DYNAMIC CHARACTERISTICS OF LIQUID MOTION IN PARTIALLY FILLED TANKS OF SPINNING SPACECRAFT

Brij N. Agrawal*
Naval Postgraduate School
Monterey, California

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Abstract

This paper presents a boundary layer model to predict dynamic characteristics of liquid motion in partially filled tanks of a spinning spacecraft. The solution is obtained by solving three boundary value problems: inviscid, the boundary layer, and viscous correction. This boundary layer solution is obtained analytically, and the inviscid and viscous correction solutions are obtained by using finite element methods. This model has been used to predict liquid natural frequencies, mode shapes, damping ratios, and nutation time constants for the INTELSAT VI spacecraft. The analytical results were compared with experimental results and are in good agreement. The results show that liquid motion in general will contain significant circulatory motion due to Coriolis forces except in the first azimuth and first elevation modes. Therefore, only these two modes can be represented accurately by equivalent pendulum models. The analytical results predict a sharp drop in nutation time constants for certain inertia ratios and tank fill fractions. This phenomenon, known as anomalous resonance, was also present during INTELSAT IV in-orbit liquid slosh tests and ground air bearing tests for INTELSAT IV and VI.

I. Introduction

A recent trend in geosynchronous spacecraft design is using liquid apogee motors, which results in liquid constituting almost half of the spacecraft mass during transfer orbit. In these spacecraft, liquid motion significantly influence the spacecraft attitude stability and control. LEASAT, a geosynchronous spacecraft with liquid apogee motor, launched in September 1984, experienced attitude control system instability[1] during the pre-apogee injection phase, immediately following the activation of despin control. The instability was found to be the result of interaction between liquid lateral sloshing modes and the attitude control. This experience demonstrated that the analysis of dynamic interaction between liquid slosh motion and attitude control is critical in the attitude control design of these spacecraft. In order to perform this analysis, accurate determination of liquid dynamic characteristics, such as natural frequencies, mode shapes, damping, and modal masses becomes important. Accurate prediction of liquid dynamic characteristics is, however, a difficult problem because of the complexity of hydrodynamical equations of motion.

Simplified models, such as rigid pendulum models, have been found to be significantly inaccurate, resulting in some cases unstable attitude control design. This has provided impetus to space industry to develop improved analytical models to predict liquid dynamic characteristics in spinning spacecraft.

Recently, a finite element model has been developed, under INTELSAT sponsorship by MBB, to predict liquid natural frequencies, mode shapes, damping ratios, nutation frequency, and nutation time constant for spinning spacecraft with partially filled tanks. This model has been extensively used to study the effects of liquid motion on the attitude dynamics and control of INTELSAT VI, a dual-spin spacecraft with a liquid apogee motor. This paper presents the boundary layer model, analytical prediction of liquid dynamic characteristics, and comparison with experimental results.

II. Equations of Motion

An analytical model is developed for the spacecraft configuration shown in Fig. 1. The tank is offset from the spin axis and is partially filled with liquid. The tank shall be of rotational symmetry but its contour may be of arbitrary shape. The symmetry axis may be inclined with respect to the spin axis. The coordinate system XYZ is fixed in the spinning spacecraft with origin at the center of mass of the spacecraft and Z axis along the spin axis. The coordinate system xyz is fixed in the tank with origin at the center of the tank and z axis along the tank symmetry axis.

General equations of motion of liquid are represented by Navier-Stokes equations as follows:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + 2\Omega \times u + \dot{\Omega} \times R + \Omega \times (\Omega \times R) = -\frac{1}{\rho}\nabla p + f + \nu\nabla^2 u \qquad (1)$$

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* Professor, Department of Aeronautics and Astronautics
Associate Fellow, AIAA

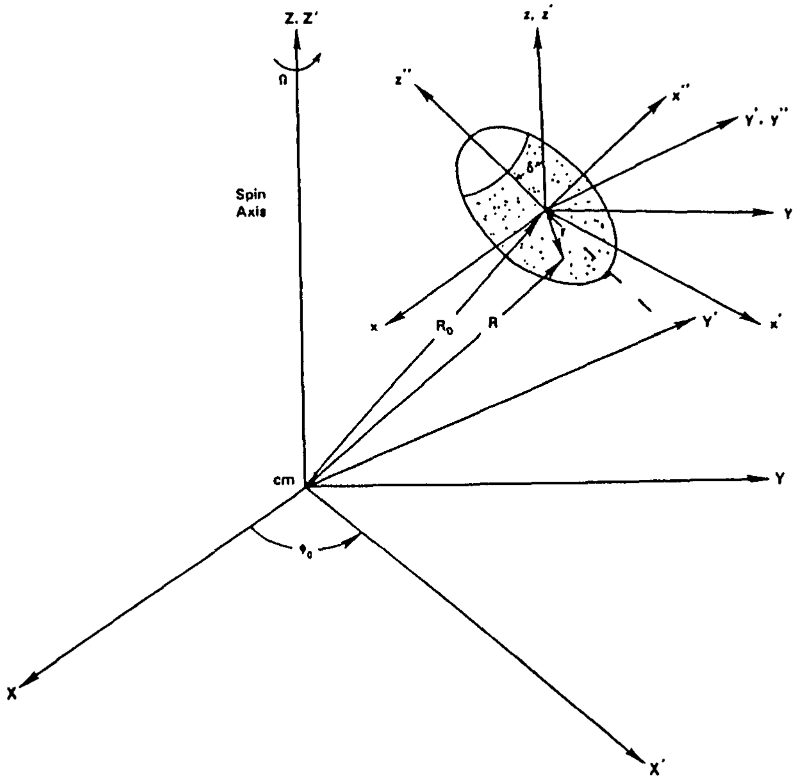

Fig. 1 Coordinate System.

where, u is the relative velocity of the liquid with respect to the tank, R is the position vector of liquid particle, Ω is the angular velocity, ρ is the liquid density, p the liquid pressure, f the body force per unit mass, and ν is the kinematic viscosity.

Assuming the liquid to be incompressible, the continuity equation is

$$\nabla \cdot u = 0 \qquad (2)$$

There are two sets of boundary conditions. The first condition enforces no flow through the tank wall.

$$u \cdot n = 0 \qquad (3)$$

where, n is a unit vector normal to the tank wall. The second set of boundary conditions enforces constant pressure and surface equation on the free surface as

$$p = \text{constant} \qquad (4)$$

$$\frac{dF}{dt} = 0 \qquad (5)$$

where F is the free surface equation.

The equation of motion of a spinning spacecraft can be written as

$$I \cdot \Omega + \Omega \times I \cdot \Omega = T \qquad (6)$$

where I is the inertia dyadic of the spacecraft with empty tanks and T is the torque exerted by the liquid on the spacecraft and is given by

$$T = \int_{S_W} R \times np \, ds + \rho\nu \int_{S_W} R \times [(n \cdot \nabla)(n \times (n \times u))] ds \qquad (7)$$

The integrals are taken over the wetted surface, S_W, where the first integral is the torque due to the liquid pressure and the second integral is the torque due to wall shear moment.

III. Analytical Model

The solution of general equations of motion of liquid, Eqs. (1) and (2), with associated boundary conditions, Eqs. (3), (4), and (5), is very complex. Therefore, several simplified analytical models have been used to study the dynamic behavior of liquids in spinning spacecraft. A simple model, commonly used by space industry, is the rigid pendulum model, where the liquid propellant is treated as a distributed mass pivoting about the center of the tank with the total liquid mass located at the mass center of the liquid. This model has been found to be very inaccurate in the prediction of liquid natural frequencies, resulting in some cases unstable attitude control design. Abramson's model[2] is based on an ideal fluid executing an irrotational motion. The centrifugal force is represented by an equivalent constant gravitational force with Coriolis effects neglected.

In the homogeneous vortex model[3], the simplifying assumptions are that the liquid vorticity is independent of the spatial coordinates and only time dependent, and the Coriolis acceleration is retained inside the liquid but is neglected in the free surface boundary condition in order to obtain an integral. In the boundary layer model, developed by A. Pohl[4], three boundary value problems are solved: inviscid, the boundary layer equation, and viscous correction of the inviscid solution. During the INTELSAT VI study, all the analytical models discussed here were compared and the conclusion was that the boundary layer model gives the most accurate prediction of liquid dynamic characteristics. The boundary layer model is briefly discussed in the following section.

IV. Boundary Layer Model

The equation of motion of the liquid, Eq. (1), is first nondimensionalized. For nondimensionalization, a fictitious velocity, $U = a \cdot \omega_n$, is defined, where, a, denotes a reference length of the tank, and ω_n is the nutation frequency of the rigid spacecraft. The following dimensionless quantities are introduced.

$$\bar{t} = t\omega_n \qquad \bar{\Omega} = \bar{\bar{\Omega}} / w_n$$

$$\bar{r} = r/a \qquad \bar{R}_O = R_O/a$$

$$\bar{R} = R/a \qquad \bar{u}\, e^{\lambda t} = u/U$$

$$\bar{p}\, e^{\lambda t} = p/(\rho U^2) \qquad \bar{\omega}\, e^{\lambda t} = \omega/\omega_n$$

$$\bar{g}_O = g/(a \cdot \omega_n^2) \qquad R_e = \omega_n^2 a^2/\nu \tag{8}$$

Here R_e denotes Reynolds number and λ is the eigenvalue to be determined. The angular velocity is written as

$$\Omega = \bar{\bar{\Omega}}\, k + \omega \tag{9}$$

where $\bar{\bar{\Omega}}$ is steady-state angular velocity. Introducing the non-dimensional parameters, Eq. (1) can be written as,

$$\lambda\, \bar{u} + 2\, \bar{\Omega} k \times \bar{u} + \bar{\omega} \times r\lambda + \bar{\Omega}[\, \bar{\omega} \times (k \times \bar{r}) + k \times (\bar{\omega} \times \bar{r})] = -\nabla P + R_e^{-1}\nabla^2\, \bar{u} \tag{10}$$

where

$$P = \bar{p} + \{\lambda\, \bar{\omega} \times \bar{R}_O + \bar{\Omega}[\, \bar{\omega} \times (k \times \bar{R}_O) + k \times (\bar{\omega} \times \bar{R}_O)]\}\, \bar{r} - \frac{\bar{\Omega}^2}{2}(k \times \bar{R})^2 + \bar{g}_O(k\, \bar{R}) \tag{11}$$

The spacecraft equation, Eq. (6), becomes,

$$\lambda\, \bar{I} \cdot \bar{\omega} + \bar{\Omega}[k \times \bar{I} \cdot \bar{\omega} + \bar{\omega} \times \bar{I} \cdot k] = \bar{T} \tag{12}$$

where $\bar{I} = I/\rho a^5$.

$$\bar{T} = T/(\rho a^5 \omega_n^2) = \int_{S_w} \bar{R} \times n\bar{p}\, ds + \frac{1}{R_e}\int_{S_w} \bar{R} \times \{(n \cdot \nabla)(n \times (n \times \bar{u}))\}\, ds$$

For a multi-tank configuration, the contributions from the tanks are summed up.

The effects of viscosity are included in the liquid motion by a boundary layer analysis. Using the procedure proposed by Handricks and Morton[5], the unknown parameters are expanded in the power of $R_e^{-1/2}$ as follows.

$$\bar{\omega} = \omega_o + R_e^{-1/2}\omega^1$$

$$\bar{u} = u_o + \tilde{u}_o + R_e^{-1/2}(u_1 + \tilde{u}_1)$$

$$P = P_o + \tilde{P}_o + R_e^{-1/2}(P_1 + \tilde{P}_1)$$

$$\lambda = \lambda_o + R_e^{-1/2}\lambda_1 \tag{13}$$

Where quantities with a tilde refer to boundary-layer variables. Substituting these perturbation expressions into Eq. (10) and equating powers of $Re^{-1/2}$ yields three boundary value problems: the inviscid, the boundary layer, and the correction to the inviscid solution. By solving these boundary conditions, the complete solution is obtained.

The unknown variables for the inviscid solution are the pressure function, P_o, the velocity, u_o, angular velocity, ω_o, and the eigenvalue, λ_o. Neglecting the viscous terms, Eq. (10) is solved for the velocity components in terms of pressure. Substituting the velocity component, the continuity equation becomes,

$$\frac{\partial^2 P_o}{\partial x^2} + \frac{\partial^2 P_o}{\partial y^2} + \frac{\lambda_o^2 + 4\ \bar{\Omega}^2}{\lambda_o^2}\ \frac{\partial^2 P_o}{\partial z^2} = 0 \qquad (14)$$

The boundary conditions are also expressed in terms of the pressure. From Eq. (14) and the boundary conditions, the pressure and the eigenvalues of the liquid motion are determined by using a finite element method. The boundary layer equations are written with respect to the wetted surface polar normal coordinate. The unknown variables $\tilde{P}_o$, $\tilde{u}_o$, $\tilde{P}_1$, and $\tilde{u}_1$ are determined analytically.

The equations for the viscous correction are obtained by comparing all terms of order $Re^{-1/2}$. The equations for the unknown viscous correction parameters P_1, u_1, and λ_1 are:

$$\frac{\partial^2 P_1}{\partial x^2} + \frac{\partial^2 P_1}{\partial y^2} + \frac{\lambda_o^2 + 4\ \bar{\Omega}^2}{\lambda_o^2}\ \frac{\partial^2 P_1}{\partial z^2}$$

$$= 8\,\frac{\lambda_1\ \bar{\Omega}^2}{\lambda_o^3}\,\frac{\partial^2 P_o}{\partial z^2} \qquad (15)$$

The finite element method is used to solve Eq. (15) with boundary conditions to determine viscous correction to the liquid natural frequencies and mode shapes.

V. Numerical Solution

The finite element computer program based on the boundary layer model calculates liquid natural frequencies, mode shapes, damping ratios, torque exerted by the liquid on the spacecraft, energy dissipation rates, spacecraft nutation frequency, and nutation time constant. This model has been used extensively on the INTELSAT VI Program[6,7] to calculate liquid natural frequencies, mode shapes, and spacecraft nutation time constants.

Liquid Natural Frequencies and Mode Shapes

The finite element model used in the analysis is shown in Fig. 2. It consists of 81 node points. The mode shape is determined from the amplitudes and phase angles of the velocity components at the node points. The liquid motion of rotating bodies can be considered as a combination of two types of natural oscillations; sloshing waves and inertial waves. The sloshing waves are characterized by the oscillation of the free surface with velocity components either in phase or out of phase, i.e., absence of the circulatory motion due to the Coriolis forces. Slosh frequencies are normally greater than twice the spin rate. Inertial waves in contrast are circulatory and there may or may not be any apparent motion at the free surface. Inertial wave frequencies are less than twice the spin rate. For a pure slosh mode, the velocity components at all node points would be either in or out or phase. For an inertial mode, representing a circulatory motion, the velocity components at a node would have a 90° phase difference. In general, a mode will have a combination of slosh and inertial wave motion.

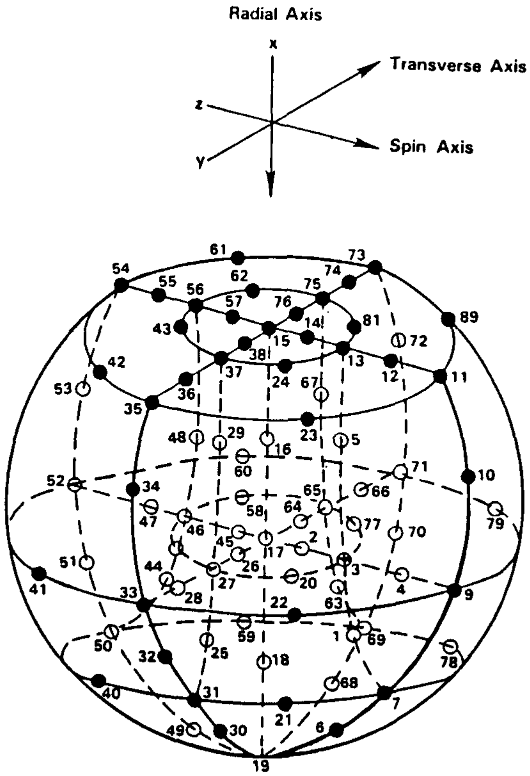

Fig. 2 Finite Element Model.

The calculated natural frequency ratios, ratios of natural frequencies to the spin rate, as a function of fraction fill for INTELSAT VI parameters are given in Table 1. The parameters are: tank radius = 0.42 m, radial distance of the tank center from the spin axis = 1.31 m, and spin rate = 30 RPM. The modes are divided into three groups: inertial, slosh and higher modes containing both inertial and slosh components. In the slosh modes, two modes are present; azimuth and elevation. In the azimuth mode, the liquid motion is in a plane normal to the spin axis with a major velocity component in the transverse direction. The azimuth mode produces reaction torque along the spin axis and therefore can cause interaction with the despin control. In the elevation mode, the motion is mainly in the plane normal to transverse direction with a major velocity component in the axial direction. The elevation mode can interact with the nutation control.

Comparison of Experimental and Analytical Results

Under the INTELSAT VI program, Hughes Aircraft company performed scaled model tests to determine liquid dynamic characteristics. the boundary layer model was also used for the scaled test parameters to analytically predict liquid dynamic characteristics. Comparison of the results is given in Table 2. The parameters used were: tank diameter = 0.146 m, radial distance of the tank center from spin axis = 0.218 m, liquid density = 1 g/ml, viscosity (cm^2/sec) = 0.009. The damping ratio is defined as a percentage of critical damping. The results show good agreement for natural frequencies and damping ratios considering the measurement errors and model approximation.

Nutation Time Constant

During several mission phases of INTELSAT VI, active nutation control is used. In order to properly design the active nutation control, the nutation dedamping due to liquid slosh needs to be determined accurately. Air bearing tests have been performed on the INTELSAT VI program to determine its nutation time constant. The critical parameters, inertia ratio and fill fraction, are kept the same and other parameters are scaled due to limitations of test vehicles and chamber. In-orbit

Table 1. Liquid Natural Frequency Ratios.

Fraction Fill, %	Inertial Modes	Slosh Modes First Azimuth	Slosh Modes First Elevation	Higher Modes		
95	1.996, 2.0, 2.009	2.876	3.04	4.114	4.42	4.81
90	1.992, 1.997, 2.002	2.57	2.784	3.778	3.998	4.255
80	1.992, 1.996, 2.0	2.314	2.556	3.366	3.694	3.792
70	1.991, 1.996, 2.0	2.118	2.434	3.287	3.552	3.580
60	1.992, 1.996, 2.0	2.109	2.351	3.167	3.396	3.53
50	1.992, 1.996, 2.0	2.056	2.287	3.07	3.278	3.507
40	1.989, 1.996, 2.0	2.023	2.233	3.008	3.17	3.512
30	1.989, 1.995, 2.0	2.007	2.187	2.95	3.09	3.536
20	1.979, 1.993, 1.997*	*2.003	2.142	2.90	3.01	3.57
10	1.993, 1.997*	*2.005	2.09	2.849	2.92	3.6

* Difficult to distinguish between inertial and first azimuth mode.

Table 2 Analytical vs. Experimental Results for Evaluation Mode.

Fill Fraction %	Frequency Ratio Analytical	Frequency Ratio Experimental	% Damping Analytical	% Damping Experimental
10	2.08	2.06	0.88	0.97
20	2.12	2.09	0.64	0.60
30	2.16	2.13	0.25	0.65
50	2.26	2.18	0.40	0.46
80	2.52	2.42	0.23	0.36
90	2.74	2.60	0.19	0.376

nutation time constants are extrapolated from the measured nutation time constant by using dimensional analysis.

Anomalous Resonances

In-orbit liquid slosh tests on INTELSAT IV exhibited sharp reduction of nutation time constants for certain tank fill fractions and inertia ratios. The air bearing tests were later performed and these tests validated the in-orbit results. These conditions were called anomalous resonances because the slosh frequencies were significantly higher than the nutation driving frequency. Therefore, the cause of these resonances was unknown.

INTELSAT VI air bearing liquid slosh test results have also exhibited sharp reductions in nutation time constants for certain fill fractions and inertial ratios. In order to validate the boundary layer finite element model, it was used to analytically predict nutation time constants for these parameters. Figure 3 shows the plots of nutation time constants determined analytically and experimentally for tank fill fractions of 20 percent and 80 percent. Both experimental and analytical results predict a sharp drop in nutation time constants for the same inertia ratio (0.45) and fill fraction (20%). The values of the nutation time constant are, however, lower than experimental results. This resonance condition could be due to excitation of inertial modes close to the nutation driving frequency.

VI. Summary and Conclusions

The boundary layer model to predict dynamic characteristics of liquid motion in spinning spacecraft with partially filled tanks is presented. The solution is obtained by solving three boundary value problems: inviscid, the boundary layer, and viscous correction. The boundary layer equations are solved analytically and the inviscid and viscous correction solutions are obtained by using finite element methods. The finite element computer program has been developed by MBB, Germany, under an INTELSAT contract. This program calculated the liquid natural frequencies and mode shapes, torque exerted by the liquid on the spacecraft, energy dissipation rates, spacecraft nutation frequencies, and time constants. The computer program has been used extensively on the INTELSAT VI program.

The results indicate that the liquid motion in rotating bodies can be considered as a combination of two types of natural oscillations: sloshing modes and inertial modes. The slosh modes are characterized by the oscillation of the free surface with velocity components either in phase or out of phase, i.e., absence of the circulatory motion due to

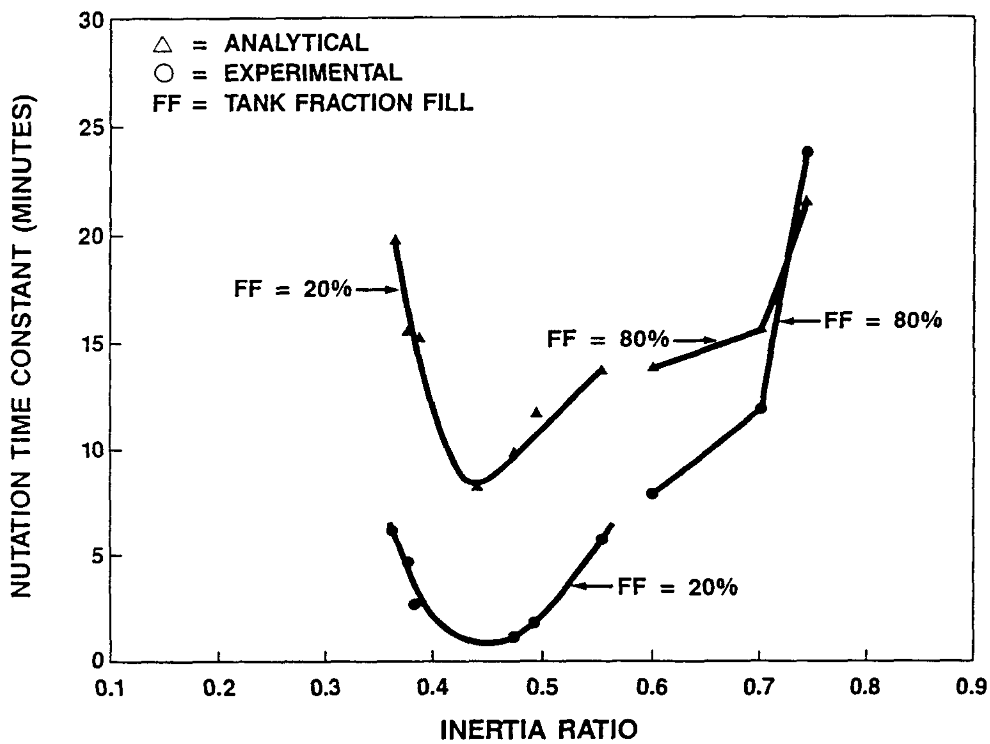

Fig. 3. Nutation Time Constant

the Coriolis forces. Inertial modes in contrast have circulatory motion and represent a dynamic interaction between Coriolis forces and the pressure forces. The numerical results show two slosh modes, first azimuth and elevation modes, and lower modes to be inertial modes and higher modes to be a combination of inertial and slosh modes. Therefore, only these two slosh modes can be accurately represented by pendulum models.

The analytical results predict a sharp drop in nutation time constant for certain inertia ratios and tank fill fractions. This phenomenon known as anomalous resonance, was also present during INTELSAT IV in-orbit liquid slosh tests and ground air bearing tests for INTELSAT IV and VI. The nutation time constant depends on two liquid characteristics, energy dissipation and resonance of normal modes due to nutation driving frequency. Since the slosh frequencies are significantly higher than the nutation driving frequencies, it appears that the resonance condition is due to excitation of inertial modes. Further work is required to identify the inertial modes which contribute to this resonance.

VII. Acknowledgements

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