Dynamic Adjustment in the U.S. Dairy Industry

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A dual model is used to examine the dynamic structure of the U.S. dairy industry. Properties implied by the theory of the competitive firm and independent adjustment of two quasi-fixed inputs, labor and herd size, are tested and not rejected. Instantaneous adjustment, however, is soundly rejected for each quasi-fixed input. Input adjustment to optimal levels is estimated to take about two years for labor and ten for cows. Quality adjustments of the labor and cow series do not fully embody the technological change that has occurred in this industry over the study period.

Key words: adjustment costs, dairy, dual, dynamic, flexible accelerator, production.

The supply side of the dairy industry has been the subject of several recent studies, motivated largely by government needs to anticipate the effects of possible policy changes. Both single-equation and multiple-equation econometric studies have estimated that milk supply is very inelastic in the short run, and a number have even found that the price of milk in the supply function is not significant (Wilson and Thompson, Prato). The long-run supply is estimated to be more elastic in these studies, as one would expect. The greater responsiveness of producers to relative price changes in the long run can be explained by the possibility of changing herd size and other quasi-fixed inputs and by genetic improvement as cows are replaced. Herd size and the factors affecting it are of particular concern in the late 1980s with the dairy herd buy-out program in progress.

Herd size has been an endogenous variable in several dairy supply models (Halvorson, Wilson and Thompson, Prato, La France and de Gorter, Chavas and Klemme). Each of these studies used some sort of lag structure to acknowledge that decisions made today about breeding or culling cows, retaining heifer calves, or genetic improvements through a breeding program take one to three years before their impact is felt. An estimate of the actual rate of adjustment from current to desired herd levels can give policy makers a time frame for the impact of policy decisions. However, dynamic models are required for such estimation.

Dynamic models have been applied in various forms, primarily in partial adjustment and flexible accelerator models. Three recent models of the dairy industry that incorporate dynamics use lag structures to model changes in the dairy herd (Dahlgran, LaFrance and de Gorter, Chavas and Klemme). These models appear to represent the dairy industry well but incorporate dynamics in a largely ad hoc nature. More than a decade ago, Nerlove noted that the applications of distributed lag models in empirical economic studies "is astounding, but what is more remarkable is the virtual lack of theoretical justification for the lag structure superimposed on basically static models" (p. 293).

Dynamic models that are consistent with the theory of the firm have also been derived from applications of optimal control theory but have not previously been used to examine the dairy industry. Primal and dual models can be derived from an intertemporal value function in the form of a Hamilton-Jacobi equation that is the present value of a stream of future profits (or costs).[1] The behavioral equations may be obtained via a primal approach using

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[1] Both the primal and the dual dynamic models are functions of prices. The primal model is obtained from the first-order conditions of an optimization problem. The dual model obtains the behavioral equations by applying the envelope theorem directly to the optimization problem.

first-order (to the Hamilton-Jacobi equation) Euler equations, or one may apply the envelope theorem to the value function to directly obtain the behavioral equations. The primal approach was developed by Treadway and has been applied to U.S. manufacturing by Berndt, Fuss, and Waverman and to the Canadian food processing industry by Lopez. The dual approach is based on results by McLaren and Cooper that have been formalized by Epstein; it has been used in empirical studies of U.S. manufacturing by Epstein and Denny and U.S. agriculture by Taylor and Monson, and Vasavada and Chambers (1982, 1986). The primal approach is limited to modeling only one quasi-fixed input or assuming independent adjustment between two or more quasi-fixed inputs. This study will use the dual approach because we wish to model more than one quasi-fixed input and to test for independent adjustment rather than assuming it.

The purpose of this study is to examine the dynamic structure of the U.S. dairy industry. Rates of adjustment of two quasi-fixed inputs, labor and herd size, are estimated using a dynamic dual approach. Properties consistent with the theory of the competitive firm are tested as are the hypotheses of independent and instantaneous adjustment of the quasi-fixed inputs. The model is estimated using annual data for 1951–82, a period of rapid technological change. Quality indexes for labor and cows are constructed to adjust the data prior to estimation, and tests are conducted to determine whether the technological change that occurred during this period is fully embodied in these indexes.

The next section gives a brief overview of the theory of the dynamic dual model. The empirical model employed in this study follows, together with a description of the data and the quality indexes. Results of theoretical and structural tests, along with short- and long-run elasticities, are presented in the final section prior to the conclusions.

The Dynamic Model

To establish the relationship between the production function, restricted profit function, and intertemporal value function, consider a profit-maximizing, competitive firm with a static restricted profit function,

$$(1)\quad \Pi(P, W, Z) = \text{Max } Pf(X, Z) - W'X,$$

where P is price of output $f(\ )$, W is a vector of prices of variable inputs X, $f(\ )$ is a "well-behaved" production function (i.e., f is twice continuously differentiable, concave over the relevant range of production, $f_x, f_z > 0$), and Z is a vector of quasi-fixed inputs. The duality between $\Pi(\ )$ and $f(\ )$ is well known: a "well-behaved" production function is sufficient to obtain the relevant input demand and output supply functions, as well as the curvature properties of $f(\ )$, from the restricted profit function.

In many cases the short-run static model is all that is necessary for analysis. In the present study, however, the objective is to examine the nature of dynamic adjustments in the quasi-fixed inputs, Z; thus the static model is inadequate. In this case an intertemporal value function must be specified. At any point in time, $t = 0$, the firm is presumed to act as though it solves the following infinite horizon problem:

$$(2)\quad J(P, W, C, r, Z_0) = \text{Max} \int_0^{\infty} e^{-rt}[PF(X, Z, \dot{Z}) - W'X - C'Z]dt$$

$$\text{subject to } X, Z > 0,\ \dot{Z}_t = I_t - \delta Z_{t-1}, \text{ and } Z(0) = Z_0 > 0,$$

where C is the rental price vector of the quasi-fixed inputs, r is the real discount rate, I is gross investment in Z, δ is the (constant) depreciation rate, Z_0 is the initial endowment of Z, and $\dot{Z}$ is the net change in Z (may be positive or negative). All variables are implicit functions of time, so time subscripts t are dropped to minimize notational clutter.

Equation (2) is termed the "value function" and is central in dynamic duality. The regularity assumptions of $f(\ )$ sufficient to establish its duality with $\Pi(\ )$ are presumed to apply also to $F(\ )$. In addition, it is assumed that $F_{\dot{z}} > 0$, that the $\lim_{t\to\infty} \dot{Z}(t) = 0$, and that $J(\ )$ is twice continuously differentiable, convex in prices, and concave in quasi-fixed inputs. The first assumption implicitly maintains positive adjustment costs, and the second assures that a steady state (long-run equilibrium) exists for $Z^*(P, W, C)$. The last three assumptions insure that both short-run and long-run solutions exist for the value function maximum; they enable us to apply the envelope theorem to establish a duality between $J(\ )$ and $F(\ )$.

Static prices are assumed in the model. This assumption of the markovian property (Hillier and Lieberman, p. 351) is that current prices contain all relevant information about future prices. As the base period changes, new expectations about prices come into being. Decisions made in period t are based on information available in that period. Reasons that a firm which recognizes the cost of acquiring information may rationally choose to formulate expectations in this manner while continuously updating decisions subject to new information are outlined by Chambers and Lopez.

The value function in (2) is the static approximation of a dynamic optimization problem. Assuming the regularity conditions on $F(X, Z, \dot{Z})$ listed above and a constant discount rate, $J(\,)$ satisfies the Hamilton-Jacobi equation for an optimal control problem which takes the form:

$$(3)\quad rJ(P, W, C, Z) = \max[PF(X, Z, \dot{Z}) - W'X - C'Z + J_z\dot{Z}],$$

where J_z is the shadow price of the quasi-fixed input.

The Hamilton-Jacobi equation allows us to transform the dynamic problem in (2) into a more manageable form. Specifically, (3) states that the value function is defined as the discounted present value of the current profit plus the marginal value of optimal change in net investment.[2] Epstein (pp. 84–86) has shown that the properties of $F(\,)$ are fully manifested in the value function $J(\,)$, given the regularity conditions maintained on $F(\,)$. The Hamilton-Jacobi equation allows us to obtain (3) from (2) and vice versa. Thus, a full dynamic duality exists between $F(\,)$, $J(\,)$, and $rJ(\,)$. Application of the envelope theorem to (3) permits variable and quasi-fixed input demand functions to be derived in a simple and direct manner. By differentiating equation (3) with respect to prices and rearranging, equations for output supply and variable and quasi-fixed input demands are obtained:

$$(4)\quad F(P, W, C, Z) = rJ_p - J_{zp}\dot{Z},$$
$$(5)\quad X(P, W, C, Z) = -rJ_w + J_{zu}\dot{Z},$$
$$(6)\quad \dot{Z}(P, W, C, Z) = J_{zc}^{-1}(rJ_c + Z).$$

If the value function has a form such that $J_{zc}(P, W, C, Z) = (M - r)^{-1}$ where M is the rate of adjustment matrix, (6) can be expressed as a multivariate flexible accelerator model,

$$(7)\quad \dot{Z}(P, W, C, Z) = M[Z - Z^*(W, C)],$$

where $Z^*(\,)$ is the desired level of the Z matrix (Epstein, p. 93). In such a case the multivariate flexible accelerator model would be consistent with the underlying theory of the firm. Further, as $M \rightarrow K$, where K is a negative identity matrix, producers adjust instantaneously to the desired level of Z, and Z shows no degree of fixity. The off-diagonal elements in K are not necessarily symmetric and measure the interdependence of the quasi-fixed inputs.

The value function (3) also assumes static technology in addition to static price expectations. However, nonstationarity caused by disembodied technological change and expected by the producers can be measured by some function $H(T)$, where T is time and appended to (4), (5), and (6). This is equivalent to including $H(T)$ in $J(\,)$. The relationship between J and H is not theoretically defined; that is, there are no presumptions on the curvature properties of H in J. Any maintained hypotheses on the way H enters F (e.g., Hicks' neutrality) impose conditions on J. Hence, any specification of H in J is ad hoc.

To account for disembodied technological change, H is included as an argument of J, and (3) is replaced by

$$(3')\quad rJ(P, W, C, Z, T) - J_h\dot{H} = \max[PF(X, Z, \dot{Z}, T) - W'X - C'Z + J_z\dot{Z}],$$

where $\dot{H}$ is the net change in H. By differentiating this equation with respect to prices, equations corresponding to (4)–(6) are obtained for output supply and variable and quasi-fixed input demand:

$$(4')\quad F(P, W, C, Z, T) = rJ_p - J_{zp}\dot{Z} - J_{hp}\dot{H},$$
$$(5')\quad X(P, W, C, Z, T) = -rJ_w + J_{zw}\dot{Z} + J_{hw}\dot{H},$$
$$(6')\quad \dot{Z}(P, W, C, Z, T) = J_{zc}^{-1}(rJ_c + Z - J_{hc}\dot{H}).$$

Because substantial technological change has prevailed in the dairy industry for many decades, it is likely that producers have expected a continuation of change. Thus, (4′)–(6′) define the output supply and input demand equations estimated in this study.

The Empirical Model

The behavior of the industry is modeled as a single representative firm using aggregate

[2] It is also assumed that producers adjust their investment decisions as prices change, so that the value function is not maximized once and the investment path followed to some terminal point, as in an optimal control problem. Producers reevaluate their investment paths as prices change, with Z_{t-1} considered as an endowment or a starting value for decisions made in period t.

data. One should consider whether it is reasonable to model the dairy industry in this fashion and whether the functional form used is consistent with aggregation. Aggregate data and models are frequently used due to the lack of firm-level data and the simplicity of an aggregate model. The dairy industry consists of many price-taking firms, and theory suggests that in long-run competitive equilibrium all such firms operate at the minimum average cost. Blackorby and Schworn (p. 600) extended this line of reasoning on aggregation to the case where there are firms with different levels of fixed factors. Given certain regularity conditions, it is necessary and sufficient for consistent aggregation across firms that the value function be affine in capital; that is, that the value function have a form such that $J_{zz} = 0$.

To express the net demand for quasi-fixed inputs in the flexible accelerator form of equation (7), the value function must also have a form such that J_{zc} is not a function of (P, W, C). This restriction on J_{zc} also facilitates determination of the curvature properties of J. Unlike static dual profit models, second-order conditions are not generally sufficient to verify the necessary curvature properties on the production technology. However, if J_z is linear in prices, convexity of J in prices is sufficient for the existence of the curvature properties (Epstein, p. 87).

Functional Form

Following Vasavada and Chambers (1982), a functional form for the Hamilton-Jacobi form of the value function (3) that meets the above requirements and also maintains linear homogeneity in prices and concavity in quasi-fixed inputs is a modified generalized leontief:

$$(8)\quad J(P, W, C, Z, T) = [PW]AZ + C'B^{-1}Z + [P^{.5}W^{.5}]EC^{.5} + C^{.5\prime}FC^{.5} + [P^{.5}W^{.5}]G[P^{.5}W^{.5}]' + TH[PWC']',$$

where P is the average blend price of fluid milk, W is the price of concentrates, Z is a (2×1) vector and includes the number of dairy cows in the United States that have calved and labor in the dairy sector,[3] C is a (2×1) vector and includes the annual average rental price of a dairy cow in the United States and the agricultural labor wage rate, and T is year. Parameters A, B^{-1}, E, G, and F are each (2×2), and H is (1×4).

Equations (4′), (5′), and (6′) are the estimation equations. They are appended with error terms to account for measurement errors and errors in optimization. They are estimated using the form specified in (8). $\dot{Z}$ is approximated discretely as $Z_t - Z_{t-1}$.[4] Lagged milk price is used as a proxy for expected milk price. Equation (6′) is nonlinear in parameters and the quasi-fixed inputs are jointly dependent variables; thus, the system is estimated using nonlinear three-stage least squares (SYSNLIN, the nonlinear estimation program in SAS). Instruments for the quasi-fixed inputs are estimated using current input prices, lagged output price, and lagged quasi-fixed input quantities. The resulting estimates are asymptotically efficient.

Data

The model was estimated using annual data for years 1951–82. The quantity of milk produced in the United States (as approximated by the combined marketings of milk and cream) and the average blend price for milk were from *Milk: Production, Disposition, and Income* (USDA 1951–83b). Pounds of concentrate fed per cow and concentrate price were from *Milk Production* (USDA 1955–83a). Concentrate prices before 1955 were computed from the milk/feed price ratio in the same publication (USDA 1951–55). Prices of dairy cows and hay were from *Agricultural Prices* (USDA 1965, 1984a). Prices of cull dairy cows sold for slaughter were from *Prices Received by Farmers* (USDA 1953–84).

The rental price of cows was computed as a

[3] In an early specification land was included as a quasi-fixed input. Because of severe collinearity between the time variable, price of milk, and land, the land variable was excluded from the model.

[4] A first-order differential equation using annual data perhaps is a simplistic way of modeling the change in U.S. dairies since it takes two years to raise a replacement heifer, and replacement and culling decisions are based on several factors excluded from the present model. However, the partial adjustment from the actual to the desired level is being modeled. This adjustment is manifested in the net change of the aggregate dairy herd. Because the aggregate U.S. replacement herd is large, and the objective is to model the rate of change in the producing herd, a first-order differential equation was regarded as an approximation to the dynamic adjustment of the aggregate dairy herd. To examine the adequacy of this approximation, the unrestricted (2SLS) model was reestimated with alternative orders on the differential equation for dairy cows, ranging from one to four years. The first-order differential equation gave the lowest sum of squared errors among the four alternatives and thus was retained as the dynamic specification. In addition, a constant rate of depreciation was assumed since the total U.S. cow herd, rather than individual cows, is subject to depreciation in the model.

discounted stream of payments on a replacement heifer kept for three lactations that would make a producer indifferent between paying three annual payments or a cash purchase price plus the discounted value of maintenance feed costs less cow salvage value, at a discount rate of 3%.[5]

The number of cows that have calved was from *Milk: Production, Disposition, and Income* (USDA 1951–83b). The productivity of cows in the United States has increased dramatically over the period in question through breeding, improved management, and feeding practices. Disembodied technological change is accounted for by $H(T)$. However, at least part of the technological change that has taken place was embodied by using a quality index on the cow numbers. The index was computed by adjusting average U.S. milk yield (1982 base) by the average predicted difference for milk (*PDM*) for Holstein (the dominant dairy breed) bulls in the United States.[6] Only the *PDM* for bulls was used rather than a weighted average of bulls and cows because the cows used to compute dam *PDM* account for less than 10% of the U.S. registered Holstein herd. The bull *PDM* is based on data from all Holstein bulls used for artificial insemination (AI) in the United States. Since about half of U.S. dairy cows are bred by AI, this constituted a much broader sample.

The bull *PDM* was lagged four years. Heifers are typically bred at approximately two years and begin milk production at about three years of age. Another year is added to allow for commercial distribution of semen. In addition, an exploratory analysis of quality indexes based on lags of three or five years produced models with unstable rates of adjustment for the alternative lag lengths.

The Holstein Association has maintained *PDM* records for AI bulls since 1950. *PDM*'s for 1947–49 were predicted by a linear regression of *PDM* on year. This OLS equation fit the data with $R^2 = .98$ and $F = 784$.

Labor quantity, in annual average number of workers per year in the U.S. dairy industry, was computed by taking the amount of labor in U.S. agriculture and multiplying it by the percentage of total U.S. agricultural labor hours used in dairy from *Economic Indicators of the Farm Sector* (USDA 1965, 1983c). Total agricultural labor was combined family and hired labor from *Agricultural Statistics* (USDA 1956–81) for the 1956–80 period and from tables provided by the National Economics Division, USDA, for 1950–55. Quarterly farm labor surveys that generated the annual labor data were discontinued after April 1981. Annual labor data for 1981–82 were extrapolated from farm labor surveys of April 1981, and July 1982. Wage rate indexes for 1951–80 were obtained by dividing the total expenditure on hired labor by the number of hired workers.[7] Expenditures for 1950–79 were from *Farm Income Statistics* (USDA 1979), and for 1980 from *Economic Indicators of the Farm Sector* (1980b). Wage rates for 1981–82 were computed by adjusting the 1980 wage rate by the rate of change of the average hourly farm labor wage rate from *Agricultural Statistics* (1980–83).

The technological change in labor quality that occurred largely from improved education over the period modeled was embodied by using a quality index on labor. Quality indexes for both family and hired labor estimated by Gollop and Jorgenson and extended by Ball were used to adjust labor. Ball's indexes are for the period 1948–79 and were extended to 1982 by predictions from a linear regression on time. This linear model fit the data with R^2's of .98 for both the hired and family labor indexes. The rental prices for cows and the wage rate for labor were computed by dividing expenditures by the quality-adjusted quantities.

The extent of collinearity in the independent variable matrix (including instruments for jointly dependent variables appearing on the right-hand side of the equations) was assessed by computing the condition index as a diagnostic. Scaling and centering the $X'X$ matrix of independent variables resulted in a condition index of 169. This suggests moderate but not strong collinearity (Hocking and Pendleton, p. 503).

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[5] There is no observable rental price for dairy cows, but amortizing the cash purchase price plus discounted value of maintenance feed cost less cow salvage value over the three-year period captures the effect of price changes on the investment decision while permitting use of a reasonable fraction of the price of capital as a proxy for rental price. Dairy cows in the United States produce milk for an average of three lactations, or slightly longer than three years. Since concentrates are fed for milk production and their demand is measured as a separate variable in the model, only maintenance feed costs are included in this variable. They are calculated using National Research Council maintenance feed requirements for dairy cows.

[6] The quality index for cows in year t is $(Average\ Milk\ Production_{1982} + PDM_t)/Average\ Milk\ Production_{1982}$, where *PDM* is calculated relative to 1982.

[7] The wage index for hired labor was also used as a proxy for the shadow price of family labor.

Results and Discussion

The parameter estimates of (8) are reported in the first column of table 1 (the unrestricted model). Nearly half of the parameters were significant at the 5% level, which was quite robust compared to other estimated dynamic dual models (e.g., Epstein and Denny; Vasavada and Chambers 1986). The model explained nearly all of the variation in the input demand equations but less than a third in the output supply equation. The R^2 for the milk supply, feed demand, cow demand, and labor demand equations were .29, .97, .996, and .98, respectively. The adjustment rates for cows ($M_{11} = B_{11} + r$) and labor ($M_{22} = B_{22} + r$) were $-.090$ and $-.397$, respectively, and each was significantly different from -1.0, indicating that both cows and labor exhibited quasi-fixity.

The estimated adjustment rates imply that cow numbers adjust 9% of the way toward long-run optimal levels in one year, and labor adjusts 40%. Vasavada and Chambers (1986) estimated a much slower labor adjustment of $-.069$, but they modeled total agricultural labor rather than labor in a specialized subsector, as in this model. The slow adjustment of cows is consistent with the very inelastic short-run milk supply found in previous studies. The rate of adjustment of cows cannot be compared to other dynamic models of the U.S. dairy industry since the others have not explicitly estimated a rate of adjustment. Other studies have estimated the rate of adjustment of capital in aggregate agricultural production; their estimates have ranged from $-.12$ for the U.S. (Vasavada and Chambers 1986) to $-.55$ in the southeastern U.S. (Taylor and Monson).

Tests of Competitive Behavior and Differentiability

The model was estimated with linear homogeneity and concavity in quasi-fixed inputs maintained by the functional form. Tests of monotonicity, symmetry, and convexity in prices were conducted. The necessary monotonicity conditions on the value function, i.e., $J(\,)$ increasing in output price and decreasing in input prices, held at nearly all observations. They were violated (not significantly) only in labor price at four observations. Symmetry and convexity of $J(\,)$ in prices were tested sequentially. The results are reported in table 2. The test statistic used was the Gallant and Jorgenson T^0, which compares the minimized distance of the residual vectors of the restricted and unrestricted models, adjusted for the sample size. The resulting statistic is approximately a chi-square, with degrees of freedom equal to the number of restrictions.

Symmetry and convexity were not rejected at the .05 level. Symmetry required that $F_{ij} = F_{ji}$ and $G_{ij} = G_{ji}$. Symmetry was maintained while convexity was tested. Global convexity is satisfied when $E_{ij} < 0$, $i, j = 1, 2$, and F_{ij}, $G_{ij} < 0$, $i \neq j$.

Because of problems in attaining convergence subject to convexity, a grid search procedure was used to select the value of F_{12} (over the interval $-.000000005$ to -5.0) that minimized SSE. Other parameters were estimated by nonlinear three-stage least squares for a given value of F_{12}. The parameters of this theoretically restricted model are reported in the second column of table 1. The restricted model yielded smaller estimates of M_{11} and M_{22}, but each of the adjustment parameters were within one standard deviation of their unrestricted values (as were all other parameters except three).

Structural Tests

Independent adjustment, instantaneous adjustment, and several technological change hypotheses are nested hypotheses that were tested while maintaining homogeneity, symmetry, and convexity of the value function. The tests are reported in table 2. Independent and instantaneous adjustment were tested sequentially.

Independence of adjustment occurs when $M_{12} = M_{21} = 0$, and means that each quasi-fixed input adjusts toward its desired level independently of the other. The null hypothesis of independence was not rejected. Instantaneous adjustment (with independence maintained) is actually a test of the dynamic nature of the model. If $M_{ii} = -1$ and $M_{ij} = 0$, the ith quasi-fixed input adjusts instantaneously to its desired level and should actually be modeled as a variable input. Instantaneous adjustment was tested separately for labor and for cows. With independent adjustment maintained, the null hypothesis of instantanous adjustment of labor, $M_{22} = -1$, was firmly rejected. Restricting $M_{11} = -1$, i.e., instantaneous adjustment of cows, caused the system to not converge.

Table 1. Nonlinear Three-Stage Least Squares Parameter Estimates of the Value Functions

Parameter	Model: Unrestricted[a]	Theoretically Restricted[b]	Theoretically and Structurally Restricted[c]
A_{11}	13.98	13.64	14.34
	(4.484)	(3.970)	(3.587)
A_{12}	1.364	1.331	1.469
	(0.4844)	(0.4979)	(0.4736)
A_{21}	0.9627	0.9283	0.8145
	(1.092)	(1.085)	(1.068)
A_{22}	−0.1072	−0.1102	−0.1278
	(0.1244)	(0.1246)	(0.1240)
B_{11}	−0.1196	−0.07684	−0.09905
	(0.05680)	(0.03544)	(0.03561)
B_{12}	−0.01545	−0.02538	
	(0.01629)	(0.01474)	
B_{21}	−0.1580	−0.1977	
	(0.3455)	(0.1930)	
B_{22}	−0.4267	−0.3878	−0.4690
	(0.1366)	(0.1280)	(0.1222)
E_{11}	−10.57	−13.46	−12.12
	(4.760)	(3.435)	(2.284)
E_{12}	−1.274	−1.414	−0.2020
	(5.189)	(1.884)	(1.419)
E_{21}	−1.233	−0.9942	−0.8569
	(0.9519)	(0.5006)	(0.4679)
E_{22}	−0.2375	−0.1061	−0.4019
	(1.676)	(0.8110)	(0.7998)
F_{11}	−4.415	2.147	0.7225
	(2.401)	(4.494)	(2.404)
F_{12}	4.505	−0.00005	−0.00005
	(2.354)		
F_{21}	1.766		
	(2.611)		
F_{22}	−23.91	−23.26	−21.30
	(3.709)	(4.053)	(1.403)
G_{11}	31.02	28.22	25.95
	(6.726)	(4.116)	(3.864)
G_{12}	−4.444	−0.00010	−0.02353
	(5.833)	(0.2353)	(0.2267)
G_{21}	−0.005152		
	(0.4745)		
G_{22}	−4.707	−4.873	−4.721
	(0.7974)	(0.7618)	(0.7576)
H_1	0.1136	0.1609	0.1847
	(0.0837)	(0.04508)	(0.04554)
H_2	−0.1631	−0.1668	−0.1674
	(0.01426)	(0.01334)	(0.01331)
H_3	0.1204	0.07845	0.09397
	(0.02286)	(0.04455)	(0.02172)
H_4	0.3877	0.4323	0.3763
	(0.05307)	(0.07535)	(0.04622)

Note: Standard errors of the estimates are in parentheses. MSE = 1.5688 with 104 degrees of freedom for the unrestricted model, 1.6987 with 107 degrees of freedom for the theoretically restricted model, and 1.8030 with 109 degrees of freedom for the structurally restricted model.

[a] Homogenous in prices.

[b] Homogenous, symmetric, and convex in prices.

[c] Homogenous, symmetric, convex with independent adjustment of cows and labor.

Consequently, no test statistic can be provided.

The last hypotheses to be tested dealt with technological change. The theoretical properties and independence of adjustment were maintained. The null hypothesis of no change

Table 2. Tests of Hypotheses

Hypothesis	Test Statistic	Critical Value
Symmetry		
$F_{12} = F_{21}, G_{12} = G_{21}$	1.347	$\chi^2_{2,.05} = 5.991$
Convexity[a]		
$E_{ij} < 0, i, j = 1, 2$		
$F_{ij}, G_{ij} < 0, i \neq j$	2.790	$\chi^2_{6,.05} = 12.592$
Independent Adjustment[b]		
$B_{12} = B_{21} = 0$	3.356	$\chi^2_{2,.05} = 5.991$
Instantaneous Adjustment of Labor[c]		
$B_{22} = -1.0$	56,186	$\chi^2_{1,.05} = 3.841$
Instantaneous Adjustment of Cows[c]		
$B_{11} = -1.0$	Did not converge	$\chi^2_{1,.05} = 3.841$
No Technological Change[c]		
$H_i = 0, i = 1, \ldots, 4$	Did not converge	$\chi^2_{4,.05} = 9.488$
No Disembodied Technological Change in Cows[c]		
$H_3 = 0$	7.802	$\chi^2_{1,.05} = 3.841$
No Disembodied Technological Change in Labor[c]		
$H_4 = 0$	41.304	$\chi^2_{1,.05} = 3.841$

[a] Symmetry maintained.
[b] Symmetry and convexity maintained.
[c] Symmetry, convexity, and independent adjustment maintained.

in technology for the period modeled, $H_i = 0$, for $i = 1, \ldots, 4$, caused the system to not converge. Null hypotheses that the quality index on cows or labor fully captured technological change for these inputs (i.e., $H_3 = 0$ or $H_4 = 0$) were rejected.[8]

Implications of Final Model

Parameter estimates for the model maintaining all nonrejected hypotheses (i.e., symmetry and convexity of prices, independent adjustment of quasi-fixed inputs, and disembodied technological change) are reported in the last column of table 1. Monotonicity conditions were satisfied at every observation. The adjustment rates were quite stable between the unrestricted and final model. Only two price parameters and one time parameter changed more than one standard deviation from the unrestricted model.

Adjustment costs were implicitly estimated along with the rates of adjustment. Applying the envelope theorem to the value function yields the marginal effect of investment on output in a derivable form: $PF_{\dot{z}} = -J_z$. From equation (8) $J_z = -\{[PW]'A + [C'M^{-1}]\}$. As $M \rightarrow 0$, i.e., quasi-fixed inputs adjust more slowly, the cost of adjustment or shadow price of the quasi-fixed input increases. For example, the rental cost of cows per head in 1982 was \$369 per year. Positive adjustment costs increase the cost of immediately reaching the desired level of cows to \$3,599 per head. The wage rate index in 1982 was .95, but adjustment costs increased the cost of immediately reaching the desired level to 3.34. Rejecting $M_{ii} = -1$ rejected the absence of positive adjustment costs for the ith input.

Short- and long-run elasticities for selected years computed from the final model are reported in table 3. All the short-run own-price input demand elasticities were negative, but the output own-price elasticity was positive for only fifteen of the thirty-two observations. Unlike static models, dynamic models do not restrict short-run output supply to be positive in order to have a profit-maximizing solution, so these results are not theoretically inconsistent (Treadway, pp. 344–45). Recall that $F_{\dot{z}} < 0$, so that if producers respond to an increase in output price by increasing their level of

[8] The upper bound on the probability of rejecting a true joint null hypothesis by sequential testing is the sum of the individual alpha levels. Thus, the probability of an instantaneous adjustment of labor and cows is at most 20%. If an upper limit of 5% were desired, that joint hypothesis still would have been rejected (under the assumption that the joint alpha level was equally divided among the sequential tests). The same conclusion applies to the joint hypothesis of symmetry, convexity, and any of the three technological conditions.

Table 3. Short- and Long-Run Output Supply and Input Demand Elasticities for the U.S. Dairy Industry, Selected Years

		Elasticity with Respect to the Price of			
Quantity	Year	Milk	Feed	Cows	Labor
Short run					
Milk	1951	0.052	−0.011	−0.043	0.002
	1962	0.014	−0.009	−0.007	0.002
	1972	−0.004	−0.007	0.010	0.002
	1982	−0.075	−0.006	0.078	0.003
Feed	1951	0.021	−0.054	0.018	0.014
	1962	0.019	−0.044	0.012	0.012
	1972	0.018	−0.046	0.016	0.012
	1982	0.021	−0.043	0.012	0.010
Cows	1951	0.043	0.003	−0.046	0.000
	1962	0.060	0.003	−0.064	0.000
	1972	0.093	0.004	−0.097	0.000
	1982	0.128	0.003	−0.131	0.000
Labor	1951	0.002	0.003	0.000	−0.005
	1962	0.002	0.003	0.000	−0.005
	1972	0.004	0.004	0.000	−0.008
	1982	0.011	0.008	0.000	−0.019
Long run					
Milk	1951	0.234	0.003	−0.235	−0.002
	1962	0.181	0.003	−0.182	−0.002
	1972	0.178	0.002	−0.179	−0.002
	1982	0.144	0.002	−0.145	−0.001
Feed	1951	−0.007	−0.055	0.049	0.013
	1962	−0.006	−0.045	0.040	0.011
	1972	−0.006	−0.046	0.041	0.011
	1982	−0.006	−0.043	0.041	0.009
Cows	1951	0.986	0.014	−1.001	0.000
	1962	1.443	0.019	−1.463	0.000
	1972	2.473	0.025	−2.498	0.000
	1982	1.777	0.011	−1.788	0.000
Labor	1951	0.003	0.006	0.000	−0.010
	1962	0.005	0.008	0.000	−0.013
	1972	0.008	0.010	0.000	−0.018
	1982	0.030	0.023	0.000	−0.053

Note: Calculated from the model maintaining symmetry, convexity, and independent adjustment of cows and labor.

quasi-fixed inputs, the short-run output supply elasticity may be negative and still consistent with profit-maximizing behavior.

The globally sufficient conditions for a maximum value function impose negative off-diagonal elements on the hessian of the value function, but they do not imply complementarity of inputs in either the short or long run. All the inputs were substitutes in the short run.

The short-run, own-price input demand elasticities for cows and labor became more elastic over time. The increasing own-price elasticity for labor was consistent with the increasing proportion of hired to family labor over the period.[9] The increasing own-price elasticity for cows was possibly due to the increased marketings of replacement heifers. Expansion in the dairy industry has been in large commercial dairies, particularly in the West and Southwest. These large dairies purchase more replacement heifers and are more responsive to price changes than small family-operated dairies. The cross-price elasticities for feed with respect to prices of cows and labor became less elastic over time. This may indicate that the amount and nutrient composition of feed per animal has become more fixed for a given milk price as information about feeding and milk production relationships has increased.

The short-run, cross-price input demand elasticities between cows and labor were effectively zero, indicating that not only did disequilibrium in the level of one quasi-fixed input not affect the other, but price changes in

[9] Hired agricultural labor as a percentage of total agricultural labor was 21.4% in 1951 and 38.0% in 1982.

one did not impact very much on the demand for the other quasi-fixed input.

Long-run elasticities are also reported in table 3. Except for four sets of cross-price elasticities, the long-run values were at least as large as the short-run elasticities, so the Le Chatelier principle held. The short-run cross-price output supply elasticities of milk with respect to price of feed were negative, while the long-run cross-price elasticities were positive. The sign changes were the opposite for input demand elasticities of feed with respect to the price of milk. The changing of signs from the short run to the long run is not theoretically inconsistent nor empirically uncommon (Berndt, Morrison, and Watkins). As previously noted, even when discounted profits are maximized, the short-run output supply elasticity may be negative; however, the long-run elasticity must be positive. Thus, both own-price output supply and several cross-price input demand elasticities changed signs in some years between the short run and long run.

Except for long-run demand elasticities of cows with respect to prices of milk and cows, all demands and supplies were highly inelastic. The inelasticity was particularly pronounced for labor demand in both the short run and long run.

With regard to long-run trends, milk supply became less elastic with respect to own price over time, and cows and labor became more elastic. The previous discussion about short-run input demand elasticity trends applies also to the long run.

Although no policy simulations were performed, several policy implications of this model suggest that short-term programs, such as the dairy diversion program, that affect economic incentives have had a very small marginal impact on the U.S. dairy industry. Short-run effects can be opposite of those desired, as evidenced by the negative short-run supply elasticity in a majority of years. In both the short and long run, milk supply was very inelastic and has become more inelastic over the last several years. Changing price supports to reduce the excess supply of dairy products is likely to have little impact in the short run. It may take only two years or so for labor to adjust in the dairy industry, but the herd level may take a decade to fully adjust. Long-term programs appear necessary if federal intervention in the dairy industry by way of price programs is to fully achieve desired results. But long-term programs are difficult to maintain when both the economic environment and policy makers change in the short term.

Summary and Conclusions

A dynamic dual model has been applied to the U.S. dairy industry in order to estimate the rate of adjustment of cows and labor to their desired levels while maintaining the underlying properties of the theory of the firm. Linear homogeneity of the value function in prices and concavity in quasi-fixed inputs were maintained. Symmetry and convexity of the value function in prices were not rejected. Independence of adjustment between cows and labor was not rejected, but instantaneous adjustment of either cows or labor was rejected. The rate of adjustment for cows and labor, while maintaining the nonrejected hypotheses, was estimated as $-.099$ and $-.469$, respectively. At these rates it took an estimated 10.1 years for cows to adjust to their optimal levels and 2.1 years for labor during the data period, 1951–82.

An important finding of this study was the documented validity of using a dynamic dual model on aggregated data for the dairy industry. Dual models are very structured because of their rigorous adherence to theoretical properties; yet these properties frequently have not been satisfied (or not examined) in reports of estimated dual models. The theoretical properties were consistent with the data used in this study, thus allowing investigation of the industry's structure while maintaining theoretical consistency.

Quality indexes on cows and labor are important for capturing exogenous changes in productivity, but additional disembodied technological change was not rejected. It may be possible to fully embody technological change in a quality index, but basing indexes of dairy cow productivity on *PDM* and labor productivity on the Gollop-Jorgenson quality index left considerable amounts of technological change disembodied.

Short-term dairy programs that do not consider the rates of adjustment of quasi-fixed inputs will likely have very small impacts on the U.S. dairy industry. Programs with at least a ten-year duration are required to fully achieve the potential effect of federal intervention in the dairy industry by way of price programs. Short-term programs probably should be limited to direct supply or quasi-fixed input controls. Programs directed toward the productive capacity of the industry, such as the herd buyout program, may be more appropriately conceived for influencing milk supplies than short-term incentive programs such as the dairy diversion program.

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References

Ball, V. E. "Output, Input, and Productivity Measurement in U.S. Agriculture, 1948–79." *Amer. J. Agr. Econ.* 67(1985):475–86.

Berndt, E. R., M. Fuss, and L. Waverman. "The Substitution Possibilities for Energy: Evidence from U.S. and Canadian Manufacturing Industries." *Model and Measuring Natural Resource Substitution,* ed. E. R. Berndt and B. C. Field, pp. 230–58. Cambridge MA: MIT Press, 1981.

Berndt, E. R., C. J. Morrison, and G. C. Watkins. "Dynamic Models of Energy Demand: An Assessment and Comparison." *Modeling and Measuring Natural Resource Substitution,* ed. E. R. Berndt and B. C. Field, pp. 259–89. Cambridge MA: MIT Press, 1981.

Blackorby, C., and W. Schworn. "Aggregate Investment and Consistent Intertemporal Technologies." *Rev. Econ. Stud.* 49(1982):595–614.

Chambers, R. G., and R. Lopez. "A General Dynamic Supply-Response Model." *Northeast J. Agr. and Resour. Econ.* 13(1984):142–54.

Chavas, J.-P., and R. M. Klemme. "Aggregate Milk Supply Response and Investment Behavior on U.S. Dairy Farms." *Amer. J. Agr. Econ.* 68(1986):55–66.

Dahlgran, R. A. "A Synthesis of Microeconomic Duality Theory and Distributed Lag Modeling with Implications for U.S. Dairy Policy." *N. Cent. J. Agr. Econ.* 7(1985):132–44.

Epstein, L. "Duality Theory and Functional Forms for Dynamic Factor Demands." *Rev. Econ. Studies* 48(1981):81–95.

Epstein, L., and M. S. Denny. "The Multivariate Flexible Accelerator Model: Its Empirical Restriction and an Application to U.S. Manufacturing." *Econometrica* 51(1983):647–74.

Gallant, A. R., and D. W. Jorgenson. "Statistical Inference for a System of Simultaneous, Non-linear, Implicit Equations in the Context of Instrumental Variable Estimation." *J. Econometrics* 11(1979):275–302.

Gollop, F., and D. W. Jorgenson. "U.S. Productivity Growth by Industry, 1949–73." *New Developments in Productivity Measurement and Productivity Analysis,* ed. J. Kendrick and B. Vaccara, pp. 17–136. National Bureau of Economic Research Studies in Income and Wealth, vol. 44, Chicago: University of Chicago Press, 1980.

Halvorson, H. W. "The Supply Elasticity of Milk in the Short Run." *J. Farm Econ.* 37(1955):1186–97.

Hillier, F. S., and G. J. Lieberman. *Operations Research,* 2nd ed. San Francisco: Holden-Day, 1974.

Hocking, R. R., and O. J. Pendleton. "The Regression Dilemma." *Comm. Statist.-Theor. Meth.* 12(1983): 497–527.

LaFrance, J. T., and H. de Gorter. "Regulation in a Dynamic Market: The U.S. Dairy Industry." *Amer. J. Agr. Econ.* 67(1985):821–31.

Lopez, R. E. "Supply Response and Investment in the Canadian Food Processing Industry." *Amer. J. Agr. Econ.* 67(1985):40–48.

McLaren, K. R., and R. J. Cooper. "Intertemporal Duality: Application to the Theory of the Firm." *Econometrica* 48(1980):1755–62.

National Research Council. *Nutrient Requirements of Dairy Cattle,* 5th ed. Washington DC: National Academy of Science, 1978.

Nerlove, M. "Lags in Economic Behavior." *Econometrica* 40(1972):221–51.

Prato, A. A. "Milk Demand, Supply, and Price Relationships, 1950–1968." *Amer. J. Agr. Econ.* 55(1973): 217–22.

Taylor, T. G., and M. J. Monson. "Dynamic Factor Demands for Aggregate Southeastern United States Agriculture." *S.J. Agr. Econ.,* no. 2 (1985), pp. 1–9.

Treadway, A. B. "Adjustment Costs and Variable Inputs in the Theory of the Competitive Firm." *J. Econ. Theory* 2(1970):329–47.

U.S. Department of Agriculture. *Agricultural Prices.* Washington DC, 1965, 1984a.

———. *Agricultural Statistics.* Washington DC, 1956–83, various issues.

U.S. Department of Agriculture, Economic Research Service. *Economic Indicators of the Farm Sector: Income and Balance Sheet Statistics.* Washington DC, 1965, 1980b.

———. *Economic Indicators of the Farm Sector: Production and Efficiency Statistics.* Washington DC, 1965, 1983c.

U.S. Department of Agriculture; Economics, Statistics, and Cooperatives Service. *Farm Income Statistics.* ESCS Statist. Bull. No. 627. Washington DC, Oct. 1979.

U.S. Department of Agriculture, Statistical Reporting Service. *Milk Production.* Washington DC, 1951–83a.

———. *Milk: Production, Disposition, and Income.* Washington DC, 1951–83b.

———. *Prices Received by Farmers.* Washington DC, 1953–84.

Vasavada, Utpal, and Robert G. Chambers. "Investment in U.S. Agriculture." *Amer. J. Agr. Econ.* 68 (1986):950–60.

———. "Testing Empirical Restrictions of the Multivariate Flexible Accelerator in a Model of U.S. Agricultural Investment." Paper presented at the AAEA meeting, Logan UT, Aug. 1982.

Wilson, R. T., and R. G. Thompson. "Demand, Supply and Price Relationships for the Dairy Sector, Post-World War II Period." *J. Farm Econ.* 49(1967):360–71.