DYNAMIC ANALYSIS OF THE NAVAL POSTGRADUATE SCHOOL OCEAN INSTRUMENT PLATFORM

Mark Francis Crane

United States
Naval Postgraduate School

THESIS

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by

Mark Francis Crane

Thesis Advisor: E. B. Thornton

September 1971

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Mark Francis Crane
Lieutenant Commander, United States Navy
B.S., United States Naval Academy, 1963

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OCEANOGRAPHY

from the

NAVAL POSTGRADUATE SCHOOL
September 1971

ABSTRACT

The dynamic and static response of the proposed NPS ocean instrument platform is investigated by developing and solving linear differential equations of motion of the tower in surge, heave, and pitch. The motion is expressed as a response spectrum which is directly proportional to a wave spectrum as the exciting force. The analysis is made for various configurations of the lateral restraining cables using both a five point and nine point mooring system. For all configurations, the heave response of the tower is found to be less than one percent of wave height. The stability of the tower in pitch is found to be considerably improved after shifting from a five point mooring system to a nine point mooring system and optimizing the location of the cable attachment points. Using this design, a significant wave height of 7.7 feet is found to produce a significant pitch of 5.4 degrees, a significant surge of .97 feet, and a 5.28 foot excursion of the lower platform on the tower. All oscillations will be superimposed on any heel angle of the tower which may exist due to steady wind forces. The angle of heel for a wind of 20 mph is evaluated to be 1.9 degrees.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

TABLE OF SYMBOLS AND ABBREVIATIONS

A	Wave amplitude
b_1	Effective width of open tower
b_2	Effective width of closed tower
C	Damping coefficient
C_a	Added mass coefficient
C_D	Drag Coefficient
C_{DL}	Linearized drag coefficient
C_H	Damping coefficient for heave
C_m	Inertia coefficient
C_p	Damping coefficient for pitch
C_s	Damping coefficient for surge
C_w	Area constant for wire cable
d	Diameter
D_1	Distance from MWL to top of buoyancy material
D_2	Distance from MWL to bottom of tower
E	Modulus of elasticity
F	Force
F_1	Vertical force due to lateral cable tension
F_4	Tension in vertical cable
F_8	Vertical force due to lateral cable tension
F_B	Buoyance force
F_D	Drag force
F_{DD}	Drag disturbing force
F_{DDL}	Linearized drag disturbing force

F_{DL}	Linearized drag force
F_{DR}	Restoring force
F_I	Inertial force
F_{ID}	Inertial disturbing force
F_R	Restoring force
F_{WM}	Total wind force on mast of tower
F_{WS}	Total wind force on tower structure
g	Acceleration due to gravity
$\bar{h}$	Depth of water
$H_{1/3}$	Significant wave height
J_a	Added moment of inertia
J_o	Moment of inertia about the center of gravity
J_p	Virtual moment of inertia
k	Wave number
K	Stiffness coefficient
K_H	Equivalent stiffness coefficient for heave
K_L	Stiffness coefficient for lateral cables
K_p	Stiffness coefficient for pitch
K_s	Stiffness coefficient for surge
L_g	Distance from MWL to center of gravity
M	Mass of tower
M_a	Added Mass
M_D	Moment due to drag
M_{DD}	Disturbing moment due to drag
M_H	Virtual mass for heave
M_I	Moment due to inertial forces
M_{ID}	Disturbing moment due to inertial forces

M_s	Virtual mass for surge
M_R	Restoring moment
R_1	Distance from MWL to center of gravity
R_2	Distance between center of gravity and center of buoyancy
R_3	Distance between center of gravity and bottom of tower
t	Time variable
u	Horizontal water particle velocity
U	Windspeed
V	Volume
w	Vertical water particle velocity
x,y,z	Space coordinates
α	Angle which lateral cables make with horizontal
γ	Specific weight
ϕ	Phase angle
η	Instantaneous height of free water surface
ρ	Mass density of water
σ	Radial frequency of waves
θ	Roll or pitch of tower
$\theta_{1/3}$	Significant roll or pitch
ω	Natural frequency of oscillation of tower
ξ	(C/M)

ACKNOWLEDGEMENTS

Dr. Edward B. Thornton proposed the topic for this thesis and guided me during my investigation. I am especially indebted to him for his assistance, encouragement and professional advice.

I am also very deeply thankful to my wife, Pat, without whose cooperation and understanding I could not have completed this thesis.

I. INTRODUCTION

A. BACKGROUND

The Naval Postgraduate School is planning to install in approximately 240 feet of water in Monterey Bay a moored ocean instrument platform supported by a tower of approximately ninety feet in length. Such a platform could be used for the mounting of instrumentation designed to measure oceanographic and meteorological data. The tower under consideration was obtained from the government as surplus and was originally designed for use as an umbilical tower prior to launching U.S. Air Force "Thor" missiles. It is the purpose of this thesis to determine the dynamic response of the proposed tower to wave action in Monterey Bay. The instrument package, appendages, proposed location, and further background is described in a Naval Postgraduate School thesis by Lt. H.H. Seibert [3].

In analyzing the dynamic response of the tower to the force of the waves, basic design parameters of the tower are varied so as to achieve an optimum design configuration which results in minimum motion of the tower due to wave action. The equations of motion are developed for the tower by direct analogy to a mechanical spring-mass system with damping and sinusoidal driving forces described by linear wave theory. The tower will be taut moored to the bottom using four one-half inch plow steel lateral cables

plus a one inch plow steel center cable. The buoyancy chambers will be below mean water level at all times so as to provide a constant buoyancy force. The tower's natural frequency of oscillation is high in comparison to the frequency around which most of the wave energy is centered. This high natural frequency results in a stiff system design thereby preventing conditions of resonance which would otherwise occur if it were a softer system with a lower natural frequency of oscillation.

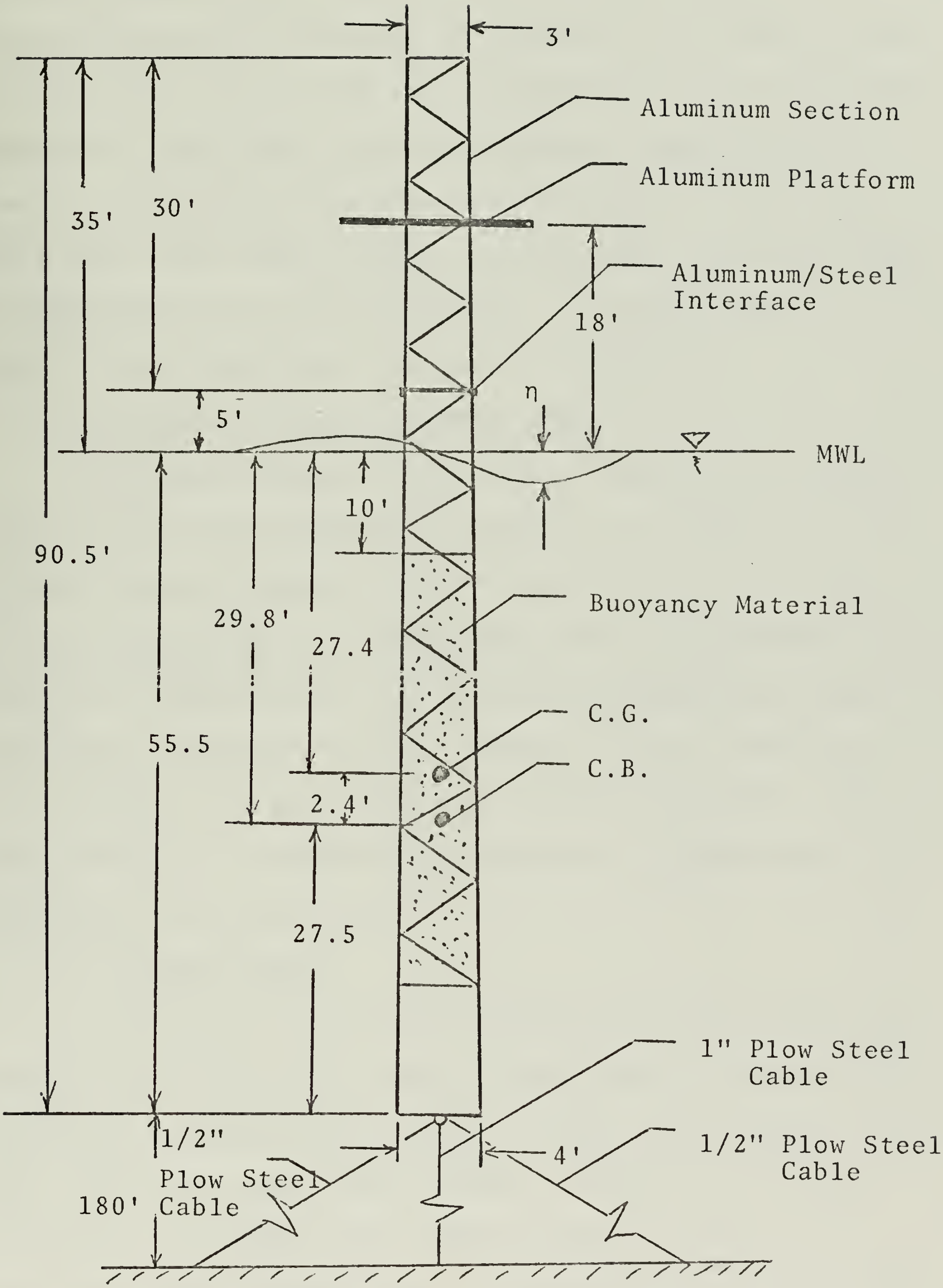
B. PLATFORM DESCRIPTION

1. General

Since a detailed description of the instrumentation and appendages to the tower was made by Lt. H.H. Seibert in his thesis, a description here is made of just the basic design of the tower, taking into consideration only those factors which affect its motion (see Figure 1). Although instrumentation and appendages are not shown in Figure 1, the location and mass of each item are considered in determining the tower's center of gravity and moment of inertia.

2. Dimensions

The overall length of the basic tower is 90.5 feet. A bottom steel section extends over a distance of 60.5 feet, and an upper aluminum section has a length of 30.5 feet. A 30.0 foot aluminum mast which is planned for installation at the top of the tower will extend the overall length to 120.5 feet. The cross section of the tower is square and has a constant width of 3.0 feet for the aluminum section.



3'
Aluminum Section
Aluminum Platform
35'
30'
Aluminum/Steel
Interface
18'
η
5'
MWL
10'
90.5'
Buoyancy Material
29.8'
27.4
C.G.
55.5
C.B.
2.4'
27.5
1" Plow Steel
Cable
1/2"
4'
1/2" Plow Steel
Cable
Plow Steel
180' Cable

FIGURE 1.

At the aluminum/steel intersection it has a width of 3.0 feet and gradually increases to a width of 4.0 feet at the bottom of the tower. The lower platform is 18.0 feet above mean water level (MWL), and the buoyancy material rises from 7.0 feet above the bottom of the tower to 10.0 feet below MWL. The steel section located just below the buoyancy material will act as ballast. It extends over a length of 7.0 feet below the buoyancy material.

3. Weights of Tower and Appendages

The steel section has a total weight (all weights taken in air unless otherwise stated) of 12,780 lbs. The aluminum section has an overall weight of 720 lbs., and all other appendages and equipment have a total weight of 8813 lbs. Table I is a list of the instrumentation and appendages giving wet and dry weights of each item and the distances of their center of gravity from MWL. The total weight of the tower including all equipment and appendages is 22,313 lbs.

4. Mooring Cables

There are four 680 ft ½ inch plow steel cables each weighing 0.37 lbs/ft (in water) amounting to a total weight (in water) of 664.0 lbs for all of the ½ inch mooring lines. The 1 inch plow steel cable weighs a total of 1.5 lbs/ft (in water), amounting to a total weight (in water) of 252.0 lbs. The assumed depth of mooring is 240 feet. This will require four lengths of ½ inch cable each extending out in directions 90 degrees apart and at an angle of 19

TABLE I

WEIGHTS OF TOWER EQUIPMENT AND APPENDAGES

Item	Dist. of C.G.* from MWL (ft.)	Dry Weight (lbs)	Wet Weight (lbs)
Mast	+50.0	60	60
Boat Fender	+ 2.0	840	400
Lwr Platform	+18.0	275	275
Upper Boom	+18.0	60	60
Rail	-17.0	600	544
Lwr Boom	-67.5	240	209
Ladder	-24.7	225	204
Wave Gauge	-23.0	93	81
Misc. Equip.	+20.0	1520	1520
Tackle	-33.0	2000	1814
Buoyancy Mat'l.**	-29.8	900	900
1/8 Sheet Metal	-29.8	2000	1740
Total Weight		8813	7907

*Plus sign indicates above MWL and negative sign indicates below MWL.

**Buoyancy considered separately.

degrees below the horizontal for a distance (slant) of 680 feet. The center 1 inch vertical mooring cable will extend from the bottom of the tower to the sea floor over a total distance of 166 feet. Each of the ½ inch cables will have a tension sufficient to exert equal vertical components of force, and the center cable will have a tension just equal to the vertical component of tension in each one of the ½ inch cables. Thus, the vertical components of force in all of the cables will be equally distributed, and their sum will be equal and opposite to the net reserve buoyancy of the tower.

5. Buoyancy

The buoyancy chambers will be filled with salvage foam which has a density of 2.0 lbs/ft^3 and a tensile strength of 70 psi. The foam will be encased in 1/8 inch steel plating which will not be water tight. Hence, the foam will absorb water at its surface. It is assumed the foam will absorb 1.0 lb/ft^3 of water which is a quite conservative assumption. Since the density of sea water is 64.0 lbs/ft^3, the net reserve buoyancy provided by the foam will be 61.0 lbs/ft^3. This result in a total buoyancy force of 33,280 lbs, giving a net reserve buoyancy (excluding mooring cables) of 13,538 lbs.

II. THEORETICAL DEVELOPMENT

A. GENERAL APPROACH

In describing the dynamic response of the NPS platform to wave action, each degree of freedom is first examined separately. There are six degrees of freedom for the tower which supports the platform. These six degrees of freedom, or modes of oscillation, are roll, pitch, yaw, heave, surge, and sway (see Figure 2). Since the tower has symmetry with respect to the x and y axes, roll may be interpreted as pitch, and sway may be interpreted as surge. And it is assumed that any motion of the tower in yaw is negligible since the tower is symmetrical with respect to the z axis. So there now remains three degrees of freedom which must be analyzed, namely, surge, pitch, and heave.

It is most logical to first analyze each degree of freedom separately, and using these results, it can be determined whether an analysis of a two or more degree of freedom system is necessary.

The motion of the tower is analyzed as a linear system in order that superposition may be used in obtaining a general response spectrum which is directly proportional to a wave spectrum. The wave spectrum itself is generated by superimposing wave components of many different frequencies and amplitudes.

B. SPRING-MASS SYSTEM ANALOGY

The simplest approach to the problem is to describe each degree of freedom separately by an equation of motion analogous to a spring-mass system with viscous damping and a sinusoidal driving term. The linear differential equation which describes the translatory motion of this system (see Figure 3) is

$$M\ddot{x} + C\dot{x} + Kx = F \sin (\sigma\tau + \varphi) \qquad (2.1)$$

or

Inertia +	Viscous Type Drag	+	Spring Restoring Force	= Sinusoidal Driving Force

x = motion being considered

M = mass

C = viscous damping coefficient

K = stiffness coefficient

F = amplitude of sinusoidal driving force

σ = circular frequency of sinusoidal driving force

φ = phase angle

It is possible to describe each degree of freedom of the NPS tower in a similar manner after drawing a general free body diagram. Figure 4 is a model of the tower for surge, pitch, and heave using the spring-mass system analogy. The horizontal components of the lateral restraining cables and the vertical cable are each represented by springs having specific stiffness coefficients. Force, F_1, is constant and is equal to the sum of the vertical

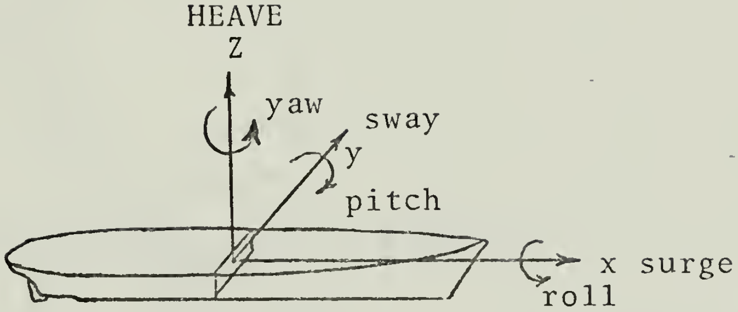


FIGURE 2.

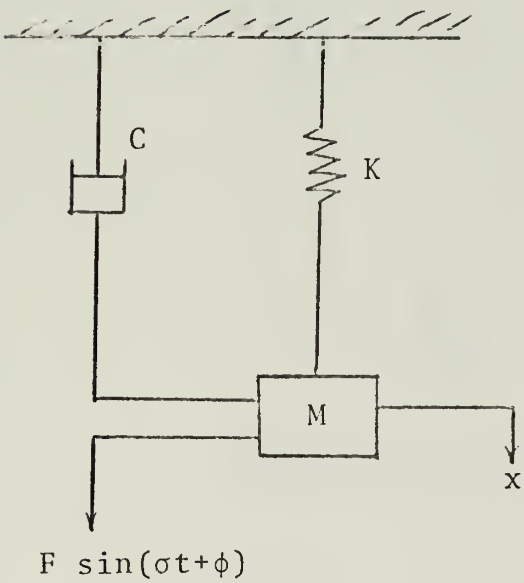


FIGURE 3.

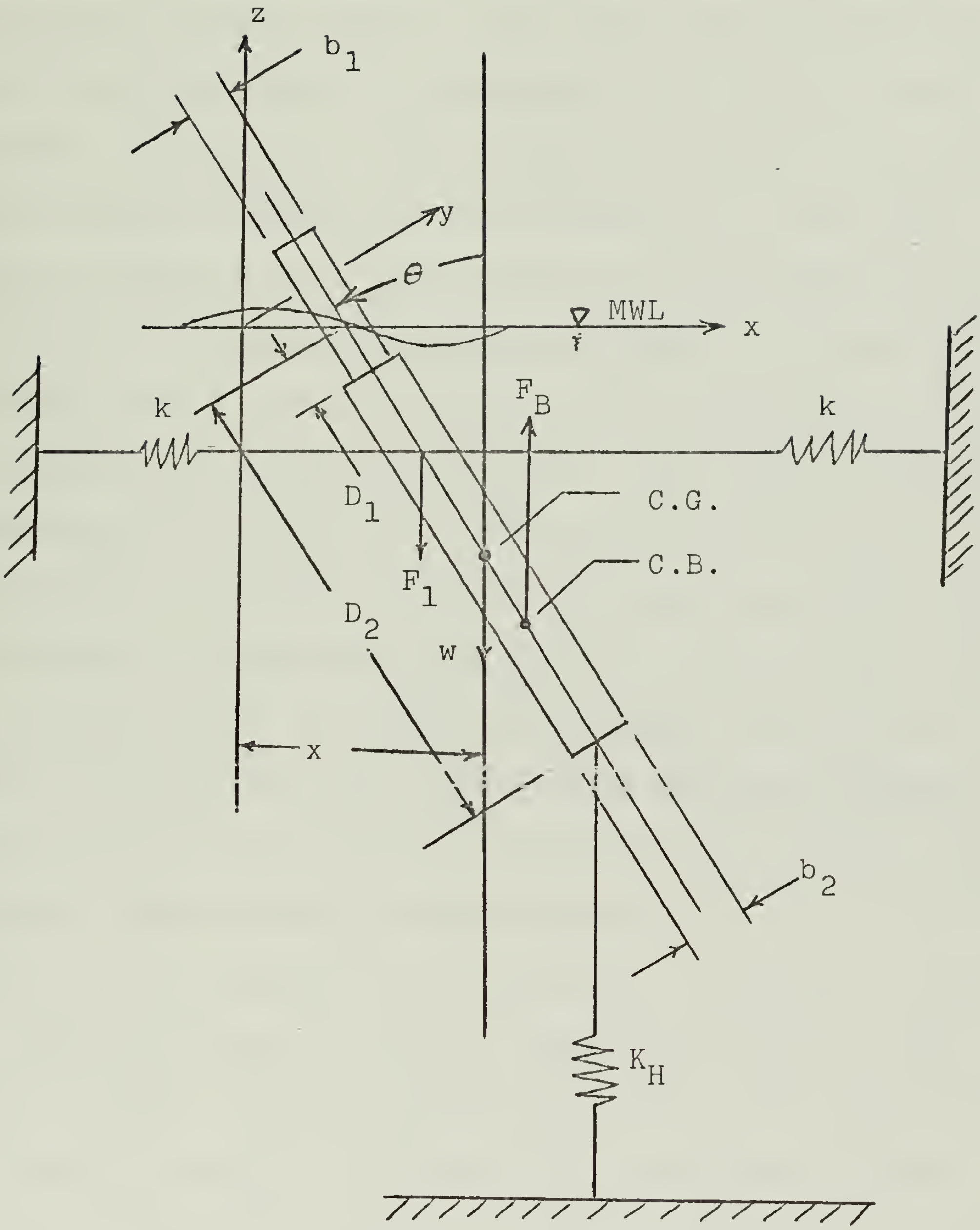


FIGURE 4.

Force Diagram-General.

components of the four horizontal restraining cables. Other forces shown are the weight of the tower acting through the center of gravity, and the buoyancy force acting through the center of buoyancy. The distance from MWL to the top of the buoyancy material is represented by D_1, and the effective width of this same section is represented by b_1. Since the area above the buoyancy material is a void, the waves will tned to pass through and be impeded only by the cross supporting members of the tower. The effective width, b_1, represents the width of a solid surface which is equivalent to the surface area of the open tower structure for purposes of determining drag and inertial forces. Likewise the width, b_2, represents the average width of the section of the tower which encases the buoyancy material. The geometry of the tower is idealized in this manner, the assumption being slightly conservative.

In order to consider only one degree of freedom, the model shown in Figure 4 must be restrained so as to move in only one mode of oscillation at a time; only those forces affecting the motion of the tower in a particular mode should be considered. Take for example the motion in surge (see Figure 5). The equation of motion for this system is

$$M_s\ddot{x} + C_s\dot{x} + K_s x = F_I \sin(-\sigma\tau) + F_D \cos(-\sigma\tau) \quad (2.2)$$

where

M_s = the virtual mass in surge, or the mass, M, plus added mass

C_s = damping or all influences which are velocity sensitive

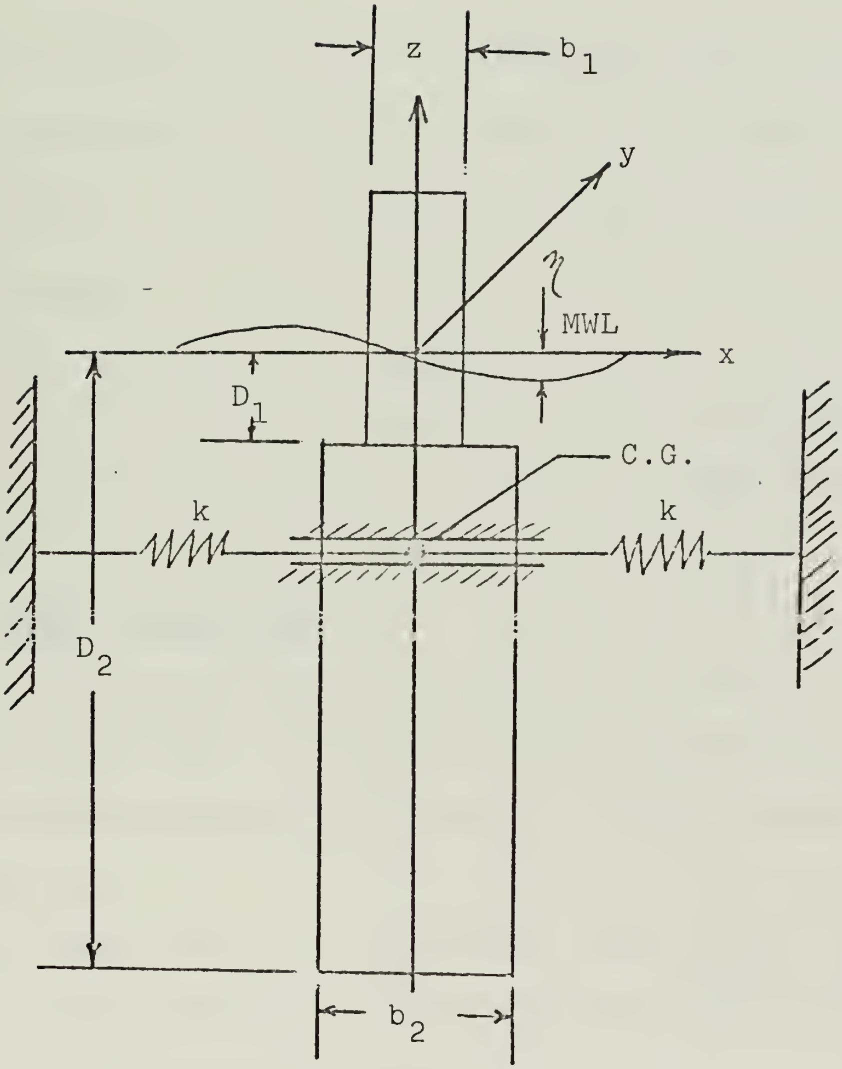


FIGURE 5.

Force Diagram-Surge.

K_s = stiffness where $K_s = 2K$, or all influences which are position sensitive.

F_D = Exciting force due to drag

F_I = Exciting force due to the unsteady motion of the waves

σ = radial frequency of exciting forces (waves)

The equations of motion for pitch and heave may be obtained using a similar analogy to a spring-mass system.

C. ASSUMPTIONS

In order to derive the equations of motion mentioned above and to facilitate obtaining a complete analysis of the motion of the tower, it is necessary to make the following assumptions:

1. Basic Assumptions

a. That the pressure field of the fluid is not affected by the tower. To satisfy this assumption it is assumed that the width of the tower is small compared to wavelength.

b. The center leg (vertical) restraining cable has a negligible effect in restraining the surge and pitch motion of the tower due to the waves.

c. All wave components are from one direction. This is a conservative assumption.

2. Assumptions Necessary for Linearization of the Problem

a. All assumptions necessary for linear wave theory which include:

(1) Fluid is inviscid. However, this assumption is made only in describing particle motion due to waves, and it is not made when drag forces on the tower are considered.

(2) Fluid is incompressible and homogeneous.

(3) Wave amplitude, A, is small compared to wavelength.

b. In accordance with linear wave theory the wave profile, η, is

$$\eta = A \cos (kx - \sigma t)$$

and water particle velocity in the x and z directions is

$$u = A\sigma \frac{\cosh k(h+z)}{\sinh kh} \cos (kx - \sigma t)$$

$$w = A\sigma \frac{\sinh k(h+z)}{\sinh kh} \sin (kx - \sigma t)$$

where σ is the radial frequency, k is the wave number and h the depth of water.

c. The roll or pitch, θ, of the tower sufficiently small such that:

(1) $\sin\theta \approx \theta$

(2) $\cos\theta \approx 1.0$

d. In accordance with linear wave theory, $\eta (x, t)$, is considered negligible and set equal to zero when integrating wave forces over the complete length of the tower.

e. The drag coefficient may be linearized for insertion into a linear differential equation.

f. The amount of catenary in the restraining cables is negligible, and the tension in the cables is sufficiently

high so as to justify using the relation

$$F_r = -Kx$$

where

F_r = restoring force of cable due to stretch

K = stiffness coefficient

x = amount of stretch

D. METHOD OF ANALYSIS

Using the method of undetermined coefficients, the steady state solution to the differential equation of motion in surge is found to be

$$X = \frac{(\omega^2-\sigma^2)\frac{F_D}{M_s} + \xi \frac{F_I}{M_s}\sigma}{(\omega^2-\sigma^2)^2+\xi^2\sigma^2}\cos(\sigma\tau) + \frac{\xi\frac{F_D}{M_s}\sigma - (\omega^2-\sigma^2)\frac{F_I}{M_s}}{(\omega^2-\sigma^2)^2+\xi^2\sigma^2}\sin(\sigma\tau) \qquad (2.3)$$

where

$F_D = F_D(\sigma, A)$ = drag force

$F_I = F_I(\sigma, A)$ = inertial force

$C_s = C_s(\sigma, A)$ = linear damping coefficient

A = wave amplitude

$\omega = \sqrt{K_s/M_s}$

$\xi = \frac{C_s}{M_s}$

or

$$x = C_1 \cos(\sigma\tau) + C_2 \sin(\sigma\tau) \qquad (2.4)$$

where

$C_1 = C_1(\sigma, A)$

$C_2 = C_2(\sigma, A)$

In terms of maximum surge where x(max) = X, the steady state solution reduces to

$$X^2 = C_1^2 + C_2^2 \qquad (2.5)$$

By factoring out the A^2 from C_1^2 and C_2^2, the solution may be expressed as

$$X^2 = \left[C_1^2 + C_2^2\right] A^2 \qquad (2.6)$$

where now

$$C_1 = C_1(\sigma)$$
$$C_2 = C_2(\sigma)$$
$$A = A(\sigma)$$

or

$$X^2(\sigma) = \left[T.F.(\sigma)\right] A^2(\sigma) \qquad (2.7)$$

where

$X^2(\sigma)$ = Response Function

$\left[T.F.(\sigma)\right] = C_1^2 + C_2^2$ = Transfer Function

$A^2(\sigma)$ = Driving Term

The response of the tower in surge, pitch, and heave can be represented as the product of a Transfer Function and Driving Term whether the motion is coupled or uncoupled. The task of determining the response of the tower to wave action now reduces to:

1. Obtaining a transfer function for each degree of freedom.
2. Determining the driving function.
3. Multiplying (1) and (2) above to obtain response.

An important characteristic of a linear system is that the frequency of the response is equal to the frequency of the driving term.

E. DETERMINATION OF DRIVING FUNCTION (Special Approach)

Neumann [2] has derived an empirical equation describing the wave energy-density spectrum which relates energy-density (which is proportional to wave amplitude squared) at a particular frequency band to frequency for a "fully developed" sea for particular wind speeds. This implies that the wind has been blowing for a sufficient time and over a sufficient distance so that the waves no longer grow in height, i.e. fully developed. This equation is based upon a great abundance of individual wave observations. The assumption of a fully developed sea is conservative for Monterey Bay, but this spectrum is used because it affords it an analytical expression for the wave spectrum.

The continuous "Neumann" spectrum in units of ft^2-sec is given by

$$A^2 = \frac{2D(2\pi)^5 g^2 e^{-2\left(\frac{g}{U\sigma}\right)}}{\sigma^6} \qquad (2.8)$$

where

$D = 2.0 \times 10^{-5}\ sec^{-1}$

g = acceleration due to gravity = 32.2 ft/sec^2

σ = natural wave frequency, sec^{-1}

U = windspeed in ft/sec necessary to develop a given sea state

It has been found from observations that the probability distribution function of wave heights is described by a Rayleigh distribution. A Rayleigh distribution is completely defined by the variance, S^2, where the variance for waves is given by

$$S^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T \eta^2(\tau)\, d\tau \tag{2.9}$$

where T is the record length. The variance can be determined either directly from equation (2.9) or from the energy-density spectrum by applying Parseval's theorem. Parsevals theorem relates the variance to the area under the spectrum such that

$$S^2 = 2\int_0^\infty A^2(\sigma)\, d\sigma \tag{2.10}$$

It is now possible to obtain the significant wave height, $H_{1/3}$, which is directly related to the variance, S^2, by the Rayleigh distribution such that

$$H_{1/3} = 2.83\sqrt{S^2} \tag{2.11}$$

The significant wave height is defined as the average of the highest one third waves. This is an important parameter because it is the wave height that one observes visually and is used continually in oceanography.

Values of significant wave height for various wind speeds were generated by the author of this thesis using equations (2.8) through (2.10). These values are listed in Table II and agree to within one percent of the values listed in a table developed by Bretschneider [2]. A

TABLE II

WAVE HEIGHTS AS A FUNCTION OF WINDSPEED FOR A FULLY DEVELOPED SEA

U knots	$H_{1/3}$ feet
10	1.4
15	3.7
20	7.6
25	13.5
30	21.4

significant wave height of 21.4 feet will be used as the maximum design wave for the NPS tower.

F. GENERAL SOLUTION

Knowing the transfer function for each degree of freedom, and describing the driving term as a "Neumann" wave spectrum, the response function can be expressed as a spectrum by using the previously derived expression:

$$X^2(\sigma) = \left[T.F(\sigma)\right] A^2(\sigma)$$

The transfer function merely acts as a filter and is a function of wave frequency and the overall geometry and physical make-up of the tower. The driving function, $A^2(\sigma)$, represents the energy-density of the waves, part of which is transferred to the tower resulting in pitch, surge, or heave motion. A typical set of curves representing a wave

spectrum (driving term), transfer function, and surge spectrum (response) is shown in Figure 6.

The area under the wave and surge spectra is proportional to the total energy-density of the waves and potential energy of the tower (in surge) respectively. The frequency at which the transfer function is a maximum corresponds to the natural frequency of oscillation, ω, of the tower where as previously defined by equation (2.3) was:

$$\omega = \sqrt{K_s/m_s} \tag{2.12}$$

It is seen in Figure 6 that the response spectrum is a product of the wave spectrum and transfer function, and it is obtained by multiplying together, ordinate by ordinate, the transfer function and wave spectrum. The wave spectrum in Figure 6 was generated by utilizing equation (2.8), and it represents a significant wave height of 21.3 feet. The transfer function in the same figure represents the surge transfer function of the tower. It can be observed that the response spectrum has two peaks. The peak which occurs at the higher frequency results from the fact that this is the natural frequency of oscillation, or resonant frequency, of the tower. The peak which occurs at the lower frequency is due to the high amount of wave energy at that particular frequency band as shown by the wave spectrum.

It is then obvious that as the peak at the transfer function approaches the peak of the wave spectrum, the response

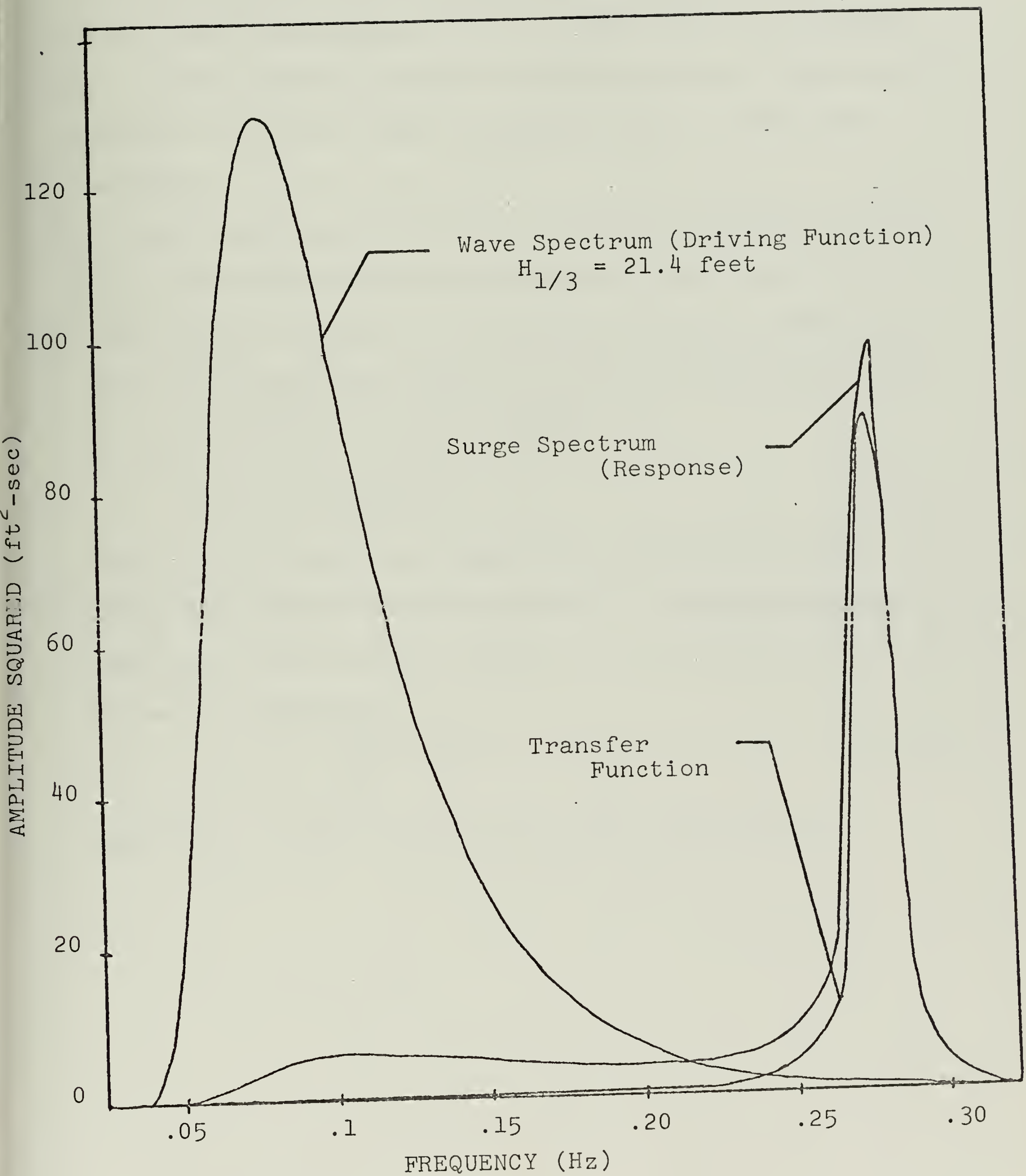


FIGURE 6. Spectrums.

spectrum of the tower increases rapidly due to resonance. It is therefore necessary that the NPS tower be designed as a stiff system in order to have the peak of the transfer function occur at a high frequency so as to eliminate resonance.

It is now assumed that the response heights will also have a Rayleigh distribution which has been found true for other similar systems. It is possible then to determine a value for significant response defined in the same manner as $H_{1/3}$ (analogous to wave heights) by

$$X_{1/3} = 2.83\sqrt{S^2} \qquad (2.13)$$

where $X_{1/3}$ in this case represents significant surge. S^2 is the variance of the surge motion. The variance can be found from the response spectrum in the same manner as for the waves such that

$$S^2(\sigma) = 2\int_0^\infty X^2(\sigma)\,d\sigma \qquad (2.14)$$

The units of $X^2(\sigma)$ in equation (2.14) are ft^2-sec.

III. DERIVATIONS AND SOLUTIONS TO EQUATIONS OF MOTION

Utilizing the theory which has been developed thus far, it is now possible to proceed in setting up and solving the differential equation of motion for each degree of freedom. The single degree of freedom motion of the tower in surge is considered first.

A. SOLUTION TO SINGLE DEGREE OF FREEDOM EQUATION-SURGE

Figure 7 shows an idealized surge model of the tower with all restoring forces. The disturbing forces on the tower due to wave action are not shown. In this analysis the vertical cable have a higher order contribution and therefore are considered negligible as a restoring force in surge.

1. General Equation

Recalling the general linear equation of motion for surge

$$M_s\ddot{x} + C_s\dot{x} + K_s x = F_I \sin(-\sigma t) + F_D \cos(-\sigma t) \qquad (2.2)$$

Each term in this equation will now be expanded and evaluated.

2. Virtual Mass

The virtual mass, M_s, is the actual mass of the tower plus the added mass due to fluid becoming entrained by the motion of the tower such that

$$M_s = M + M_a \qquad (3.1)$$

where

M = mass of tower

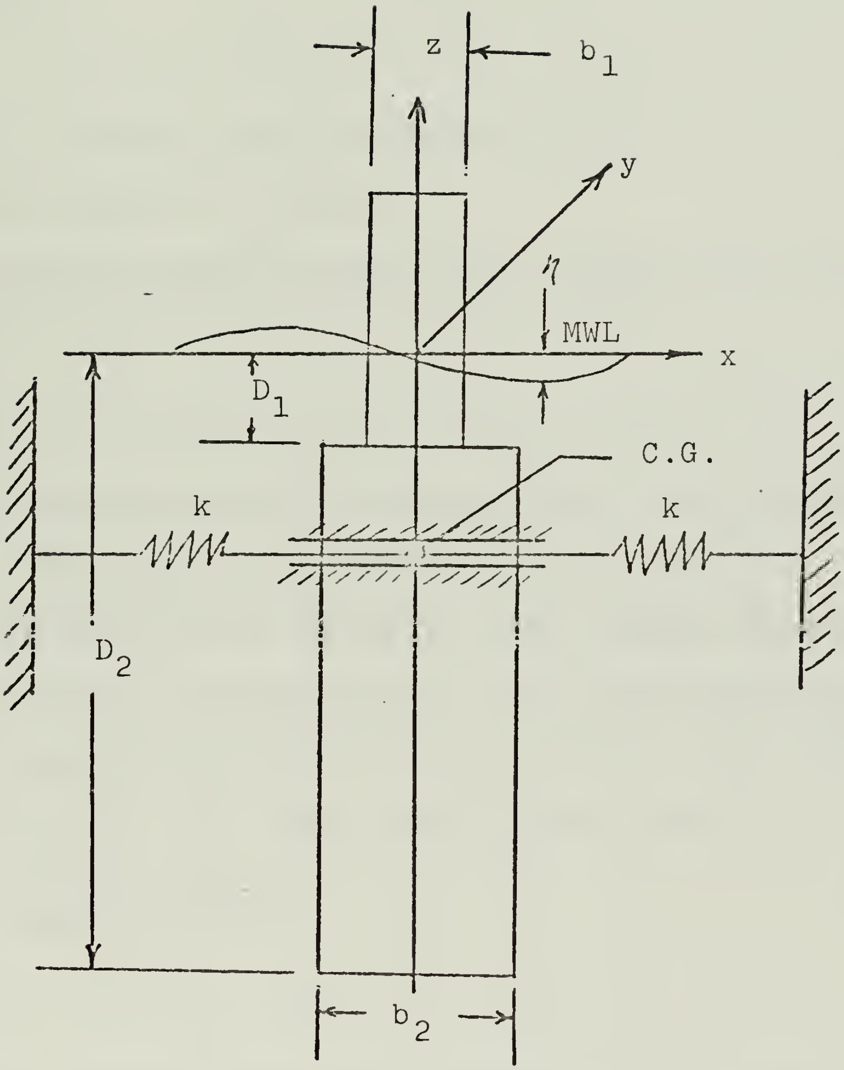


FIGURE 7.

Force Diagram-Surge.

$= 693.0$ slugs

and

M_s = added mass

$= C_a \rho V$

where

$C_a = 0.5$ = added mass coefficient

= mass density of water

V = volume of water which would be displaced by the idealized solid model of the tower,

or

$M_s = M + M_a = 693.0 + 830.0 = 1523.0$ slugs.

3. Disturbing and Restoring Forces Due to Drag

When water moves past the tower as a result of wave action and the tower's own motion, an overall drag force resulting from a combination of form drag, wake drag, and viscous drag will be exerted on the tower. The combined effect of these drag forces can be represented by

$$dF_D = \frac{C_d \rho (u - \dot{x})^2 \, dA}{2} \qquad (3.2)$$

where

dF_d = drag force on vertical element, dz, of tower

C_d = drag coefficient = 1.05

ρ = mass density of water

u = instantaneous horizontal water particle velocity

$\dot{x}$ = instantaneous velocity of tower in surge.

dA = element of projected area exposed to wave force.

Equation (3.2) may be re-written as

$$dF_D = \frac{C_D \rho u^2 dA}{2} - \frac{C_D \rho \dot{x}^2 dA}{2} \qquad (3.3)$$

$$= dF_{DD} - dF_{DR}$$

= disturbing force - restoring force.

Considering first the disturbing force due to drag, F_{DD}, where

$$dF_{DD} = \frac{C_D \rho u^2 dA}{2} \qquad (3.4)$$

It is apparent that this term must be linearized for insertion into a linear differential equation. As illustrated by Thomson [4], this can be done by defining a new drag coefficient, C_{DL}, evaluated such that by using C_{DL}, the same amount of work per wave cycle would be done as when using the non-linear drag term, C_D, where energy, E, per cycle is

$$E = \int_0^T F_{DD} \cdot u \, d\tau \qquad (3.5)$$

and where

$$u = U_c \sin(\sigma\tau - \epsilon)$$

$$F_{DD} = \frac{1}{2} \rho C_D A u^2$$

A = area of tower normal to wave propagation and

$$\int_0^T F_{DD} \cdot u \, d\tau = -\frac{4 \rho C_D A U_c^3}{3\sigma} \qquad (3.6)$$

And it is assumed that the linearized drag force, F_{DDL}, is of the form

$$F_{DDL} = \frac{1}{2} \rho C_{DL} A u \qquad (3.7)$$

where C_{DL} represents the linearized drag coefficient. Now substituting equation (3.7) into equation (3.5), and integrating once more to obtain energy per wave cycle due to F_{DDL},

$$E = \int_0^T F_{DDL}\, u \, d\tau$$

$$= \frac{\rho C_{DL} A U_o^2 \pi}{2\sigma} \quad (3.8)$$

By requiring that the work cycle done by F_{DD} equal the work done per cycle by F_{DDL}, equation (3.8) can be set equal to equation (3.6) to yield

$$C_{DL} = \frac{8 C_D U_o}{3\pi} \quad (3.9)$$

From linear wave theory, the horizontal water particle velocity amplitude is

$$U_o = \frac{A\sigma \cosh k(h+z)}{\sinh kh}$$

It is now possible to re-write equation (3.3) in a linearized form to yield

$$dF_{DL} = \frac{C_{DL}\rho u\, dA}{2} - \frac{C_{DL}\rho \dot{x}\, dA}{2} \quad (3.10)$$

Expanding now the first term of equation (3.10) and integrating from the water surface to the bottom of the tower

$$F_{DDL} = \frac{\rho}{2}\int_{-D_2}^{0}\int_{-\frac{b(z)}{2}}^{\frac{b(z)}{2}} C_{DL}\, u\, dy\, dz$$

$$= F_D \cos(-\sigma\tau) \quad (3.11)$$

where

$$F_D = \frac{4 C_D A^2 \sigma^2 \rho}{3\pi k \sinh^2 kh}\left[b_2\left(\frac{\sinh 2k(h-D_1)}{4} - \frac{kD_1}{2} - \frac{\sinh 2k(h-D_2)}{4} + \frac{kD_2}{2}\right) + b_1\left(\frac{\sinh 2kh}{4} - \frac{\sinh 2k(h-D_1)}{4} + \frac{kD_1}{2}\right)\right] \tag{3.12}$$

Now consider the second term of equation (3.10). Since it is a restoring force due to drag, it may be equated to the damping term of equation (2.2) such that

$$C_s \dot{x} = \int_A \frac{C_{Dw} \rho \, dA}{2} \dot{x}$$

or

$$C_s = \int \frac{C_{Dw} \rho \, dA}{2} \tag{3.13}$$

And integrating from MWL to the bottom of the tower

$$C_s = \frac{\rho}{2}\int_{D_2}^{0} C_{Dw} \, b(z) \, dz$$

$$= \frac{4 C_D A \sigma \rho}{3\pi k \sinh kh}\left[b_1\left(\sinh kh - \sinh k(h-D_1)\right) + b_2\left(\sinh k(h-D_1) - \sinh k(h-D_2)\right)\right] \tag{3.14}$$

Referring again to equation (2.2), the virtual mass, M_s, damping coefficient, C_s, and drag force, F_D, have now been determined. The remaining terms must now also be expanded and evaluated.

4. Stiffness Coefficient

Considering again the free body diagram in Figure 7, the equivalent stiffness coefficient is

$$K_{eq} = K_s = 2K \tag{3.15}$$

where it is assumed that the stiffness coefficient, K, for each cable is

$$K = \frac{EA}{L} \cos \alpha \tag{3.16}$$

and

α = angle of cables with respect to horizontal = 19°

E = elastic modulus = 13×10^6 psi

L = length of cable = 680 feet

A = $C_w d^2$ = effective cross sectional area of cable = 0.101^2in

C_w = area constant for wire cable = 0.405

d = diameter of cable.

Various values for C_w have been developed by Wilson [1]. And in its final expanded form, equation (3.15) becomes

$$K_s = \frac{2EC_w d^2 \cos \alpha}{L} = 3650 \ lb/ft \tag{3.17}$$

5. Inertial Forces

The task now remains to expand and evaluate the inertial force driving term due to the wave water particle acceleration, F_{ID}, where $F_{ID} = F_I \sin(-\sigma t)$. In evaluating the inertial forces on an elemental section, dZ, of the tower, the following relationship will be utilized:

$$dF_{ID} = C_m \rho A \dot{u} \, dz \tag{3.18}$$

where

C_m = inertia coefficient = 1.5

ρ = mass density of water

A = cross sectional area of tower

$\ddot{u}$ = instantaneous horizontal water particle acceleration.

It is noted that the inertial driving force is formulated in terms of the water particle acceleration, $\ddot{u}$, and not the relative water particle acceleration, $(\ddot{u}-\ddot{x})$. The term involving the acceleration of the tower, $\ddot{x}$, has been included as the inertia term on the left hand side of equation (3.1) and includes the "added mass." This can be shown by considering the inertial force, F_I, that results from the relative acceleration of fluid about the tower:

$$dF_I = C_m \rho A(\ddot{u} - \ddot{x})\,dz \tag{3.18a}$$

which describes the inertial force, F_I, that results from the relative acceleration of fluid about the tower. It is obvious then that the elemental force, dF_{ID}, can be obtained by expanding equation (3.18a). However, one additional term will appear in the expansion of this equation, and that can be considered as a restoring component of inertial force, dF_{IR}, where

$$\begin{aligned} dF_{IR} &= -C_m \rho A \ddot{x}\,dz \\ &= -C_m \rho\, dV \ddot{x} \\ &= -C_m\, dM \ddot{x} \\ &= -1.5\, dM \ddot{x} \end{aligned}$$

or

$$\begin{aligned} dF_{IR} &= -(dM + 0.5\,dM)\ddot{x} \\ &= -(\text{ACTUAL MASS} + \text{ADDED MASS}) \\ &= -M_s \ddot{x} \end{aligned}$$

and

M_s = virtual in surge

= actual mass + added mass

which is exactly as previously stated in equation (3.1). Integrating equation (3.18) from MWL to the bottom of the tower

$$F_{ID} = C_m \rho \int_{-D_2}^{0} A(z)\, \dot{u}\, dz \qquad (3.19)$$

or

$$F_{ID} = C_m \rho \int_{-D_2}^{0} \int_{-\frac{b(z)}{2}}^{\frac{b(z)}{2}} \dot{u}\; b(z)\, dx\, dz$$

$$= F_I \sin(-\sigma t) \qquad (3.20)$$

where

$$F_I = \frac{2\rho C_m A \sigma^2}{k^2 \sinh kh} \left[\left(\sinh k(h-D_1) - \sinh k(h-D_2) \right) b_2 \sin\left(\frac{k b_2}{2}\right) + \right.$$

$$\left. \left(\sinh kh - \sinh k(h-D_1) \right) b_1 \sin\left(\frac{k b_1}{2}\right) \right] \qquad (3.21)$$

6. Complete Solution

Each term of the equation of motion for surge

$$M_s \ddot{x} + C_s \dot{x} + K_s x = F_I \sin(-\sigma t) + F_D \cos(-\sigma t) \qquad (2.2)$$

has now been evaluated completely, and by using the method of undetermined coefficients, the steady state solution to equation (2.2) is found to be

$$x = \frac{(\omega^2-\sigma^2)\frac{F_D}{M_s} + \xi\frac{F_I}{M_s}\sigma}{(\omega^2-\sigma^2)^2 + \xi^2\sigma^2} \cos(-\sigma t)$$

$$+ \frac{\xi\frac{F_D}{M_s}\sigma - (\omega^2-\sigma^2)\frac{F_I}{M_s}}{(\omega^2-\sigma^2)^2 + \xi^2\sigma^2} \sin(-\sigma t) \qquad (3.22)$$

or

$$x = C_1 \cos(-\sigma t) + C_2 \sin(-\sigma t) \qquad (3.23)$$

In terms of maximum surge, X, the solution becomes

$$X^2 = C_1^2 + C_2^2. \qquad (3.24)$$

And by factoring the A^2 out of C_1^2 and C_2^2, it becomes

$$X^2(\sigma) = [T.F.(\sigma)]A^2(\sigma) \qquad (3.25)$$

where

$$[T.F.(\sigma)] = (C_1^2 + C_2^2)/A^2. \qquad (3.26)$$

B. SOLUTION TO SINGLE DEGREE OF FREEDOM EQUATION-PITCH

Having solved the single degree of greedom equation for surge, the motion of the tower in pitch is now analyzed. As in surge, it is necessary to consider a free body diagram which shows the restoring forces which must be considered in the pitch problem (see Figure 8). In this analysis, the downward force, F_4, of the vertical cable will cause a restoring moment, and the buoyancy force will cause a disturbing moment about the center of gravity. The lateral restraining cables have no effect on the motion of the tower in pitch since

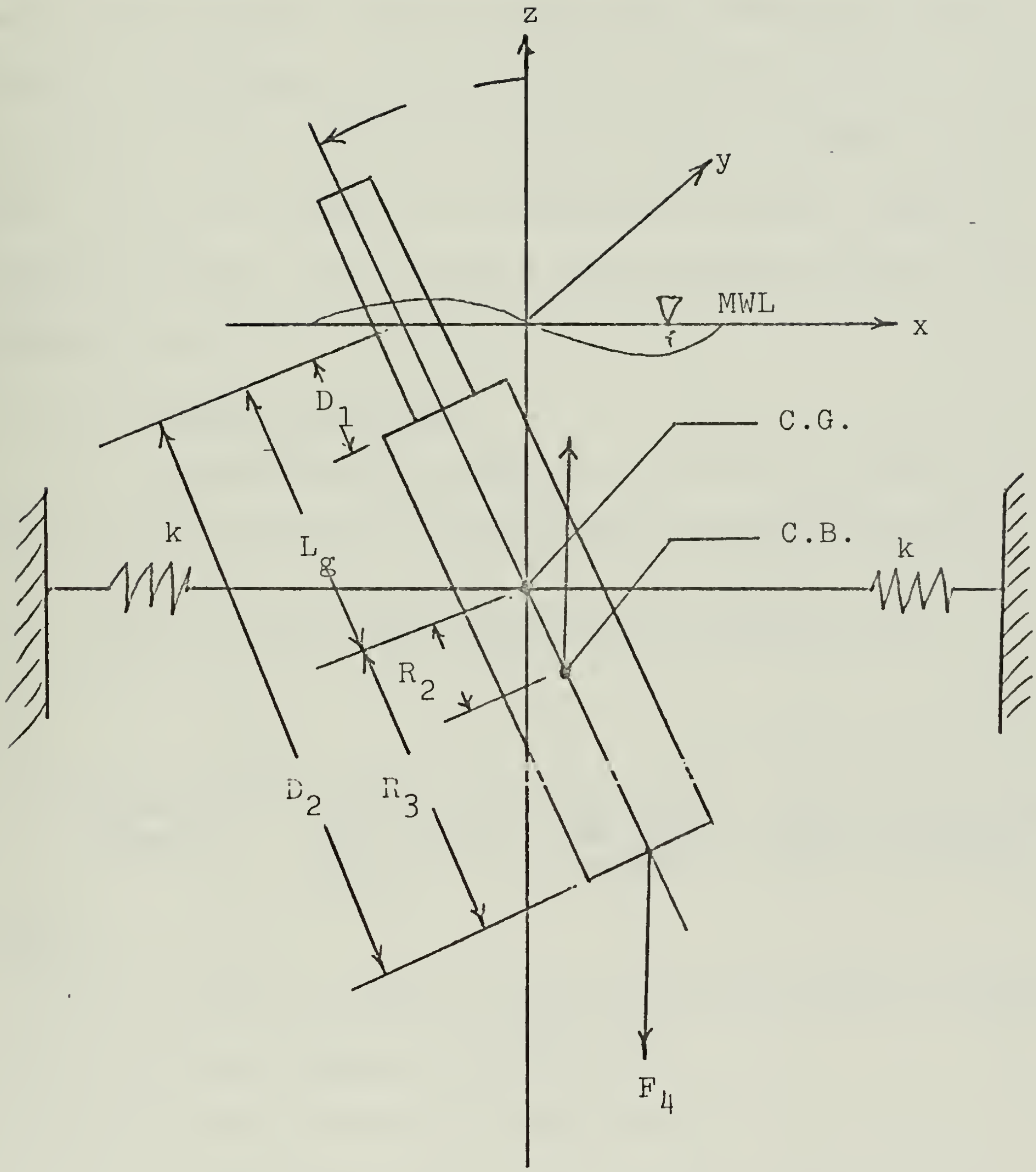


FIGURE 8.

Force Diagram-Pitch.

they act through the center of gravity, and when the tower oscillates in pitch motion alone, the rotation is about the center of gravity. If they did not act through the center of gravity, the motion of pitch would be coupled to surge and it would no longer be considered a single degree of freedom problem. The coupled pitch-surge problem is considered later.

1. General Equation

A differential equation of motion for pitch is similar to the equation for surge and may be written as

$$J_p \ddot{\theta} + C_p \dot{\theta} + K_p \theta = M_I \sin(-\sigma t) + M_D \cos(-\sigma t) \tag{3.27}$$

or

Inertial Moment + Moment due to Viscous Drag + Spring Restoring Moment = Disturbing Moments Due to Waves

where

J_p = virtual moment of inertia about center of gravity.

C_p = pitch damping coefficient.

K_p = pitch stiffness coefficient.

M_I = disturbing moment due to the unsteady motion of the waves.

M_D = disturbing moment due to drag.

Each term in the above equation must now be evaluated.

2. Virtual Moment of Inertia

Like the virtual mass, M_s, in equation (2.2), there is a virtual moment of inertia for pitch, J_p, such that

$$J_p = J_o + J_a \tag{3.28}$$

where

J_o = moment of inertia of tower about center of gravity

J_a = added moment of inertia due to entrained fluid.

or

$$J_p = J_o + J_a$$

$$= 231{,}500.0 + 400{,}000.0$$

$$= 631{,}500 \text{ slug-ft}^2.$$

3. Damping Coefficient

It has already been determined from evaluation of equation (3.14) that the damping force, $C_s\dot{x}$, on an elemental section, dz, of the tower, is given by

$$C_s\dot{x} = \frac{\rho}{2}\int C_{DL}\, b(z)\, dz\, \dot{x} \qquad (3.30)$$

or

$$C_s = \frac{\rho}{2}\int_{-D_L}^{L} C_{DL}\, b(z)\, dz \qquad (3.31)$$

Now the damping moment, $C_p\dot{\theta}$, can be evaluated by simply integrating from MWL to the bottom of the tower the product of damping force on an element, dz, of the tower and the moment arm, $(L_g - z)$, of that element. But first it is necessary to express x in terms of θ and z, such that for small angles of (see Figure 8),

$$x = (L_g - z)\theta \qquad (3.32)$$

and

$$\dot{x} = (L_g - z)\dot{\theta} \qquad (3.33)$$

Multiplying equation (3.30) by the moment arm $(L_g - z)$, inside the integral of equation (3.30), the damping moment, $C_p\dot{\theta}$, becomes

$$C_p \dot{\theta} = \frac{\rho}{2}\int_{-D_2}^{0} C_{DL}\, b(z)(L_g - z)\, dz\, \dot{x} \tag{3.34}$$

Substituting equation (3.33) into equation (3.34) so as to have everything in terms of θ,

$$C_p \dot{\theta} = \frac{\rho}{2}\int_{-D_2}^{0} C_{DL}(L_g - z)^2\, b(z)\, dz\, \dot{\theta} \tag{3.35}$$

or

$$C_p = \frac{\rho}{2}\int_{-D_2}^{0} C_{DL}(L_g - z)^2\, b(z)\, dz \tag{3.36}$$

$$= \left(\frac{4 C_D \rho A \sigma}{3\pi \sinh kh}\right)\Bigg\{ b_1\Bigg[L_g^2 \frac{\cosh kh \sinh kD_1}{k} + \frac{L_g^2 \sinh kh}{k}(1 - \cosh kD_1) + \frac{2 L_g \cosh kh}{k^2}(1 + kD_1 \sinh kD_1 + \cosh kD_1)$$

$$- \frac{2 L_g \sinh kh}{k^2}(kD_1 \cosh kD_1 - \sinh kD_1) + \frac{\cosh kh}{k^3}(k^2 D_1^2 \sinh kD_1 - 2kD_1 \cosh kD_1 + 2\sinh kD_1) + \frac{\sinh kh}{k^3}(2 - k^2 D_1^2 \cosh kD_1$$

$$+ 2kD_1 \sinh kD_1 - 2\cosh kD_1)\Bigg] + b_2\Bigg[\frac{L_g^2 \cosh kh}{k}(-\sinh kD_1 + \sinh kD_2) + L_g^2 \frac{\sinh kh}{k}(\cosh kD_1 - \cosh kD_2)$$

$$- \frac{2 L_g \cosh kh}{k^2}(kD_1 \sinh kD_1 - \cosh kD_1 - kD_2 \sinh kD_2 + \cosh kD_2)$$

$$+ 2\frac{L_g \sinh kh}{k^2}(kD_1 \cosh kD_1 - \sinh kD_1 - kD_2 \cosh kD_2 + \sinh kD_2)$$

$$- \frac{\cosh kh}{k^3}(k^2 D_1^2 \sinh kD_1 - 2kD_1 \cosh kD_1 + 2\sinh kD_1 - k^2 D_2^2 \sinh kD_2 + 2kD_2 \cosh kD_2 - 2\sinh kD_2) + \frac{\sinh kh}{k^3}(k^2 D_1^2 \cosh kD_1$$

$$- 2kD_1 \sinh kD_1 + 2\cosh kD_1 - k^2 D_2^2 \cosh kD_2 + 2kD_2 \sinh kD_2 - 2\cosh kD_2)\Bigg]\Bigg\}$$

4. Stiffness Coefficient

Considering again Figure 8, it can be seen that the buoyancy force, F_B, will cause a disturbing moment, $F_B R_2 \theta$, whereas the force, F_4, will act as a restoring moment, $F_4 R_3 \theta$, such that the total restoring moment, $K_p \theta$, becomes

$$K_p = (F_4 R_3 - F_B R_2)\theta \tag{3.39}$$

and

$$K_p = F_4 R_3 - F_3 R_2 \tag{3.40}$$

5. Exciting Moment Due to Inertial Forces

In evaluating the disturbing moment due to inertial forces, M_{ID}, it is necessary to refer again to equation (3.20) to determine the relation

$$dF_{ID} = C_m \rho \dot{u}\, b(z)\, dx\, dt \tag{3.41}$$

which defines the inertial force on a section, dz, of the tower. Now multiplying this by its moment arm, $(L_g - z)$, and then integrating from the water surface to the bottom of the tower, we obtain

$$\begin{aligned} M_{ID} &= \rho C_m \int_{-D_2}^{0} \int_{-\frac{b(z)}{2}}^{\frac{b(z)}{2}} \dot{u}\,(L_g - z)\, b(z)\, dx\, dz \\ &= M_I \sin(-\sigma t) \end{aligned} \tag{3.42}$$

where

$$M_I = \frac{2\rho C_m A \sigma^2}{k \sinh kh} \Big\{ b_1 \sin\left(\frac{k b_1}{2}\right) \Big[\frac{L_g}{k} \cosh kh \sinh kD_1 + \frac{L_g}{k} \sinh kh (1 - \cosh kD_1) + \frac{\cosh kh}{-k^2} (1 + kD_1 \sinh kD_1 - \cosh kD_1) - \frac{\sinh kh}{k^2} (kD_1 \cosh kD_1 - \sinh kD_1) \Big] + b_2 \sin\left(\frac{k b_2}{2}\right) \Big[\frac{L_g}{k} \cosh kh (\sinh kD_2 -$$

$$\sinh kD_1\Big) + \frac{L_g}{k}\sinh kh\Big(\cosh kD_1 - \cosh kD_2\Big) - \frac{\cosh kh}{k^2}$$

$$\Big(kD_1 \sinh kD_1\Big) - \cosh kD_1 - kD_2 \sinh kD_2 + \cosh kD_2\Big) +$$

$$\frac{\sinh kh}{k^2}\Big(kD_1 \cosh kD_1 - \sinh kD_1 - kD_2 \cosh kD_2$$

$$+ \sinh kD_2\Big)\Big]\Big\}$$

6. Disturbing Moment Due to Drag

Likewise the disturbing moment due to drag, M_{DD}, may be determined by multiplying equation (3.11) by a variable moment arm, $(L_g - z)$, to obtain

$$M_{DD} = \frac{\rho}{2}\int_{-D_2}^{0}\int_{-\frac{C(z)}{2}}^{\frac{C(z)}{2}} C_{DL}\, u\,(L_g - z)\, dy\, dz$$

$$- M_D \cos(-\sigma T) \qquad (3.14)$$

where

$$M_D = \frac{\rho\, 0.892\, A^2 \sigma^2}{4 \sinh^2 kh}\Big\{ b_1\Big[\frac{L_g \cosh 2kh \sinh 2kD_1}{2k} + \frac{L_g \sinh 2kh}{2k}\Big(1 -$$

$$\cosh 2kD_1\Big) + L_g D_1 + \frac{\cosh 2kh}{4k^2}\Big(1 + 2kD_1 \sinh 2kD_1 +$$

$$\cosh 2kD_1\Big) - \frac{\sinh 2kh}{4k^2}\Big(2kD_1 \cosh 2kD_1 - \sinh 2kD_1\Big) +$$

$$\frac{D_1^2}{2}\Big] + b_2\Big[\frac{L_g \cosh 2kh}{2k}\Big(\sinh 2kD_2 - \sinh 2kD_1\Big) + \frac{L_g \sinh 2kh}{2k}$$

$$\Big(\cosh 2kD_1 - \cosh 2kD_2\Big) + L_g\Big(D_2 - D_1\Big) - \frac{\cosh 2kh}{4k^2}\Big(2kD_1 \sinh 2kD_1$$

$$- \cosh 2kD_1 - 2kD_2 \sinh 2kD_2 + \cosh 2kD_2\Big) + \frac{\sinh 2kh}{4k^2}$$

$$\Big(2kD_1 \cosh 2kD_1 - \sinh 2kD_1 - 2kD_2 \cosh 2kD_2 +$$

$$\sinh 2kD_2\Big) - \frac{D_1^2}{2} + \frac{D_2^2}{2}\Big]\Big\}$$

7. Complete Solution

Now that each term of equation (3.27) has been evaluated, the steady state solution may be evaluated to give

$$\Theta = \frac{(\omega^2-\sigma^2)\frac{M_o}{J_P} + \xi\left(\frac{M_I}{J_P}\right)\sigma}{(\omega^2-\sigma^2)^2 + \xi^2\sigma^2}\cos(-\sigma\tau)$$

$$+ \frac{\xi\left(\frac{M_o}{J_P}\right)\sigma - (\omega^2-\sigma^2)\left(\frac{M_I}{J_P}\right)}{(\omega^2-\sigma^2)^2 + \xi^2\sigma^2} SIN(-\sigma\tau) \quad (3.46)$$

$$= C_3 \cos(-\sigma\tau) + C_4 SIN(-\sigma\tau) \quad (3.47)$$

where the natural frequency oscillation in pitch is given by

$$\omega = \sqrt{\frac{K_P}{J_P}}$$

$$\xi = \frac{C_P}{J_P}$$

In terms of maximum roll or pitch, , the solution becomes

$$\Theta^2 = C_3^2 + C_4^2 \quad (3.48)$$

or

$$\Theta^2(\sigma) = [T.F.(\sigma)]\, A^2(\sigma) \quad (3.49)$$

where

$$[T.F.(\sigma)] = (C_3^2 + C_4^2)/A^2 \quad (3.50)$$

C. SOLUTION TO SINGLE DEGREE OF FREEDOM EQUATION-HEAVE

Considering first the free body diagram for heave as shown in Figure 9, it is apparent that all of the restraining

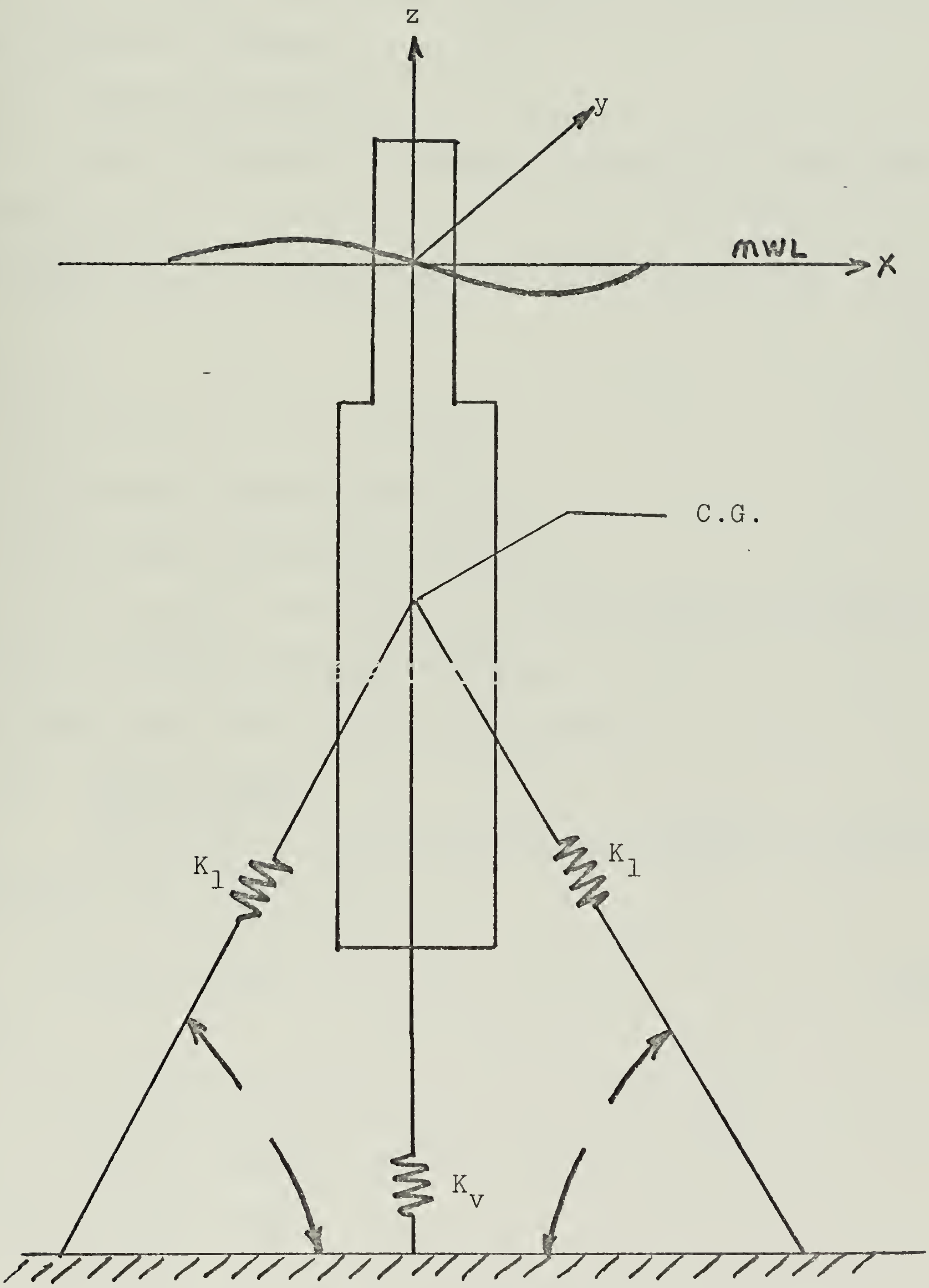


FIGURE 9.

Force Diagram-Heave.

cables will act as restoring forces in heave. All of the cables act through the center of gravity for the single degree of freedom problem.

1. General Equation

The differential equation of motion for heave may be written as

$$M_H \ddot{Z} + C_H \dot{Z} + K_H Z = F_I \sin(-\sigma t) + F_D \cos(-\sigma t) \tag{3.51}$$

where

M_H = virtual mass in heave.

C_H = heave damping coefficient.

K_H = heave stiffness coefficient.

F_I = exciting force due to unsteady motion of waves.

F_D = exciting force due to drag.

Again each term above must be evaluated.

2. Virtual Mass

The equation for virtual mass in heave is the same as for the surge analysis

$$M_H = M + M_a \tag{3.52}$$

where

M = mass of tower

= 693.0 slugs

M_s = added mass

= $C_a \rho V$

= 830.0 slugs

and

M_H = 1523.0 slugs.

3. <u>Heave Damping Coefficient</u>

The heave damping coefficient is evaluated in the same manner as the surge damping coefficient except that the vertical velocity and acceleration of water particles are involved instead of the horizontal water particle motion. Proceeding in a manner similar to the surge analysis, the equation for C_H is found to be

$$C_H = \rho \frac{C_{DL} A}{2} \tag{3.53}$$

where

C_{DL} = linearized drag coefficient for heave

A = cross sectional area of tower

or

$$C_H = \frac{64\, C_{D} \text{[illegible]}\, \rho_T \sinh k(D\ \text{[illegible]})}{3\pi\ \sinh kh}$$

4. <u>Stiffness Coefficient-Heave</u>

Considering the cables shown in Figure 9, the relation for K_H can be written as

$$K_H = 4 K_1 \sin\alpha + K_V \tag{3.54}$$

where

$K_1 = (EA_1)/L_1$.

α = angle of cables with respect to horizontal.

E = elastic modulus.

L_1 = length of side cables.

$A_1 = C_w d_1^2$ = effective cross sectional area of vertical cable.

d_1 = side cable diameter.

$K_V = (EA_2)/L_2$

$A_2 = C_w d_2^2$ = effective cross sectional area of vertical cable.

d_2 = vertical cable diameter.

L_2 = length of vertical cable.

5. Inertial Forces-Heave

Proceeding as previously done in the surge problem, the inertial disturbing force for heave becomes

$$F_{ID} = \rho C_m \int_{-D_2}^{0} \int_{-\frac{b(z)}{2}}^{\frac{b(z)}{2}} \dot{u}\, b(z)\, dx\, dz$$

$$= F_I \sin(-\sigma t) \qquad (3.55)$$

where

$$F_I = -\frac{2A\sigma^2 \rho C_m}{k^2 \sinh kh}\Big[b_1\big(\cosh(kh) - \cosh(kh - kD_1)\big)\sin\Big(\frac{kb_1}{2}\Big) +$$

$$b_2\big(\cosh(kh - kD_1) - \cosh(kh - kD_2)\big)\sin\Big(\frac{kb_2}{2}\Big)\Big] \qquad (3.56)$$

6. Drag Forces-Heave

Using the linearized drag coefficient for heave, the drag disturbing force, F_{DD}, may be written as

$$F_{DD} = \frac{\rho}{2}\int_{-\frac{b(z)}{2}}^{\frac{b(z)}{2}} C_{DL}\, w\, b(z)\, dx$$

$$= F_D \cos(-\sigma t) \qquad (3.57)$$

where

$$F_D = 0.892\rho A^2 \sigma^2 \sinh^2 k(h - D_2)\Big[b_1 \sin\Big(\frac{kb_1}{2}\Big) + b_2 \sin\Big(\frac{kb_2}{2}\Big)\Big] \qquad (3.58)$$

7. Complete Solution-Heave

With each term now evaluated, the steady state solution for heave amplitude, Z, of the tower may be expressed exactly by equations (3.22) through (3.26) with the exception that the subscript "s" indicating surge be changed to "H" for heave.

D. SOLUTION TO THE COUPLED SYSTEM-SURGE AND PITCH

Referring once more to a free body diagram as shown in Figure 10, it is apparent that the problem of tower motion should also be considered for cases in which the lateral restraining cables are attached at some point above or below the center of gravity. By analyzing the motion of the tower for all possible configurations, it is possible to arrive at an optimum design configuration. If the cables are in fact attached above or below the center of gravity, any surge of the tower results in the lateral cables causing a moment about the center of gravity. This indicates that the surge motion is coupled to the motion of the tower in pitch. From the results obtained in the heave analysis which is shown later, the heave motion of the tower is found to be small even for large significant wave heights; it is therefore considered negligible in comparison to pitch and surge and not accounted for in the coupled problem.

With this in mind, the two degree of freedom problem for surge and pitch is now analyzed for the general case in which the lateral cables are attached at any point of distance, R_1, above or below the center of gravity.

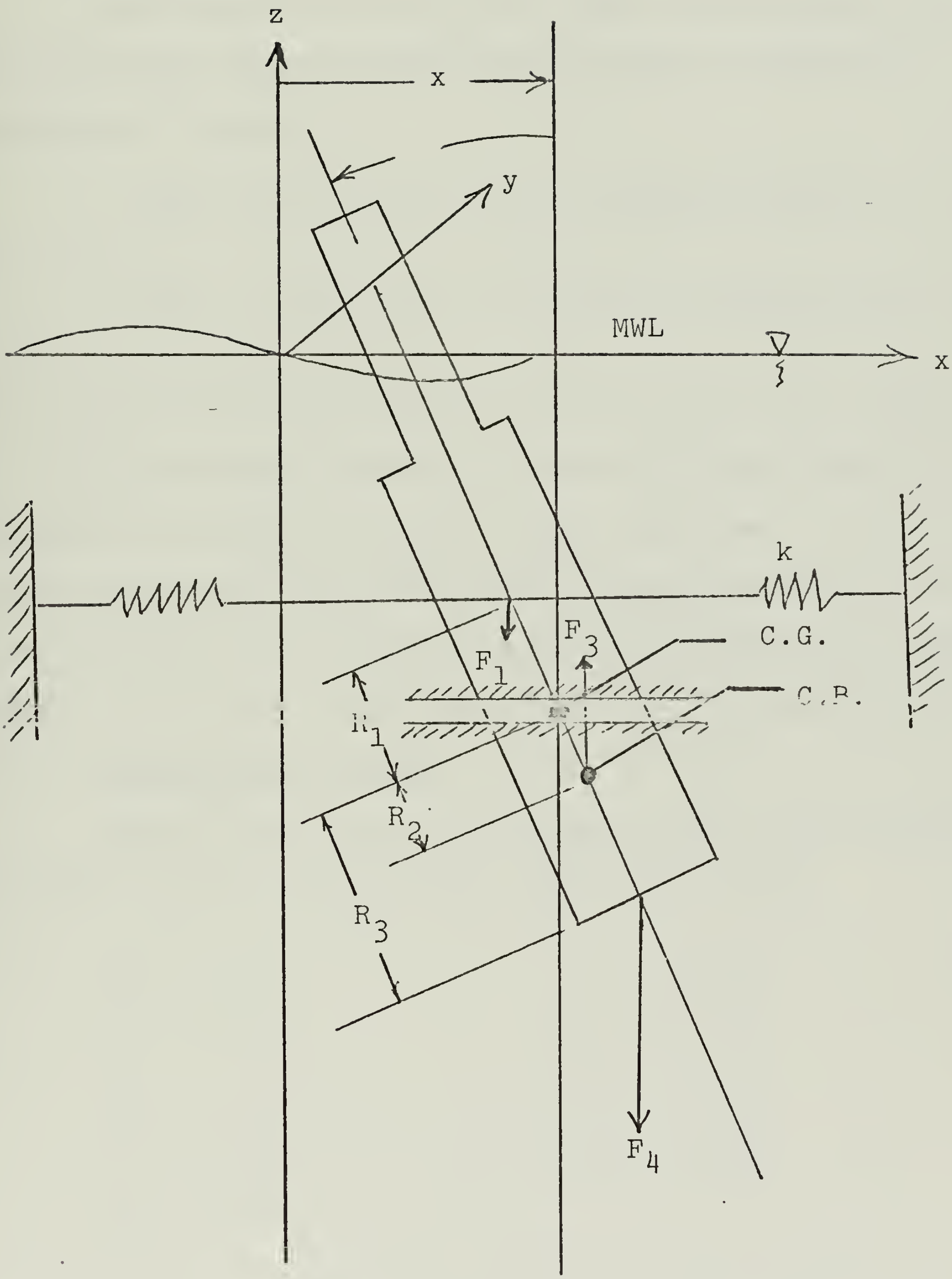


FIGURE 10.

Force Diagram-Coupled System.

1. <u>General Equations-Coupled Motion</u>

The coupled motion of the tower in surge and pitch may be described by two simultaneous linear differential equations as follows:

$$M_s \ddot{x} + C_s \dot{x} + K_s x - F\theta = F_I \sin(-\sigma t) + F_D \cos(-\sigma t) \quad (3.59)$$

$$J_p \ddot{\theta} + C_p \dot{\theta} + k_p \theta - Fx = M_I \sin(-\sigma t) + M_D \cos(-\sigma t) \quad (3.60)$$

The term which couples equations (3.59(and (3.60) together is F. As the lateral cables are attached more closely to the center of gravity, this term approaches zero. As would be expected, when these cables are attached exactly at the center of gravity, F equals zero and the motion is no longer coupled.

2. <u>Evaluation of Terms</u>

Each of the terms listed below have been evaluated previously by the corresponding equations listed:

M_s - (3.1)	J_p - (3.28)
C_s - (3.14)	C_p - (3.38)
K_s - (3.17)	M_I - (3.43)
F_I - (3.21)	M_D - (3.45)
F_D - (3.12)	

It is now necessary to evaluate the coupling term, F, and stiffness coefficient, K_p. The value of K_p has changed from previous calculations since moving the lateral cables up or down introduces new disturbing or restoring forces in pitch. In the case of surge, no new restoring or disturbing

forces are added, but those which did exist in the single degree of freedom problem are now modified by the coupling term, F. Therefore the value for K_s has not changed from the uncoupled situation.

Considering now the total restoring force, F_r, in surge (see Figure 10) which is found to be

$$
\begin{aligned}
F_R &= -2K(x - R_1\theta) \\
&= -2Kx + 2KR_1\theta \\
&= -K_s x + F\theta
\end{aligned}
\tag{3.61}
$$

where

$$F = 2KR_1 \quad = \text{coupling term.}$$

Proceeding one step further to evaluate the restoring moments in pitch, it can be shown by again referring to Figure 10 that the summation of restoring moments, M_r, is

$$
\begin{aligned}
M_R &= 2K(x - R_1\theta)R_1 + F_1R_1\theta + F_3R_2\theta - F_4R_3\theta \\
&= -(2KR_1^2 - F_1R_1 - F_3R_2 + F_4R_3)\theta + (2KR_1)x \\
&= -K_p\theta + Fx
\end{aligned}
\tag{3.62}
$$

where

K_p = new stiffness coefficient for pitch

$F = 2kR_1$ = coupling term.

A useful check in the analysis is that the coupling term, F, must be equal in both differential equations, and in this case it checks.

3. Solution to Coupled Equations

Referring again to equations (3.59) and (3.60), a steady state solution may be assumed as

$$X = A \sin(-\sigma\tau) + B \cos(-\sigma\tau) \tag{3.63}$$

$$\Theta = C \sin(-\sigma\tau) + D \cos(-\sigma\tau) \tag{3.64}$$

Substituting the above equations back into equations (3.59) and (3.60), and using the method of undetermined coefficients a set of four simultaneous equations with four unknowns, A, B, C, and D is obtained such that

$$g_{11}A + g_{12}B + g_{13}C + g_{14}D = F_I$$
$$g_{21}A + g_{22}B + g_{23}C + g_{24}D = F_O$$
$$g_{31}A + g_{32}B + g_{33}C + g_{34}D = M_I$$
$$g_{41}A + g_{42}B + g_{43}C + g_{44}D = M_O$$

where the elements g_{ij} are coefficients. The solution to the above set of equations may be expressed conveniently in matrix form as

$$S = G^{-1}F \tag{3.65}$$

where

$$S = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \tag{3.66}$$

G = 4 X 4 coefficient matrix of elements g_{ij}

$$= \begin{bmatrix} (-M_s\sigma^2+K_s) & (C_s\sigma) & (-F) & 0 \\ (-C_s\sigma) & (-M_s\sigma^2+K_s) & 0 & (-F) \\ (-F) & 0 & (-J_p\sigma^2+K_p) & (C_p\sigma) \\ 0 & (-F) & (-C_p\sigma) & (-J_p\sigma^2+K_p) \end{bmatrix} \tag{3.67}$$

$$F = \begin{bmatrix} F_I \\ F_D \\ M_I \\ M_D \end{bmatrix} \quad (3.68)$$

Now that the unknowns of the assumed solution have been determined, the values of matrix S may be substituted back into equations (3.63) and (3.64) for a complete steady state solution to the coupled problem. A convenient tool which may be used in obtaining the solution matrix S is the NPS computer subroutine GELG. This subroutine was used extensively by the author of this thesis.

As a check on the previous single degree of freedom solutions, the coupled solutions were compared to the uncoupled solutions after setting the value of the coupling term, F, in the coupled system equal to zero. The results were identical.

4. Nine Point Mooring System

An analysis of coupled motion of the tower was also made for a nine point mooring configuration as shown in Figures 11 and 12. The analysis and solution to the nine point mooring system is identical to that obtained for the five point mooring system with the exception that the values of K_p, K_s, and F are changed such that now

$$K_s = 4K \quad (3.69)$$

$$K_p = 2kR_1^2 - F_8 R_1 - F_3 R_2 + F_9 R_3 + 2kR_3^2 - F_1 R_3 \quad (3.70)$$

$$F = 2KR_1 + 2KR_3 \quad (3.71)$$

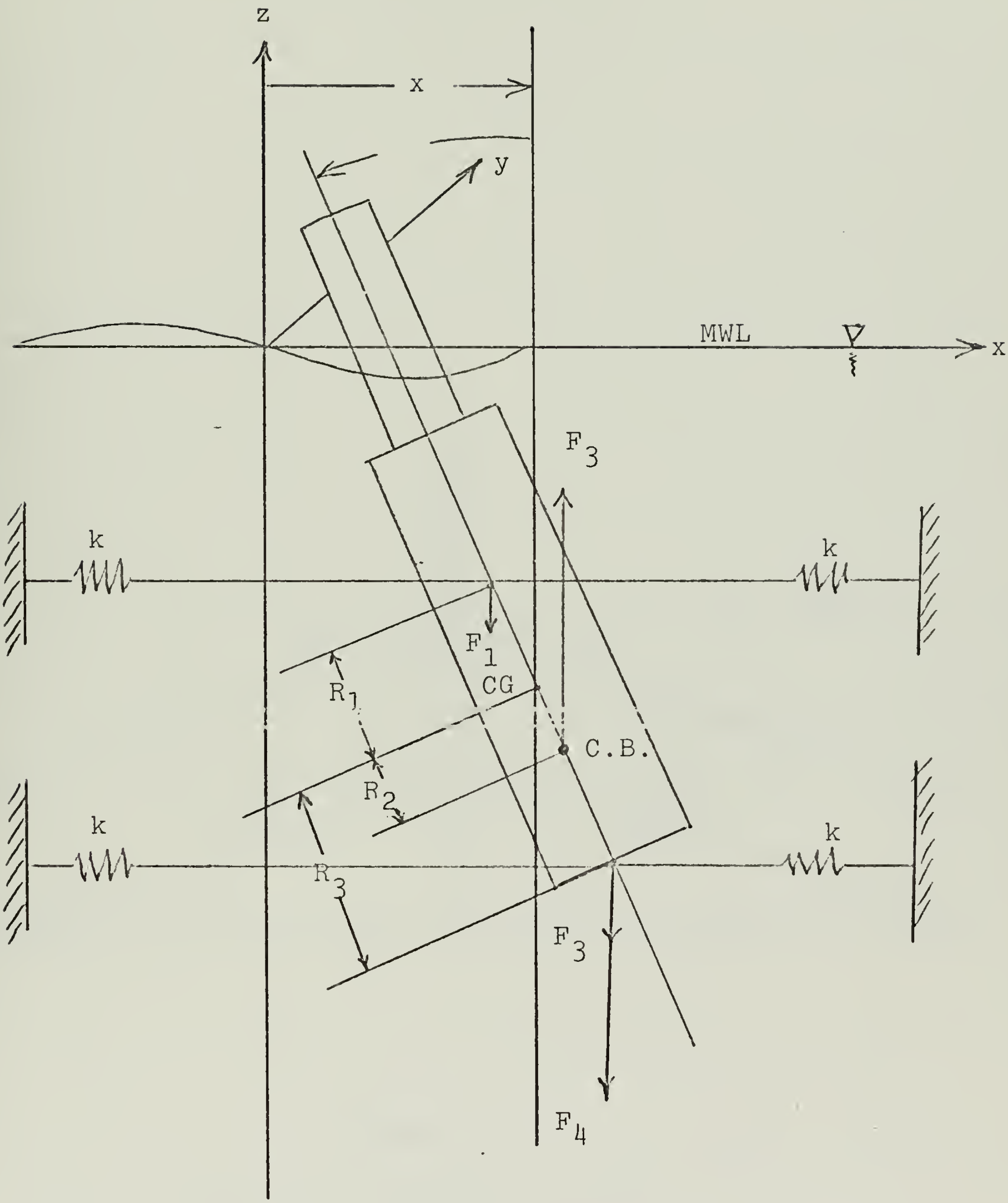


FIGURE 11.

Force Diagram-Nine Point Mooring.

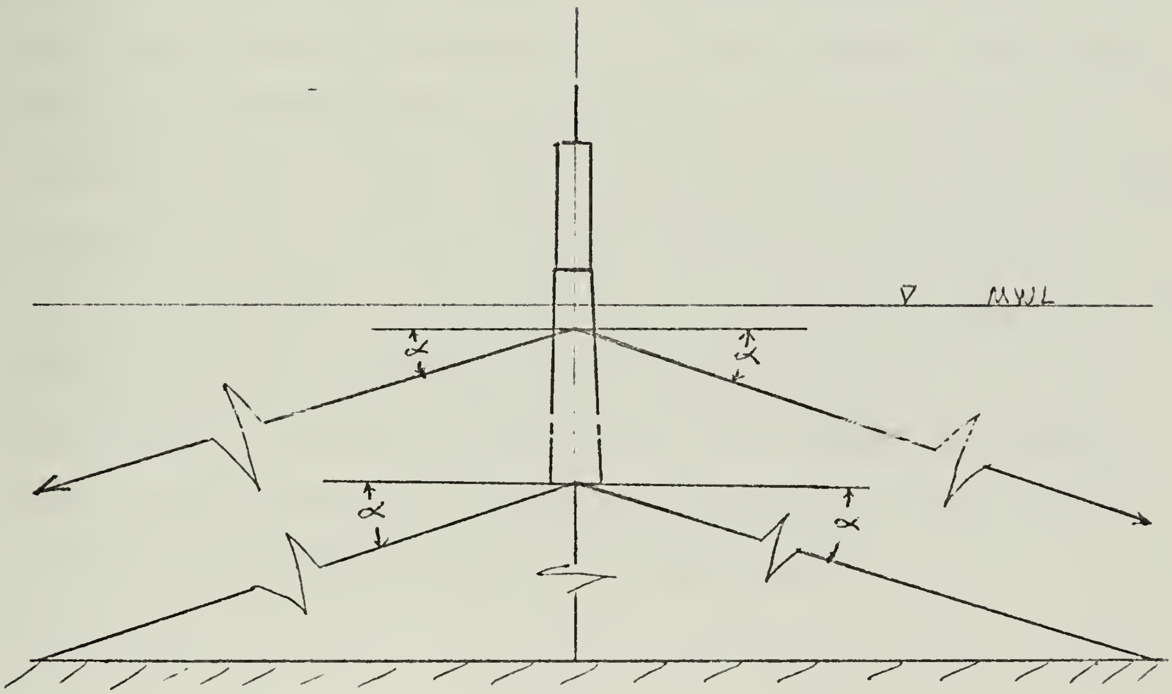


FIGURE 12.

Nine Point Mooring Schematic.

The analysis can now be made for various points of attachment of the upper cable to the tower so as to achieve an optimum configuration for this mooring system.

It is important that the tension in the cables of the nine point mooring system be distributed such that the force, F_1, in Figure 11 is a minimum. Yet it is also important that there be sufficient tension in the upper cables such that they can be treated as linear springs. By referring to a standard stress-strain curve for wire cables by Wilson [1], it is found that a cable tension of 7,500.0 lbs is sufficient to ensure a linear response in the cables.

A tension of 7,500.0 lbs in each of the upper cables results in a force, F_1, equal to 9,800.0 lbs. Accordingly the tension in the lower five cables is changed such that the vertical component of tension in each cable is reduced by (9800/5)lbs, or 1,960.0 lbs.

IV. STATIC WIND FORCE ANALYSIS

In view of the fact that wind may cause a significant amount of heel to the tower due to its force on the mast and tower superstructure, an analysis will now be made to determine the heel of the tower as a function of wind speed. This will be done for both the five point and nine point mooring configurations.

A. FIVE POINT MOORING SYSTEM

Referring now to Figure 13, but disregarding the upper cables and force, F_1, since they apply only to the nine point mooring system, it can be observed that for static equilibrium in heel, it is necessary that

$$\sum M_B = 0$$

$$= F_{WM} d_1 + F_{WS} d_2 + W d_4 \sin\theta - F_B d_5 \sin\theta \quad (4.1)$$

where the force due to the steady wind is given by

$$F = \frac{1}{2} \rho C_D U^2 A$$

and

F_{WM} = wind drag force on mast.

C_D = drag coefficient

= 1.2 (for cylindrical mast)

= 1.5 (for tower)

ρ = density of air = .00237 slugs/ft^3

U = uniform wind velocity in ft/sec

A = projected area of structure

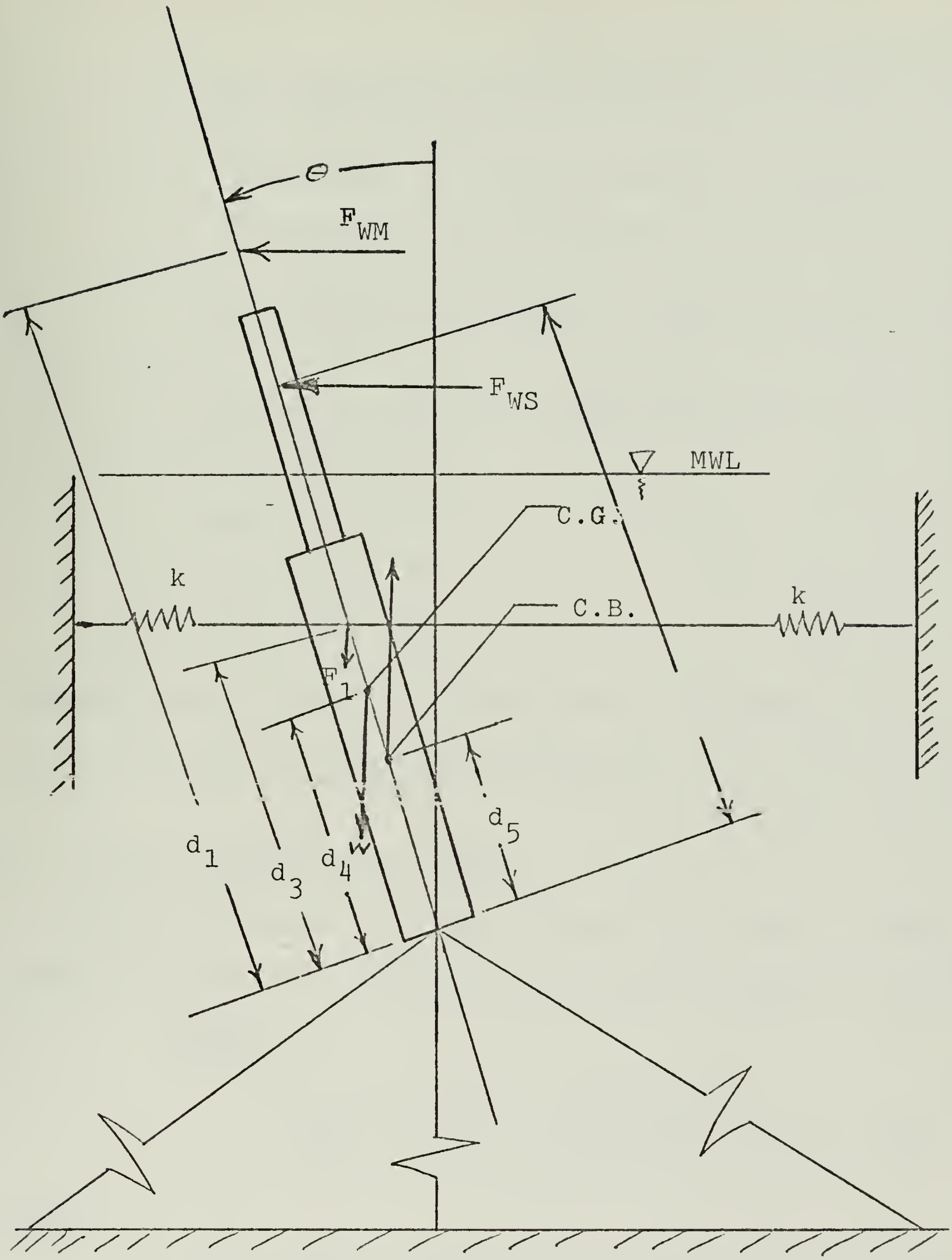


FIGURE 13.

Wind Force Diagram.

F_{WS} = wind drag force on portion of tower structure above MWL.

W = weight of tower in water = 19,742.0 lbs

θ = angle of heel of tower using five point moor

M_B = summation of moments about the bottom of the tower

F_B = buoyancy force = 33,280.0 lbs

d_1 = 105.5 ft

d_2 = 73.0 ft

d_4 = 28.12 ft

d_5 = 25.7 ft.

Solving equation (4.1) for θ in terms of U gives

$$\theta = \sin^{-1}(1.15 \times 10^{-4} U^2) \qquad (4.2)$$

B. NINE POINT MOORING SYSTEM

Referring again to Figure 13 and this time including all cables and forces, the equilibrium equation may be written as

$$\sum M_B = 0$$

$$= F_{WM} d_1 + F_{WS} d_2 + W d_4 \sin\theta - F_B d_5 \sin\theta - 2K d_3 \sin\theta + F_1 d_3 \sin\theta \qquad (4.3)$$

where

θ = heel of tower using nine point mooring system

F_1 = vertical component of forces due to upper cable tension

K = stiffness coefficient = 1825.0 lbs/ft

d_3 = distance from the bottom of the tower to the point of attachement of the upper cables.

And again solving for θ but expressing it this time in terms of d_3 such that

$$\theta = \sin^{-1}\left(\frac{34.7\,U^2}{30.2\times10^4 + 2342\,d_3}\right) \qquad (4.4)$$

Using equations (4.2) and (4.4), the heel of the tower for various windspeeds can be determined.

V. SUMMARY OF RESULTS AND CONCLUSIONS

Utilizing the results obtained from each analysis, a series of response spectra (see Figure 6) for various modes of oscillation, sea states, and design configurations are developed from the following relations:

$$X^2(\sigma) = \left[T.F.(\sigma)\right] A^2(\sigma) \tag{3.72}$$

$$Z^2(\sigma) = \left[T.F.(\sigma)\right] A^2(\sigma) \tag{3.73}$$

$$\theta^2(\sigma) = \left[T.F.(\sigma)\right] A^2(\sigma) \tag{3.74}$$

Then the values of significant surge, pitch, and heave for each of the above situations are obtained using equation (3.13).

A. MOTION OF THE TOWER IN HEAVE

The motion of the tower in heave was found to be very slight, and almost negligible when compared to surge and pitch, thus justifying the assumption for a two degree of freedom analysis. The results of the heave analysis for various sea states for both the five and nine point mooring systems are tabulated in Table III. The results for heave are independent of cable location.

B. MOTION OF TOWER IN SURGE AND PITCH-FIVE POINT MOORING SYSTEM

The results for surge and pitch are presented together since they both depend on the lateral cable location. Assuming a five point mooring configuration, various distances,

Mooring System	$H_{1/3}$ ft	$Z_{1/3}$ ft
5 Pt.	1.3	0.024
5 Pt.	3.7	0.066
5 Pt.	7.7	0.119
5 Pt.	13.5	0.177
5 Pt.	21.3	0.232
9 Pt.	1.3	0.019
9 Pt.	3.7	0.061
9 Pt.	7.7	0.111
9 Pt.	13.5	0.165
9 Pt.	21.3	0.215

TABLE III. SIGNIFICANT HEAVE FOR VARIOUS VALUES OF SIGNIFICANT WAVE HEIGHT.

d, of cable attachment point from the bottom of the tower were used to obtain corresponding values of significant surge and pitch as shown in Figure 13. A significant wave height of 7.6 feet was assumed.

It becomes obvious from the results presented in Figure 14 that the best point of attachment for the cable in minimizing tower motion is at the bottom of the tower, This was found to be the best point of attachment for all other values of significant wave height as well. The values of significant heave, surge, and pitch for various sea states using this design configuration are presented in Table IV.

C. MOTION OF THE TOWER IN SURGE AND PITCH-NINE POINT MOORING SYSTEM

If a nine point mooring system is utilized in the design of the ocean platform, it is necessary to determine the optimum distance from the bottom of the tower for the lateral cable attachment point as in the case of the five point mooring system.

Assuming that four of the lateral restraining cables would be attached to the bottom of the tower, an analysis was made to determine the response of the tower for various distances, d, to the points of attachment of the remaining four lateral cables. The results of this analysis are shown in Figure 15.

D. LOWER PLATFORM EXCURSION

Although Figures 14 and 15 are useful in illustrating surge and pitch separately, it is also important to determine

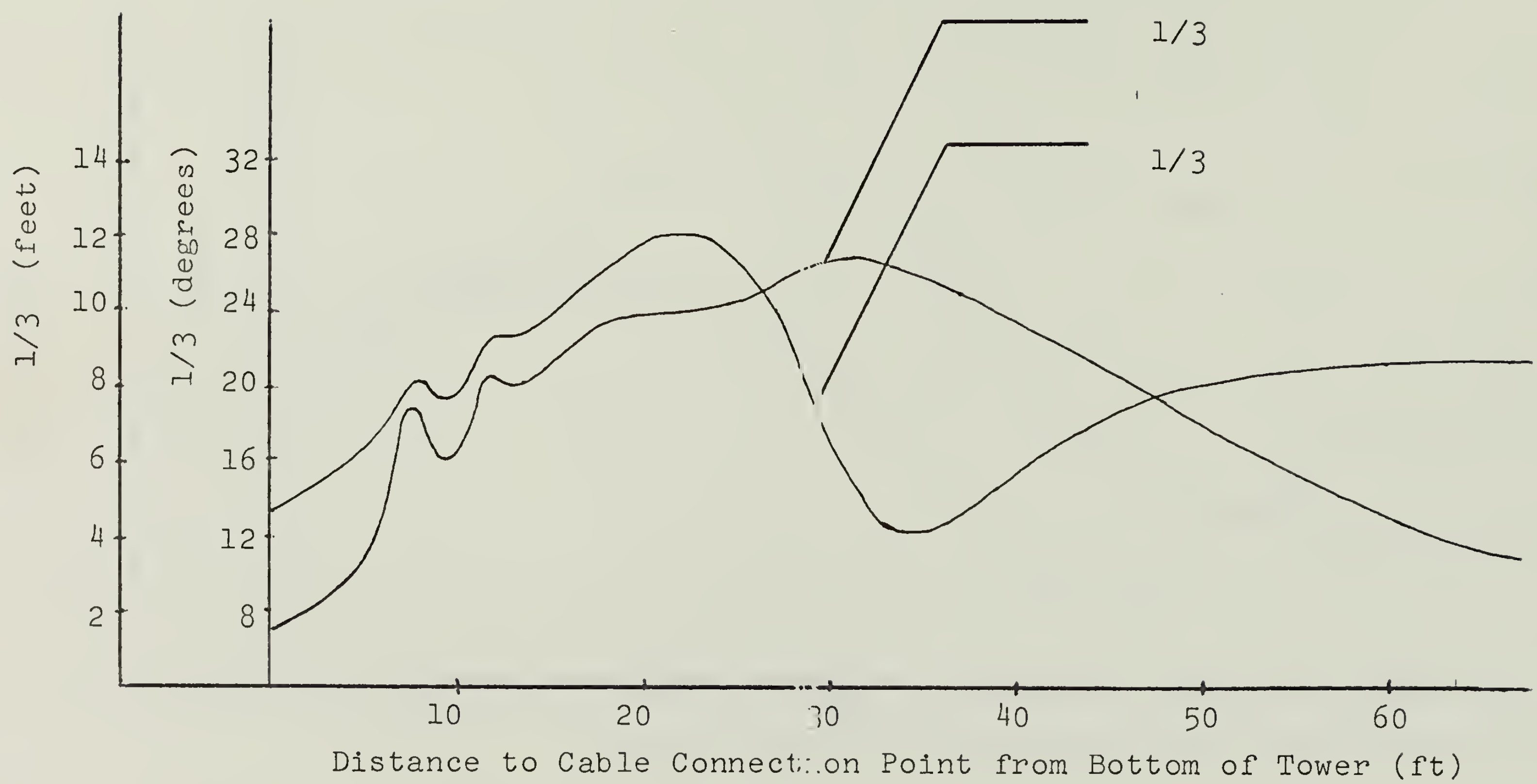


FIGURE 14. Significant Surge and Pitch as a Function of Cable Connection Point Using a Five Point Mooring System and H 1/3 = 7.6 feet.

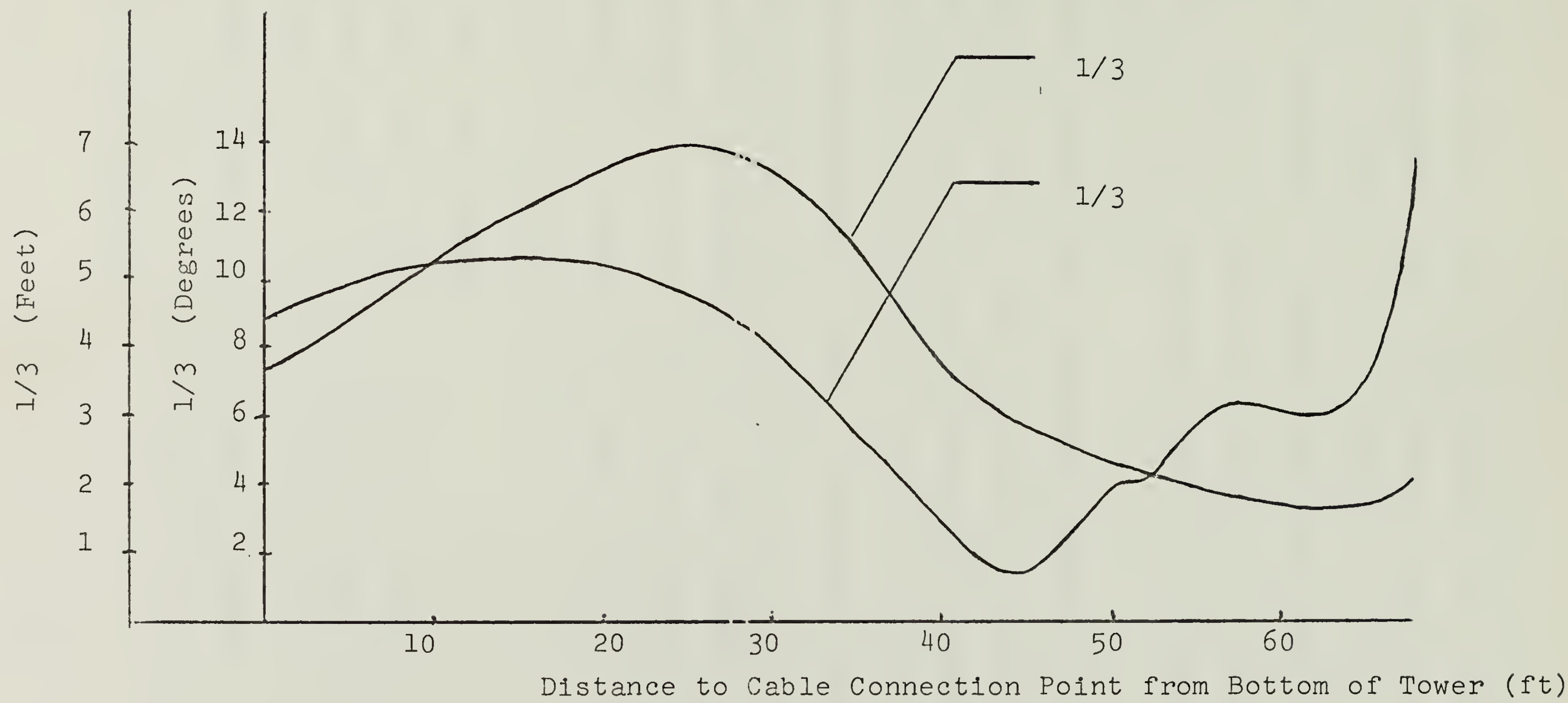


Figure 15. Significant Surge and Pitch as a Function of Cable Connection Point Using a Nine Point Mooring System.

the total excursion of the lower platform due to surge and pitch. It is necessary that the platform excursion be known for various sea states in order to determine the type of measurements which can be validly taken from the platform. The graph is Figure 16 shows the lower platform excursion as a function of cable attachment point for both the five and nine point mooring system.

It becomes obvious from the results shown in Figure 16 that the optimum distance from the bottom of the tower for attachement of the second set of restraining cables is 46 feet. This point of attachment is still 9.5 feet below MWL and therefore acceptable in terms of preventing small craft interference with the cables. This was also found to be the best point of attachment for all other values of significant wave height as well. The values of significant heave, surge, and pitch for various sea states using this design configuration are presented in Table V.

E. HEEL DUE TO WIND FORCES

Utilizing equations (4.2) and (4.4), the graph in Figure 17 was developed to illustrate the heel angle of the tower as a function of wind speed for both the five point mooring system and the nine point mooring system. It is obbious from equation (4.4) that the angle of heel decreases as the distance, d_3, increases.

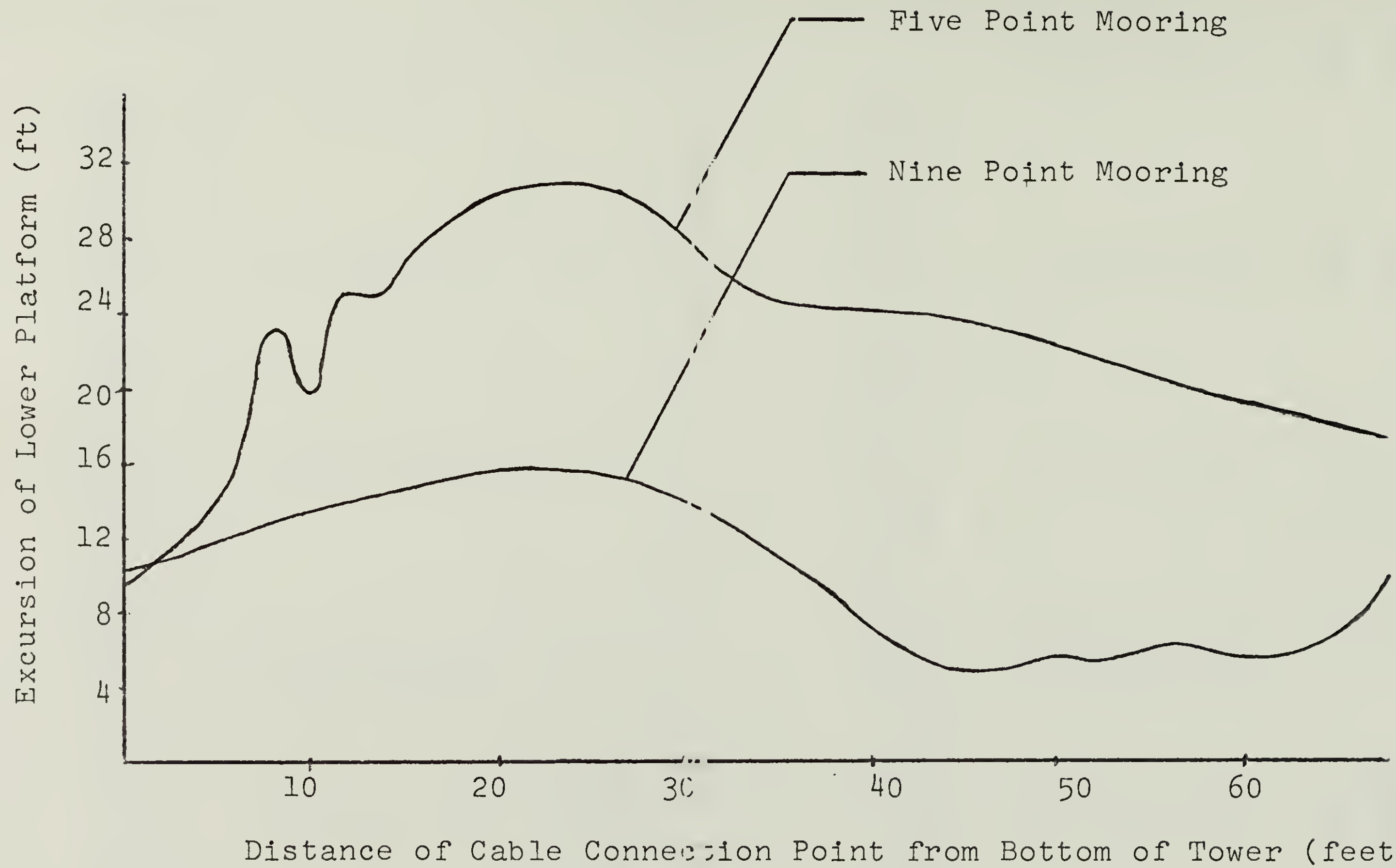


FIGURE 16. Total Excursion of Lower Platform due to Surge and Pitch as a Function of Cable Connection Point for both Five and Nine Point Mooring Systems

$H_{1/3}$ (ft)	$X_{1/3}$ (ft)	$Z_{1/3}$ (ft)	1/3 (°)	HORIZONTAL EXCURSION (ft)
1.3	0.36	.024	0.78	0.98
3.7	1.78	.066	2.00	3.37
7.7	4.8	.119	6.33	9.82
13.5	6.37	.117	8.51	13.12
21.3	7.65	.232	10.47	15.94

TABLE IV. SIGNIFICANT HEAVE, SURGE, PITCH, AND EXCURSION FOR VARIOUS VALUES OF SIGNIFICANT WAVE HEIGHTS USING FIVE POINT MOORING SYSTEM

$H_{1/3}$ (ft)	$X_{1/3}$ (ft)	$Z_{1/3}$ (ft)	1/3 (°)	HORIZONTAL EXCURSION (ft)
1.3	.608	.019	1.34	1.67
3.7	.755	.061	3.04	3.16
7.7	.973	.111	5.43	5.28
13.5	1.27	.165	8.22	7.79
21.3	1.73	.215	11.51	10.84

TABLE V. SIGNIFICANT HEAVE, SURGE, PITCH, AND EXCURSION FOR VARIOUS VALUES OF SIGNIFICANT WAVE HEIGHTS USING NINE POINT MOORING SYSTEM

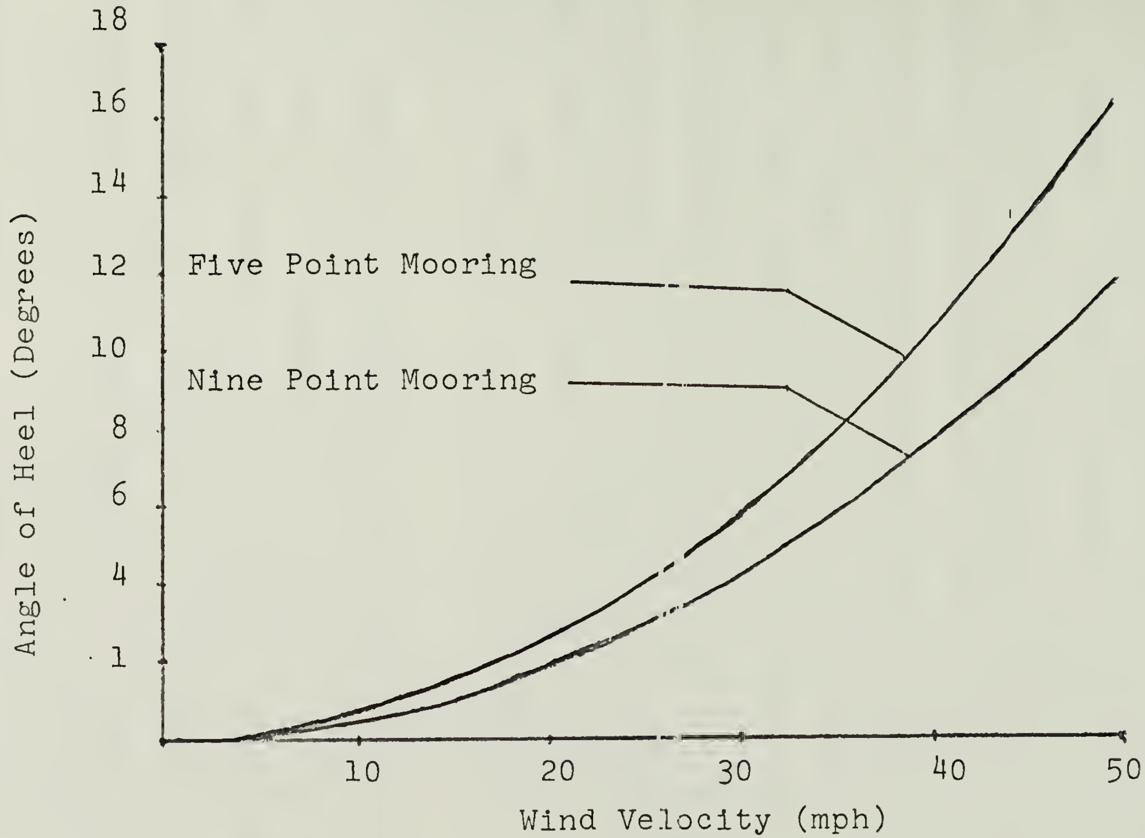


FIGURE 17. Angle of Heel of Tower as a Function of Wind Velocity.

F. CONCLUSIONS AND RECOMMENDATIONS

Based on the results obtained in this study of the dynamic response of the NPS ocean instrument platform to wave action, it can be concluded that the surge and pitch as given in Table IV for a five point mooring system with all cables attached at the bottom of the tower may be excessive for certain types of instrumentation. However, it is felt that a nine point mooring system will considerably decrease the platform motion as shown in Table V to the extent that most of the desired observations and measurements may be taken with accuracy. Considering the factors of dynamic wave forces, steady wind forces, and safe clearance for boating, the optimum design for a nine point mooring system is such that the lower set of cables are attached at the bottom of the tower and the upper set of lateral cables are attached at a point 46 feet above the bottom of the tower.

A number of assumptions were made in the development, some of which were conservative, others of which were possibly non-conservative but considered necessary in linearizing the equations of motion. The conservative assumptions were that all of the waves approach the tower from one direction, the estimates of the geometric configuration of the tower model, and the use of a wave spectrum which represents a fully developed sea. The assumption that waves approach the tower from one direction is conservative for Monterey Bay because most of the large swell in the Bay approaches

Taking into consideration all of the assumptions, it is felt that the results of this analysis give a conservative prediction of exactly how the NPS ocean instrument platform will respond to environmental forces using various design configurations. Hence, the motion of the tower is expected to be less than predicted by the analysis presented.

It is therefore recommended that the NPS ocean instrument be first installed using a five point mooring system. This will provide at least minimum stability. If the motion does in fact prove to be excessive, then the additional four cables may be added to improve stability. One benefit from this particular sequence is that the tension may be gradually increased on the last four cables as they are being installed so as to "tune" the platform restraining system to its best design configuration.

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

DYNAMIC ANALYSIS OF THE NAVAL POSTGRADUATE SCHOOL OCEAN INSTRUMENT PLATFORM

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

September 1971; Master's Thesis

5. AUTHOR(S) (First name, middle initial, last name)

Mark F. Crane

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1971	80	4

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

The dynamic and static response of the proposed NPS ocean instrument platform is investigated by developing and solving linear differential equations of motion of the tower in surge, heave, and pitch. The motion is expressed as a response spectrum which is directly proportional to a wave spectrum as the exciting force. The analysis is made for various configurations of the lateral restraining cables using both a five point and nine point mooring system. For all configurations, the heave response of the tower is found to be less than one percent of wave height. The stability of the tower in pitch is found to be considerably improved after shifting from a five point mooring system to a nine point mooring system and optimizing the location of the cable attachment points. Using this design, a significant wave height of 7.7 feet is found to produce a significant pitch of 5.4 degrees, a significant surge of .97 feet, and a 5.28 foot excursion of the lower platform on the tower. All oscillations will be superimposed on any heel angle of the tower which may exist due to steady wind forces. The angle of heel for a wind of 20 mph is evaluated to be 1.9 degrees.

DD FORM 1 NOV 65 1473 (PAGE 1)
S/N 0101-807-6811

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A-31408

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	ROLE	WT	ROLE	WT	ROLE	WT
Ocean Platform						
Stable Ocean Platform						
Ocean Instrument Platform						
Moored Ocean Platform						

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Dynamic analysis of the Naval Postgraduate School ocean instrument platform.