

**Duration Dependence in Stock Prices:
An Analysis of Bull and Bear Markets***

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Duration Dependence in Stock Prices: An Analysis of Bull and Bear Markets*

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ABSTRACT

This paper proposes a new approach to studying time series dependence in stock prices by modelling the probability that a bull or bear market terminates as a function of its age and a set of underlying state variables. Controlling for time-varying volatility and the state of the economy, the termination probability of a bull market declines as a function of its age. Bear market termination probabilities follow a U-shaped pattern, assigning low probability to very long bear markets. Interest rates are found to have an important effect on the drift in stock prices. Increasing interest rates are associated with an increased termination probability in bull markets and a decreased probability in bear markets.

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1. INTRODUCTION

Since the classical work of Samuelson (1965) and Leroy (1973), the random walk and martingale models of stock prices have formed the cornerstone of modern finance. Hence it is not surprising that an extensive empirical literature has considered deviations from these benchmark models. Several authors, including Lo & MacKinlay (1988), Fama & French (1988), Poterba & Summers (1988), Richardson & Stock (1989) and Boudoukh & Richardson (1994) study long-run serial correlations in stock returns. Although this literature finds indications of a slowly mean reverting component in stock prices, deviations from normally distributed returns, time-varying volatility and small sample sizes have plagued existing tests and made it difficult to conclusively reject the random walk model.

This paper proposes a new approach to modelling time series dependence in stock prices that allow bull and bear hazard rates, i.e. the probability that a bull or bear market terminates next period, to depend on the age of the market. Inspection of these hazard rates yields new insights into long-run dependencies and deviations from parametric models of asset prices proposed in the literature, including the simple random walk model with a constant drift and models that allow for volatility persistence. By explicitly focusing on duration dependence in stock prices, the proposed tests are very different from the tests based on autocorrelations previously adopted in the literature. Our approach does not require that stock prices follow a low-order Markov process although this is a special case of our model when termination probabilities are memoryless.¹

We formalize bull and bear states in terms of movements between local peaks and troughs. Earlier studies such as Fabozzi & Francis (1977), Kim & Zumwalt (1979) and Chen (1982) consider definitions of bull markets based simply on returns in a given month exceeding a certain threshold value.² Such definitions do not reflect long-run dependencies in stock prices and ignore information about the trend in stock price levels. By our definition the stock market switches from a bull to a bear state if stock prices have declined by a certain percentage since their previous (local) peak within that bull state. Likewise, a switch from a bear to a bull state occurs if stock prices experience a similar percentage increase since

¹ Faust (1992) demonstrated that existing tests for autocorrelation based on variance ratios have optimal power in testing the random walk hypothesis against certain classes of stationary ARMA models. However, these models only form a small subset of the alternatives that are interesting from an economic point of view, such as nonlinear speculative bubble processes or processes where the drift depends on past cumulated returns within a state. There is no result for the power of autocorrelation tests against nonlinear alternatives or processes with long memory. This is important since there is mounting evidence of nonlinearities such as a switching factor in the mean and volatility of stock returns, c.f. Maheu & McCurdy (1999) and Perez-Quiros & Timmermann (2000).

² Fabozzi & Francis (1977) consider a definition of bull markets based on 'substantial' up and down movements. In this definition, a substantial move in stock prices occurs whenever the absolute value of stock returns in a given month exceeds half of one standard deviation of the return distribution.

their local minimum within that state. This definition does not rule out sequences of negative (positive) price movements in stock prices during a bull (bear) market as long as their cumulated value does not exceed a certain threshold. By abstracting from the small unsystematic price movements that dominate time series as noisy as daily price changes this definition is better suited to capture long-run dependencies in the underlying drift in stock prices.

Most closely related to the current study is Pagan & Sossounov (2000) who also consider a definition of bull and bear states based on cumulated changes. They use a pattern recognition dating algorithm for bull and bear states that filters monthly returns through a sequence of censoring operations. Their study is concerned with modeling overall characteristics such as average durations and amplitudes of bull and bear markets. Our study does not impose minimal duration constraints on bull and bear markets and jointly models the full duration distribution of both short and long bull and bear states.

We find evidence of distinctly different duration dependence in bull and bear states. The longer a bull market has lasted, the lower its hazard rate and hence the lower the probability that it terminates next period. While the hazard rate of bear markets also declines initially, it has a U-shaped pattern and thus increases at longer horizons. Interest rates are also found to have an important effect on hazard rates. Higher interest rates are associated with an increase in the bull market hazard rate and a decrease in the bear hazard rate. They are therefore associated with a higher probability of being in a bear market and a lower bull market probability.

The finding of a hazard function that depends on age suggests that stock prices do not follow a low order Markov process but that the drift and the effect of interest rates on stock prices is related to the market's memory of the time spent in the current state. This means that the effect of an interest rate change on stock prices depends on the age and type of the state where the change occurs. In addition it is necessary to account for the entire sequence of hazard rates.

The plan of the paper is as follows. Section 2 presents our definition of bull and bear market states. Section 3 characterizes the unconditional distribution of the durations and returns in bull and bear markets using more than a century of daily stock prices from the US. Section 4 discusses estimation of bull and bear markets whose hazard rate may depend on age. Section 5 reports empirical results while Section 6 undertakes a scenario analysis to investigate the effect of an interest rate change on stock prices. Section 7 briefly discusses further economic interpretation of our findings.

2. DEFINITION OF BULL AND BEAR MARKETS

Financial analysts and stock market commentators frequently classify the underlying trend in stock prices into bull and bear markets. The duration of bull and bear markets is a key component of the risk and mean of stock returns so it is clearly important to understand its determinants. Yet, despite their importance, little work has been done on formalizing these concepts and investigate whether bull and bear states provide a useful way of characterizing long-run dependencies in stock prices. It is not clear, for example, whether such states serve a purely descriptive purpose or whether the knowledge that stock prices have been in a particular state for a certain length of time affects the conditional distribution of future price movements. In the latter case, investment performance could conceivably be improved by conditioning on the type and age of the current state.

There is no generally accepted formal definition of bull and bear markets in the finance literature. One of the few sources that attempts a definition of bull and bear markets is Sperandeo (1990) who defines bull and bear markets as follows:

"Bull market: A long-term ... upward price movement characterized by a series of higher intermediate ... highs interrupted by a series of higher intermediate lows.

Bear market: A long-term downtrend characterized by lower intermediate lows interrupted by lower intermediate highs". (p. 102).

To formalize the idea of a series of increasing highs interrupted by a series of higher intermediate lows, let I_t be a bull market indicator variable taking the value 1 if the stock market is in a bull state at time t , and zero otherwise. We assume that time is measured on a discrete scale and that the stock price at the end of period t is P_t . Suppose that at t_0 the stock market is at a local maximum and define the stochastic process $P_{t_0}^{\max} = P_{t_0}$, where P_{t_0} is the stock price at time t_0 . Let λ be a scalar defining the threshold of the movements in stock prices that trigger a switch between bull and bear markets. Also let τ_{\max} and τ_{\min} be stopping time variables defined by the following conditions:

$$\begin{aligned}\tau_{\max}(P_{t_0}^{\max}, t_0, \lambda) &= \inf\{t_0 + \tau : P_{t_0+\tau} \geq P_{t_0}^{\max}\}, \\ \tau_{\min}(P_{t_0}^{\max}, t_0, \lambda) &= \inf\{t_0 + \tau : P_{t_0+\tau} < (1 - \lambda)P_{t_0}^{\max}\},\end{aligned}\tag{1}$$

where $\tau \geq 1$. Then $\min(\tau_{\max}, \tau_{\min})$ is the first time the price process crosses one of the two barriers $\{P_{t_0}^{\max}, (1 - \lambda)P_{t_0}^{\max}\}$. If $\tau_{\max} < \tau_{\min}$, we update the local maximum in the current bull market state:

$$P_{t_0+\tau_{\max}}^{\max} = P_{t_0+\tau_{\max}},\tag{2}$$

and the bull market continued between $t_0 + 1$ and $t_0 + \tau_{\max}$: $I_{t_0+1} = \dots = I_{t_0+\tau_{\max}} = 1$.

Conversely, if $\tau_{\min} < \tau_{\max}$ so that the stock price at $t_0 + \tau_{\min}$ has declined by a fraction λ since its local peak

$$P_{t_0+\tau_{\min}} < (1 - \lambda)P_{t_0}^{\max}, \quad (3)$$

then the bull market has switched to a bear market which prevailed from $t_0 + 1$ to $t_0 + \tau_{\min}$: $I_{t_0+1} = \dots = I_{t_0+\tau_{\min}} = 0$. In the latter case we set $P_{t_0+\tau_{\min}}^{\min} = P_{t_0+\tau_{\min}}$.

If the starting point at t_0 is a bear market state, the stopping times get defined as follows:

$$\begin{aligned} \tau_{\min}(P_{t_0}^{\min}, t_0, \lambda) &= \inf\{t_0 + \tau : P_{t_0+\tau} \leq P_{t_0}^{\min}\}, \\ \tau_{\max}(P_{t_0}^{\min}, t_0, \lambda) &= \inf\{t_0 + \tau : P_{t_0+\tau} > (1 + \lambda)P_{t_0}^{\min}\} \end{aligned} \quad (4)$$

This definition of bull and bear states partitions the data on stock prices into mutually exclusive and exhaustive bull and bear market subsets based on sequences of first passage times. The resulting indicator function, I_t , gives rise to a random variable, T , which measures the duration of bull or bear markets. This is simply given as the time between successive switches in I_t .

The focus on local peaks and troughs allows us to concentrate on the systematic up and down movements in stock prices and to filter out short term noise. This is an important consideration for data as noisy as daily stock price changes. While some arguments can be made in favor of imposing an additional minimum duration constraint, this also adds an extra layer of complexity and means that the data has to be filtered through a complicated recursive pattern recognition algorithm, as carefully explained by Pagan & Sossounov (2000). Instead we take a flexible approach to modeling the entire distribution of durations that allows their risk characteristics to differ across short and long durations.

Naturally such filters are related to a long literature on technical trading rules that models local trends in stock prices, c.f. Brock, Lakonishok & LeBaron (1992), Brown, Goetzmann & Kumar (1998) and Sullivan, Timmermann & White (1999). However, the similarities between technical trading rules and duration measures are only superficial. Technical trading rules search for patterns in prices conditional on a time horizon that is typically quite short. For example, the value of a 100-day moving average of prices may be compared to the value of a 25 day moving average. In contrast, we do not condition on the time of a particular movement but instead explicitly treat this as a random variable whose distribution we are interested in modelling.

Our definition of bull and bear states focuses on the direction or drift of the stock prices and provides an observable bull market indicator. Another approach would be to treat the state as an unobserved variable as proposed by Maheu & McCurdy (1999). They classify monthly stock returns into two latent states based on a Markov switching model. However, while our regimes are defined according to the

cumulated movements in stock prices, the Markov switching approach effectively identifies high and low volatility states.

3. DURATIONS OF BULL AND BEAR MARKETS

3.1. Data

To investigate the properties of bull and bear market states along the definition proposed in Section 2, we construct a data set of daily stock prices in the US from 2/17/1885 to 12/31/1997. From 2/17/1885 to 2/7/1962 the nominal stock price index is based on the daily returns provided by Schwert (1990). These returns include dividends. From 3/7/1962 to 12/31/1997 the price index is constructed from daily returns on the Standard & Poors 500 price index, again including dividends and obtained from the CRSP tapes. Combining these series generates a time series of 31,412 daily nominal stock prices.

Inflation has varied considerably over the sample period and the drift in nominal prices does not have the same interpretation during low and high inflation periods. To deal with this issue, we construct a daily inflation index as follows. We use monthly data on the consumer price index taken from Shiller (2000) and convert it into daily inflation rates by solving for the daily inflation rate such that the daily price index grows smoothly - and at the same rate - between subsequent values of the monthly consumer price index.³ Finally we divide the nominal stock price index by the consumer price index to get a daily index for real stock prices.

We will also consider the effect of time-varying interest rates on hazard rates. Since there is no continuous data series on daily interest rates from 1885 to 1997, we construct our data from four separate sources. From 1885 to 1889 the source is again Shiller (1989). From 1890 to 1925, we use the interest rate on 90-day stock exchange time loans as reported in Banking and Monetary Statistics, Board of Governors of the Federal Reserve System (1943). These rates are provided on a monthly basis and we convert them into a daily series by simply applying the interest rate reported for a given month to each day of that month. From 1926 to 1954, we use the one-month T-bill rates from the risk-free rates file on the CRSP tapes, again reported on a monthly basis and converted into a daily series. Finally, from July 1954 to 1997, we use the daily Federal Funds rate. These three sets of interest rates are concatenated to form one time series covering the full sample.

Much of standard survival analysis in economics and finance assumes continuously measured data.

³ Since the volatility of daily inflation rates is likely to be only a fraction of that of daily stock returns, normalizing by the inflation rate has the effect of a time-varying drift adjustment. Lack of access to daily inflation data is unlikely to affect our results in any important way.

However, since we use daily data and do not follow price movements continuously, our data is interval censored and the termination of our durations is only known to lie between consecutive follow ups. Effectively the measurement of T , the duration of bull and bear markets, is divided into A intervals

$$[a_0, a_1), [a_1, a_2), \dots, [a_{q-1}, a_q), [a_q, \infty) \text{ where } q = A - 1,$$

and only the discrete time duration $T \in \{1, \dots, A\}$ is observed, where $T = t$ denotes termination within the interval $[a_{t-1}, a_t)$.⁴

3.2. Volatility Clustering

The volatility of daily stock price changes is strongly serially correlated and such clustering will undoubtedly affect the duration distribution of our data. We are interested in documenting possible time-series dependencies in the drift of stock prices and hence want to control for the effects of time-varying conditional volatility. To accomplish this and account for both short- and long-run persistence in volatility, we estimate the following components GARCH model proposed by Engle & Lee (1999) and extended to include an ARCH-in-mean effect:

$$\begin{aligned} r_t &= \mu + \beta r_{t-1} + \gamma \sigma_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^2), \\ \sigma_t^2 &= q_t + \alpha(\varepsilon_{t-1}^2 - q_{t-1}) + \beta(\sigma_{t-1}^2 - q_{t-1}) \\ q_t &= \omega + \rho(q_{t-1} - \omega) + \phi(\varepsilon_{t-1}^2 - \sigma_{t-1}^2). \end{aligned} \tag{5}$$

Using this specification, we compute ARCH-adjusted, normalized returns $r_i^* = \hat{\varepsilon}_i / \hat{\sigma}_i$ where $\hat{\varepsilon}_i = r_i - \hat{\mu} - \hat{\beta}r_{i-1} - \hat{\gamma}\hat{\sigma}_i$ is the residual from the ARCH model and $\hat{\sigma}_i$ its estimated volatility. We then construct a new price index that is adjusted for first-order autocorrelation (reflecting the effects of asynchronous trading) and ARCH effects in the volatility and drift. Such adjustments ensure that the new price index has the same average drift and volatility as the original one.

3.3. Bull and Bear Durations

Insight into how our definition partitions real stock prices into bull and bear spells is gained from Figures 1a and 1b which use the unadjusted price index to show the sequence of consecutive bull and bear market durations over the full sample period 1885-1997. These figures use a barrier, λ , of 15 percent that splits

⁴ Although we will draw on approaches from the literature on economic duration data (see, e.g., Kiefer (1988), Kalbfleisch & Prentice (1980) and Lancaster (1990)) this also means that we have to be careful in modifying the standard tools from continuous time analysis.

the sample into 52 bull and 52 bear markets. To better illustrate the individual episodes, we plot in eight separate windows the natural logarithm of the real stock price index. Many of the bull markets are very long, the longest lasting from 1948 to 1956. Most of the short durations occur from 1929 to 1935 around the Great Depression.

Table 1 presents descriptive statistics for the distribution of bull and bear market durations. Properties of bull and bear market states are reported in weeks although it should be recalled that our analysis was carried out using daily data. For comparison purposes we report results for filter sizes, λ , of 10, 15 and 20 percent, but our discussion focuses on the 15% filter. With a filter of 15 percent the mean bull market duration is 85 weeks against 36 weeks for bear market durations. The corresponding median values are 57 and 23 weeks for bull and bear markets, respectively. While the shortest bull and bear markets each lasted for only a week, the longest bull market, at 454 weeks, lasted more than three times longer than the longest bear market (136 weeks). Partly as a result of this, the dispersion of bull market durations is about three times greater than that of bear markets. Overall, the stock market spends roughly two thirds of the time in the bull state and one third in the bear state. On a volatility adjusted basis the shortest bull and bear states last a more reasonable 10 and seven weeks. The very short bull and bear spells in the unadjusted price index thus reflect episodes of high volatility clustering such as during the Great Depression. Generally the mean duration goes up while its standard deviation declines as a result of the volatility adjustment.

A more systematic picture of bull and bear durations is provided in Figure 2 which plots the estimated densities of bull and bear market durations using a Gaussian kernel smoother and a filter of 15 percent. We plot the density both for unadjusted and volatility adjusted prices and for the benchmark random walk model. From the bottom windows, clearly there are significant differences in the duration profile of bull and bear markets relative to what one would expect under the null of a random walk, even after adjusting for time-varying volatility. In bear markets the random walk density starts off much lower and peaks at a higher point than the densities for the actual or volatility-adjusted returns data. Very young bear markets thus appear to be at greater risk of instantaneous termination than predicted by the random walk model.

Figure 2 was drawn for a specific choice of filter size, λ . Since λ is arbitrarily chosen, it is important to investigate the effect of λ on the results. In Figure 3 we plot densities of bull and bear market durations normalized by their individual standard deviations and using filter sizes of $\lambda = 0.10, 0.15, 0.20$. The basic shapes of the densities, which summarize the duration information, are very similar. This suggests that results on duration dependence are robust across filter sizes.

To see how much returns vary across bull and bear states, Table 2 reports return statistics for these

states. Mean returns are 2.6 and -3.3 percent per week in bull and bear markets, respectively.⁵ A larger asymmetry shows up in the median return which is 0.7 and -1.2 percent per week for bull and bear markets, respectively. Although bear states last much shorter than bull markets, the downward drift in bear markets is thus stronger than the upward drift during bull markets. The effect of volatility adjustment is to lower mean returns in bull states and increase them in bear states. On a volatility adjusted basis the standard deviation of returns is lower in bull states and higher in bear states than under the random walk model. This indicates that the random walk model does not capture the basic properties of the data on stock prices.

4. MODELS OF DURATION DEPENDENCE IN BULL AND BEAR MARKETS

Section 3 characterized the unconditional distribution of bull and bear market spells. However, if the age of a bull or bear market affects future price movements, investors will want to calculate expectations conditional on the path followed by stock prices up to a given point in time. For instance, during the long bull market of the nineties, the concern was often expressed that this bull market was at greater risk of coming to an end because it had lasted 'too long' by historical standards. Translated into statistical terms, this indicates a belief that the bull market hazard rate depends positively on its duration. The opposite view is that bull markets gain momentum: the longer a bull market has lasted, the more robust it is, and hence the lower its hazard rate.

Testing these hypotheses requires that we go well beyond inspecting the unconditional probability of termination for the bull or bear markets. Instead the duration data needs to be characterized in terms of the conditional probability that the bull or bear state ends in a short time interval following some period t , given that the state lasted up to t . For the i 'th duration, T_i , this is measured by the discrete hazard function

$$\lambda_i(t|\mathbf{X}_{it}) = \Pr(T_i = t | T_i \geq t, \mathbf{X}_{it}), \quad t = 1, \dots, A, \quad (6)$$

which is the conditional probability of termination in interval $[a_{t-1}, a_t)$ given that the interval was reached in the first place. $\mathbf{X}_{it} = \{x_{i1}, \dots, x_{it}\}$ is a vector of additional conditioning information, which will depend on the particular duration. Hypotheses on the probability that a bull or bear market is terminated as a function of its age are naturally expressed in terms of the shape of this hazard function. For example, the natural null hypothesis is that the duration of the current state does not affect the hazard

⁵ These figures are computed as the mean return per bull or bear market converted into a weekly number. The unconditional real return of six percent per annum can be computed as the duration-weighted average return per bull or bear market spell.

rate.

The probability that a bull or bear market lasts for a certain period of time can still be derived from these hazard rates. This is given as the discrete survivor function which measures the probability that a bull or bear market survives on the interval $[a_{t-1}, a_t]$:⁶

$$S_i(t|\mathbf{X}_{it}) = \Pr(T_i > t|\mathbf{X}_{it}) = \prod_{s=1}^t (1 - \lambda_i(s|\mathbf{X}_{is})), \quad t = 1, \dots, A. \quad (7)$$

Common choices of hazard models are the Probit, Logit and Double Exponential link.⁷ Throughout the paper we use a Logit link, i.e.,

$$\lambda_i(t|\mathbf{X}_{it}) = F(\mathbf{x}'_{it}\boldsymbol{\beta}) = \frac{\exp(\mathbf{x}'_{it}\boldsymbol{\beta})}{1 + \exp(\mathbf{x}'_{it}\boldsymbol{\beta})}. \quad (8)$$

We consider two separate models for these hazard rates. The first, static, model assumes that the underlying parameters linking the covariates or state variables to the hazard rate do not vary over time and that the covariates are fixed from the point of entry into a state. This makes our results directly comparable to the large literature on univariate dynamics in stock prices.⁸ Under this assumption, the data takes the form of $\{t_i, \mathbf{x}_i; i = 1, \dots, n\}$, where t_i is the survival time and \mathbf{x}_i is a covariate (or state variable) observed at the beginning of the interval $[a_{i-1}, a_i]$.

However, switches between bull and bear market states are likely to be caused by changes in the underlying economic environment. For example, the drift in stock prices may turn from positive to negative as a result of increased interest rates or worsening economic prospects. The effect of covariates may well depend on the age of the current bull or bear market. To account for this possibility, we need to extend the setup from the previous section and allow \mathbf{x}_{it} to be a vector that incorporates time-varying covariates. Now the data for the i 'th duration spell takes the form

$$\left\{ \underbrace{t_i}_{\text{duration}}, \overbrace{\mathbf{x}_i(a_0), \mathbf{x}_i(a_1), \dots, \mathbf{x}_i(a_{i-1}), \mathbf{x}_i(a_i)}^{= \mathbf{x}_i: \text{time-varying covariates}} \right\}.$$

⁶ Using this definition it is clear that the unconditional probability of termination is related to the conditional termination probability (6) by

$$\Pr(T_i = t|\mathbf{X}_{it}) = \lambda_i(t|\mathbf{X}_{it}) \prod_{s=1}^{t-1} (1 - \lambda_i(s|\mathbf{X}_{is})) = \lambda_i(t|\mathbf{X}_{it}) S_i(t-1|\mathbf{X}_{it-1}).$$

⁷ An extensive comparison of such link functions is found in Sueyoshi (1995).

⁸ Chapter 2 in Campbell, Lo & MacKinlay (1997) surveys this extensive literature.

Since our data is discretely measured, the covariates follow a step function with jumps at the follow-up times, a_t . Within the interval $[a_{t-1}, a_t)$ the history of covariates

$$\mathbf{X}_{it} = (\mathbf{x}_{i1}, \mathbf{x}_{i2}, \dots, \mathbf{x}_{it}),$$

is allowed to influence the hazard rate $\lambda_i(t|\mathbf{X}_{it})$.

To allow for the possibility that the effect on the hazard rate of these covariates could depend on the age of the current state, we consider an approach that allows the parameters to vary with duration:

$$\lambda_i(t|\mathbf{X}_{it}) = F(\mathbf{x}'_{it}\boldsymbol{\alpha}_t). \quad (9)$$

Here the vector $\boldsymbol{\alpha}_t = (\gamma_{0t}, \boldsymbol{\gamma}'_t)'$ comprises both the baseline and the covariance parameters. We use the first-order random walk as our choice of transition equation determining the evolution in $\boldsymbol{\alpha}_t$:

$$\boldsymbol{\alpha}_t = \Phi\boldsymbol{\alpha}_{t-1} + \boldsymbol{\xi}_t, \Leftrightarrow \begin{pmatrix} \gamma_{0t} \\ \boldsymbol{\gamma}_t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma_{0t-1} \\ \boldsymbol{\gamma}_{t-1} \end{pmatrix} + \begin{pmatrix} \xi_{0t} \\ \boldsymbol{\xi}_{1t} \end{pmatrix}, \quad (10)$$

$$\text{where } \begin{pmatrix} \xi_{0t} \\ \boldsymbol{\xi}_{1t} \end{pmatrix} \sim \mathcal{N}_{p+1}(\mathbf{0}, \text{diag}(\sigma_0^2, \sigma_1^2, \dots, \sigma_p^2)), \quad p = \dim(\boldsymbol{\gamma}_t), \quad \boldsymbol{\alpha}_0 \sim \mathcal{N}_{p+1}(\mathbf{a}_0, \mathbf{Q}_0),$$

and $\mathcal{N}_{p+1}(\cdot, \cdot)$ is the $(p+1)$ -dimensional standard normal distribution. This random walk specification has the advantage of not imposing mean reversion on the parameters which are allowed to differ across durations (although neighboring points cannot be too far from each other) if the data supports such variation.

4.1. Estimation

The log-likelihood function can be conveniently set up using notation from the literature on discrete choice models. Consider the following discrete indicator variable:

$$y_{is} = \begin{cases} 1, & i\text{'th bull or bear market terminates in } [a_{s-1}, a_s) \\ 0, & i\text{'th bull or bear market survives through } [a_{s-1}, a_s) \end{cases}$$

for $s = 1, \dots, t_i$, and $i = 1, \dots, n$. Each bull or bear spell, i , thus generates a string

$$\mathbf{y}_i = (y_{i1}, \dots, y_{it_i}) = (0, \dots, 0, 1), \quad i = 1, \dots, n. \quad (11)$$

Using this notation, the contribution to the likelihood function from the i 'th observation is

$$\mathcal{L}_i \propto \prod_{s=1}^{t_i} \lambda_i(s|\mathbf{x}_i)^{y_{is}} (1 - \lambda_i(s|\mathbf{x}_i))^{1-y_{is}}. \quad (12)$$

For every spell the bull or bear market lives through it therefore contributes to the likelihood with the survivor probability $1 - \lambda_i(s|\mathbf{x}_i)$. Summing across duration spells, the total log-likelihood function for the model $\lambda_i(t|\mathbf{x}_i) = F(\mathbf{x}'_{it}\boldsymbol{\theta})$ is given by

$$\ln \mathcal{L} \propto \sum_{i=1}^n \sum_{s=1}^{t_i} y_{is} \ln(\lambda_i(s|\mathbf{x}_i)) + (1 - y_{is}) \ln(1 - \lambda_i(s|\mathbf{x}_i)). \quad (13)$$

Often the covariate parameters are treated as fixed effects. However, this is only appropriate if the number of intervals is very small. In applications such as ours without enough intervals to apply continuous time techniques, maximum likelihood estimates of the large number of parameters in the hazard functions of such unrestricted models can have very poor properties.

To get around this problem, we follow Fahrmeir (1994) and adopt state space techniques that treat the hazard function as the measurement equation. Since our measurement equation is non-normal it can be complicated to solve for the posterior density of the hazard function conditional on the data, $p(\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_q | y_1, \dots, y_q, \mathbf{x}_1, \dots, \mathbf{x}_q)$, which is required for writing down the likelihood function. We adopt the strategy, advocated by Fahrmeir (1992), of basing estimation on posterior modes subject to smoothing priors which penalize large changes in neighboring parameters, $\boldsymbol{\alpha}_t - \boldsymbol{\alpha}_{t-1}$. As detailed in appendix A, repeatedly applying Bayes' theorem to the posterior density, estimation of $\boldsymbol{\alpha}_t$ by posterior modes is equivalent to maximizing the following penalized log-likelihood function:

$$\begin{aligned} \ln \mathcal{L}(\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_q) &= \sum_{i=1}^n \sum_{t=1}^{t_i} l_{it}(\boldsymbol{\alpha}_t) - \frac{1}{2}(\boldsymbol{\alpha}_0 - \mathbf{a}_0)' \mathbf{Q}_0^{-1} (\boldsymbol{\alpha}_0 - \mathbf{a}_0) \\ &\quad - \frac{1}{2} \sum_{t=1}^q (\boldsymbol{\alpha}_t - \boldsymbol{\alpha}_{t-1})' \mathbf{Q}^{-1} (\boldsymbol{\alpha}_t - \boldsymbol{\alpha}_{t-1}), \end{aligned} \quad (14)$$

where,

$$l_{it}(\boldsymbol{\alpha}_t) = y_{it} \ln(F(\mathbf{x}'_{it}\boldsymbol{\alpha}_t)) + (1 - y_{it}) \ln(1 - F(\mathbf{x}'_{it}\boldsymbol{\alpha}_t)) \quad (15)$$

is the log-likelihood contribution of the i 'th duration spell. The first term measures the goodness of fit of the model. The second and third terms - both of which are introduced by the smoothness priors, \mathbf{Q}_0, \mathbf{Q} , specified by the transition model - penalize large deviations between successive parameters and lead to smoothed estimates. Appendix B provides details on the numerical optimization of this penalized likelihood function through a generalized extended Kalman filter and smoother.

5. EMPIRICAL RESULTS

Using the estimation techniques and hazard models from Section 4, we first estimate the hazard function for bull and bear markets in a model without time-varying covariates. The output from this exercise is

the baseline hazard rates plotted in Figures 4 (bull market) and 5 (bear market). These measure the pure age dependence of the bull and bear market termination probabilities. The upper windows show the hazard rate estimated for the unadjusted price index, while the bottom windows plot the estimates for the volatility-adjusted prices. To establish a benchmark for the hazard rate under no serial dependence in returns, we also plot the hazard rate for the memoryless random walk model. Because it takes some time before the market moves the full distance of the filter (assumed to be 15 percent in the figure), the hazard rate is initially close to zero under the random walk model but it rapidly increases to a level of 1.5 percent in the case of a bull market or three percent in bear markets. Once the effect of conditioning on a short duration wears off, the random walk hazard rates are constant as expected.

For the unadjusted price series, the baseline hazard in bull markets is initially three percent per week but it quickly drops to one percent for markets that have lasted 30 weeks only to peak again after 130 weeks. Comparing the hazard rates of the raw and volatility adjusted price indexes, it is clear that the high hazard rate for short-lived bull markets entirely reflects volatility clustering. Similar findings emerge for the baseline hazard in bear markets. While the hazard rate of the unadjusted price series is essentially flat, it increases over time on a volatility adjusted basis. Once again the random walk hazard rates at the very short and long ends of the duration distribution are too low to be consistent with the data.

Long run stock returns depend crucially on the *difference* between bear and bull market hazard rates which are plotted in Figure 6 using a bivariate logit link model. This setup allows us to directly evaluate the relative bear and bull market hazards since we can compute standard errors for their difference. A bear market always has a higher probability of instantaneous termination than a bull market of the same age. In fact, the excess hazard rate of bear over bull markets appears to be increasing as a function of duration and is about four times higher for long durations compared with short ones. When combined with the insights from Table 1, this suggests that it is the absence of very long bear markets that account for the historically high mean returns on US stocks as opposed to differences between bull and bear markets at the short end of the duration distribution. On a volatility adjusted basis (the bottom window in Figure 6) it is also clear that the excess hazard rates of bear markets are higher for very short and long durations than what one would expect under the random walk model.⁹

As a means of providing a single summary measure of the attrition rates in bull and bear markets, Figure 7 plots the survivor functions (7). For a given duration—and both on a raw return and volatility

⁹ These findings are consistent with Pagan & Sossounov (2000)'s analysis. In simulations they find that the random walk model would never have generated a bear market with the same amplitude and duration as observed in the US data at the end of the 1920s.

adjusted basis—the bull state always has the highest survival probability and the wedge between the survivor probabilities increases as a function of duration. 45 percent of the bull markets survive through their first year while only 25 percent of the bear markets last this long.

5.1. Interest Rate Effects

To shed light on how the hazard rates depend on the underlying state of the economy, we next include interest rates as a time-varying covariate. Interest rates have been widely documented to closely track the state of the business cycle and appear to be a key determinant of stock returns at the monthly horizon.¹⁰ Interest rate *levels*, i_t , may be affected by a low frequency component and therefore might not contain the same information over a sample as long as ours, while interest rate *changes*, Δi_t , are more likely to track business cycle variation across the full sample. For this reason we include both levels and changes in interest rates so the set of covariates is $\mathbf{x}'_{it} = (1, i_t, \Delta i_t)$. Our hazard specification—which allows interest rate effects to vary with the age of the current state—is

$$\lambda_i(t|\mathbf{x}_i(t)) = F(\gamma_{0i} + \gamma_{1i}i_t + \gamma_{2i}\Delta i_t). \quad (16)$$

Figure 8 shows the sequence of baseline hazards and interest rate effects using volatility adjusted stock prices. Compared to Figure 4, it is clear that controlling for interest rates has a significant effect on the shape of the volatility adjusted bull market baseline hazard. Controlling for interest rate effects, the baseline hazard drops sharply from five to one percent per week as the bull market duration extends beyond six months and remains flat at longer durations. Young bull markets thus appear substantially more at risk of termination than bull markets that have lasted for six months or longer.

Panel (b) shows that at very short durations, higher interest rates are associated with a lower bull market hazard rate. However, the sign of this parameter switches so higher interest rates become associated with a higher hazard rate. The initial negative sign should be interpreted with caution since interest rates tend to be high towards the beginning of a new expansion state and this is often the beginning of a bull market in stock prices. More importantly, perhaps, positive interest rate changes are consistently associated with large increases in the bull hazard rate, c.f. panel (c).

Turning next to the bear market hazard estimates and comparing Figures 9 and 5, once again the shape of the baseline hazard changes as a result of controlling for interest rate effects. The baseline hazard now declines from four to one percent per week as the duration extends from one to ten weeks. Thereafter it increases relatively smoothly to a level of four percent. There is no systematic effect of

¹⁰ See, e.g., Kandel & Stambaugh (1990), Fama & French (1989), Glosten, Jagannathan & Runkle (1993), Pesaran & Timmermann (1995) and Whitelaw (1994).

interest rate levels on the bear market hazard rate, while there is a systematically negative, but weak, effect of interest rate changes on bear hazards. On balance a bear market thus tends to last longer in an environment with increasing interest rates.

6. STOCK PRICE MOVEMENTS AFTER AN INTEREST RATE CHANGE

A large literature has found strong negative effects of nominal interest rates on stock returns. Mostly such effects have been documented within single-period regressions of stock returns on interest rates. While interest rate effects at longer horizons could be captured from a vector autoregression, this approach is no longer valid when the hazard rate varies at long horizons so that a low-order Markov representation fails to properly capture the dynamics of stock prices. Furthermore, the findings in Section 5 suggest that such effects will depend both on the type of state—bull versus bear—and the age of the market where the shock occurs. Our modeling approach allows us to compute the effect of an interest rate change on stock prices at all time horizons while accounting for the evidence of duration dependence.

To demonstrate the effect on stock prices of a change in interest rates, we study a scenario where the current interest rate level is permanently raised from 5% to 7% after 52 weeks in a bull market. In the baseline or non-raise scenario the covariate matrix is

$$\mathbf{X}_i^{\text{non-raise}} = \begin{pmatrix} 1 & \dots & 1 & 1 & 1 & \dots & 1 \\ 5 & \dots & 5 & 5 & 5 & \dots & 5 \\ 0 & \dots & 0 & 0 & 0 & \dots & 0 \end{pmatrix},$$

$\underbrace{\hspace{10em}}$
 $\underbrace{\hspace{10em}}$
 $\underbrace{\hspace{10em}}$

obs 1 to 51
52
obs 52 to 90

while in the raise scenario the matrix of covariates is

$$\mathbf{X}_i^{\text{raise}} = \begin{pmatrix} 1 & \dots & 1 & 1 & 1 & \dots & 1 \\ 5 & \dots & 5 & 7 & 7 & \dots & 7 \\ 0 & \dots & 0 & 2 & 0 & \dots & 0 \end{pmatrix}.$$

$\underbrace{\hspace{10em}}$
 $\underbrace{\hspace{10em}}$
 $\underbrace{\hspace{10em}}$

obs 1 to 51
52
obs 53 onwards

In both cases the hazard rates are computed as

$$\lambda_i(t|\mathbf{X}_{it}) = F(\gamma_{0t} + \gamma_{1t}i_t + \gamma_{2t}\Delta i_t) = \frac{\exp(\gamma_{0t} + \gamma_{1t}i_t + \gamma_{2t}\Delta i_t)}{1 + \exp(\gamma_{0t} + \gamma_{1t}i_t + \gamma_{2t}\Delta i_t)}. \quad (17)$$

To derive the impact on stock prices of this raise, we first consider how the bull and bear hazard rates

change. This means computing

$$\begin{aligned}\frac{\partial \lambda_i(t|\mathbf{X}_{it})}{\partial i_t} &= \gamma_{1t} \lambda_i(t|\mathbf{X}_{it})(1 - \lambda_i(t|\mathbf{X}_{it})) \\ \frac{\partial \lambda_i(t|\mathbf{X}_{it})}{\partial \Delta i_t} &= \gamma_{2t} \lambda_i(t|\mathbf{X}_{it})(1 - \lambda_i(t|\mathbf{X}_{it})).\end{aligned}\quad (18)$$

Notice the duration dependence of these effects through their t subscripts. Figure 10 shows the effect on the bull market hazard rate of the two percent increase in the interest rate after 26 and 52 weeks, respectively. The spike in the hazard rate arises because of the one-off change in Δi_t . Following this impact, the hazard rate remains higher than in the base scenario due to the higher value of i_t . Therefore the bull market survival probability in the interest rate raise scenario is also much lower.

The interest rate raise causes less of a one-off decline in the bear market hazard rate than the corresponding increase in the bull market hazard, c.f. Figure 11. Furthermore, the permanent effect on bear market hazard or survival rates are clearly also smaller than in the bull market.

These changes in the sequence of hazard rates do not show the effect on the expected stock returns resulting from a higher interest rate. To study this, we conducted the following Monte Carlo experiment. Let $\mathfrak{S}_t = \{I_t, I_{t-1}, \dots\}$ be the sequence of indicator functions up to point t . The expected return i periods from now is

$$E_t[r_{t+i}|\mathfrak{S}_t] = \sum_{j=0}^1 E[r_{t+i}|I_{t+i} = j] \Pr(I_{t+i} = j|\mathfrak{S}_t), \quad (19)$$

where $E[r_{t+i}|I_{t+i} = j]$ is the expected return in state j and $\Pr(I_{t+i} = j|\mathfrak{S}_t)$ is the probability that state j occurs in period $t+i$. This will depend on the time spent in the state prevailing at time t and thus in principle not just on I_t but on the whole sequence, \mathfrak{S}_t . The probability of being in state j in period $t+i$ can be updated recursively from the hazard rate given the age of the current state. Let $\lambda_0(t|\mathbf{X}_t)$ and $\lambda_1(t|\mathbf{X}_t)$ be the hazard rates for bull and bear markets that have lasted t periods. The state probabilities then follow from a pair of equations

$$\begin{aligned}\Pr(I_{t+i} = 0) &= (1 - \lambda_0(t+i|\mathbf{X}_{t+i})) \Pr(I_{t+i-1} = 0) + \lambda_1(t+i|\mathbf{X}_{t+i}) \Pr(I_{t+i-1} = 1) \\ \Pr(I_{t+i} = 1) &= \lambda_0(t+i|\mathbf{X}_{t+i}) \Pr(I_{t+i-1} = 0) + (1 - \lambda_1(t+i|\mathbf{X}_{t+i})) \Pr(I_{t+i-1} = 1)\end{aligned}\quad (20)$$

In the absence of duration dependence, $\lambda_i(t+i|\mathbf{X}_{t+i}) = \lambda_i$, so the system reduces to a two-state Markov process. However, we relax this assumption and allow for time-varying state transitions. Figure 12 shows the outcome of 100,000 Monte Carlo simulations for the scenario where the interest rate is raised by two percent after 26 or 52 weeks starting from a bull state (panels (a) and (b)) or a bear state (panels (c) and (d)). A higher interest rate in a bull state leads to a higher bull hazard and a lower bear hazard.

This increases the probability of a low mean state and decreases the probability of a high mean state relative to the no change scenario. Consequently the mean return in future periods is consistently lower in the high interest rate scenario initiating from the bull state. There is much less of an effect on future mean returns if the initial state is a bear market, although the lower mean returns come through towards the end.

7. CONCLUSION

This paper has used a new approach to document dependence in the direction of stock prices based on the probability of exiting from bull or bear states. Since the length of time spent in these states is a key determinant of the mean and risk of stock returns, it is important to study the determinants of bull and bear durations. We find strong evidence contradicting standard models of stock prices even after adjusting for time-varying volatility and state variables. On a volatility adjusted basis the bull and bear hazard rates drop sharply in the beginning. However, while the hazard rate in bull markets is flat at longer horizons, it reverts to its initially high level in bear markets, making long bear spells very unlikely.

Evidence of deviations from the random walk model does not imply a rejection of the efficient market hypothesis. On the other hand, long-run dependencies in stock prices have important implications for both long-run risk management and for interpretation of the sources of movements in stock prices. It is beyond this paper to propose an economic model that can explain duration dependence in stock prices. Instead we briefly consider explanations of duration dependence based on speculative bubbles or fundamentals.

McQueen & Thorley (1994) study speculative bubbles that take the form of sequences of small positive abnormal returns interrupted by rare but large negative abnormal returns in a crash state. They show that the bubble implies that the probability that a run of positive abnormal returns comes to an end declines with the length of the sequence. In empirical tests on monthly stock returns over the period 1927-1991, they find evidence of negative duration dependence for positive runs while there appears to be no duration dependence in negative runs. Data limitations mean that they consider runs of at most six months' duration. We use daily data over a much longer period (1885-1997) which allows us to consider hazard rates at both much shorter and longer durations. While our finding of a declining bull market hazard rate is consistent with McQueen and Thorley's result, it is more difficult to appeal to a bubble-related explanation for the U-shaped pattern in the bear market hazard rate.

Duration dependence in stock prices may alternatively be driven by information effects or by fun-

damentals such as dividend payoffs and time-varying risk premiums. Wang (1993) models asymmetry of information between noise traders and rational investors which leads uninformed traders to rationally behave like price chasers. This introduces serial correlation in stock returns. If such effects are linked to the underlying state of the economy it is possible that they could affect the duration distribution of stock returns. Campbell & Cochrane (1999) propose an asset pricing model in which consumption growth follows a lognormal process with habit formation effects. Pagan & Sossounov (2000) find that this model has some promise for matching the average duration of bull and bear states, although matching the hazard function may be a more difficult test to pass. Cecchetti, Lam & Mark (2000) introduce belief distortions that vary over expansions and contractions and leads to systematic predictability in returns. These models all seem to have some promise for explaining bull and bear durations which we intend to explore in future work.

Bull and bear markets ought also be related to recession and expansion states. In the most systematic work to date, Diebold & Rudebusch (1990) and Diebold, Rudebusch & Sichel (1993) investigate duration dependence in the US business cycle. Although duration analyses of aggregate data must be tempered by the infrequency of such data, these papers nevertheless find evidence of positive duration dependence in pre-war expansions and post-war contractions. Their finding of a very strong rise in the hazard rate for post-war contractions is likely to be closely related to the rise in the bear market hazard rate documented for stock prices.

APPENDIX A: ASSUMPTIONS OF THE ESTIMATIONS

The assumptions underlying our estimation approach are most appropriately stated by reordering the observations and using risk set notation. Suppose that all bull or bear durations have been lined up so that they start at the same point in time and let risk indicators r_{it} ($i, t \geq 1$) be defined by

$$r_{it} = \begin{cases} 1, & \text{if the } i\text{'th bull or bear market is at risk in } [a_{t-1}, a_t) \\ 0, & \text{otherwise.} \end{cases} \quad (\text{A1})$$

Furthermore, define the risk vector $\mathbf{r}_t = (r_{it}, i \geq 1)$, and the risk set $\mathcal{R}_t = \{i : t \leq t_i\}$ at time t , i.e., the set of duration spells that are at risk in the interval $[a_{t-1}, a_t)$. Using this notation the log-likelihood function can be written as

$$\ln \mathcal{L} \propto \sum_{t=1}^q \sum_{i \in \mathcal{R}_t} y_{it} \ln(\lambda(t|\mathbf{X}_{it})) + (1 - y_{it}) \ln(1 - \lambda(t|\mathbf{X}_{it})). \quad (\text{A2})$$

As t increases, fewer bull or bear markets continue to survive and thus the dimension of \mathbf{r}_t declines.

Covariates and failure indicators are collected in the vectors

$$\begin{aligned}\mathbf{x}_t &= (x_{it}, i \in \mathcal{R}_t) \\ \mathbf{y}_t &= (y_{it}, i \in \mathcal{R}_t).\end{aligned}\tag{A3}$$

Finally the histories of covariates, failure and risk indicators up to period $t - 1$ are given by

$$\begin{aligned}\mathbf{x}_{t-1}^* &= (\mathbf{x}_1, \dots, \mathbf{x}_{t-1}) \\ \mathbf{y}_{t-1}^* &= (\mathbf{y}_1, \dots, \mathbf{y}_{t-1}) \\ \mathbf{r}_{t-1}^* &= (\mathbf{r}_1, \dots, \mathbf{r}_{t-1}).\end{aligned}\tag{A4}$$

The following set of assumptions are required for the maximum likelihood estimation:

(A1) Given α_t , \mathbf{y}_{t-1}^* , \mathbf{r}_t^* and \mathbf{x}_t^* , current \mathbf{y}_t is independent of $\alpha_{t-1}^* = (\alpha_1, \dots, \alpha_{t-1})$:

$$p(\mathbf{y}_t | \alpha_t^*, \mathbf{y}_{t-1}^*, \mathbf{r}_t^*, \mathbf{x}_t^*) = p(\mathbf{y}_t | \alpha_t, \mathbf{y}_{t-1}^*, \mathbf{r}_t^*, \mathbf{x}_t^*), \quad t = 1, 2, \dots$$

This assumption is standard in state space modelling. It simply states that the conditional information in α_t^* about \mathbf{y}_t is exclusively contained in the current parameter α_t .

(A2) Conditional on \mathbf{y}_{t-1}^* , \mathbf{r}_{t-1}^* and \mathbf{x}_{t-1}^* , the covariate \mathbf{x}_t and risk vector \mathbf{r}_t are independent of α_{t-1}^* :

$$p(\mathbf{x}_t, \mathbf{r}_t | \alpha_{t-1}^*, \mathbf{y}_{t-1}^*, \mathbf{r}_{t-1}^*, \mathbf{x}_{t-1}^*) = p(\mathbf{x}_t, \mathbf{r}_t | \mathbf{y}_{t-1}^*, \mathbf{r}_{t-1}^*, \mathbf{x}_{t-1}^*), \quad t = 1, 2, \dots$$

Thus we assume that the covariate processes contain no information on the parameter process.

(A3) The parameter transitions follow a Markov process:

$$p(\alpha_t | \alpha_{t-1}^*, \mathbf{y}_{t-1}^*, \mathbf{x}_t^*) = p(\alpha_t | \alpha_{t-1}), \quad t = 1, 2, \dots$$

This assumption is implied by the transition model and the assumption on the error sequence.

(A4) Given α_t , \mathbf{y}_{t-1}^* , \mathbf{r}_t^* and \mathbf{x}_t^* , individual responses y_{it} within \mathbf{y}_t are conditionally independent:

$$p(\mathbf{y}_t | \alpha_t, \mathbf{y}_{t-1}^*, \mathbf{x}_t^*, \mathbf{r}_t^*) = \prod_{i \in \mathcal{R}_t} p(y_{it} | \alpha_t, \mathbf{y}_{t-1}^*, \mathbf{x}_t^*, \mathbf{r}_t^*), \quad t = 1, 2, \dots$$

This assumption is much weaker than an assumption of unconditional independence.

To estimate the parameters, α_q^* , we repeatedly apply Bayes' theorem to get the posterior density:

$$p(\alpha_q^* | y_q^*, x_q^*, r_q^*) = \prod_{t=1}^q p(y_t | y_{t-1}^*, x_t^*, r_t^*; \alpha_t^*) \prod_{t=1}^q p(\alpha_t | \alpha_{t-1}^*, y_{t-1}^*, x_t^*, r_t^*) \quad (A5)$$

$$\cdot \prod_{t=1}^q p(x_t, r_t | \alpha_{t-1}^*, y_{t-1}^*, x_{t-1}^*, r_{t-1}^*) \frac{p(\alpha_0)}{p(y_q^*, x_q^*, r_q^*)}.$$

Under assumptions (A1)-(A4), this now simplifies to

$$p(\alpha_q^* | y_q^*, x_q^*, r_q^*) \propto \prod_{t=1}^q \prod_{i \in \mathcal{R}_t} p(y_{it} | y_{t-1}^*, x_t^*, r_t^*; \alpha_t) \prod_{t=1}^q p(\alpha_t | \alpha_{t-1}) \cdot p(\alpha_0), \quad (A6)$$

which is the expression used in the calculation of the log-likelihood function (15).

APPENDIX B: MAXIMIZATION OF THE GENERALIZED KALMAN FILTER AND SMOOTHER

This appendix briefly explains some of the details of the numerical optimizations. To perform numerical optimization of the penalized log-likelihood function, we use the generalized extended Kalman filter and smoother suggested by Fahrmeir (1992). Let $d_{it}(\alpha_t)$ denote the first derivate $\partial F(\eta)/\partial \eta$ of the response function $F(\eta)$ evaluated at $\eta = x_{it}'\alpha_t$. The contribution to the score of the failure indicator y_{it} is given by

$$\mathbf{u}_{it}(\alpha_t) = \frac{\partial l_{it}(\alpha_t)}{\partial \alpha_t} = x_{it} \frac{d_{it}(\alpha_t)}{F(x_{it}'\alpha_t)\{1 - F(x_{it}'\alpha_t)\}} \{y_{it} - F(x_{it}'\alpha_t)\}, \quad (B1)$$

and the contribution of the expected information matrix is

$$\mathbf{U}_{it}(\alpha_t) = -E \left[\frac{\partial^2 l_{it}(\alpha_t)}{\partial \alpha_t' \partial \alpha_t} | \alpha_t, y_{t-1}^*, x_{t-1}^* \right] = x_{it} x_{it}' \frac{(d_{it}(\alpha_t))^2}{F(x_{it}'\alpha_t)\{1 - F(x_{it}'\alpha_t)\}}. \quad (B2)$$

The contributions of the risk set to the score vector and the expected information matrix in the interval $[a_{t-1}, a_t]$ can be obtained by summing over the durations in the risk set at time t . This means computing

$$\mathbf{u}_t(\alpha_t) = \sum_{i \in \mathcal{R}_t} \mathbf{u}_{it}(\alpha_t) \text{ and } \mathbf{U}_t(\alpha_t) = \sum_{i \in \mathcal{R}_t} \mathbf{U}_{it}(\alpha_t).$$

Let $\mathbf{a}_{t|q}$ ($t = 0, \dots, q$) denote the smoothed estimates of α_t . These estimates can be obtained as numerical approximations to posterior modes given all the data (y^*, x^*, r^*) up to q . Approximate error covariance matrices $\mathbf{V}_{t|q}$ are obtained as the corresponding numerical approximations to curvatures, i.e. inverses of expected negative second derivatives of $\ln \mathcal{L}(\alpha^*)$, evaluated at the mode. Finally $\mathbf{a}_{t|t-1}$ and $\mathbf{a}_{t|t}$ are the prediction and filter estimates of α_t , given the data up to $t-1$ and t , with corresponding error matrices $\mathbf{V}_{t|t-1}$ and $\mathbf{V}_{t|t}$.

Filtering and smoothing of our sample data proceed in the following steps:

1. INITIALIZATION:

$$\begin{aligned}\mathbf{a}_{0|0} &= \mathbf{a}_0, \\ \mathbf{V}_{0|0} &= \mathbf{Q}_0.\end{aligned}\tag{B3}$$

2. FILTER PREDICTION STEPS:

For $t = 1, \dots, q$:

$$\begin{aligned}\mathbf{a}_{t|t-1} &= \Phi \mathbf{a}_{t-1|t-1}, \\ \mathbf{V}_{t|t-1} &= \Phi \mathbf{V}_{t-1|t-1} \Phi + \mathbf{Q}.\end{aligned}\tag{B4}$$

3. FILTER CORRECTION STEPS:

For $t = 1, \dots, q$ use the global scoring steps:

$$\begin{aligned}\mathbf{a}_{t|t} &= \mathbf{a}_{t|t-1} + \mathbf{V}_{t|t} \mathbf{u}_t, \\ \mathbf{V}_{t|t} &= (\mathbf{V}_{t|t-1}^{-1} + \mathbf{U}_t)^{-1}.\end{aligned}\tag{B5}$$

4. BACKWARD SMOOTHING STEPS:

For $t = 1, \dots, q$:

$$\begin{aligned}\mathbf{a}_{t-1|q} &= \mathbf{a}_{t-1|t-1} + \mathbf{B}_t (\mathbf{a}_{t|q} - \mathbf{a}_{t|t-1}), \\ \mathbf{V}_{t-1|q} &= \mathbf{V}_{t-1|t-1} + \mathbf{B}_t (\mathbf{V}_{t|q} - \mathbf{V}_{t|t-1}) \mathbf{B}_t',\end{aligned}\tag{B6}$$

where

$$\mathbf{B}_t = \mathbf{V}_{t-1|t-1} \Phi' \mathbf{V}_{t|t-1}^{-1}.\tag{B7}$$

The algorithm relies on having initial values \mathbf{a}_0 , \mathbf{Q}_0 and error covariances \mathbf{Q} of the transition equation. In practice hyper-parameters $\alpha_0, \dots, \alpha_q$, \mathbf{a}_0 , \mathbf{Q}_0 and \mathbf{Q} can be jointly estimated by the following EM-type algorithm, applied until some convergence point is reached:

1. Select initial values $\mathbf{a}_0^{(0)}$, $\mathbf{Q}_0^{(0)}$ and $\mathbf{Q}^{(0)}$.

Iterate on steps 2 and 3 for $p = 1, 2, \dots$

2. SMOOTHING:

Compute $\mathbf{a}_{t|q}^{(p)}$ and $\mathbf{V}_{t|q}^{(p)}$ ($t = 1, \dots, q$) by the generalized Kalman filter and smoother replacing the unknown parameters by the current estimates. These are given by $\mathbf{a}_0^{(p)}$, $\mathbf{Q}_0^{(p)}$ and $\mathbf{Q}^{(p)}$.

3. EM STEP:

Compute $\mathbf{a}_0^{(p+1)}$, $\mathbf{Q}_0^{(p+1)}$ and $\mathbf{Q}^{(p+1)}$ as follows

$$\begin{aligned} \mathbf{a}_0^{(p+1)} &= \mathbf{a}_{0|q}^{(p)} \\ \mathbf{Q}_0^{(p+1)} &= \mathbf{V}_{0|q}^{(p)} \\ \mathbf{Q}^{(p+1)} &= \frac{1}{q} \sum_{t=1}^q \{ (\mathbf{a}_{t|q}^{(p)} - \Phi \mathbf{a}_{t-1|q}^{(p)}) (\mathbf{a}_{t|q}^{(p)} - \Phi \mathbf{a}_{t-1|q}^{(p)})' + \mathbf{V}_{t|q}^{(p)} \\ &\quad - \Phi \mathbf{B}_t^{(p)} \mathbf{V}_{t|q}^{(p)} - \mathbf{V}_{t|q}^{(p)} \mathbf{B}_t^{(p)'} \Phi + \Phi \mathbf{V}_{t-1|q}^{(p)} \Phi' \} \end{aligned} \quad (\text{B8})$$

where $\mathbf{B}_t^{(p)}$ is defined in (B7).

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Table 1: Summary Statistics of Bull and Bear Market Durations.

Series	Filter	Origin	Mean	Median	Standard deviation	Min.	Max.	
Bull market durations (weeks)	10%	Actual ^a	37.52	18	50.98	1	354	
		Volatility adj. ^b	37.75	26	33.32	4	168	
		Random walk ^c	30.35	22	27.58	1	261	
	15%	Actual	85.25	57	100.78	1	454	
		Volatility adj.	68.07	50	62.24	10	354	
		Random walk	73.92	51	68.81	4	501	
	20%	Actual	115.42	85	125.51	5	522	
		Volatility adj.	125.48	105	85.58	21	354	
		Random walk	143.65	102	136.30	5	856	
	Bear market durations (weeks)	10%	Actual	18.89	12	18.56	1	88
			Volatility adj.	22.37	18	15.82	3	77
			Random walk	20.40	16	15.51	1	143
15%		Actual	36.46	23	35.06	1	136	
		Volatility adj.	41.09	37	26.17	7	99	
		Random walk	41.66	34	30.05	3	217	
20%		Actual	50.76	38	42.58	4	190	
		Volatility adj.	65.73	57	43.81	12	209	
		Random walk	67.06	54	46.00	7	382	

^a The durations were defined from the inflation adjusted S&P500 stock price index. The 10%, 15%, and 20% filters split the sample into 114, 52, and 38 bull and bear markets, respectively.

^b The volatility adjustment were done using the component ARCH model given in equation (5). The estimated parameters (standard errors in brackets) are:

$$r_t = \frac{-0.00001}{(0.00012)} + \frac{0.086}{(0.0061)} r_{t-1} + \frac{0.080}{(0.017)} \sigma_t + \varepsilon_t, \quad \sigma_t^2 = q_t + \frac{0.093}{(0.0028)} (\varepsilon_{t-1}^2 - q_{t-1}) + \frac{0.83}{(0.0060)} (\sigma_{t-1}^2 - q_{t-1})$$

$$q_t = \frac{0.000080}{(0.000005)} + \frac{0.996}{(0.0004)} (q_{t-1} - 0.000080) + \frac{0.0267}{(0.0020)} (\varepsilon_{t-1}^2 - \sigma_{t-1}^2)$$

From this specification ARCH-adjusted residuals are given by $\hat{\varepsilon}_i^* = \hat{\varepsilon}_i / \sqrt{\hat{\sigma}_i}$ where $\hat{\varepsilon}_i = r_i - \hat{\mu} - \hat{\beta} r_{i-1} - \hat{\gamma} \hat{\sigma}_i$ is the residual from the ARCH model and $\hat{\sigma}_i$ its estimated volatility. These residuals define normalized returns, $r_i^* = \hat{\varepsilon}_i$, and the new price index were constructed as: $P_i^* = \exp(\sum_{i=1}^t r_i^* + t(\bar{r}_T - \bar{r}_T^*))$, where $\bar{r}_T = T^{-1} \sum_{i=1}^T r_i$ and $\bar{r}_T^* = T^{-1} \sum_{i=1}^T r_i^*$. The 10%, 15%, and 20% filters split the sample into 106, 58 and, 33 bull and bear markets, respectively.

^c The random walk with drift model was estimated from the inflation adjusted S&P500 stock price index, and the following result were obtained: $p_t = \exp(0.000222 + \ln(p_{t-1}) + \varepsilon_t)$, $\varepsilon_t \sim N(0, 0.010123^2)$. The results for this model are based on a Monte Carlo simulation with 1, 000, 000 observations.

Table 2: Summary Statistics of Bull and Bear Market Returns.

Series	Filter	Origin	Mean	Median	Standard deviation	Min.	Max.
Log-return (%) Bull markets	10%	Actual	3.10	1.00	6.01	0.25	41.85
		Volatility adj.	1.04	0.95	0.52	0.30	3.04
		Random walk	1.14	0.95	0.69	0.18	6.44
	15%	Actual	2.62	0.72	6.31	0.19	41.85
		Volatility adj.	0.83	0.72	0.38	0.34	2.43
		Random walk	0.79	0.65	0.49	0.13	3.93
	20%	Actual	1.26	0.59	1.43	0.21	5.55
		Volatility adj.	0.57	0.53	0.21	0.29	1.18
		Random walk	0.61	0.48	0.41	0.13	3.93
Log-return (%) Bear markets	10%	Actual	-3.31	-1.64	5.46	-39.22	-0.27
		Volatility adj.	-1.39	-1.20	0.83	-6.00	-0.45
		Random walk	-1.27	-1.06	0.76	-6.87	-0.22
	15%	Actual	-3.32	-1.16	6.56	-39.22	-0.23
		Volatility adj.	-1.02	-0.90	0.59	-2.87	-0.32
		Random walk	-0.90	-0.76	0.52	-5.57	-0.20
	20%	Actual	-1.50	-0.92	1.54	-6.4	-0.14
		Volatility adj.	-0.85	-0.76	0.41	-2.23	-0.40
		Random walk	-0.78	-0.68	0.34	-1.89	-0.32

The construction of the actual, volatility adjusted and random walk time series is explained in table 1. Returns are reported per week and are based on the distribution of returns per bull or bear spell.

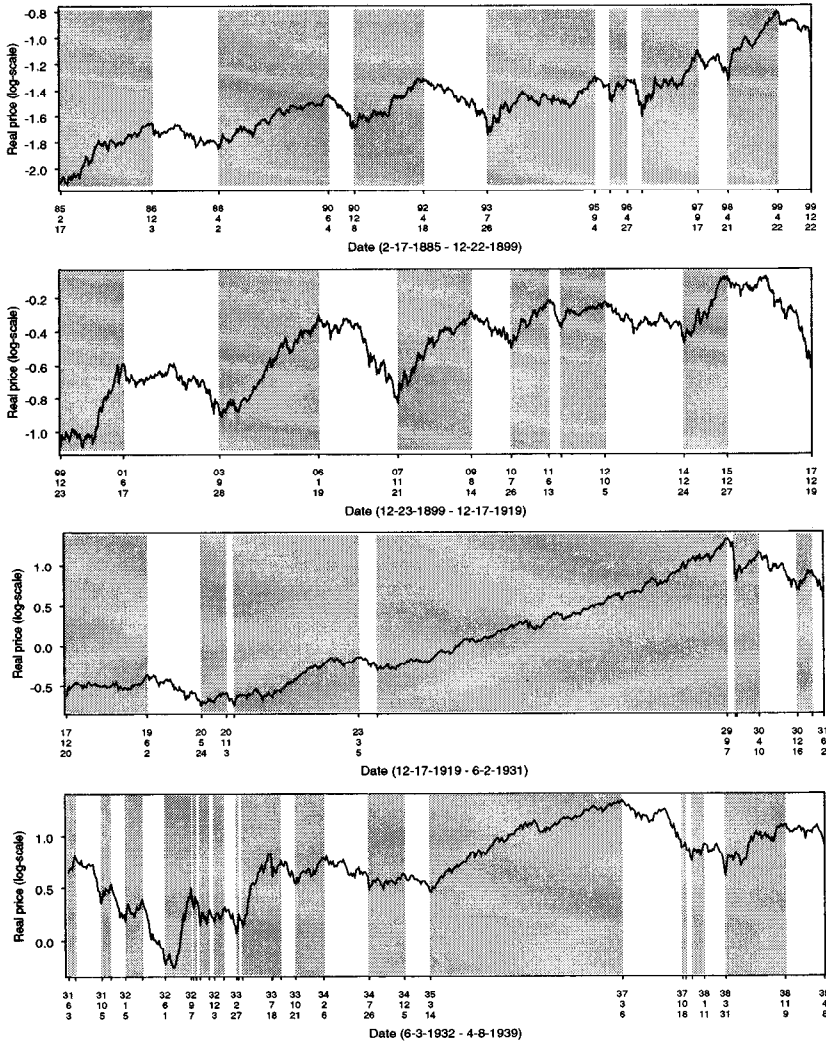


Figure 1a: Bull and Bear markets based on the real S&P500 stock price index and a 15% filter.

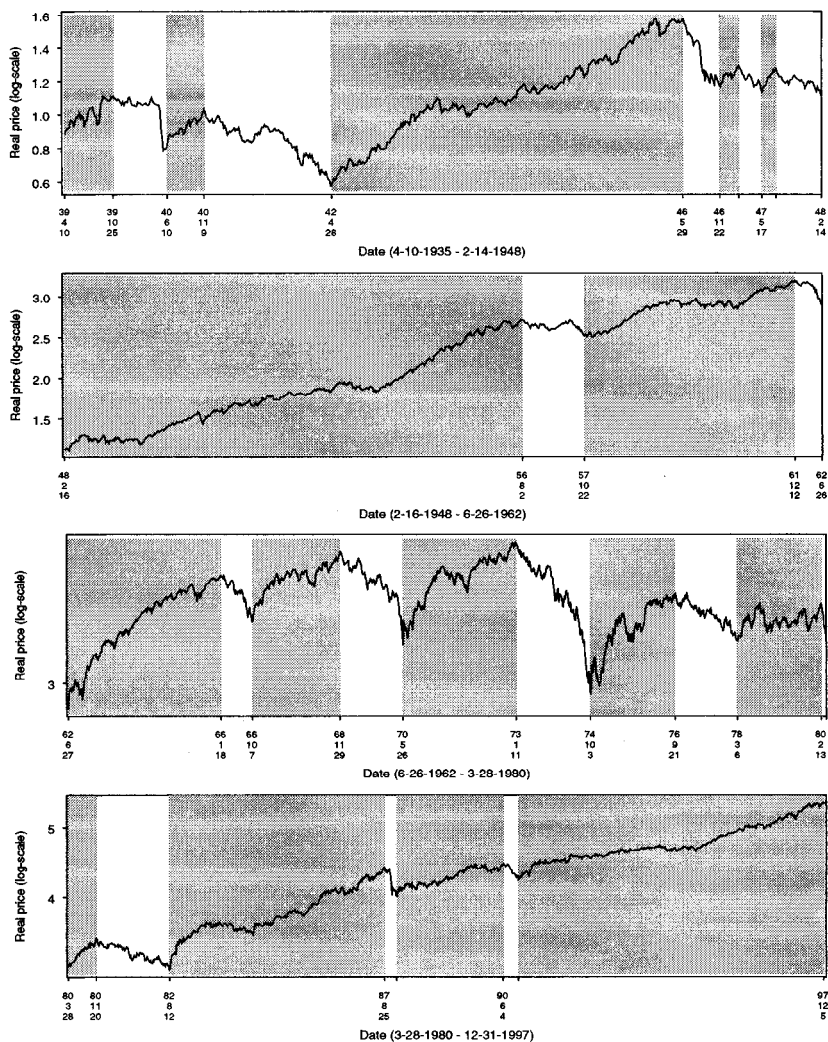


Figure 1b: Bull and Bear markets based on the real S&P500 stock price index and a 15% filter.

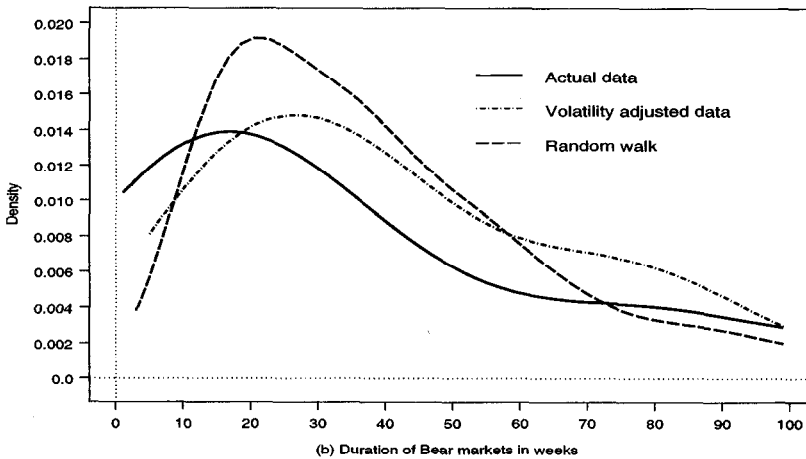
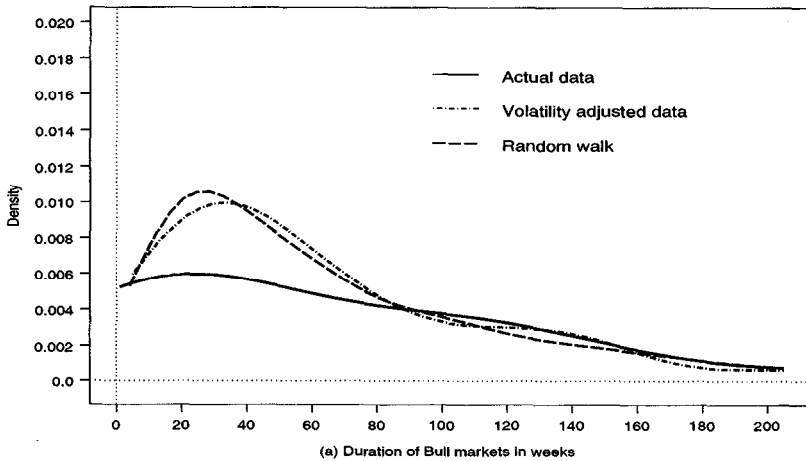


Figure 2 : Smoothed densities of (a) Bull and (b) Bear market durations based on a Gaussian kernel. The actual durations are based on the inflation adjusted S&P500 stock price index using a 15% filter. The volatility adjusted durations were corrected for ARCH effects using an GARCH(1,1) components model. The random walk density is based on a simulation of 1,000,000 stock index prices.

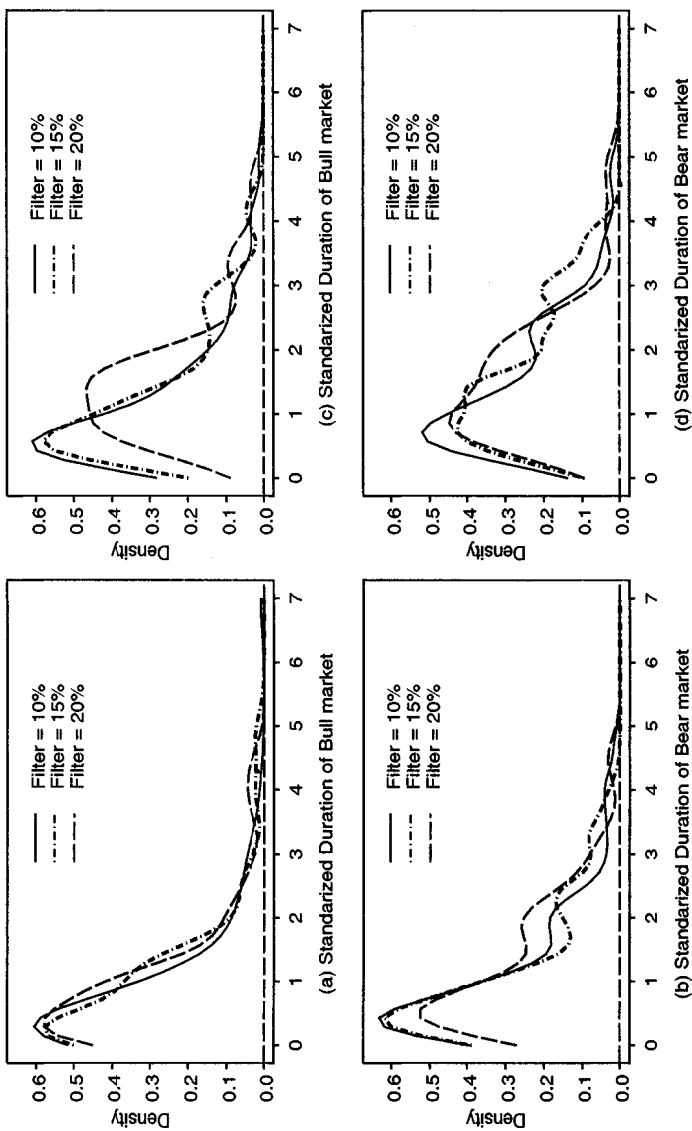


Figure 3 : This figure shows kernel smoothed densities of (a) Bull and (b) Bear market durations. The actual durations were based on the inflation adjusted S&P500 stock price index with 10%, 15% and 20% filters. Panels (c) and (d) present density plots for the volatility adjusted price series.

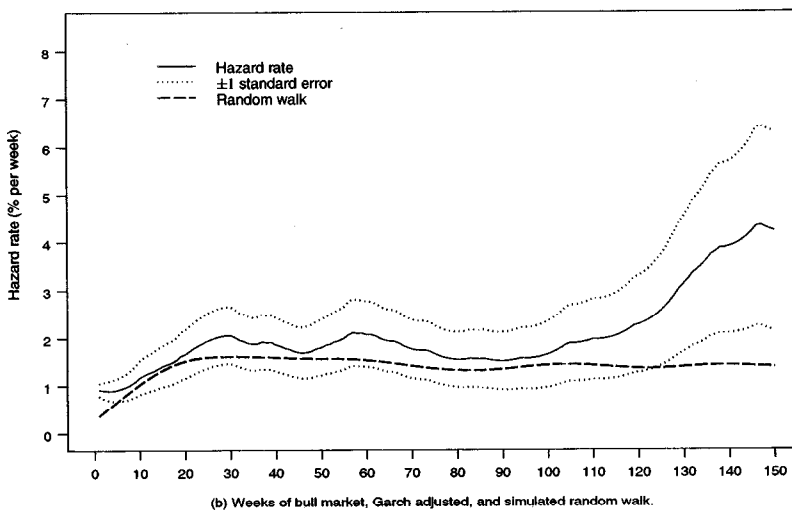
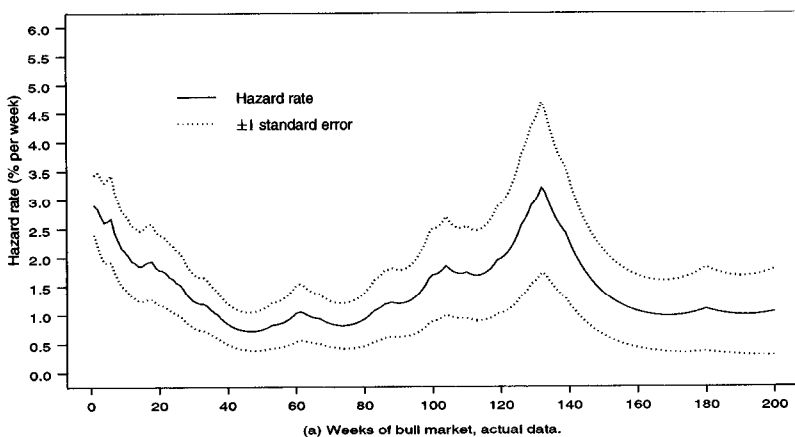
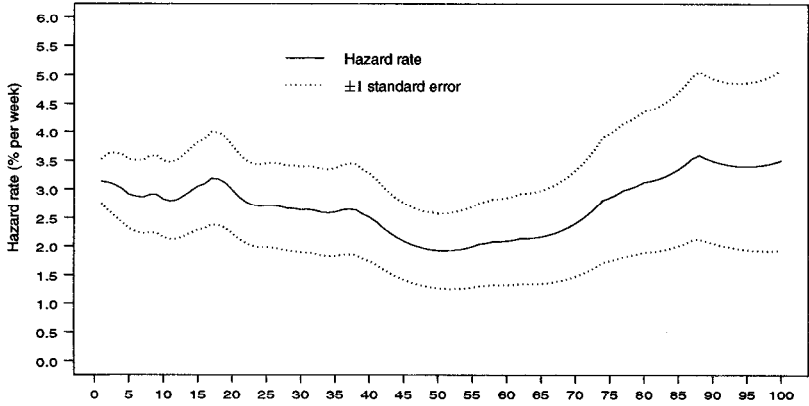
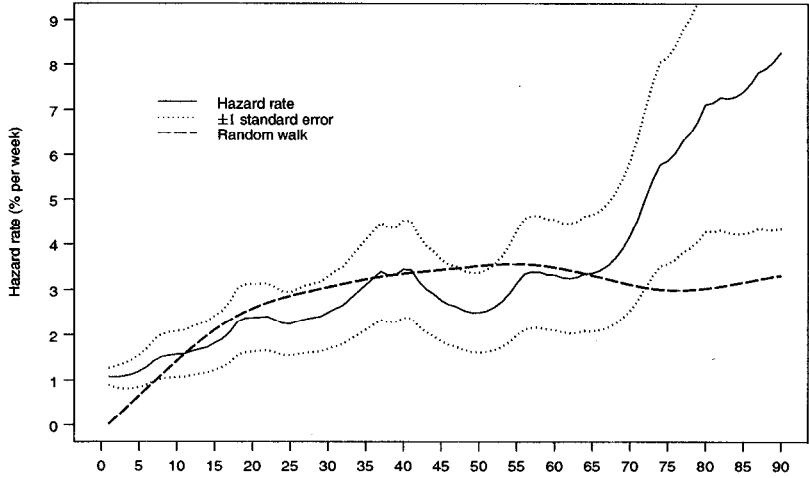


Figure 4 : Unconditional hazard rates in Bull markets. Panel (a) presents the bull market hazard rate for the S&P500 stock price index with a 15% filter. Panel (b) shows hazard rates based on the volatility adjusted durations and the random walk model simulated from 1,000,000 stock prices. In both panels the hazard rate model assumes a logit link function: $\lambda(t|X_t(t)) = F(x'_{it}\alpha_t)$, where $x'_{it} = 1$ and $\alpha_t = \gamma_{0t}$, $\gamma_{0t} = \gamma_{0t-1} + \xi_{0t}$, $\xi_{0t} \sim N(0, \sigma_1^2)$, and $\gamma_{00} \sim N(g_0, \sigma_0^2)$.

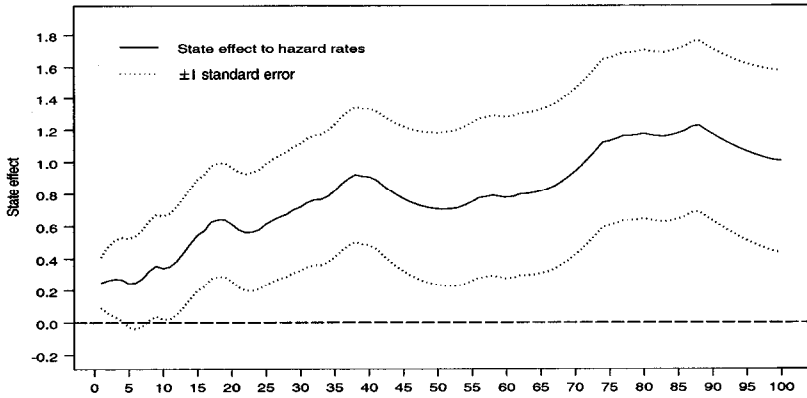


(a) Weeks of bear market, actual data.

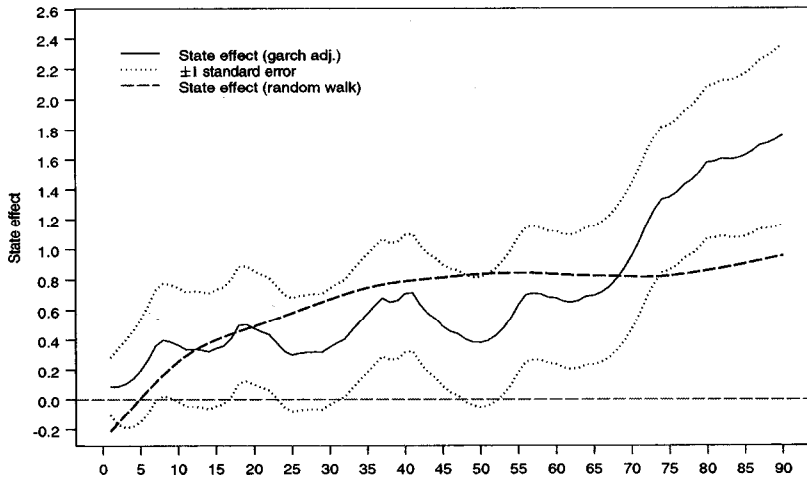


(b) Weeks of bear market, Garch adjusted, and simulated random walk.

Figure 5 : Unconditional hazard rates in Bear markets. Panel (a) presents the bear market hazard rate for the S&P500 stock price index with a 15% filter. Panel (b) shows hazard rates based on the volatility adjusted durations and the random walk model simulated from 1,000,000 stock prices. In both panels the hazard rate model assumes a logit link function: $\lambda(t|X_1(t)) = F(x'_{it}\alpha_t)$, where $x'_{it} = 1$ and $\alpha_t = \gamma_{0t}$, $\gamma_{0t} = \gamma_{0t-1} + \xi_{0t}$, $\xi_{0t} \sim N(0, \sigma_1^2)$, and $\gamma_{00} \sim N(g_0, \sigma_0^2)$.



(a) Weeks of duration, actual data.



(b) Weeks of bear market, Garch adjusted, and simulated random walk.

Figure 6 : The difference between Bear and Bull market hazard rates. Panel (a) presents the hazard rate of the S&P500 stock price index with a 15% filter. Panel (b) shows hazard rates for the volatility adjusted durations and the random walk simulated from 1,000,000 prices. The hazard rate assumes a logit link function: $\lambda(t|X_t(t)) = F(x'_t \alpha_t)$, where $x'_t = (1, w_{it})$ and $\alpha'_t = (\gamma_0, \beta_t)$, $\alpha_t = \alpha_{t-1} + \xi_t$, $\xi_t \sim N(0, Q)$, and $\alpha_0 \sim N(g_0, Q_0)$. β_t gives the difference between Bull markets ($w_{it} = 0$), and Bear markets ($w_{it} = 1$).

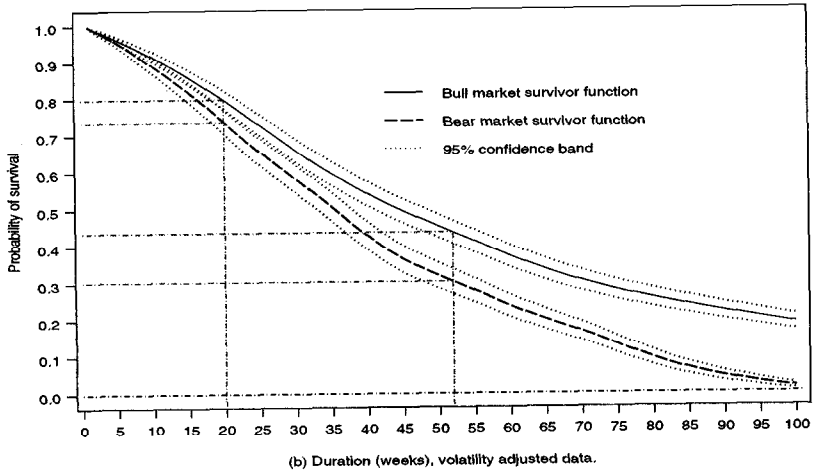
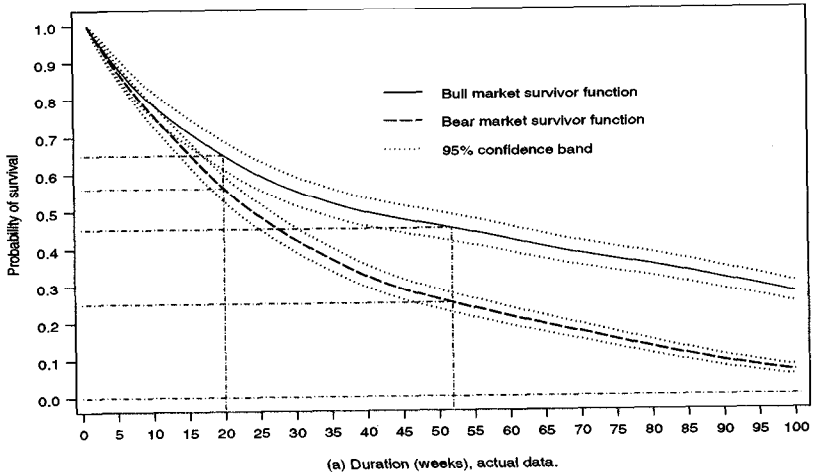


Figure 7: Survivor functions for (a) Bull and (b) Bear markets estimated from the simple unconditional hazard rates shown in Figures 4 and 5. Panel (a) presents the survivor functions for the S&P500 stock price index with a 15% filter. Panel (b) shows the survivor functions based on the volatility adjusted durations.

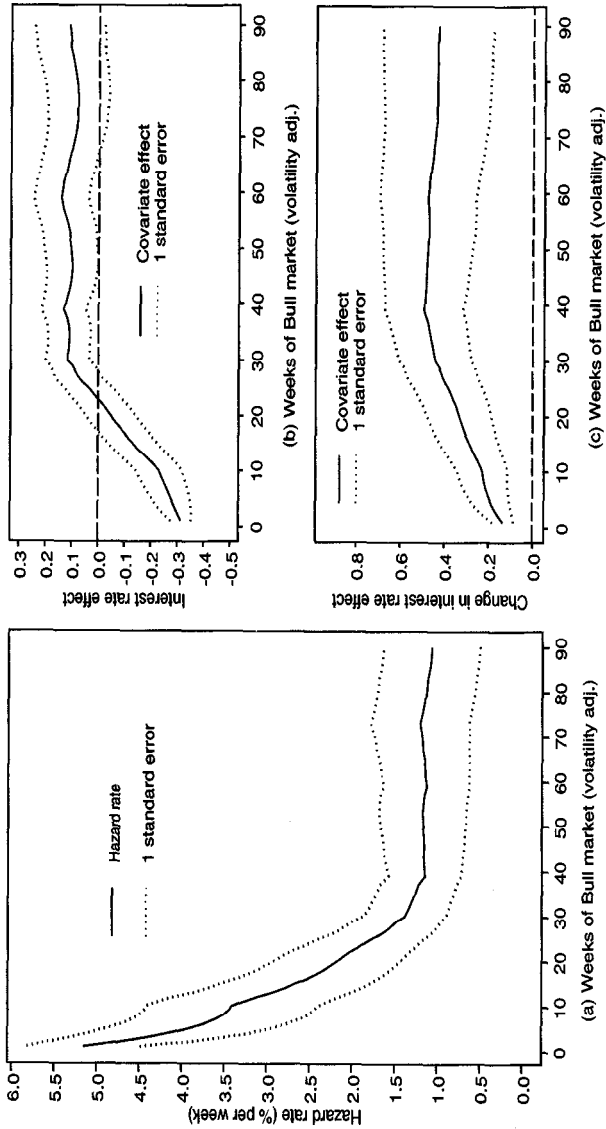


Figure 8 : Interest rate effects on bull market hazard rate. Panel (a) presents the baseline hazard rates for Bull markets, controlling for interest rate and interest rate change effects. Panel (b) shows the interest rate effect on the Bull hazard rate, and panel (c) plots the interest rate change effect on the Bull hazard rate. The confidence bands are ± 1 standard error. Durations were based on the volatility-adjusted S&P-500 stock price index with a stopping rule of 15%. The hazard rate model is the logit link function: $\lambda(t|X_t(t)) = F(x_t' \alpha_t)$, where $x_t' = (1, i_t, \Delta i_t)$ and $\alpha_t' = (\gamma_0, \beta)$, $\alpha_t = \alpha_{t-1} + \xi_t$, $\xi_t \sim \mathcal{N}(0, Q)$, and $\alpha_0 \sim \mathcal{N}(g_0, \Omega_0)$. i_t is the interest rate at the beginning of the week in question, and Δi_t is the change in the interest rate.

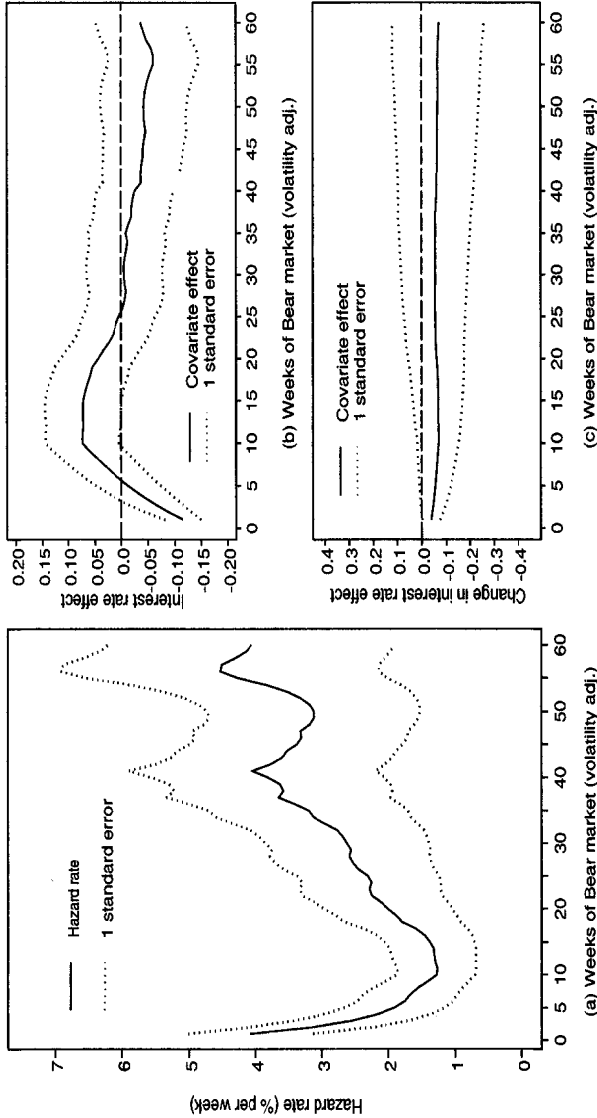


Figure 9: Interest rate effects on bear market hazard rate. Panel (a) presents the baseline hazard rates for Bear markets, controlling for interest rate and interest rate change effects. Panel (b) shows the interest rate effect on the Bear hazard rate, and panel (c) plots the interest rate change effect on the Bear hazard rate. The confidence bands are ± 1 standard error. Durations were based on the volatility-adjusted S&P-500 stock price index with a stopping rule of 15%. The hazard rate model is the logit link function: $\lambda(t|X_t(t)) = F(x_t' \alpha_t)$, where $x_t' = (1, i_t, \Delta i_t)$ and $\alpha_t' = (\gamma_0, \beta_t)$, $\alpha_t = \alpha_{t-1} + \xi_t$, $\xi_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$, and $\alpha_0 \sim \mathcal{N}(\xi_0, \mathbf{Q})$. i_t is the interest rate at the beginning of the week in question, and Δi_t is the change in the interest rate.

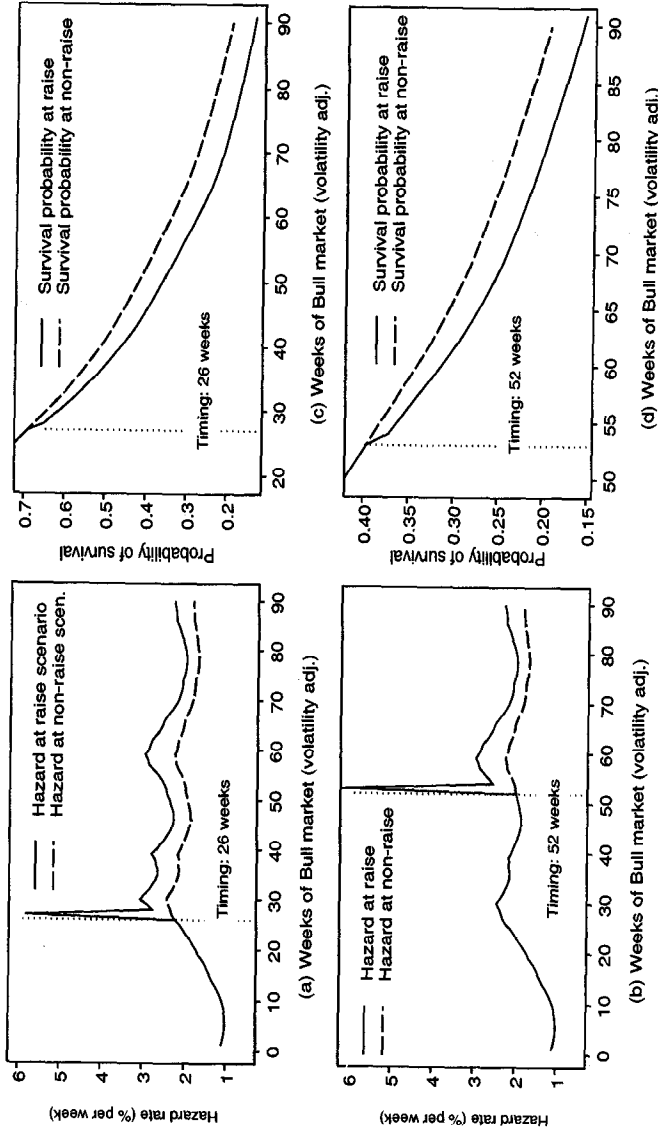


Figure 10 : Scenario analysis for the effects interest rate increase on bull hazard rates. The figures present scenarios where the interest rate is raised from 5% to 7% in a bull market. Panel (a) shows the effect on the hazard rate for a raise occurring after 26 weeks and panel (b) have the raise occurring after 52 weeks. Panels (c) and (d) present the corresponding survivor functions.

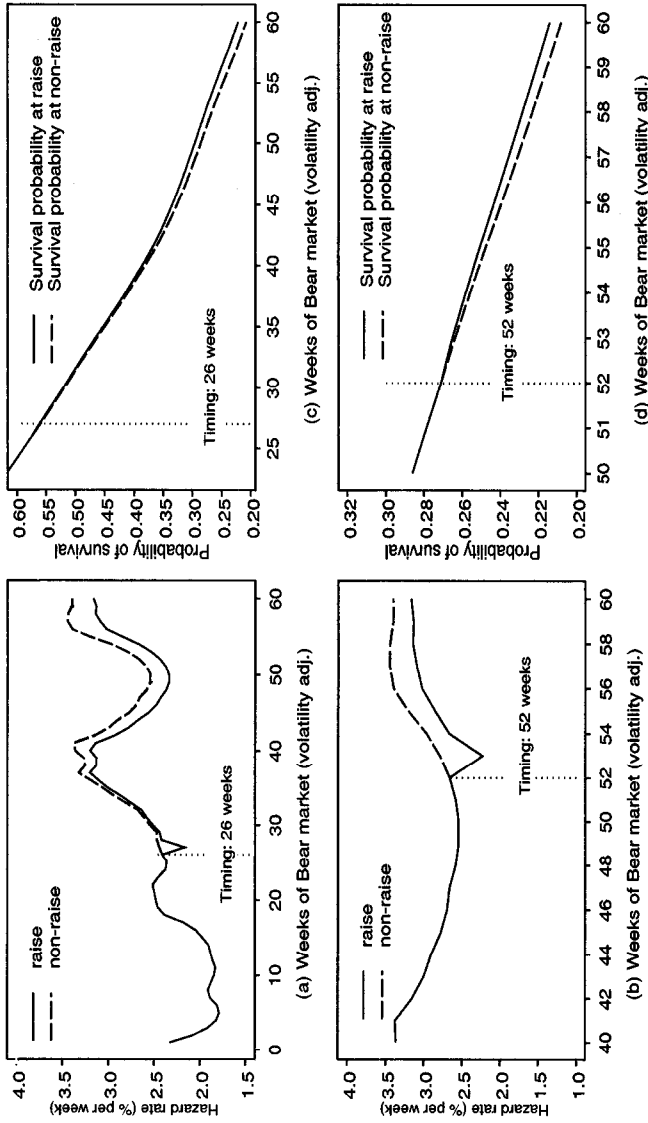


Figure 11 : Scenario analysis for the effects interest rate increase on bear hazard rates. The figures present scenarios where the interest rate is raised from 5% to 7% in a bear market. Panel (a) shows the effect on the hazard rate for a raise occurring after 26 weeks and panel (b) have the raise occurring after 52 weeks. Panels (c) and (d) present the corresponding survivor functions.

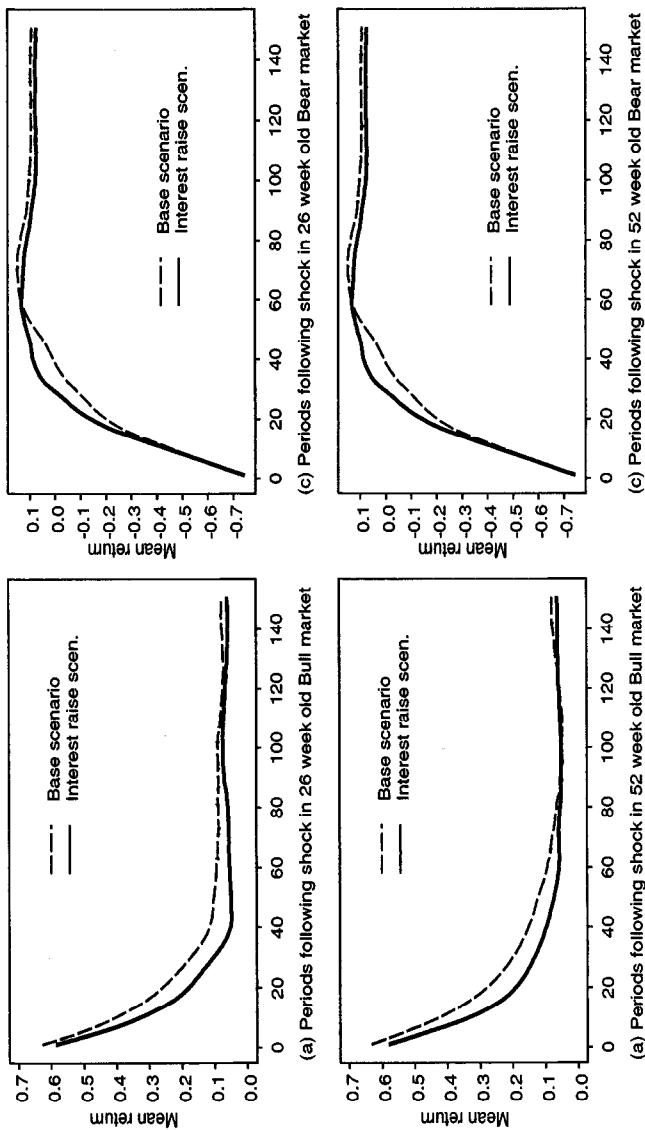


Figure 12. Mean effect of a raise in the interest rate. The figures present the mean return effect of a permanent 2% increase in the interest rate as a function of the time (in weeks) since the raise. Panel (a) assumes that the increase occurs after 26 weeks in a bull market, while panel (b) assumes the increase happens after 52 weeks. Panels (c) and (d) assume that the initial state is a bear market. All effects are calculated using 100,000 simulated realizations of stock price series.