

# Dynamic Analysis of Modified Composite Helicopter Blade

**Dimitrios Garinis**

Design Engineer  
Pratt and Whitney Canada

**Mirko Dinulović**

Assistant Professor  
University of Belgrade  
Faculty of Mechanical Engineering

**Boško Rašuo**

Full Professor  
University of Belgrade  
Faculty of Mechanical Engineering

*In the present study, modal analysis has been performed on modified Gazelle helicopter blade. The construction of the blade is fully composite with the honeycomb core. The approach to determining structure mode shapes and natural frequencies is presented. Modified blade consists of core material, 3D unidirectional composite spar and thin carbon composite facesheets as blade skin. To determine the stiffness of the honeycomb core, the equivalent mass approach was used. Several methods of eigenvalue extraction have been investigated in order to find optimal method which can be used in dynamic analysis of composite structures containing honeycomb cores. Among all extraction methods investigated, it was found that combined Lanczos method is most effective in terms of accuracy and CPU time for eigenvalue extraction in composite structures with honeycomb core having large number of degrees of freedom. Strain energies for first four mode shapes of modified helicopter blade have been calculated using numerical approach and results are presented.*

**Keywords:** normal mode, natural frequencies, helicopter composite blade, hexagonal honeycomb core, carbon fiber.

## 1. INTRODUCTION

Because of the very serious effects that unwanted vibrations can have on rotating helicopter blades, it is essential that vibration analysis be carried out as an inherent part of their design; there are two factors that control the amplitude and frequency of vibration in such a structure: the excitation applied and the response of the structure to that particular excitation. This is because vibration creates dynamic stresses and strains which can cause fatigue and failure of the complete structure, fretting corrosion between contacting elements and noise in the environment; also it can impair the function and life of the blade itself.

It is necessary to analyze the vibration in order to predict the natural frequencies and the response to the expected excitation.

The natural frequencies of the structure must be found because if the structure is excited at one of these frequencies resonance occurs, resulting in high vibration amplitudes, dynamic stresses and noise levels [1]. Resonance should be avoided and the structure designed so that it is not encountered during normal conditions; this often means that the structure needs to be analyzed over the expected frequency range of excitation.

Vibration analysis of helicopter blades can be carried out most conveniently by adopting the following three-stage approach:

Stage I Devise a mathematical or physical model of the structure to be analyzed.

Stage II From the model, write the equations of motion.

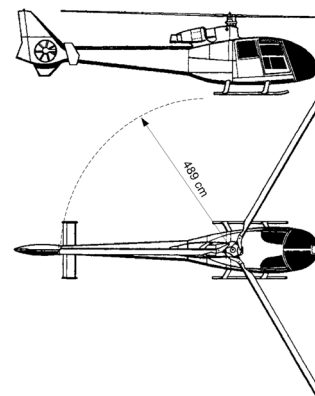
Stage III Evaluate the structure response to a relevant specific excitation.

Natural frequencies and corresponding mode shapes are functions of the structural properties and boundary conditions.

Computation of the natural frequencies and mode shapes is performed by solving an Eigen value problem. In order to assess the dynamic interaction between a component and its supporting structure natural frequencies must be computed. Decisions regarding subsequent dynamic analyses (transient response, frequency response, response spectrum analysis, etc.) can be based on the results of a natural frequency analysis [2,3].

## 2. MODIFIED BLADE CONSTRUCTION

The overall dimensions of the helicopter blade are given on the following picture (Fig. 1).



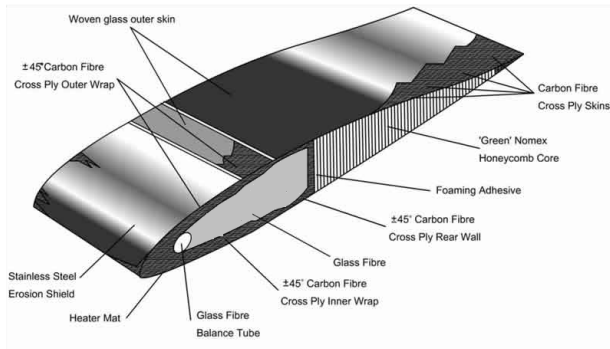
**Figure 1. Helicopter blade dimensions**

The original Gazelle helicopter blade [4] was modified and consists of Nomex honeycomb core, carbon fiber  $\pm 45^\circ$  crossply wrap, stainless steel corrosion shield, glass fiber balancing tube, carbon fiber

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Correspondence to: Dr Boško Rašuo  
Faculty of Mechanical Engineering,  
Kraljice Marije 16, 11120 Belgrade 35, Serbia  
E-mail: brasuo@mas.bg.ac.rs

+/-45° crossply inner wrap, glass fiber thick spar and carbon fiber crossply skins. The structure of the modified rotor blade investigated in this paper is presented in the following picture (Fig. 2).



**Figure 2. Modified helicopter blade construction**

### 3. ANALYTIC SOLUTIONS

The solution of the equation of motion for natural frequencies and normal modes requires a special reduced form of the equation of motion. If there is no damping and no applied loading, the equation of motion in matrix form reduces to [5]:

$$[M]\{\ddot{u}\} + [K][u] = 0, \quad (1)$$

where  $[M]$  is the mass matrix and  $[K]$  is the stiffness matrix. This is the equation of motion for undamped free vibration. To solve previous equation, harmonic solution is assumed in the following form:

$$\{u\} = \{\phi\} \sin \omega t, \quad (2)$$

where  $\{\phi\}$  are the mode shapes and  $\omega$  is the circular natural frequency. Aside from this harmonic form being the key to the numerical solution of the problem, this form also has a physical importance. The harmonic form of the solution means that all the degrees-of-freedom of the vibrating structure move in a synchronous manner.

The structural configuration does not change its basic shape during motion; only its amplitude changes. If differentiation of the assumed harmonic solution is performed and substituted into the equation of motion, the following is obtained:

$$-\omega^2 [M]\{\phi\} \sin \omega t + [K]\{\phi\} \sin \omega t. \quad (3)$$

This after simplification becomes:

$$([K] - \omega^2 [M])\{\phi\} = 0. \quad (4)$$

This represents the eigenequation, which is a set of homogeneous algebraic equations for the components of the eigenvector and forms the basis for the eigenvalue problem.

The basic form of an eigenvalue problem is:

$$[A - \lambda I]x = 0, \quad (5)$$

where  $[A]$  is a square matrix,  $\lambda$  are eigenvalues,  $[I]$  is identity matrix and  $x$  eigenvector. Assuming non-trivial solution, the  $\det([K] - \omega^2 [M])$  is zero only at a set of

discrete eigenvalues  $\omega_i^2$ , therefore the equation can be rewritten:

$$([K] - \omega_i^2 [M])\{\phi_i\} = 0, \quad i = 1, 2, 3, \dots \quad (6)$$

Each eigenvalue and eigenvector define free vibration mode of the structure. The  $i$ -th eigenvalue  $\lambda_i$  is related to the  $i$ -th natural frequency  $f_i = \omega_i / 2\pi$ .

Modal quantities can be used to identify problem areas by indicating the more highly stressed elements. Elements that are consistently highly stressed across many or all modes will probably be highly stressed when dynamic loads are applied. Modal strain energy is a useful quantity in identifying candidate elements for design changes to eliminate problem frequencies. Elements with large values of strain energy in a mode indicate the location of large elastic deformation (energy). These elements are those which most directly affect the deformation in a mode. Therefore, changing the properties of these elements with large strain energy should have more effect on the natural frequencies and mode shapes than if elements with low strain energy were changed.

### 4. METHODS OF COMPUTATION

In present analysis two groups of methods for eigenvalue extraction are investigated in order to determine the most efficient method that can be used when composite structures with solid or honeycomb cores are analyzed.

Methods analyzed are Transformation methods and Tracking methods [6,7]. In the transformation method, the eigenvalue equation is transformed into a special form from which eigenvalues may easily be extracted. In the tracking method, the eigenvalues are extracted one at a time using an iterative procedure.

In the present work four transformation methods are analyzed: Givens method, Householder method, modified Givens method and modified Householder method. Two tracking methods analyzed are inverse power method and Sturm modified inverse power method. The Givens and Householder modal extraction methods require a positive definite mass matrix (all degrees-of-freedom must have mass). There is no restriction on the stiffness matrix except that it must be symmetric. These matrices always result in real (positive) eigenvalues.

The Givens and Householder methods are the most efficient methods for small problems and problems with dense matrices when large portions of the eigenvectors are needed. These methods find all of the eigenvalues and as many eigenvectors as requested. While these methods do not take advantage of sparse matrices, they are efficient with the dense matrices sometimes created using dynamic reduction. The Givens and Householder methods fail if the mass matrix is not positive definite.

To minimize this problem, degrees-of-freedom with null columns are removed by the application of static condensation. The modified Givens and modified Householder methods are similar to their standard methods with the exception that the mass matrix can be singular. Although the mass matrix is not required to be

nonsingular in the modified methods, a singular mass matrix can produce one or more infinite eigenvalues.

Due to round off error, these infinite eigenvalues appear in the output as very large positive or negative eigenvalues. To reduce the incidence of such meaningless results, degrees-of-freedom with null masses are eliminated by static condensation as in the case of the unmodified methods.

The modified methods require more computer time than the standard methods. The inverse power method is a tracking method since the lowest eigenvalue and eigenvector in the desired range are found first. Then their effects are “swept” out of the dynamic matrix, the next higher mode is found, and its effects are “swept” out, and so on. In addition, each root is found via an iterative procedure. However, the inverse power method can miss modes, making it unreliable.

Sturm sequence logic ensures that all modes are found. The Sturm sequence check determines the number of eigenvalues below a trial eigenvalue, and then finds all of the eigenvalues below this trial eigenvalue until all modes in the designed range are computed. This process helps to ensure that modes are not missed. The Sturm modified inverse power method is useful for models in which only the lowest few modes are needed. This method is also useful as a backup method to verify the accuracy of other methods.

The Lanczos method overcomes the limitations and combines the best features of the other methods. It requires that the mass matrix is positive semidefinite and the stiffness is symmetric.

Like the transformation methods, it does not miss roots, but has the efficiency of the tracking methods, because it only makes the calculations necessary to find the roots requested by the user. This method computes accurate eigenvalues and eigenvectors. Unlike the other methods, its performance has been continually enhanced since its introduction giving it an advantage. The Lanczos method is the preferred method for most medium to large-sized problems, since it has a performance advantage over other methods.

The analysis of extraction methods is presented in the following table (Table 1).

## 5. MODELLING

Finite element model of the analyzed helicopter blade is presented in the following figure (Fig. 3).

Modelling of 3D structure is generally more complex and tedious. In the present model eight nodal hexahedron elements are being used for meshing the

sandwich structure. Plate elements, based on Kirchoff thin plate theory are used for modelling top and bottom skins of the analyzed helicopter blade.



Figure 3. FEA of the helicopter blade

Detailed mesh construction at the blade rotor tip is presented in the following picture (Fig. 4):

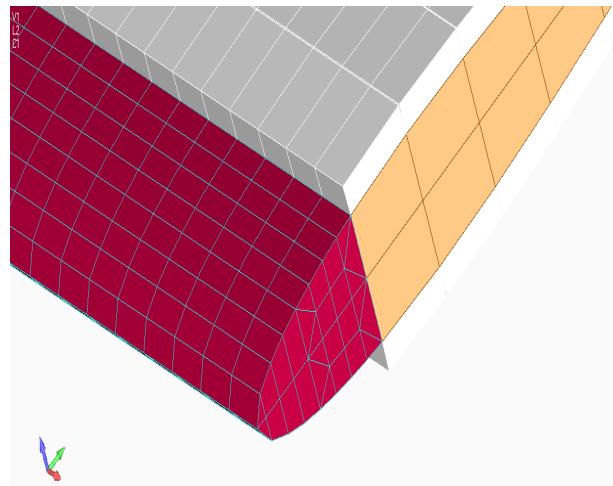


Figure 4. Detailed FEA Mesh at the rotor tip (leading edge top skin removed for clarity)

### 5.1 Honeycomb core model

It is assumed that the core can resist the transverse shearing deformation and has some in-plane stiffness, while the top and bottom surface layer cannot resist the shearing deformation but satisfy the Kirchoff hypothesis (Fig. 5). Under the above assumption the honeycomb core can be regarded as an orthotropic layer.

For the hexagon honeycomb core with  $\theta=30^\circ$ , the equivalent elastic parameters are as follows [8,9,10].

Table 1. Comparison of methods for eigenvalue extraction

	Method				
	Givens, Householder	Modified Givens, Householder	Inverse power	Strum modified Inverse power	Lanczos
Reliability	High	High	Low	High	High
Cost:					
Few modes	Medium	Medium	Low	Low	Medium
Many modes	High	High	High	High	Medium
Limitations	Cannot analyze singular [M]	-	Can miss modes	-	Difficulties with mechanisms
Application	Small models (DOF)	Small models (DOF)	Few modes	Few modes	Large (DOF) models

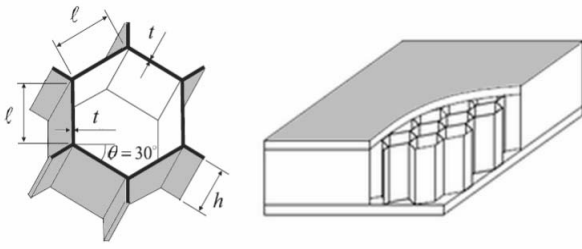


Figure 5. Honeycomb structure of the core with face sheets

$$\begin{aligned} E_{11} = E_{22} &= \frac{4}{\sqrt{3}} \left( \frac{t}{l} \right)^3 E, \quad G_{12} = \frac{\sqrt{3}}{2} \cdot \xi \cdot \left( \frac{t}{l} \right)^3 E, \\ G_{13} &= \frac{1}{\sqrt{3}} \cdot \xi \cdot \left( \frac{t}{l} \right) G, \quad G_{23} = \frac{\sqrt{3}}{2} \cdot \xi \cdot \left( \frac{t}{l} \right) G, \end{aligned} \quad (7)$$

where  $E$  and  $G$  are the engineering constants of the core material;  $l$  and  $t$  are the length and thickness of the honeycomb cell. In the previous equations coefficient  $\xi$  is the technology correction coefficient. The value of correction coefficient varies between 0.4 and 0.6 when the honeycomb is fabricated by the expansion method.

Stress strain analysis of a single hexagon [11] cell has been performed and the deformed state is presented in the following picture (Fig. 6).

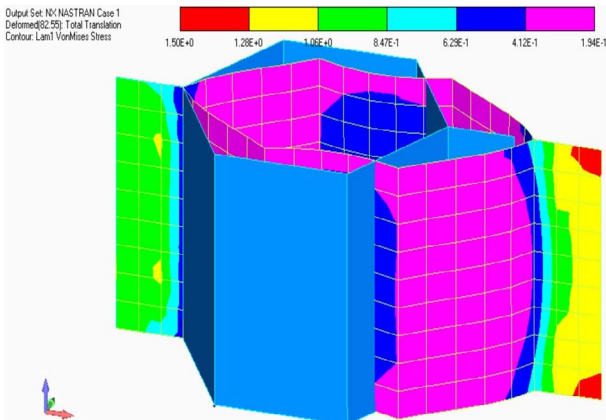


Figure 6. Deformed honeycomb cell

## 5.2 Facesheets model

Facesheets are considered to be thin composite laminates with continuous fibers. The stress strain relation for composite facesheet lamina can be expressed as [12]:

$$\begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_{11}} & -\nu_{21} & -\nu_{31} & \eta_{2,11} & \eta_{23,11} & \eta_{31,11} \\ E_{11} & E_{22} & E_{33} & G_{12} & G_{23} & G_{31} \\ -\nu_{12} & 1 & -\nu_{32} & \eta_{2,22} & \eta_{23,22} & \eta_{31,22} \\ E_{11} & E_{22} & E_{33} & G_{12} & G_{23} & G_{31} \\ -\nu_{13} & -\nu_{23} & 1 & \eta_{2,33} & \eta_{23,33} & \eta_{31,33} \\ E_{11} & E_{22} & E_{33} & G_{12} & G_{23} & G_{31} \\ \eta_{1,12} & \eta_{22,12} & \eta_{33,12} & 1 & \mu_{23,11} & \eta_{31,12} \\ E_{11} & E_{22} & E_{33} & G_{12} & G_{23} & G_{31} \\ \eta_{1,23} & \eta_{22,23} & \eta_{33,23} & \eta_{2,23} & 1 & \eta_{31,23} \\ E_{11} & E_{22} & E_{33} & G_{12} & G_{23} & G_{31} \\ \eta_{1,31} & \eta_{22,31} & \eta_{33,31} & \mu_{2,31} & \mu_{23,31} & 1 \\ E_{11} & E_{22} & E_{33} & G_{12} & G_{23} & G_{31} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} \quad (8)$$

Each lamina can be considered as orthotropic (three mutually perpendicular planes of material symmetry exist) and it can be regarded as transversely isotropic (one of the material planes is plane of isotropy). For the facesheet lamina, in previous relation coefficients of mutual influence ( $\eta_{ij,ji}$ ) and Chentsov coefficients ( $\mu_{ij,ji}$ ) are equal to zero since there is no interaction between normal stresses and shear strains, shear stresses and normal (axial) strains nor interaction between shear stresses and shear strains on different planes.

The remaining nine elastic coefficients in relation (8) can be determined as follows: The mechanics of materials predictions are adequate for longitudinal properties such as Young's modulus  $E_{11}$  and major Poisson ratio  $\nu_{12}$ . These properties are not sensitive to fiber shape and distribution.

In order to predict transverse and shear moduli Semi-empirical Tsai-Hahn relations (9,10) were used, since the mechanics of materials approach underestimates the transverse and shear properties. Both  $E_{22}$  and  $G_{12}$  are sensitive to void content, fiber anisotropy and the matrix Poisson's ratio.

$$E_{22} = \frac{(V_f + \eta'_{22} \cdot V_m) \cdot E_f \cdot E_m}{(E_m \cdot V_f + \eta'_{12} \cdot V_m \cdot E_f)}, \quad (9)$$

$$G_{12} = \frac{(V_f + \eta'_{12} \cdot V_m) \cdot G_f \cdot G_m}{(G_m \cdot V_f + \eta'_{12} \cdot V_m \cdot G_{12})}. \quad (10)$$

In Tsai-Hahn relations coefficients  $\eta'_{ij}$  are stress partitioning parameters. They are experimentally obtained. Typical values for  $\eta'_{22}$  and  $\eta'_{12}$  for epoxy matrix composites are given in the following table (Table 2).

Table 2 stress partitioning parameters for epoxy matrix composites

fiber	$\eta'_{22}$	$\eta'_{12}$
Carbon	0.500	0.400
Glass	0.516	0.316
Kevlar 49	0.516	0.400

Using Betti's reciprocal law according to which transverse deformation due to a stress applied in the longitudinal direction is equal to the longitudinal direction due to an equal stress applied in the transverse direction, the following relation for composite laminate facesheets can be written:

$$\frac{\nu_{ij}}{E_{ii}} = \frac{\nu_{ji}}{E_{jj}} \quad (i, j = 1, 2, 3). \quad (11)$$

In the case of transversely isotropic material with the 2-3 plane as the plane of isotropy elastic parameters are:

$$\begin{aligned} E_{22} = E_{33}, \quad G_{13} = G_{12}, \\ \nu_{12} = \nu_{13}, \quad G_{23} = \frac{E_{22}}{2(1 + \nu_{23})}. \end{aligned} \quad (12)$$

The Poisson ratio coefficient  $\nu_{23}$ , as a function of constituents' properties and volume fractions can be expressed as:

$$v_{23} = v_f \cdot V_f + v_m \cdot (1 - V_f) \cdot \left[ \frac{1 + v_m - v_{12} \cdot \frac{E_m}{E_{11}}}{1 - v_m^2 + v_m \cdot v_{12} \cdot \frac{E_m}{E_f}} \right] \quad (13)$$

In the previous equation  $V_f$  is the fiber volume fraction in the composite,  $v_f$  and  $v_m$  are fiber and matrix Poisson ratios respectively,  $E_m$  is matrix modulus of elasticity and  $E_f$  is fiber longitudinal modulus of elasticity [13]. Based on equations (9) to (13) and rule of mixtures, all elastic coefficients in relation (8) can be determined for each lamina in the composite laminate facesheet based on known properties of constituents.

### 5.3 Adhesive model

It is assumed that the attachment of facesheets to honeycomb core is made by bonding at node bonds locations and that perfect bonding is achieved. Taking into consideration the elasto-plastic adhesive behaviour, shear stress-strain relation for a ductile adhesive is modelled by a two-parameter exponential fitting curve [14]. The adhesive non-linear model is added as the contact layer between blade skins, leading edge and honeycomb.

$$\tau_a = (G_a - kB_1)\gamma + B_1(1 - e^{-k\gamma}) \quad (14)$$

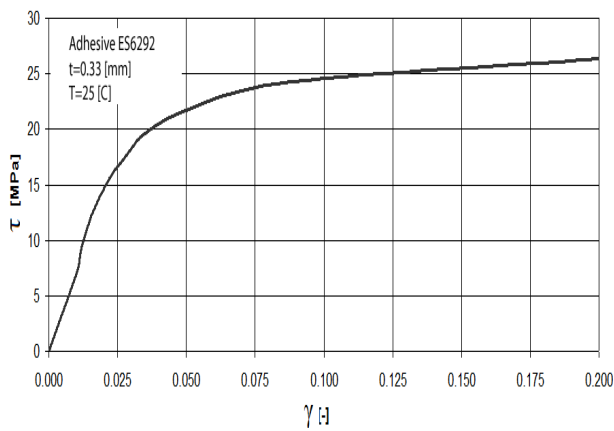


Figure 7. Adhesive stress-strain curve

In the previous equation  $\gamma$  is the strain in the adhesive and  $G_a$  adhesive shear modulus. The parameters,  $k$  and  $B_1$ , are chosen based on the following conditions: (a) the final stress at ultimate strain should equal the average between the ultimate and final stress and (b) the area of the fitting curve should match the area of the experimental data (Fig. 7).

## 6. NUMERICAL RESULTS

Using material models described in previous section for each component of the modified composite helicopter blade (blade skins, blade spar and core) modal analysis is performed. Eigenvalue extraction is performed using Lanczos method and the results for first four modes

with corresponding strain energies are presented in Figures 8,9,10 and 11 for each computed mode.

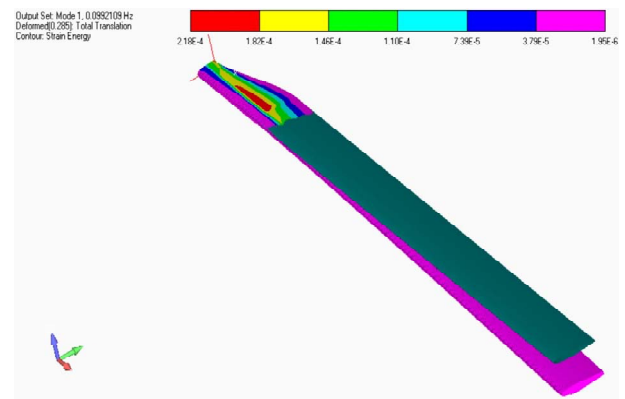


Figure 8. First mode, strain energy

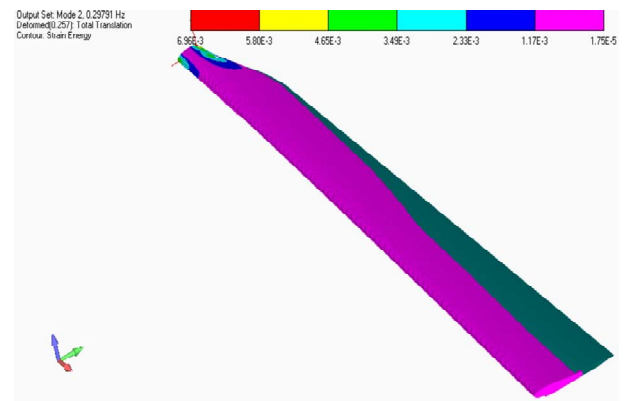


Figure 9. Second mode, strain energy

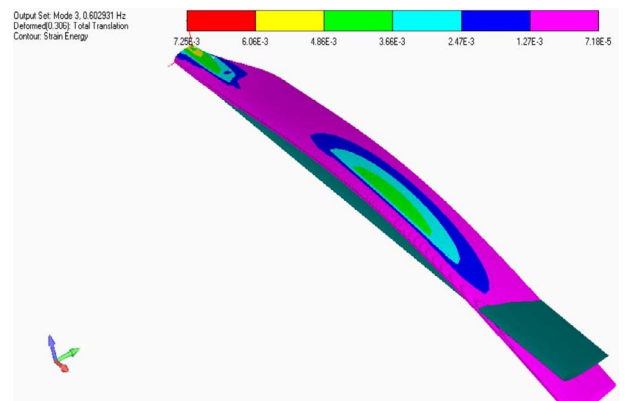


Figure 10. Third mode, strain energy

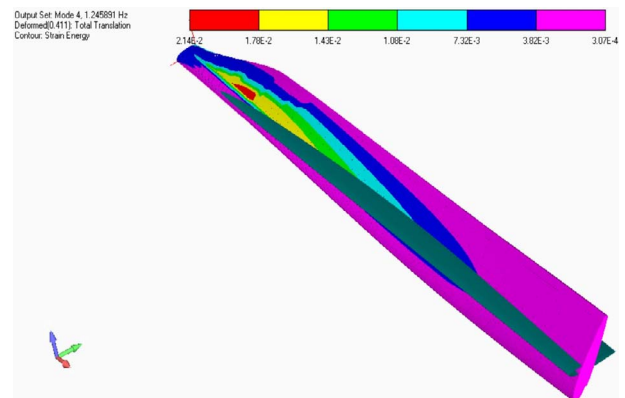


Figure 11. Fourth mode, strain energy

## CONCLUSION

In the present work, the new modeling approach for determining natural frequencies and normal modes of helicopter composite rotor blade is presented. It was found that the most effective method for matrix decomposition and eigenvalue extraction is Lanczos method. The accuracy and computing time is highly influenced by proper mesh creation and material models. The structure that contains honeycomb structure can be modeled as a continuum, using equivalent plate theory since the exact modeling of honeycomb at cell level is tedious and does not affect results accuracy. When composite structures with many degrees of freedom are analyzed, modeling of honeycomb core at cell level is not even possible and equivalent plate theories have to be used.

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### ДИНАМИЧКА АНАЛИЗА МОДИФИКОВАНЕ ХЕЛИКОПТЕРСКЕ ЛОПАТИЦЕ ОД КОМПОЗИТНИХ МАТЕРИЈАЛА

Димитриос Гаринис, Мирко Динуловић,  
Бошко Рашуо

У овом раду извршена је модална анализа модификоване лопатице хеликоптера „Газела“. Модификована лопатица је комплетно композитна са саћастом испуном. Приказан је метод одређивања модова осциловања и сопствених фреквенција. Модификована лопатица састоји се од саћасте испуне, рамењаче од 3Д усмереног композита и танких карбонских плоча као оплате. Да би се одредила матрица крутости испуне коришћен је метод еквивалентних маса. У циљу налажења оптималног метода за одређивање сопствених фреквенција испитано је неколико познатих метода. Метод Ланцоса показао је најтачније резултате кроз умерено процесорско време када је у питању одређивање сопствених фреквенција и модова осциловања код структура од композитних материјала са саћастим испунама. Овом методом израчуната су прва четири мода осциловања модификоване композитне лопатице, и приказани су резултати модова осциловања и деформационе енергије лопатице.