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Dynamic control of electromagnetic wave propagation with the equivalent principle inspired tunable metasurface

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Transmission and reflection are two fundamental properties of the electromagnetic wave propagation through obstacles. Full control of both the magnitude and phase of the transmission and reflection independently are important issue for free manipulation of electromagnetic wave propagation. Here we employed the equivalent principle, one fundamental theorem of electromagnetics, to analyze the required surface electric and magnetic impedances of a passive metasurface to produce either arbitrary transmission magnitude and phase or arbitrary reflection magnitude and phase. Based on the analysis, a tunable metasurface is proposed. It is shown that the transmission phase can be tuned by 360° with the unity transmissivity or the transmissivity can be tuned from 0 to 1 while the transmission phase is kept around 0°. The reflection magnitude and phase can also been tuned similarly with the proposed metasurface. The proposed design may have many potential applications, such as the dynamic EM beam forming and scanning.

ontrolling light has a long history in human civilization. The techniques to control the light originate from the observation of light propagation, such as focusing light with lens and decomposing light with prisms, and has been developed continuously to date with the deeper understanding of electromagnetic (EM) waves after the discovery of Maxwell equations.

Transmission and reflection are two fundamental properties of the EM wave propagation through obstacles. Both of them can be characterized as complex quantities with both magnitude and phase. Controlling wave propagation means to have a full control for these four factor independently, i.e. magnitude and phase of transmission and reflection. The reflectarray, which is a non-penetrable EM impedance surface, can be used to control the reflection phase of the incident waves under the lossless case¹⁻³. It can also be designed to work as the EM wave absorbers to control the reflection magnitude by adding lossy material into the surface^{4,5}. The frequency selective surface (FSS) essentially works as a spatial filter to control either the transmission or reflection magnitude conventionally⁶⁻⁸. It has been studied to control the transmission phase recently⁹⁻¹¹. However, in order to obtain full transmission phase, multi-layer FSS has to be used, which increases the total thickness¹¹. Transmitarray is another way to control the transmission phase by employing receiving and transmitting elements connected by phase delay line^{12,13}. Its working principle is similar to that of optical lens¹⁴. Active version of the reflectarray, transmitarray and FSS has also been reported to achieve various functions, such as beam forming and scanning in either reflection or transmission mode, and controllable spatial filtering of EM waves¹⁵⁻²¹. Recently, the metasurface, which are thin metamaterial structure generally created by assembling arrays of subwavelength resonant sactters, could produce abrupt phase change associated with the reflection or transmission at the metasurface, providing new ways to control EM wave propagation^{22,23}. For example, it has been found that the nonconventional far-field EM wave responses of ultrathin metasurfaces could deviate from classical reflection and refraction laws^{23,24}.

However, all these techniques can not provide independent control for the magnitude and phase of transmission and reflection respectively. From the Smith Chart, we know that in order to independently control the reflection phase and magnitude, the real and imaginary part of the surface impedance shall be controlled independently. However, the real part representing the surface resistance is not easy to tune. As to the transmission coefficient control, for single layer FSS characterized by the penetrable electric surface impedance, it is known that $|T| = \cos(\phi)$, where |T| and ϕ are the transmission magnitude and phase respectively¹¹. Hence, it is difficult to obtain independent control for these two factors.



Figure 1 | Schematic of the metasurface under EM wave incidence. The induced electric and magnetic current on the metasurface under normal incidence produces the scattered field E', H', E^{M} and H^{M} in region 1 and 2. **k** is the wave vector.

The equivalent principle is one of the fundamental theorem of electromagnetics²⁵. It allows arbitrary electromagnetic field at both sides of a surface by introducing electric and magnetic current on the surface to satisfy the boundary condition of the field. Traditionally, the equivalent principle is mainly used in theoretical analysis because the effective magnetic current can not be easily generated. However,

it inspires an new way to manipulate the transmission and reflection of the EM waves when combined with the metasurface concept. In²⁶, a metasurface composed of electric and magnetic dipoles are proposed with designed surface electric and magnetic impedances. Under the plane wave illumination, surface electric and magnetic current are induced, which can produce the scattered field so that total transmitted field has the designed wave front and the reflected field is null. In fact, the metasurface employing the equivalent principle allows the independent design of the magnitude and phase in either transmission or reflection with its physical thickness much smaller than working wavelength.

In practice, the passive and lossless metasurface, which is characterized only by surface electric and magnetic reactance, is more general and easy to realize. In this work, without loss of generality, we first analyze the achievable transmission and reflection coefficients of such kind of metasurface. Then, we propose the tunable metasurface design so that the induced surface electric and magnetic currents can be tuned dynamically. By this way, we can independently and dynamically control either transmission magnitude or phase with the other one kept constant. Such function is also available in reflection mode. Comparing with conventional RF techniques, the proposed design provides more freedom to arbitrarily manipulate the EM wave propagation, and can be applied in reconfigurable beam forming in either transmission or reflection mode.

Results

Theorectical analysis. Assuming the metasurface is located at z = 0, and illuminated by the normal incident plane wave propagating



Figure 2 | The required surface electric and magnetic reactance for various transmission and reflection coefficients. Contour of surface electric and magnetic reactance with respect to the magnitude and phase of (a) transmission and (b) reflection.



Figure 3 | The tunable metasurface structure and its surface electric and magnetic impedance. (a) The structure. The calculated (b) surface electric impedance and (c) surface magnetic impedance with different varactor capacitance and junction resistance respectively.

along \hat{z} with the electric field along \hat{x} direction, as illustrated in Fig. 1. In order to investigate the induced surface electric current J_s and surface magnetic current M_s on the metasurface, we can define the penetrable surface electric impedance Z_e and penetrable surface magnetic impedance Z_m as

$$\mathbf{E} = Z_e \mathbf{J}_{\mathbf{s}},\tag{1}$$

$$\mathbf{H} = \frac{1}{Z_m} \mathbf{M}_{\mathbf{s}},\tag{2}$$

where E and H are the total field including both the incident and the scattered field²⁷. To obtain arbitrary transmission and reflection fields, surface electric and magnetic currents are required to satisfy the boundary condition. For the passive metasurface, these surface currents can only be excited by the incident waves. For the given incident wave, the surface currents are solely determined by the surface electric and magnetic impedance. Hence, the transmission and reflection coefficients are the functions of the surface electric and magnetic impedance,

$$R = \frac{E^{J} + E_{1}^{M}}{E^{inc}} = \frac{-\eta}{2Z_{e} + \eta} + \frac{Z_{m}}{Z_{m} + 2\eta},$$
(3)

$$T = \frac{E^{J} + E_{2}^{M} + E^{inc}}{E^{inc}} = \frac{2Z_{e}}{2Z_{e} + \eta} - \frac{Z_{m}}{Z_{m} + 2\eta},$$
 (4)

where η is the free space wave impedance. The detailed derivation can be found in the Supplementary Information. The transmission and reflection coefficients depend on both the surface electric and magnetic impedance. By inverting equations (3) and (4), we get

$$Z_e = \frac{\eta}{2} \frac{1 + (R+T)}{1 - (R+T)},$$
(5)

$$Z_m = 2\eta \frac{1 + (R - T)}{1 - (R - T)},$$
(6)

which gives the required surface electric and magnetic impedance of the metasurface for the given transmission and reflection coefficients. This can be used to design the metasurface to achieve certain transmission and reflection property.

Assuming the metasurface is passive and lossless, then the reflection coefficient is $R = re^{i\phi_r}$, and the transmission coefficient is $T = \sqrt{1 - r^2}e^{i\phi_r}$, where $0 \le r \le 1$. Moreover, the electric impedance and magnetic impedance shall be purely imaginary numbers for passive and lossless case. To satisfy this requirement, it can be shown that $|R \pm T| = 1$ from equations (5) and (6), which leads to

$$r\sqrt{1-r^2}\cos(\phi_r-\phi_t)=0\tag{7}$$

Detailed derivation can be founded in the Supplementary Information. This equation reveals the achievable transmission and reflection coefficients with passive and lossless metasurface. If transmission (reflection) magnitude is unity, then arbitrary transmission (reflection) phase $\phi_r(\phi_t)$ can be achieved. If the transmission



Figure 4 | The required Z_e and Z_m for transmission magnitude and phase independent tuning. (a) Various transmission phase with unity transmissivity. (b) Various transmissivity with zero transmission phase.



Figure 5 | Independent transmission phase tuning under lossless case. (a), (c), (e), (g) show the various combinations of surface electric and magnetic reactance to realize the different transmission phase with unity transmissivity at 3 GHz. (b), (d), (f), (h) show the corresponding transmission magnitude and phase. The solid lines are the theoretical calculation, and the scatterers are the simulation results. The arrows indicate the corresponding vertical coordinate axis for the circled curves.

(reflection) magnitude is between zero and unity, then ϕ_r or ϕ_t can still take arbitrary value, but must fulfill $\phi_r - \phi_t = \pi/2 + n\pi$, n = 0, 1.

Inserting the achievable transmission and reflection coefficients into equations (5) and (6), we get the corresponding surface electric and magnetic impedance

$$Z_e = i \frac{\eta}{2} \cot\left(\frac{\phi_t \pm \alpha}{2}\right),\tag{8}$$

$$Z_m = -i2\eta \tan\left(\frac{\phi_t \mp \alpha}{2}\right),\tag{9}$$

where $\alpha = \arctan\left(r/\sqrt{1-r^2}\right)$. The upper and lower signs of \pm in equation (8) and \mp in equation (9) correspond to n = 0, 1 respectively. Note that the surface electric and magnetic impedance are purely imaginary numbers under this case. This result is important because it indicates that arbitrary transmission coefficient can be obtained with passive metasurface with only surface electric and magnetic reactance.

Fig. 2(a) shows the surface electric reactance and magnetic reactance required to produce certain transmission magnitude and phase according to equations (8) and (9) with n = 0. Using the relation $|R|^2 + |T|^2 = 1$ and $\phi_r - \phi_t = \pi/2$, we can also plot the electric and



magnetic reactance for certain reflection magnitude and phase, as shown in Fig. 2(b).

Metasurface with tunable surface impedance. We have just analyzed the surface electric and magnetic reactance needed to generate either transmission or reflection coefficient arbitrarily for an incident plane EM waves. As observed in Fig. 2, both the electric and the magnetic reactance range from negative to positive. In order to realize such a penetrable impedance surface, we propose the two dimensional periodic structure, as illustrated in Fig. 3(a). Each unit cell is composed of a parallel LC resonant tank and metallic small loops so as to provide the needed surface electric and magnetic reactance. Varactors, which are represented as the purple square blocks, are integrated into the unit cells so that the surface impedance of the proposed metasurface can be dynamically tuned. The black lines stand for the high resistive bias lines to supply bias voltage on the varactors. They are transparent to the RF EM waves. The small green and blue blocks are the constant capacitors to tune the loop resonance frequency and to decouple the DC bias voltage from shorting respectively.

The surface electric reactance is

$$Z_e = \frac{2g}{2b} Z_{LC} = \frac{g}{b} \frac{i\omega L Z_d}{i\omega L + Z_d},\tag{10}$$

where Z_{LC} is the circuit impedance of a single LC tank, Z_d is the impedance of the varactor, L is parallel inductance, and 2g and 2b are the width and height of the unit cell. Fig. 3(b) shows the typical surface electric impedance for both lossy and lossless case. In the lossy case, the junction resistance of the varactor is considered.

The surface magnetic impedance is derived as

$$Z_m = \frac{M_s}{H} = i\omega\mu a,\tag{11}$$

where, ω is the angular frequency and μ is the effective permeability of the metasurface. The detailed derivation can be found in the Supplementary Information. Fig. 3(c) shows the typical surface magnetic impedance.

By designing LC tanks and metallic loops, the whole metasurface can have the surface electric and magnetic impedance required for arbitrary transmission or reflection which can not be easily achieved with a single layer FSS. Furthermore, by controlling the varactor capacitance to dynamically adjust the resonance frequency of both the LC tanks and the loops, the impedance curves can be shifted with respect to the frequency such that the metasurface's property can be adjusted. This feature can be used to easily generate the wanted scattering EM field distribution according to the EM equivalent principle.

Tunable transmission phase with unity transmissivity. Two applications of the proposed active metasurface will be presented in the following. One is to dynamically control the transmission phase with fixed transmissivity, and the other one is to dynamically control the transmissivity with fixed transmission phase.

Theoretical analysis, numerical simulation and experiment are performed to characterize the proposed tunable metasurface. The numerical simulation is performed by using the commercial EM full wave solver. The dimensional parameter of the unit cell is: a = 4 mm, b = 6 mm, g = 10 mm, t = 1.5 mm. The thickness is much smaller than the free space wavelength of the designed 3 GHz operation frequency. The substrate relative permittivity is 2.2. The constant capacitor denoted by the green block in the magnetic loop is 0.5 pF, and the one denoted by the blue block is 10 μ F. The purple blocks represent the varactor, which is modeled as the parasitic inductance (0.7 nH), the variable junction capacitance (0.5 pF to 2.5 pF), and the junction resistance (2.5 Ω) in serial.



Figure 6 | Transmissivity variation at 3 GHz when the transmission phase is tuned under both lossless and lossy cases.

Figure 4(a) shows the required surface electric and magnetic reactance of the metasurface for unity transmissivity with various transmission phase from -180° to 180° .

First, we consider the lossless case by setting the varactor junction resistance to 0 Ω . Fig. 5 shows the Z_e and Z_m combinations by tuning the varactor capacitance in order to obtain different transmission phase at 3 GHz with unity transmissivity. The Z_e and Z_m are calculated using equations (10) and (11). The transmission phase and magnitude are calculated using equation (4), and also compared with the simulation results with good agreement. As can be seen, the transmission phase can be tuned to take various values at 3 GHz with the transmissivity close to unity.

In practice, the lossless case can not be achieved since the varactor diode always has a certain junction resistance. The detailed study on the effect of the junction resistance on the performance of the proposed metasurface can be found in the Supplementary Information.

To verify both the theoretical and simulation results, we fabricated the prototype metasurface and conducted the experiment. The experiment details can be found in the Supplementary Information. The transmission phase tuning performance under both lossy and lossless cases are compared in Fig. 6 at 3 GHz. For the lossless case, the transmission phase can be tuned within 360° with nearly unity transmissivity. For the lossy case, 360° full phase tuning can still be achieved. However, the transmissivity is not a constant. It is low near the $\pm 180^{\circ}$ transmission phase where the loss effect is strong due to the magnetic resistance, and high around 0° transmission phase. Under both cases, the theoretical, simulation and experiment results are in good agreement.

Tunable transmissivity with zero transmission phase. The proposed metasurface also permits us to dynamically control the transmissivity while keeping the transmission phase fixed. Fig. 4(b) plots the Z_e and Z_m required for different transmissivity with 0° transmission phase. Fig. 7 shows the theoretical calculation results for both lossless and lossy case. At the 3 GHz, the transmissivity can be tuned from 0.1 to 0.9 with the transmission phase close to 0° for both cases. The transmissivity tuning versus transmission phase at 3 GHz is extracted and plotted in Fig. 8, where the experiment results are also shown. These three results are in good agreement, which indicates that for the transmissivity has a large tuning range almost from 0 to 1. This is because for the 0° transmission phase, the required Z_m is small for any transmissivity according to Fig. 2 so that the loss effect is small.

Discussion

We have theoretically proved that either arbitrary transmission or reflection can be obtained by the passive metasurface with only the surface electric and magnetic reactance. To enhance the capability to





Figure 7 | Tunable transmissivity with nearly 0° transmission phase at 3 GHz. (a) and (b) correspond to the lossless case, and (c) and (d) to the lossly case. The four curves correspond to different biasing on varactors.

control the EM wave propagation, an electrically ultra thin metasurface composed of both tunable electric and magnetic resonators are presented, which can be characterized by the surface electric and magnetic impedance. The surface impedances can be tuned dynamically, so that the magnitude or phase of either the transmission or the reflection can be dynamically and independently controlled. As the examples, it is shown under the lossless case that the transmission phase can be tuned by 360° with unity transmissivity or the transmissivity can be tuned from 0 to 1 while the transmission phase is kept around 0°. The conventional transmitarray techniques based on FSS or phase delay line may also provide such full tuning range for either transmission magnitude or phase, but the independent control of these two factors are not easily achievable as discussed in the Introduction part of this paper. However, it is demonstrated that the loss effect will have a large influence on the performance of the metasurface, mainly due to the surface magnetic resistance. Hence, low loss tunable component, e.g. the ferroelectric varactors¹⁹, should be used in the proposed design to improve its performance. The proposed method provide new ways to manipulate EM propagation,



Figure 8 | Transmission phase variation at 3 GHz when the transmissivity is tuned.

and can be applied in dynamic beam forming and scanning in both reflection and transmission modes. It may also be used to do mathematical operations as suggested in²⁸ if tunable surface loss is added and carefully designed to remove the unwanted reflection while keeping the transmission coefficient control function.

Methods

To verify both the theoretical and simulation results, we fabricated the active metasurface and conducted experiment. The substrate is low loss F4B board. The fabricated metasurface is composed of 5 by 12 unit cells. The varactor is SMV1231-079. The metasurface is connected to a DC bias source to tune the electric impedance and magnetic impedance actively. One patch antenna is used to transmit the incident waves onto the metasurface, and a magnetic loop probe is placed behind the metasurface to detect the transmitted field. The transmission coefficients are measured using a vector network analyzer. The experiment details can be found in the Supplementary Information.

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Author contributions

B.O.Z. conceived the original idea, did the theoretical analysis and designed the metasurface. K.C., N.J. and L.S. fabricated the prototype metasurface. B.O.Z., J.Z. and T.J. performed the experiment. Y.F. supervised the project. B.O.Z. and Y.F. had deep discussion about this work and produced the manuscript.

Additional information

Supplementary information accompanies this paper at http://www.nature.com/ scientificreports

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