V^2 \cos^2 \beta + m_V^2 \sin^2 \beta = m_P^2 \cos^2 \theta + m_{A^{(*)}}^2 \sin^2 \theta$$

$$= \frac{1}{2} (m_P^2 + m_{A^{(*)}}^2). \quad (4.13)$$

(When $\beta=0$, this is of course reduced to Eq. (4.7).) The left-hand side with

Table I. Predictions for the matrix elements $|X(0)|^2$ and $|X(1)|^2$ together with corresponding expressions in terms of mixing angles and experimental data.

- a) The value cited from Ref. 1).
 b) The value corresponding to the total decay width of δ in Ref. 8).
 c) World average cited in Ref. 9).

a) $\lambda=0$

Transitions		$ X(0) ^2$					
		mixing angles	Case I	$C=0.2$	Case II	$C^{V-T}=0.2$	exp.
$V \rightarrow P$	$\rho \rightarrow \pi\pi$	$\cos^2 \theta \cos^2 \beta$	$\frac{1}{2} \frac{1}{1+C}$	0.42	$\frac{1}{2}$	0.5	0.50 ^{a)}
	$K^* \rightarrow K\pi$	$\frac{1}{2} \cos^2 \theta \cos^2 \beta$		0.21		0.25	0.23 ^{a)}
$T \rightarrow P$	$f \rightarrow \pi\pi$	$\sin^2 \theta \sin^2 \phi$	$\frac{1}{2} \frac{C}{1+C}$	0.083	$\frac{1}{2} C^{V-T}$	0.10	0.099 ^{a)}
	$A_2 \rightarrow \eta\pi$	$\frac{1}{3} \sin^2 \theta \sin^2 \phi$		0.028		0.033	0.032 ^{a)}
	$K_N \rightarrow K\pi$	$\frac{1}{2} \sin^2 \theta \sin^2 \phi$		0.042		0.050	0.045 ^{a)}
$S \rightarrow P$	$\delta \rightarrow \eta\pi$	$\frac{1}{3} \sin^2 \theta \cos^2 \phi$	$\frac{1}{3} \frac{1}{2} \frac{1}{1+C}$	0.14	$\frac{1}{3} \frac{1}{2} (1-C^{V-T})$	0.13	0.10 $\begin{smallmatrix} +0.08^b) \\ -0.05 \\ \text{(total)} \end{smallmatrix}$
$A^{(*)} \rightarrow S$	$D \rightarrow \delta\pi$	$\frac{1}{3} \cos^2 \theta \cos^2 \phi$	$\frac{1}{3} \frac{1}{2} \frac{1}{1+C}$	0.14	$\frac{1}{3} \frac{1}{2} (1-C^{V-T})$	0.13	
$A^{(*)} \rightarrow V$	$A_1 \rightarrow \rho\pi$	$\sin^2 \theta \cos^2 \beta$	$\frac{1}{2} \frac{1}{1+C}$	0.42	$\frac{1}{2}$	0.50	
$A^{(*)} \rightarrow V$	$B \rightarrow \omega\pi$	$\sin^2 \alpha \sin^2 \beta$	$\frac{1}{2} \frac{C}{1+C}$	0.083	0	0	0.064 \pm 0.006 ^{c)}

b) $\lambda=1$

Transitions		$ X(1) ^2$					
		mixing angles	Case I	$C=0.2$	Case II	$C^{V-T}=0.2$	exp.
$V \rightarrow V$	$\omega \rightarrow \rho\pi$	$\cos^2 \phi^{(-)}$	$\frac{1}{4}$	0.25	$\frac{1}{4}$	0.25	(0.50) ^{a)}
$T \rightarrow V$	$A_2 \rightarrow \rho\pi$	$\sin^2 \phi^{(-)} \sin^2 \phi^{(+)}$	$\frac{1}{4} C$	0.050	$\frac{1}{4} C^{V-T}$	0.050	0.080 ^{a)}
	$K_N \rightarrow K^* \pi$	$\frac{1}{2} \sin^2 \phi^{(-)} \sin^2 \phi^{(+)}$		0.025		0.025	0.039 ^{a)}
$A^{(*)} \rightarrow V$	$A_1 \rightarrow \rho\pi$	$\sin^2 \phi^{(-)} \cos^2 \phi^{(+)}$	$\frac{1}{2} \left(1 - \frac{C}{2}\right)$	0.45	$\frac{1}{2} \left(1 - \frac{C^{V-T}}{2}\right)$	0.45	
$A^{(*)} \rightarrow V$	$B \rightarrow \omega\pi$	$\sin^2 \phi^{(-)} \cos^2 \phi^{(-)}$	$\frac{1}{4}$	0.25	$\frac{1}{4}$	0.25	0.22 \pm 0.02 ^{c)}

$m_{\rho'} = 1.6 \text{ GeV}$ gives $\sim 0.92 \text{ (GeV)}^2$, while the right-hand side is $\sim 0.62 \text{ (GeV)}^2$. The discrepancy is obviously due to the largeness of $\sin^2 \beta$. Weinberg's mass relation is consistent with Eq. (4.13) only when $\sin^2 \beta \sim 0$. So we next consider the case $\beta = 0$.

(II) The case $\beta = 0$.

With λ -independent C^{V-T} ($=C_0^{V-T} = C_1^{V-T}$) for simplicity, Eqs. (4.3), (4.4) and (4.6) are rewritten as

$$C^{V-T} = \sin^2 \phi = 2 \sin^2 \phi^{(+)}, \quad (4.14)$$

$$\beta = C_0^{P-A^{(-)}} = C_0^{A^{(+)-PT} = 0. \quad (4.15)$$

α is not determined because of Eqs. (4.5) and (4.15). So there remain two parameters C^{V-T} and α . The pionic couplings are presented in Table I. As for the $\lambda=0$ meson decays, this Case II solution shows nice agreements with the data, though there is not much to choose between Case II and Case I. Firstly, as stated in the beginning of this section, the so-called KSFR⁹⁾ relation holds in Case II. Therefore the predicted reduced matrix elements for decays $\rho \rightarrow \pi\pi$, $K^* \rightarrow K\pi$ agree with experiment as follows:

$$|X(0)_{\rho \rightarrow \pi\pi}|^2 = \frac{1}{2}, \quad (0.50 \text{ exp.}) \quad (4.16)$$

$$|X(0)_{K^* \rightarrow K\pi}|^2 = \frac{1}{4}. \quad (0.23 \text{ exp.}) \quad (4.17)$$

Secondly, if $C^{V-T} = 0.2$ is taken as input, $T \rightarrow P$ decays ($f \rightarrow \pi\pi$, $A_2 \rightarrow \eta\pi$, $K_N \rightarrow K\pi$) are fitted within 10% error. Thirdly for the decay $\delta \rightarrow \eta\pi$

$$|X(0)_{\delta \rightarrow \eta\pi}|^2 = 0.13 \quad (4.18)$$

is predicted, while the experiment gives the value

$$|X(0)_{\delta \rightarrow \eta\pi}|^2_{\text{exp.}} = 0.10^{+0.08}_{-0.05} \quad (4.19)$$

if δ decays only into $\eta\pi$. This is consistent with the prediction (4.18). Finally the decay $B \rightarrow \omega\pi$ needs some comments. The world average and BNL/LBL data¹⁰⁾ show appreciable amount of the $\lambda=0$ amplitude ($|X(0)_{B \rightarrow \omega\pi}|^2_{\text{world average}} \sim 0.064 \pm 0.006$) and they seem to prefer the Case I solution ($|X(0)_{B \rightarrow \omega\pi}|^2 = 0.083$), while in Weizmann's data¹⁰⁾ with a fairly small $\lambda=0$ amplitude, we have $|X(0)_{B \rightarrow \omega\pi}|^2 \sim 0.006$ (and $|X(1)_{B \rightarrow \omega\pi}|^2 \sim 0.28$), which favours Case II.

Also as stated in the beginning, Weinberg's mass relation holds in this Case II solution. Further the strong suppression of $\rho' \rightarrow \pi\pi$ mode compared with $\rho' \rightarrow \pi\pi\pi\pi$ is consistent with the small β . Therefore the Case II solution nicely agrees with experimental values of masses and π -couplings for the case $\lambda=0$.

Finally we comment on some difficulties for both Cases I and II we have in the case $\lambda=1$. If we wish to fit all the couplings of $\omega \rightarrow \rho\pi$, $A_2 \rightarrow \rho\pi$, $K_N \rightarrow K^*\pi$ and $B \rightarrow \omega\pi$ together with the mass relation¹⁾

$$m_V^2 \sin^2 \phi^{(-)} + m_A^2 \cos^2 \phi^{(-)} = m_T^2 \sin^2 \phi^{(+)} + m_A^2 \cos^2 \phi^{(+)}, \quad (4.20)$$

the values¹⁾

$$\sin^2 \phi^{(-)} \sim 0.27, \quad (4.21)$$

$$\sin^2 \phi^{(+)} \sim 0.29 \quad (4.22)$$

are preferable. On the other hand our results are

$$\sin^2 \phi^{(-)} = \frac{1}{2}, \quad (4.23)$$

$$\sin^2 \phi^{(+)} = \frac{C_{\lambda=1}^{V-T}}{2} = 0.1. \quad (4.24)$$

Then as for the mass relation (4.20), the left-hand side gives ~ 1.06 , while the right-hand side with the value $\sin^2 \phi^{(+)} = 0.1$ ($C_{\lambda=1}^{V-T} = 0.2$) is ~ 1.25 , showing a discrepancy of about 20%. As for the decays $\omega \rightarrow \rho\pi$, $A_2 \rightarrow \rho\pi$, $K_N \rightarrow K^*\pi$, our predictions are 40~50% smaller than the experimental width as shown in Table I. These discrepancies can be reduced to the ones between the values (4.21), (4.22) and (4.23), (4.24). The discrepancy between (4.22) and (4.24) may be evaded by setting $C_{\lambda=1}^{V-T} = 2 \times 0.29$ because $C_{\lambda=1}$ has not necessarily to be equal to $C_{\lambda=0}$. The disagreement between (4.21) and (4.23) or in general the deviations of our solutions from experiments, if taken seriously, may be due to our neglect of mixings with other states, e.g., V' , $PT^{(\pm)}$, T' and so on. Another possibility is to take the dependence of $\tilde{\beta}_\lambda(u)$ on the external particles into consideration, but it will reduce our nice predictions for $\lambda=0$ to a mere accident.

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Appendix I

Mixing angles between octets are as shown in Fig. 1. Those with primes are between singlets. Octet-singlet mixing is denoted by θ : e.g., θ_V for vector nonet.

1) $\lambda=0$

$$V_8: \cos \beta | (8, 1)^+ \rangle + \sin \beta | (3, 3^*)'_8{}^+ \rangle$$

$$V_1: \cos \beta' | (1, 1)^+ \rangle + \sin \beta' | (3, 3^*)'_1{}^+ \rangle$$

$$P_8: \cos \theta | (8, 1)^- \rangle + \sin \theta | (3, 3^*)'_8{}^- \rangle$$

$$P_1: \cos \theta' | (1, 1)^- \rangle + \sin \theta' | (3, 3^*)'_1{}^- \rangle$$

$$A_8^{(+)}: -\sin \theta | (8, 1)^- \rangle + \cos \theta | (3, 3^*)'_8{}^- \rangle$$

$$\begin{aligned}
A_1^{(+)}: & -\sin \theta' |(1, 1)^-\rangle + \cos \theta' |(3, 3^*)_1^-\rangle \\
S_8: & \cos \psi |(3, 3^*)_8^+\rangle + \sin \psi |(8, 1)^{+\prime}\rangle \\
S_1: & \cos \psi' |(3, 3^*)_1^+\rangle + \sin \psi' |(1, 1)^{+\prime}\rangle \\
T_8: & -\sin \psi |(3, 3^*)_8^+\rangle + \cos \psi |(8, 1)^{+\prime}\rangle \\
T_1: & -\sin \psi' |(3, 3^*)_1^+\rangle + \cos \psi' |(1, 1)^{+\prime}\rangle \\
A_8^{(-)}: & \cos \alpha |(8, 1)^{-\prime}\rangle + \sin \alpha |(3, 3^*)_8^-\rangle \\
A_1^{(-)}: & \cos \alpha' |(1, 1)^{-\prime}\rangle + \sin \alpha' |(3, 3^*)_1^-\rangle \\
PT_8: & -\sin \alpha |(8, 1)^{-\prime}\rangle + \cos \alpha |(3, 3^*)_8^-\rangle \\
PT_1: & -\sin \alpha' |(1, 1)^{-\prime}\rangle + \cos \alpha' |(3, 3^*)_1^-\rangle \\
V_8': & -\sin \beta |(8, 1)^{+\prime}\rangle + \cos \beta |(3, 3^*)_8'^+\rangle \\
V_1': & -\sin \beta' |(1, 1)^{+\prime}\rangle + \cos \beta' |(3, 3^*)_1'^+\rangle.
\end{aligned}$$

2) $\lambda=1$

$$\begin{aligned}
V_8: & \cos \phi^{(-)} |(3, 3^*)_8\rangle + \sin \phi^{(-)} \frac{1}{\sqrt{2}} [| (8, 1)\rangle - | (1, 8)\rangle] \\
V_1: & \cos \phi^{(-)\prime} |(3, 3^*)_1\rangle + \sin \phi^{(-)\prime} \frac{1}{\sqrt{2}} [| (1, 1)\rangle - | (1, 1)'\rangle] \\
A_8^{(-)}: & -\sin \phi^{(-)} |(3, 3^*)_8\rangle + \cos \phi^{(-)} \frac{1}{\sqrt{2}} [| (8, 1)\rangle - | (1, 8)\rangle] \\
A_1^{(-)}: & -\sin \phi^{(-)\prime} |(3, 3^*)_1\rangle + \cos \phi^{(-)\prime} \frac{1}{\sqrt{2}} [| (1, 1)\rangle - | (1, 1)'\rangle] \\
T_8: & \cos \phi^{(+)} |(3, 3^*)_8\rangle + \sin \phi^{(+)} \frac{1}{\sqrt{2}} [| (8, 1)\rangle + | (1, 8)\rangle] \\
T_1: & \cos \phi^{(+)\prime} |(3, 3^*)_1\rangle + \sin \phi^{(+)\prime} \frac{1}{\sqrt{2}} [| (1, 1)\rangle + | (1, 1)'\rangle] \\
A_8^{(+)}: & -\sin \phi^{(+)} |(3, 3^*)_8\rangle + \cos \phi^{(+)} \frac{1}{\sqrt{2}} [| (8, 1)\rangle + | (1, 8)\rangle] \\
A_1^{(+)}: & -\sin \phi^{(+)\prime} |(3, 3^*)_1\rangle + \cos \phi^{(+)\prime} \frac{1}{\sqrt{2}} [| (1, 1)\rangle + | (1, 1)'\rangle].
\end{aligned}$$

Appendix II

1) $P_8 P_8 \rightarrow P_8 P_8$ $\pi\pi \rightarrow \pi\pi$:

$$\tau_f R_f \frac{1}{\tilde{\beta}(u_f)} \sin^2 \theta \left(\sqrt{\frac{2}{3}} \cos \theta_r \sin \psi' + \sqrt{\frac{1}{3}} \sin \theta_r \sin \psi \right)^2 + \tau_{f'} R_{f'} \frac{1}{\tilde{\beta}(u_{f'})}$$

$$\begin{aligned} & \times \sin^2 \theta \left(-\sqrt{\frac{2}{3}} \sin \theta_T \sin \psi' + \sqrt{\frac{1}{3}} \cos \theta_T \sin \psi \right)^2 \\ & + \tau_\rho R_\rho \frac{1}{\tilde{\beta}(u_\rho)} \cos^2 \theta \cos^2 \beta = 0. \end{aligned} \quad (\text{A} \cdot 1)^*)$$

$K\pi \rightarrow K\pi$:

$$\tau_{K^*} R_{K^*} \frac{1}{\tilde{\beta}(u_{K^*})} \cos^2 \theta \cos^2 \beta + \tau_{K^{**}} R_{K^{**}} \frac{1}{\tilde{\beta}(u_{K^{**}})} \sin^2 \theta \sin^2 \psi = 0. \quad (\text{A} \cdot 2)$$

$KK \rightarrow KK$:

$I_u = 0$

$$\begin{aligned} & \tau_\phi R_\phi \frac{1}{\tilde{\beta}(u_\phi)} \frac{3}{4} \cos^2 \theta_V \cos^2 \theta \cos^2 \beta + \tau_\omega R_\omega \frac{1}{\tilde{\beta}(u_\omega)} \frac{3}{4} \sin^2 \theta_V \cos^2 \theta \cos^2 \beta \\ & + \tau_f R_f \frac{1}{\tilde{\beta}(u_f)} \left(\sqrt{\frac{2}{3}} \cos \theta_T \sin \psi' - \frac{1}{2\sqrt{3}} \sin \theta_T \sin \psi \right)^2 \sin^2 \theta \\ & + \tau_{f'} R_{f'} \frac{1}{\tilde{\beta}(u_{f'})} \left(-\sqrt{\frac{2}{3}} \sin \theta_T \sin \psi' - \frac{1}{2\sqrt{3}} \cos \theta_T \sin \psi \right)^2 \sin^2 \theta = 0, \end{aligned} \quad (\text{A} \cdot 3)$$

$I_u = 1$

$$\tau_\rho R_\rho \frac{1}{\tilde{\beta}(u_\rho)} \cos^2 \theta \cos^2 \beta + \tau_{A_2} R_{A_2} \frac{1}{\tilde{\beta}(u_{A_2})} \sin^2 \theta \sin^2 \psi = 0. \quad (\text{A} \cdot 4)$$

2) $V_8 P_8 \rightarrow V_8 P_8$

$\rho\pi \rightarrow \rho\pi$:

$$\begin{aligned} & \tau_\pi R_\pi \frac{1}{\tilde{\beta}(u_\pi)} \cos^2 \beta \cos^2 \theta + \tau_{A_1} R_{A_1} \frac{1}{\tilde{\beta}(u_{A_1})} \cos^2 \beta \sin^2 \theta \\ & + \tau_H R_H \frac{1}{\tilde{\beta}(u_H)} \left(\sqrt{\frac{2}{3}} \cos \theta_{A^{(\rightarrow)}} \sin \alpha' + \sqrt{\frac{1}{3}} \sin \theta_{A^{(\rightarrow)}} \sin \alpha \right)^2 \sin^2 \beta \\ & + \tau_{H'} R_{H'} \frac{1}{\tilde{\beta}(u_{H'})} \left(-\sqrt{\frac{2}{3}} \sin \theta_{A^{(\rightarrow)}} \sin \alpha' + \sqrt{\frac{1}{3}} \cos \theta_{A^{(\rightarrow)}} \sin \alpha \right)^2 \sin^2 \beta \\ & + \tau_{(PT)_0} R_{(PT)_0} \frac{1}{\tilde{\beta}(u_{(PT)_0})} \left(\sqrt{\frac{2}{3}} \cos \theta_{PT} \cos \alpha' + \sqrt{\frac{1}{3}} \sin \theta_{PT} \cos \alpha \right)^2 \sin^2 \beta \\ & + \tau_{(PT)_0'} R_{(PT)_0'} \frac{1}{\tilde{\beta}(u_{(PT)_0'})} \left(-\sqrt{\frac{2}{3}} \sin \theta_{PT} \cos \alpha' \right. \\ & \quad \left. + \sqrt{\frac{1}{3}} \cos \theta_{PT} \cos \alpha \right)^2 \sin^2 \beta = 0. \end{aligned} \quad (\text{A} \cdot 5)$$

*) R_i stands for the factor $s^{\alpha_i(u)} \tilde{\beta}_{i,1}(u) K(u)$. Troublesome suffices are omitted from $\tilde{\beta}_{i,1}(u_n)$.

$K^*\pi \rightarrow K^*\pi$: K_α and K_β are physical (1^{++}) and (1^{+-}) states

$$K_\beta = K_{+-} \cos \delta + K_{++} \sin \delta,$$

$$K_\alpha = -K_{+-} \sin \delta + K_{++} \cos \delta,$$

$$\begin{aligned} & \tau_K R_K \frac{1}{\tilde{\beta}(u_K)} \left\{ \frac{1}{2} \cos \theta \cos \beta \right\}^2 + \tau_{K(PT)} R_{K(PT)} \frac{1}{\tilde{\beta}(u_{K(PT)})} \left\{ \frac{1}{2} \sin \beta \cos \alpha \right\}^2 \\ & + \tau_{K_\beta} R_{K_\beta} \frac{1}{\tilde{\beta}(u_{K_\beta})} \left\{ \frac{1}{2} \cos \delta \sin \beta \sin \alpha - \frac{1}{2} \sin \delta \cos \beta \sin \theta \right\}^2 \\ & + \tau_{K_\alpha} R_{K_\alpha} \frac{1}{\tilde{\beta}(u_{K_\alpha})} \left\{ -\frac{1}{2} \sin \delta \sin \beta \sin \alpha - \frac{1}{2} \cos \delta \cos \beta \sin \theta \right\}^2 = 0. \quad (\text{A} \cdot 6) \end{aligned}$$

$\rho K \rightarrow \rho K$: the same as for $K^*\pi \rightarrow K^*\pi$.

$K^*K \rightarrow K^*K$:

$$\begin{aligned} & \tau_\pi R_\pi \frac{1}{\tilde{\beta}(u_\pi)} \left\{ \frac{1}{2} \cos \beta \cos \theta \right\}^2 + \tau_{A_1} R_{A_1} \frac{1}{\tilde{\beta}(u_{A_1})} \left\{ \frac{1}{2} \cos \beta \sin \theta \right\}^2 \\ & + \tau_B R_B \frac{1}{\tilde{\beta}(u_B)} \left\{ \frac{1}{2} \sin \beta \sin \alpha \right\}^2 + \tau_{(PT)_1} R_{(PT)_1} \frac{1}{\tilde{\beta}(u_{(PT)_1})} \left\{ \frac{1}{2} \sin \beta \cos \alpha \right\}^2 = 0. \quad (\text{A} \cdot 7) \end{aligned}$$

Appendix III

$\rho\pi \rightarrow \rho\pi$:

$$\begin{aligned} & \tau_\omega R_\omega \frac{1}{\tilde{\beta}(u_\omega)} \cos^2 \phi^{(-)} \left\{ \sqrt{\frac{2}{3}} \cos \theta_V \cos \phi^{(-)'} + \sqrt{\frac{1}{3}} \sin \theta_V \cos \phi^{(-)} \right\}^2 \\ & + \tau_\phi R_\phi \frac{1}{\tilde{\beta}(u_\phi)} \cos^2 \phi^{(-)} \left\{ -\sqrt{\frac{2}{3}} \sin \theta_V \cos \phi^{(-)'} + \sqrt{\frac{1}{3}} \cos \theta_V \cos \phi^{(-)} \right\}^2 \\ & + \tau_{A_1} R_{A_1} \frac{1}{\tilde{\beta}(u_{A_1})} \sin^2 \phi^{(-)} \sin^2 \phi^{(+)} = 0. \quad (\text{A} \cdot 8) \end{aligned}$$

$K^*\pi \rightarrow K^*\pi$:

$$\tau_{K^*} R_{K^*} \frac{1}{\tilde{\beta}(u_{K^*})} \left\{ \frac{1}{2} \cos^2 \phi^{(-)} \right\}^2 + \tau_{K^{**}} R_{K^{**}} \frac{1}{\tilde{\beta}(u_{K^{**}})} \left\{ \frac{1}{2} \sin \phi^{(-)} \sin \phi^{(+)} \right\}^2 = 0. \quad (\text{A} \cdot 9)$$

$A_1\pi \rightarrow A_1\pi$:

$$\begin{aligned} & \tau_\rho R_\rho \frac{1}{\tilde{\beta}(u_\rho)} \{ \cos \phi^{(+)} \sin \phi^{(-)} \}^2 \\ & + \tau_f R_f \frac{1}{\tilde{\beta}(u_f)} (-\sin \phi^{(+)})^2 \left\{ \sqrt{\frac{2}{3}} \cos \theta_T \cos \phi^{(+)'} + \sqrt{\frac{1}{3}} \sin \theta_T \cos \phi^{(+)} \right\}^2 \end{aligned}$$

$$\begin{aligned}
 & + \tau_{K'} R_{K'} \frac{1}{\tilde{\beta}(u_{K'})} (-\sin \phi^{(+)})^2 \left\{ -\sqrt{\frac{2}{3}} \sin \theta_T \cos \phi^{(+)} \right. \\
 & \left. + \sqrt{\frac{1}{3}} \cos \theta_T \cos \phi^{(+)} \right\}^2 = 0. \quad (\text{A} \cdot 10)
 \end{aligned}$$

$K^{*-}\pi \rightarrow K^{*-}\pi$:

$$\begin{aligned}
 & \tau_{K^*} R_{K^*} \frac{1}{\tilde{\beta}(u_{K^*})} \left\{ -\frac{1}{2} \sin \phi^{(-)} \cos \phi^{(-)} \right\}^2 \\
 & + \tau_{K^{**}} R_{K^{**}} \frac{1}{\tilde{\beta}(u_{K^{**}})} \left\{ \frac{1}{2} \cos \phi^{(-)} \sin \phi^{(+)} \right\}^2 = 0. \quad (\text{A} \cdot 11)
 \end{aligned}$$

$K^{*+}\pi \rightarrow K^{*+}\pi$:

$$\begin{aligned}
 & \tau_{K^*} R_{K^*} \frac{1}{\tilde{\beta}(u_{K^*})} \left\{ \frac{1}{2} \cos \phi^{(+)} \sin \phi^{(-)} \right\}^2 \\
 & + \tau_{K^{**}} R_{K^{**}} \frac{1}{\tilde{\beta}(u_{K^{**}})} \left\{ -\frac{1}{2} \sin \phi^{(+)} \cos \phi^{(+)} \right\}^2 = 0. \quad (\text{A} \cdot 12)
 \end{aligned}$$

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