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DYNAMIC DEBT RUNS

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ABSTRACT

We develop a dynamic model of debt runs on a firm, which invests in an illiquid asset by rolling over staggered short-term debt contracts. We derive a unique threshold equilibrium, in which creditors coordinate their asynchronous rollover decisions based on the firm's publicly observable and time-varying fundamental. Fear of the firm's future rollover risk motivates each maturing creditor to run ahead of others even when the firm is still solvent. Our model provides implications on the roles played by volatility, illiquidity and debt maturity in driving debt runs, as well as on firms' capital adequacy standards and credit risk.

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1 Introduction

There is a long history of panic runs on banks. The panic of 1907 led to the creation of the Federal Reserve and the wave of bank failures during the Great Depression led to the establishment of deposit insurance. With the creation of these safeguards, runs on banks have become uncommon in the modern time. However, the structure of the financial system has changed in the 21st century, with rapid growth of the share of assets held by non-bank financial institutions such as investment banks, special investment vehicles, conduits, and hedge funds. According to U.S. Treasury Secretary Timothy Geithner (2008), the size of the non-bank financial system had already surpassed that of the traditional banking system by early 2007. Non-bank institutions do not accept deposits like banks; instead, they rely on short-term debt contracts such as commercial paper and repo transactions to finance their long-term risky and relatively illiquid assets. Without the protection of deposit insurance, the non-bank financial institutions are intrinsically vulnerable to panic runs. In fact, many regulators and researchers, e.g., Bernanke (2008), Cox (2008), Geithner (2008), Brunnermeier (2009), Gorton (2008), Krishnamurthy (2009), and Shin (2009), describe runs on the non-bank financial institutions as one of the main causes of the credit crisis of 2007-2008.[1]

This crisis raises many important policy and academic questions, which require a theoretical framework to analyze panic runs on financial institutions. Diamond and Dybvig (1983) provide a classic bank-run model to show that failure of depositors to coordinate their withdrawal decisions could lead to a self-fulfilling bank-run equilibrium. In this equilibrium, all depositors choose to withdraw from a solvent but illiquid bank, thus causing it to fail prematurely.

Two particular features of financial institutions motivate new perspectives on the coordination problem between their creditors. First, the assets held by financial institutions are mostly financial securities whose fundamentals change over time. The fact that runs on the financial institutions started in 2007 after their losses from mortgage related holdings became publicly known, e.g., Gorton (2008), suggests that fluctuating fundamentals played a potentially important role in triggering the runs in the credit crisis. Second, non-bank financial institutions have a different financing structure from banks. They are mostly financed by short-term debt contracts. While demand deposits allow bank depositors to run at any time, debt contracts lock in creditors until contract expirations. Furthermore, in practice, firms

[1]The freeze of the U.S. asset backed commercial paper (ABCP) markets in 2007 provides a vivid illustration of runs by creditors on the financial institutions. Prompted by concerns about the mounting delinquencies of subprime mortgages, the ABCP outstandings fell by a staggering $400 billion (one third of the existing amount) during the second half of 2007, e.g., Covitz, Liang, and Suarez (2009).

typically spread out their debt expirations over time to reduce liquidity risk. In other words, a firm's debt contracts mature at different times.[2] This staggered debt structure together with time-varying fundamentals leads to a dynamic coordination problem between creditors, in contrast to the static one formulated by Diamond and Dybvig (1983).

In this paper, we develop a parsimonious model in continuous time to analyze this problem. A firm finances its long-term asset holding by rolling over short-term debt with a continuum of small creditors. While one can interpret this firm as any firm, either financial or non-financial, financial firms are especially appealing because of their tendency to use higher leverage and more short-term debt. We assume that the firm's debt expirations are uniformly spread out across time. This staggered debt structure implies that the fraction of debt maturing in a short period is small, insulating our model from the coordination problem between creditors maturing at the same time. Instead, we focus on the coordination problem between creditors maturing at different times. On the asset side, we assume that the firm asset has publicly observable and time-varying fundamentals. The asset is also illiquid. When some maturing creditors choose to run and the firm fails to raise new funds to repay them, it has to prematurely liquidate the asset at a fire-sale price equal to a fraction of its fundamental value.

We derive in closed form a unique threshold equilibrium, in which each maturing creditor chooses to run on the firm if the firm fundamental falls below a certain endogenously determined threshold. To protect himself against the firm's future rollover risk caused by other creditors, each maturing creditor will choose to roll over his debt if and only if the current fundamental provides a sufficient safety margin. Each creditor's optimal threshold choice depends on that of others—if a creditor anticipates that the creditors maturing during his next contract period are more likely to run (i.e., using a higher rollover threshold), he has a greater incentive to run ahead of them (i.e., using an even higher threshold) when he gets the chance now. In this way, creditors engage in a preemptive "rat race," which leads all creditors to choose a rollover threshold substantially higher than he would in the absence of the coordination problem.

Our model naturally integrates two distinct and long-standing views about runs. The first view, advocated by Friedman and Schwartz (1963) and Kindleberger (1978), attributes many historical banking crises to unwarranted panics by arguing that the banks that were

[2]For example, on February 10, 2009, the data from Bloomberg show that Morgan Stanley, one of the major U.S. investment banks, had short-term debt (with maturities less than 1.5 years) expiring on almost every day throughout February and March 2009. If we sum up the total value of Morgan Stanley's expiring short-term debt in each week, the values for the following five weeks are 62 million, 324 million, 339 million, 239 million, and 457 million, respectively. The Federal Reserve Release also shows that the commercial paper issued by financial firms in aggregate has maturities well spread out over time.

forced to liquidate in such episodes were illiquid rather than insolvent. The alternative view, proposed by Mitchell (1941) and others, suggests that runs occur when depositors have fundamental concerns about the health of banks. There are a large number of models building on these views, which we briefly review in Section 6.1. In our model, the possibility of the firm's future fundamental deterioration generates creditors' fear of the firm's future rollover risk. The coordination problem between creditors further amplifies this fear and leads each creditor to choose a substantially higher rollover threshold. This intricate interaction between the firm's rollover risk and fundamental risk explains why it is often difficult to identify a financial crisis as a fundamental crisis or liquidity crisis.

By incorporating both fundamental and liquidity factors, our model provides several testable implications. First, firms with deteriorating fundamentals are more likely to experience runs. Second, firms with higher fundamental volatilities are more exposed to runs. This is because a higher volatility makes a firm's fundamental more likely to hit below the other creditors' rollover threshold during a creditor's contract period, thus motivating him to use a higher rollover threshold. Third, the more illiquid a firm's asset is, the more likely the runs on the firm. This is because a deeper discount of the firm's asset in the secondary market exposes each creditor to a greater loss in the event of a forced liquidation in the future. Fourth, under a wide range of parameter values, firms with shorter debt maturities are more exposed to runs, because they face greater rollover risks. Our numerical illustration in Section 5 also shows that the aforementioned rat race mechanism can dramatically amplify the effects of higher volatility, lower liquidity and shorter maturity on each creditor's incentive to run.

As a further highlight of the strong impact of the dynamic coordination problem, our model implies higher capital adequacy standards necessary for preventing inefficient panic runs than those suggested by static bank-run models. It is intuitive that in any static setting, if a firm is sufficiently capitalized so that it can still pay back its liability after a forced liquidation, there is no need for any creditor to worry about runs by others. However, this criterion breaks down in our dynamic model with debt contracts maturing throughout all periods. The capacity of its liquidation value to pay back its liability now is not a guarantee for future periods when the fundamental may deteriorate. As a result, each creditor is still concerned about the firm's future rollover risk. When the firm's fundamental volatility is sufficiently large, this concern becomes so strong that each maturing creditor chooses to run ahead of future maturing creditors despite the firm's ample capital cushion now.

Our model also provides useful implications for evaluating credit risk—i.e., the risk that a firm defaults on its debt—in an illiquid market environment. While the standard credit modeling approach focuses on insolvency risk, the risk that a firm's fundamental value drops below its liability, our model shows that insolvency risk and rollover risk, a form of liquidity risk, are intertwined and operate jointly to determine the firm's credit risk. In particular, our model calls for more attention on debt maturity structure in credit modeling, because a firm's credit risk is determined not only by its fundamental risk and leverage, but also by its debt maturity.

The emergence of the unique threshold equilibrium derived in our model is reminiscent of the global games models developed by Carlsson and van Damme (1993) and Morris and Shin (1998) in static coordination games. The mechanism in our model is different and works as follows. When the firm fundamental is sufficiently high (or low), each creditor's dominant strategy is rollover (or run), regardless of other creditors' future decisions. In other words, the equilibrium is uniquely determined in these regions, which are often referred to as the upper and lower dominance regions. When the firm fundamental is in the intermediate region between the two dominance regions, the Diamond-Dybvig type self-fulfilling multiple equilibria would arise if the fundamental is constant. However, when the fundamental is time-varying (either deterministically or stochastically) and could reach the dominance regions in the future, the creditors' anticipation of future maturing creditors' uniquely determined rollover strategy inside the dominance regions allows them to induce their optimal strategy in the intermediate region. Thus, a unique subgame perfect equilibrium emerges.

Unlike the global games models in which agents use noisy private information to coordinate their synchronous actions, creditors in our model coordinate their asynchronous rollover decisions based on the publicly observable and time-varying fundamental. This insight builds on Frankel and Pauzner (2000) and Burdzy, Frankel, and Pauzner (2001), who show that in dynamic coordination games with strategic complementarities, random fundamental shocks can act as a coordination device.[3] The realistic debt payoffs in our model prevent the use of the standard iterated deletion of dominated strategies approach to derive the equilibrium. Instead, we use a guess-and-verify approach.[4]

Our model also shares some spirit of the static bank-run models of Rochet and Vives (2004) and Goldstein and Pauzner (2005). By allowing the bank fundamental to be unobservable and depositors to have noisy private signals about the fundamental, these models adopt the global games approach to extend the Diamond-Dybvig bank-run setting. In these

[3] The same insight is also used by Guimaraes (2006) and Plantin and Shin (2008) to study coordinated currency attacks and speculative dynamics in carry trades.

[4] By making the firm fundamental publicly observable, our model does not have strategic uncertainty generated by agents' higher order beliefs, and thus differs in emphasis from several other dynamic coordination models, e.g., Abreu and Brunnermeier (2003), Chamley (2003), Angeletos, Hellwig, and Pavan (2007), Dasgupta (2007), and Toxvaerd (2008).

models, the coordination problem between depositors amplifies the effect of uncertainty about bank fundamentals and leads to inefficient runs on banks. In contrast to these models, our model incorporates time-varying fundamentals and staggered debt structures, which allow us to analyze the effects of volatility and debt maturity in driving runs.

Our model is related to the quickly growing literature analyzing various theoretical issues motivated by the credit crisis. Acharya, Gale, and Yorulmzer (2009) also study financial institutions' rollover risk and show that certain information structures can lead to credit freezes as rollover frequency increases. Brunnermeier and Oehmke (2009) study the conflict between long-term and short-term creditors during debt crises. He and Xiong (2009) analyze the role played by market illiquidity and short-term debt in exacerbating the conflict between debt and equity holders during debt crises. Morris and Shin (2009) build a global games model to analyze the liquidity component of financial institutions' credit risk. Diamond and Rajan (2009) show that anticipation of future fire sales can motivate banks to hoard liquidity and thus lead to a credit freeze. Shleifer and Vishny (2009) develop a model to explain the instability of banks as they use high leverage to take advantage of market sentiment.

The paper is organized as follows. Section 2 describes the model setup. We derive a unique monotone debt-run equilibrium in Section 3, and provide a single-creditor benchmark in Section 4. Section 5 analyzes the determinants of the creditors' equilibrium rollover threshold, and Section 6 discusses the implications for various issues related to dynamic debt runs. Finally, we conclude in Section 7. All technical proofs are given in the Appendix.

2 Model

We consider a continuous-time model with an infinite time horizon. A firm invests in a long-term asset by rolling over short-term debt. One can interpret this firm as any firm, either financial or non-financial. Our model is particularly appealing for financial firms because they tend to use higher leverage and more short-term debt. To make debt runs a relevant concern for the firm, we assume that the capital markets are imperfect along the following dimensions. First, the firm cannot find a single creditor with "deep pockets" to finance all of its debt and has to rely on a continuum of small creditors. Second, if some of the creditors choose not to roll over their debt, the firm needs to draw on its credit lines, which are not perfectly reliable and could be withdrawn by its issuer with a probability. Third, the secondary market for the firm asset is illiquid and the firm incurs a price discount if forced to liquidate the asset prematurely. We also impose two realistic assumptions about the firm: the fundamental value of the firm asset changes randomly over time and is publicly observable; and the firm has a staggered debt structure.

2.1 Asset

We normalize the firm's asset holding to be 1 unit. The firm borrows \$1 at time 0 to acquire its asset. Once the asset is in place, it generates a constant stream of cash flow, i.e., rdt over the time interval $[t, t+dt]$. At a random time τ_ϕ, which arrives according to a Poisson process with intensity $\phi > 0$, the asset matures with a final payoff. An important advantage of assuming a random asset maturity with a Poisson process is that at any point before the maturity, the expected remaining maturity is always $1/\phi$.

The asset's final payoff is equal to the time-τ_ϕ value of a stochastic process y_t, which follows a geometric Brownian motion with constant drift μ and volatility $\sigma > 0$:

$$\frac{dy_t}{y_t} = \mu dt + \sigma dZ_t,$$

where $\{Z_t\}$ is a standard Brownian motion. We assume that the value of the fundamental process is publicly observable at any time.

Taken together, the firm asset generates a constant cash flow of rdt before τ_ϕ and a final value of y_{τ_ϕ} at τ_ϕ. Then, by assuming that agents in this economy (including the firm creditors) are risk-neutral and have a discount rate of $\rho > 0$, we can compute the fundamental value of the firm asset as its expected discounted future cash flows:

$$F(y_t) = E_t \left[\int_t^{\tau_\phi} e^{-\rho(s-t)} r ds + e^{-\rho(\tau_\phi - t)} y_{\tau_\phi} \right] = \frac{r}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_t, \tag{1}$$

where the two components, $\frac{r}{\rho+\phi}$ and $\frac{\phi}{\rho+\phi-\mu} y_t$, correspond to the present values of the asset's constant cash flow and final payoff, respectively. Since the asset's fundamental value increases linearly with y_t, we will conveniently refer to y_t as the firm fundamental.

The assumption that the firm fundamental is time-varying is natural. It is somewhat strong to assume that the fundamental is publicly observable. This assumption mainly serves to insulate our model from further complications caused by agents' private information about the firm fundamental. In fact, our model would stay intact if we assume that the fundamental is unobservable and instead all agents only observe the same noisy public signals about the unobservable fundamental.

2.2 Debt Financing

The firm finances its asset holding by issuing short-term debt. Short-term debt is a natural response of outside creditors to a variety of agency problems inside the firm. By choosing short-term financing, creditors keep the option to pull out if they discover the firm managers are pursuing value-destroying projects.[5] While this exit option puts a short leash on the firm, it also creates a coordination problem between creditors. In this paper, we take a realistic debt structure as given and focus on analyzing the coordination problem between creditors. By demonstrating the potentially strong impact of this coordination problem, our model highlights the importance of incorporating it in future studies of firms' optimal debt structures.[6]

We emphasize an important feature of real-life firms' debt structure: firms tend to spread out their debt expirations over time to reduce liquidity risk. For example, the data from Bloomberg show that on February 10, 2009, Morgan Stanley, one of the major U.S. investment banks, had short-term debt (with maturities less than 1.5 years) expiring on almost every day throughout February and March 2009. If we sum up the total value of Morgan Stanley's expiring short-term debt in each week, the values for the following five weeks are 62 million, 324 million, 339 million, 239 million, and 457 million, respectively.[7] Furthermore, the Federal Reserve Release also shows that the commercial paper issued by financial firms in aggregate have maturities well spread out over time.[8]

Specifically, we assume that the firm finances its asset holding by issuing one unit of debt divided uniformly among a continuum of small creditors with measure 1. The promised interest rate is r so that the cash flow from the asset exactly pays off the interest payment until the asset matures or until the firm is forced to liquidate the asset prematurely. Once a creditor lends money to the firm, the debt contract lasts for a random period, which ends upon the arrival of an independent Poisson shock with intensity $\delta > 0$. In other words, the duration of each debt contract has an exponential distribution and the distribution is independent across different creditors. Once the contract expires, the creditor chooses whether to roll over the debt or to withdraw money (i.e., to run).

While the random duration assumption appears different from the standard debt contract with a predetermined maturity, it captures the aforementioned staggered debt structure of a typical firm—in aggregate, the firm has a fixed fraction δdt of its debt maturing over

[5] See Kashyap, Rajan, and Stein (2008) for a recent review of this agency literature and capital regulation issues related to the recent financial crisis.

[6] Cheng and Milbradt (2009) extend our model to allow the firm manager to freely switch between two projects, a good one with high drift and low volatility and an inferior one with low drift and high volatility. They show that the use of short-term debt can discipline the manager from choosing the inferior project when the firm fundamental is high.

[7] Our conversations with several bankers confirm that financial institutions prefer to spread out their debt expirations so that they do not have to roll over a large fraction of their debt on a single day.

[8] Almeida et al. (2009) summarize the debt maturity structure of all U.S. non-financial firms and show that most firms' debt expirations are spread out across different years.

time, where the parameter δ represents the firm's rollover frequency. This random duration assumption simplifies the complication of the debt's maturity effect, because at any time before the debt maturity the expected remaining maturity is always $1/\delta$. By matching $1/\delta$ with the fixed maturity of a real-life debt contract, this assumption captures the first order effect of debt maturity when a creditor makes his rollover decision.[9]

While we treat the rollover frequency as given for most of our analysis, we will analyze the creditors' preference over debt maturity in Section 6.3. To focus on the coordination problem between creditors, we also take the interest payment of the firm debt as given and leave a more elaborate analysis of the effects of endogenous interest payments for future research.[10]

2.3 Runs and Liquidation

When the maturing creditors choose to run, they expose the firm to bankruptcy risk if it cannot raise new funds to repay the running creditors. Of course, the firm can acquire credit lines from other institutions to protect itself against such an adverse event.[11] However, credit lines are never fully reliable. As experienced by many financial institutions during the credit crisis of 2007-2008, credit lines were frequently withdrawn by the issuers, either because they also faced funding problems or because they anticipated future funding problems and thus chose to hoard liquidity.[12]

Our model incorporates the imperfect reliability of the firm's credit lines by an exogenous parameter. More specifically, over a short time interval $[t, t + dt]$, δdt fraction of the firm's debt contracts expire. If these creditors choose to run, the firm will draw on its credit lines to raise new funds to pay off the running creditors. We assume that with probability $\theta\delta dt$, the issuer of the firm's credit lines fails to provide liquidity and the firm is thus forced into

[9] This assumption also generates an artificial second-order effect: If the debt contracts have a fixed maturity, a creditor, after rolling over his contract, will go to the end of the maturity queue. The random maturity assumption makes it possible for the creditor to be released early and therefore to run before other creditors when the asset fundamental deteriorates. This possibility makes the creditor less worried about the firm's rollover risk than he would if the debt contract has a fixed maturity. This in turn makes him more likely to roll over his debt. Thus, by assuming the random debt maturity, our model underestimates the firm's rollover risk.

[10] One might argue that when facing rollover difficulties, the firm can attract the maturing creditors by promising higher interest rates. However, doing so dilutes the stakes of other creditors in the firm and would motivate earlier maturing creditors to demand higher interest rates preemptively, similar to the preemptive runs highlighted in our model. In other words, promising higher interest rates could become a self-enforcing tightening mechanism on the firm, instead of a way to bail out.

[11] In fact, Ivashina and Scharfstein (2008) document that during the peak of the credit crisis in September-November 2008, many firms had drawn on their credit lines from banks.

[12] In the experience of the runs in the asset-backed commercial paper (ABCP) market in 2007, Covitz, Liang, and Suarez (2009) find that across different ABCP programs, the reliability of their credit lines is an important determinant of the likelihood of runs.

liquidation. The parameter $\theta > 0$ measures the unreliability of the firm's credit lines.[13] The higher the value of θ, the less reliable the firm's credit lines, and therefore the more likely the firm will be forced into liquidation given the same creditor outflow rate. With probability $1 - \theta\delta dt$, the firm is able to raise new funds through the credit lines to pay off the running creditors. For simplicity, we assume that the new funds raised from the credit lines have the same debt contract as the existing ones. Taken together, if every maturing creditor chooses to run, the firm can survive on average for a period of $\frac{1}{\theta\delta}$.[14]

Once the firm fails to raise new funds to pay off the running creditors, it falls into bankruptcy and has to liquidate its asset in an illiquid secondary market.[15] We assume that the firm can only recover a fraction $\alpha \in (0,1)$ of its fundamental value. That is, the firm obtains a discounted price of

$$\widetilde{L}(y_t) = \alpha F(y_t) = L + l y_t, \tag{2}$$

where

$$L = \frac{\alpha r}{\rho + \phi} \quad \text{and} \quad l = \frac{\alpha \phi}{\rho + \phi - \mu}. \tag{3}$$

For simplicity, we rule out partial liquidations in this paper.

The liquidation value will then be used to pay off all creditors on an equal basis. In other words, both the running creditors and the other creditors who are locked in by their current contracts, get the same payoff $\min\left(\widetilde{L}(y), 1\right)$.[16]

Due to the staggered debt structure in our continuous-time setting, the fraction of maturing creditors over a small time interval (i.e., δdt) is small. This implies that an individual creditor's running decision is not affected by the concurrent decisions of other maturing creditors. This feature insulates our model from the Diamond-Dybvig type of static coordination

[13] We also consider the special case when the firm does not have any credit line ($\theta = \infty$), i.e., the firm fails immediately when any maturing creditor chooses to run, in footnote 21.

[14] One could also interpret θ as inversely related to the firm's cash reserve. If the firm has more cash reserve, it can survive the creditors' runs for a longer period. Since outside creditors usually cannot perfectly observe the balance of a firm's cash reserve, from their perspective the failure of the firm under creditors' runs will occur at a random time.

[15] Our model implicitly assumes that once in distress, the firm cannot raise more capital by issuing new equity. This assumption is consistent with the existence of the conflict of interest between debt and equity holders, a la endogenous default in Leland (1994). When a firm faces liquidity problems in the debt market, equity holders could find it optimal not to inject more equity. By injecting equity they bear all the financial burden of keeping the firm from bankruptcy, but the benefit is shared by both debt and equity holders. See He and Xiong (2009) for an analysis of the effects of this distortion on short-term debt crises.

[16] From the view of any running creditor, his expected payoff from choosing run is still 1 because the probability of the firm failure $\theta\delta dt$ is in a higher dt order. This observation implies that in our model the sharing rule in the event of bankruptcy is inconsequential. We can also assume that during bankruptcy those maturing creditors who have chosen to run get a full payoff 1, while the remaining creditors who are locked in by their current contracts get $\min\left(\widetilde{L}(y), 1\right)$. This alternative assumption gives a greater incentive for maturing creditors to run. However, since the probability of the firm failure is $\theta\delta dt$, the difference in incentive is negligible.

problems, in which agents make simultaneous decisions, and instead allows us to focus on the coordination problem between creditors whose contracts mature at different times.

2.4 Parameter Restrictions

To make our analysis meaningful, we impose several parameter restrictions. First, we bound the interest payment by

$$\rho < r < \rho + \phi. \tag{4}$$

The first part $r > \rho$ makes the interest payment attractive to the creditors, who have a discount rate of ρ. The second part $r < \rho + \phi$ rules out the scenario where the interest payment is so attractive that rollover becomes the dominant strategy even when the firm fundamental y_t is close to zero. Essentially, this condition ensures the existence of the lower dominance region in which each creditor's dominant strategy is to run if the firm fundamental y_t is sufficiently low.

Second, we limit the growth rate of the firm fundamental by

$$\mu < \rho + \phi. \tag{5}$$

Otherwise, the firm's fundamental value in equation (1) would explode.

Third, we also limit the premature liquidation recovery rate of the firm asset:

$$\alpha < \frac{1}{\frac{r}{\rho+\phi} + \frac{\phi}{\rho+\phi-\mu}}, \tag{6}$$

so that $L + l < 1$ (see equation (3)). Under this condition, the asset liquidation value is not enough to pay off all the creditors when $y_t = 1$. This condition is sufficient for ensuring that each creditor is concerned about the firm's future rollover risk when the firm fundamental y_t is in an intermediate region.

Finally, we assume that the parameter θ is sufficiently high:

$$\theta > \frac{\phi}{\delta (1 - L - l)}, \tag{7}$$

so that the firm faces a serious bankruptcy probability when some creditors choose to run.

3 The Debt-Run Equilibrium

Given the firm's asset and financing structures described in the previous section, we now analyze the debt-run equilibrium. We limit our attention to monotone equilibria, equilibria in which each creditor's rollover strategy is monotonic with respect to the firm fundamental

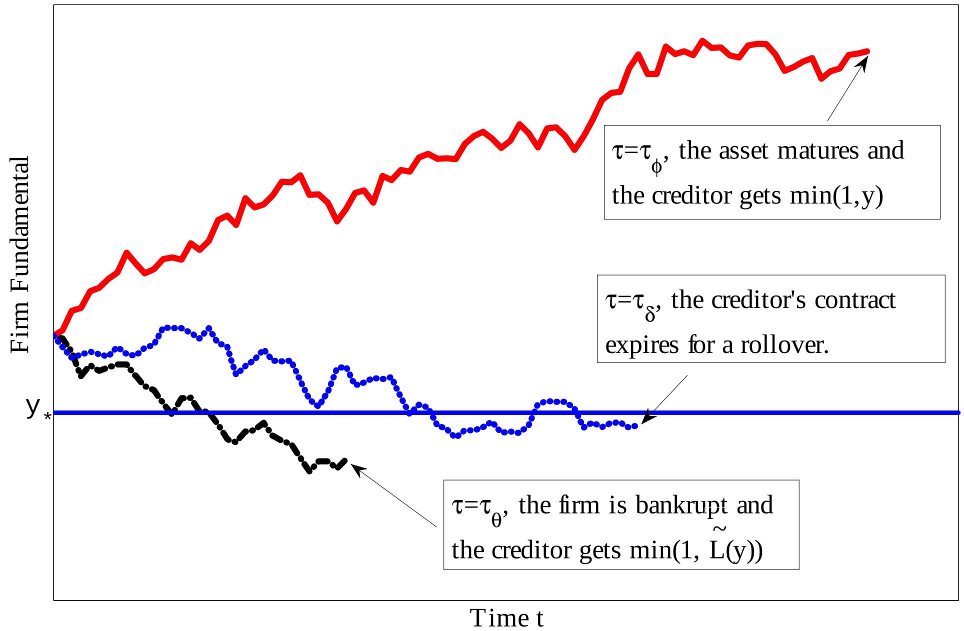

Figure 1: Three possible outcomes to a creditor.

y_t (i.e., to roll over if and only if the firm fundamental is above a threshold). In making his rollover decision, a creditor rationally anticipates that once he rolls over the debt, he faces the firm's rollover risk during his contract period. This is because volatility could cause the firm fundamental to fall below the other creditors' rollover threshold. As a result, the creditor's optimal rollover threshold depends on the other creditors' threshold choice.

In this section, we first set up an individual creditor's optimization problem in choosing his optimal threshold. We then construct a unique monotone equilibrium in closed form. Finally, we characterize the key ingredients that lead to the unique equilibrium.

3.1 An Individual Creditor's Problem

We first analyze the optimal rollover decision of an individual creditor who holds a small fraction of the firm's outstanding debt. In analyzing the individual creditor's problem, we take as given that all other creditors use a monotone strategy with a rollover threshold y_* (i.e., other creditors will roll over their debt if and only if the firm fundamental is above y_* when their debt contracts mature). During the creditor's contract period, his value function depends directly on the firm fundamental y_t, and indirectly on the other creditors' rollover threshold y_*. Since the creditor's future payoff is proportional to the unit of debt he holds, we denote $V(y_t; y_*)$ as the creditor's value function normalized by the debt unit.

For each unit of debt, the creditor receives a stream of interest payments r until

$$\tau = \min(\tau_\phi, \tau_\delta, \tau_\theta),$$

which is the earliest of the following three events, illustrated in Figure 1 at the end of three different fundamental paths. On the top path, the firm stays alive until its asset matures at τ_ϕ. At this time, the creditor gets a final payoff of $\min(1, y_{\tau_\phi})$, i.e., the face value 1 if the asset's maturity payoff y_{τ_ϕ} is sufficient to pay all the debt, and y_{τ_ϕ} otherwise. The possibility that the asset's maturity value may be insufficient to pay off the debt represents the firm's insolvency risk. On the bottom path, the firm fundamental drops below the other creditors' rollover threshold and the firm is eventually forced to liquidate its asset prematurely at τ_θ. At this time, the creditor gets $\min(1, L + l y_{\tau_\theta})$. This outcome represents the firm's rollover risk faced by each creditor. On the middle path, the firm stays alive (although its fundamental dips below the other creditors' rollover threshold) until τ_δ when the creditor's contract expires. At this time, the creditor has an option, i.e., he can choose whether to roll over depending on whether the continuation value $V(y_{\tau_\delta}; y_*)$ is higher than getting the one dollar back.

Due to risk neutrality, the individual creditor's value function is given by

$$\begin{aligned} V(y_t; y_*) &= E_t \Bigg\{ \int_t^\tau e^{-\rho(s-t)} r ds + e^{-\rho(\tau-t)} \Big[\min(1, y_\tau) \mathbf{1}_{\{\tau=\tau_\phi\}} \\ &\quad + \min(1, L + l y_\tau) \mathbf{1}_{\{\tau=\tau_\theta\}} + \max_{\text{rollover or run}} \{V(y_\tau; y_*), 1\} \mathbf{1}_{\{\tau=\tau_\delta\}} \Big] \Bigg\} \end{aligned} \tag{8}$$

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function that takes a value of 1 if the statement in the bracket is true or zero otherwise. The individual creditor's future payoff during his contract period depends on other creditors' rollover choices because other creditors' runs might force the firm to liquidate its asset prematurely, as illustrated by the bottom path of Figure 1. This dependence gives rise to strategic complementarity in the creditors' rollover decisions, and therefore a coordination problem between the creditors whose contracts mature at different times.[17]

[17] It is important to note that our model is substantially different from the standard game theoretical frameworks for analyzing dynamic binary action coordination problems. For example, consider the framework developed in Frankel and Pauzner (2000) and Burdzy, Frankel, and Pauzner (2001). Their framework consists of a sequence of repeated stage games. In each period, each agent receives a flow payoff, which satisfies an exogenous form of strategic complementarity, i.e., the agent receives a higher flow payoff if his current-period strategy overlaps with that of a greater fraction of the population. In contrast, in our model each creditor's flow payoff, which is given by the debt contract (interest payment r and possible asset maturity payoff $\min(y, 1)$), does not exhibit strategic complementarity. Instead, the strategic complementarity between the creditors emerges from the implicit dependence of a creditor's continuation value function on other creditors' rollover decisions, as shown in Figure 1 and equation (8). This important difference in model framework

By considering the change of the creditor's value function over a small time interval $[t, t + dt]$, we can derive his Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} \rho V(y_t; y_*) &= \mu y_t V_y + \frac{\sigma^2}{2} y_t^2 V_{yy} + r + \phi [\min(1, y_t) - V(y_t; y_*)] \\ &\quad + \theta \delta \mathbf{1}_{\{y_t < y_*\}} [\min(L + l y_t, 1) - V(y_t; y_*)] + \delta \max_{\text{rollover or run}} \{0, 1 - V(y_t; y_*)\}. \end{aligned} \tag{9}$$

The left-hand side term $\rho V(y_t; y_*)$ represents the creditor's required return. This term should be equal to the expected increment in his value function, as summarized by the terms on the right-hand side.

- The first two terms $\mu y_t V_y + \frac{\sigma^2}{2} y_t^2 V_{yy}$ capture the expected change in the value function caused by the fluctuation in the firm fundamental y_t.

- The third term r is the interest payment per unit of time.

The next three terms capture the three events illustrated in Figure 1:

- The fourth term $\phi [\min(1, y_t) - V(y_t; y_*)]$ captures the possibility that the asset matures during the time interval, which occurs at a probability of ϕdt and generates an impact of $\min(1, y_t) - V(y_t; y_*)$ on the creditor's value function.

- The fifth term $\theta \delta \mathbf{1}_{\{y_t < y_*\}} [\min(L + l y_t, 1) - V(y_t; y_*)]$ represents the expected effect of premature liquidation from other creditors' runs, which occurs at a probability of $\theta \delta \mathbf{1}_{\{y_t < y_*\}} dt$ (other maturing creditors will run only if $y_t < y_*$) and generates an impact of $\min(L + l y_t, 1) - V(y_t; y_*)$ on the creditor's value function.

- The last term $\delta \max_{\text{rollover or run}} \{0, 1 - V(y_t; y_*)\}$ captures the expected effect from the creditor's own contract expiration, which arrives at a probability of δdt. Upon its arrival, the creditor chooses whether to rollover or to run: $\max_{\text{rollover or run}} \{0, 1 - V(y_t; y_*)\}$.[18]

It is obvious that a maturing creditor will choose to roll over his contract if and only if $V(y_t; y_*) > 1$, and to run otherwise. This implies that if the value function V only crosses 1 at a single point y', i.e., $V(y'; y_*) = 1$, then y' is the creditor's optimal threshold.

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prevents us from readily applying the method of iterated deletion of dominated strategies used by Burdzy, Frankel, and Pauzner (2001) to our model. Instead, we derive the equilibrium by invoking a guess-and-verify approach detailed in the proof of Theorem 1.

[18]From each creditor's view, the probability of the event that his contract expires and the firm is forced into a premature liquidation is in the second order of $(dt)^2$. As a result, whether the creditor gets 1 or the asset's premature liquidation value in such an event is inconsequential. See another related discussion in footnote 16.

Externality on Future Maturing Creditors The rollover decision of current-period maturing creditors affects not only their own payoffs, but also future maturing creditors'. In particular, their decision to run adds to the firm's bankruptcy probability and thus imposes an implicit cost on future maturing creditors. Since they do not internalize the cost of their actions on others, this externality is the ultimate source of debt runs in our model. To see this point precisely, we summarize the payoff (or continuation value) of the current-period maturing creditors and future maturing creditors depending on the choice of the current-period maturing creditors in Table 1. For simplicity, we treat all the current-period maturing creditors as one identity in this illustration.

Table 1. Externality on future maturing creditors.

Choice of current-period maturing creditors	Run		Rollover
Possible firm outcomes	failed	survived	survived
Probabilities	$\theta\delta dt$	$1-\theta\delta dt$	1
Payoff of current-period maturing creditors	$\widetilde{L}(y)$	1	$V(y)$
Payoff of future maturing creditors	$\widetilde{L}(y)$	$V(y)$	$V(y)$

The maturing creditors will choose run if $1\cdot(1-\theta\delta dt)+\widetilde{L}\cdot\theta\delta dt > V$, which is $V<1$ after ignoring the higher order dt term. Their runs reduce the remaining creditors' continuation value function by

$$V-\left[V\cdot(1-\theta\delta dt)+\widetilde{L}\cdot\theta\delta dt\right]=\left(V-\widetilde{L}\right)\theta\delta dt.$$

While this effect is of the dt order, a remaining creditor needs to bear the accumulative externality effect of all maturing creditors before him, which, in expectation, could be significant.[19]

Dominance Regions When the firm fundamental y_t is sufficiently low (i.e., close to zero), an individual creditor's dominant strategy is run. This is because even if all other creditors choose to roll over in the future, the expected asset payoff at the maturity plus the interest payments before the asset maturity are not as attractive as getting one dollar back now. On the other hand, when the firm fundamental y_t is sufficiently high (i.e., close to infinity), the creditor's dominant strategy is rollover. Even if all other creditors choose to run in the future, the asset's liquidation value is sufficient to pay off the debt in the event of a forced

[19] Note that the current-period maturing creditors' runs also impose externality effects on each other. But these effects are one time and of the dt order, thus can be ignored.

liquidation. These two regions are called the lower and upper dominance regions. Their existence is important for ensuring a unique equilibrium.

3.2 The Unique Monotone Equilibrium

We first focus our attention on symmetric monotone equilibria, and then show that there cannot be any asymmetric monotone equilibrium. In a symmetric monotone equilibrium, each creditor's optimal threshold choice y' must be equal to the other creditors' threshold y_*. Thus, we obtain the condition for determining the equilibrium threshold:

$$V(y_*; y_*) = 1.$$

We employ a guess-and-verify approach to derive a unique monotone equilibrium in four steps. First, we derive an individual creditor's value function $V(y_t; y_*)$ from the HJB equation in (9) by assuming that every creditor (including the creditor under consideration) uses the same monotone strategy with a rollover threshold y_*. Due to terms $\min(1, y_t)$ and $\min(L + l y_t, 1)$ in (9), the value function depends on the value of y_* in the three cases:

1. If $y_* < 1$,

$$V(y_t; y_*) = \begin{cases} \frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} + \frac{\phi+\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu} y_t + A_1 y_t^{\eta_1} & \text{when } 0 < y_t \le y_* \\ \frac{r}{\rho+\phi} + \frac{\phi}{\rho+\phi-\mu} y_t + A_2 y_t^{-\gamma_2} + A_3 y_t^{\eta_2} & \text{when } y_* < y_t \le 1 \\ \frac{r+\phi}{\rho+\phi} + A_4 y_t^{-\gamma_2} & \text{when } y_t > 1 \end{cases};$$

2. If $1 \le y_* < \frac{1-L}{l}$,

$$V(y_t; y_*) = \begin{cases} \frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} + \frac{\phi+\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu} y_t + B_1 y_t^{\eta_1} & \text{when } 0 < y_t \le 1 \\ \frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} + \frac{\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu} y_t + B_2 y_t^{-\gamma_1} + B_3 y_t^{\eta_1} & \text{when } 1 < y_t \le y_* \\ \frac{r+\phi}{\rho+\phi} + B_4 y_t^{-\gamma_2} & \text{when } y_t > y_* \end{cases};$$

3. If $y_* \ge \frac{1-L}{l}$,

$$V(y_t; y_*) = \begin{cases} \frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} + \frac{\phi+\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu} y_t + C_1 y^{\eta_1} & \text{when } 0 < y_t \le 1 \\ \frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} + \frac{\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu} y_t + C_2 y_t^{-\gamma_1} + C_3 y_t^{\eta_1} & \text{when } 1 < y_t \le \frac{1-L}{l} \\ \frac{r+\phi+\theta\delta+\delta}{\rho+\phi+(1+\theta)\delta} + C_4 y_t^{-\gamma_1} + C_5 y_t^{\eta_1} & \text{when } \frac{1-L}{l} < y_t \le y_* \\ \frac{r+\phi}{\rho+\phi} + C_6 y_t^{-\gamma_2} & \text{when } y_t > y_* \end{cases}.$$

The coefficients η_1, η_2, γ_1, γ_2, A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, C_1, C_2, C_3, C_4, C_5, and C_6 are given in Appendix A.1 and are expressions of the model parameters and y_*.

Second, based on the derived value function, we show that there exists a unique fixed point y_* such that $V(y_*; y_*) = 1$. Third, we prove the optimality of the threshold y_* for any individual creditor, i.e., $V(y; y_*) > 1$ for $y > y_*$ and $V(y; y_*) < 1$ for $y < y_*$. Finally, we show that there cannot be any asymmetric monotone equilibrium.

We summarize the main results in the following theorem.

Theorem 1 *There exists a unique monotone equilibrium, in which each maturing creditor chooses to roll over his debt if y_t is above the threshold y_* and to run otherwise. The equilibrium threshold y_* is uniquely determined by the condition that $V(y_*, y_*) = 1$.*

The equilibrium threshold y_* could fall into any one of the three cases listed above, depending on the values of the model parameters. The third case is particularly interesting as $\widetilde{L}(y_*) = L + l y_* \geq 1$, i.e., creditors start to run on the firm even though the firm's current liquidation value is sufficient to pay off its liability. The emergence of this type of frantic run reflects creditors' strong fear of the firm's future rollover risk, which we will discuss in more detail in Section 6.2.

3.3 Understanding the Uniqueness of the Equilibrium

Like the classic bank run model of Diamond and Dybvig (1983), our model also features the externality of one creditor's run on other creditors. However, there is a unique threshold equilibrium, instead of multiple self-fulfilling equilibria. What leads to the unique equilibrium? In this section, we discuss the roles of two important ingredients, staggered debt structure and time-varying fundamental, by studying several variations of our model.

3.3.1 Synchronous Debt Expirations

To highlight the role of the staggered debt structure, we consider the following model variation. Suppose that the firm's debt contracts all expire at time 0, and the current firm fundamental is y_0. At this time, each creditor decides whether to run or to roll over into a perpetual debt contract lasting until the firm asset matures at τ_ϕ. We also assume that if all creditors choose to run, the firm might fail with a probability of $\theta_s \in (0, 1)$. This setting closely resembles that in Diamond-Dybvig model, because all creditors simultaneously choose their rollover decisions at time 0 and the firm does not face any future rollover risk. We formally characterize this coordination problem below.

Proposition 2 *Given the aforementioned setting, there exist* $y_h > y_l > 0$ *such that if* $y_0 > y_h$ *(the upper dominance region), an individual creditor's dominant strategy is to roll over; if* $y_0 < y_l$ *(the lower dominance region), the creditor's dominant strategy is to run. However, if* $y_0 \in [y_l,\ y_h]$ *(the intermediate region), the creditor's optimal choice depends on the others', i.e., it is optimal to run if the others choose to run and it is optimal to roll over if the others choose to roll over.*

Proposition 2 shows that when the firm fundamental is in an intermediate region, multiple self-fulfilling equilibria emerge, like those in Diamond and Dybvig (1983). By using a staggered debt structure, the firm can mitigate the Diamond-Dybvig type of coordination problems. This is because the fraction of contracts maturing over a small interval of time (say a day) is small and the collective choice of these creditors is too insignificant to affect the firm. However, there exists another coordination problem between creditors whose contracts mature at different times. This problem is the core of our model.

3.3.2 Constant Fundamental and Staggered Debt Structure

To highlight the role of the time-varying fundamental, we let the firm fundamental be constant and the firm have a staggered debt structure. The following proposition shows that in this setting, the coordination problem between creditors whose contracts mature at different times can still lead to multiple self-fulfilling equilibria.

Proposition 3 *Suppose that* $y_t = y$ *is constant (i.e.,* $\sigma = 0$ *and* $\mu = 0$*) and the firm has a staggered debt structure. There exist* $y_h^c > y_l^c > 0$ *such that when* $y > y_h^c$ *(the upper dominance region), an individual creditor's dominant strategy is to roll over; when* $y < y_l^c$ *(the lower dominance region), the creditor's dominance strategy is to run; and when* $y \in [y_l^c, y_h^c]$ *(the intermediate region), the creditor's optimal choice depends on the others', i.e., it is optimal to run if the others will choose to run in the future and it is optimal to roll over if the others will choose to roll over in the future.*

Proposition 3 shows that when the firm fundamental is constant and between the upper and lower dominance regions, multiple self-fulfilling equilibria again emerge despite the firm's staggered debt structure. In this intermediate region, once each creditor believes that other maturing creditors in the future will all choose to roll over, this "no-future-rollover-risk" belief is self-fulfilling because the firm fundamental always stays above the lower dominance region and thus will never contradict the no-future-rollover-risk belief. Similarly, once each creditor believes that other maturing creditors in the future will all choose to run, this belief is also self-fulfilling because the firm fundamental is always below the upper dominance region.

The staggered debt structure in our model provides a natural debt maturity line for creditors to sequentially withdraw funds from the firm. This line plays a role similar to the sequential service constraint in the standard bank-run models. In this sense, one can interpret the setting in Proposition 3 as an infinite-period version of the Diamond-Dybvig model, in which each depositor keeps rolling over term deposits in a bank and the bank fundamental stays constant over time. The emergence of the self-fulfilling bank-run equilibrium in this setting contrasts the finding of Green and Lin (2003). They consider a finite-agent version of the Diamond-Dybvig model, and show that the self-fulfilling bank-run equilibrium does not exist under the assumption that the bank's service line is finite and each depositor knows his relative position in the line when he contacts the bank. This is because the depositor at the end of the line will rationally choose not to run on the bank, then earlier depositors by backward induction will choose not to run either. This backward induction scheme does not work in our setting. Since the debt maturity line is recurring—i.e., after a creditor rolls over his debt, he goes back to the line—there is not any end of the line to start the backward induction.

3.3.3 Time-Varying Fundamental and Staggered Debt Structure

The self-fulfilling multiple equilibria in Proposition 3 break down if the firm fundamental changes over time and could reach the upper and lower dominance regions in the future. Instead, a unique (subgame perfect) equilibrium emerges, because anticipation of future creditors' uniquely determined rollover strategy inside the dominance regions allows the creditors to induce their optimal strategy in the intermediate region between the dominance regions.

It is easy to see this mechanism in the case where the firm fundamental changes deterministically (i.e., $\sigma = 0$ and $\mu \neq 0$). Suppose that $\mu < 0$, i.e., the fundamental continues to deteriorate until the asset matures. Knowing that once the fundamental is in the lower dominance region other creditors will always choose run, each maturing creditor right before the fundamental enters the region will choose run. This in turn motivates earlier maturing creditors to choose run too. This backward induction amplifies the creditors' incentive to run, and thus generating excessive rollover risk to the firm. Rollover is optimal only when the current firm fundamental is sufficiently high, i.e., above a threshold $y_{\mu-} > 1$, so that it provides enough cushion against the firm's future rollover risk. Otherwise, when $y \leq y_{\mu-}$ run is optimal for each creditor. A similar reasoning works in determining a unique equilibrium for the case $\mu > 0$. The following proposition formally derives this unique equilibrium.

Proposition 4 *Suppose that the firm fundamental is deterministic with a nonzero drift μ and the firm has a staggered debt structure.*

1. *If $\mu > 0$, there is a unique monotone equilibrium, in which each creditor chooses rollover if the firm fundamental is above a threshold $y_{\mu+} < 1$, and run otherwise.*

2. *If $\mu < 0$, there is a similar unique monotone equilibrium with a threshold $y_{\mu-} > 1$.*

As a special case of Theorem 1, the same backward induction mechanism also applies to the case where the firm fundamental is only subject to random shocks (i.e., $\sigma > 0$ and $\mu = 0$). That is, random shocks can serve the same role as deterministic drifts, i.e., allowing the creditors to backwardly induce the equilibrium in the intermediate region. This key insight follows Frankel and Pauzner (2000) and Burdzy, Frankel, and Pauzner (2001), who show that in dynamic coordination games with strategic complementarities, random fundamental shocks allow agents to coordinate their asynchronous actions and to induce a unique equilibrium. As the realistic debt payoffs in our model prevent the use of the standard iterated deletion of dominated strategies approach to solve for the equilibrium, our model demonstrates that this insight is robust even in a rather complex and realistic setting.

The emergence of the unique equilibrium in Theorem 1 is analogous to that in the global games models developed by Carlsson and van Damme (1993) and Morris and Shin (1998). In the global games models, agents possess noisy signals about a fundamental variable and each agent uses his private signal to form expectations of other agents' signals and simultaneous actions. In our model, creditors have the same information but make their rollover decisions at different times. Since the firm fundamental is time-varying and persistent, the current fundamental allows each maturing creditor to form expectations of future maturing creditors' rollover decisions.

The following proposition shows that the unique monotone equilibrium derived in Theorem 1 holds even as $\delta \to \infty$, i.e., the maturity of each debt contract converges to zero, just like demand deposits in Diamond and Dybvig (1983).

Proposition 5 *When $\delta \to \infty$, the unique equilibrium rollover threshold y_* converges to $\frac{1-L}{l}$.*

This proposition further shows that it is the asynchronous timing of the creditors' rollover decisions, rather than the non-zero debt maturity, that drives the unique equilibrium in our model.[20] As $\delta \to \infty$, the debt maturity goes down to zero, but the asynchronous timing of the creditors' rollover decisions still remains.[21]

4 The Single-Creditor Benchmark

To facilitate our discussion of the coordination problem between creditors, it is useful to establish a benchmark case, in which a single creditor holds all the debt of the firm. Like the main model described in Section 2, we assume that the single creditor faces a contract period which expires upon the arrival of a Poisson shock with intensity δ. When the contract expires, the single creditor decides whether to roll over the debt for another random contract period or not. If he decides not to roll over, the firm is forced into a premature liquidation. In this event, the creditor's payoff is $\min(L + ly_t, 1)$. Because the single creditor does not need to worry about the firm's future rollover risk with other creditors, his rollover decision is free of the coordination problem with other creditors. As a result, he would internalize the cost of a premature firm liquidation. The following proposition shows that he will always roll over his debt if the liquidation cost is sufficiently high.

Proposition 6 *Suppose that a single creditor finances all the debt of the firm. If the cost of a premature liquidation is sufficiently high, i.e., α is sufficiently low, then the single creditor will always roll over his debt.*

Given that the single creditor will not choose to run in the benchmark case, the runs derived in our main model are ultimately caused by interactions between creditors.

[20] The firm's liquidation value at the limiting running threshold, i.e., $L + ly_*$, is exactly the firm's liability 1. When $\delta \to \infty$, each creditor's contract will mature instantaneously. This means that a creditor will be locked in by his contract only for a short period. However, as other creditors are also kept loose, the firm could fail at any time if they choose to run. As a result, each creditor will choose to roll over his debt if and only if the firm's current liquidation value is sufficient to pay off its liability.

[21] Another special case to consider is when $\theta = \infty$ (i.e., the firm does not have any credit line.) In this case, the firm fails immediately if any maturing creditor chooses to run. Because of the frailty of the firm, the sharing rule between the running creditor and the other creditors during the firm bankruptcy becomes important. Suppose that the running creditor gets paid in full, while the other creditors divide the liquidation value of the firm asset. Then, we can show that there is still a unique threshold equilibrium, in which each maturing creditor chooses to run if the fundamental drops below $\frac{1-L}{l}$. Deriving this equilibrium follows a similar procedure as outlined in Sections 3.1 and 3.2, except for some minor differences in the formulas. In particular, due to the absence of credit lines, the rollover risk term $\min(1, L + ly_\tau)\, \mathbf{1}_{\{\tau=\tau_\theta\}}$ in equation (8) is replaced by a boundary condition that when $y = y_*$, $V(y, y_*) = L + ly_*$. It is direct to see that the equilibrium condition $V(y_*, y_*) = 1$ implies that $y_* = \frac{1-L}{l}$ is the unique equilibrium threshold.

5 Determinants of Equilibrium Rollover Threshold

Despite the absence of self-fulfilling multiple equilibria in our model, preemptive debt runs could still occur through a rat race between the creditors in choosing higher and higher rollover thresholds. In this section, we analyze the effects of this rat race and the dependence of the creditors' equilibrium rollover threshold on various model parameters, such as the firm's liquidation recovery rate α, fundamental volatility σ, and rollover frequency δ.

For illustration, we will use a set of baseline values for the model parameters:

$$\rho = 5\%, \ r = 10\%, \ \delta = 10, \ \phi = 0.2, \ \theta = 2, \ \mu = 5\%, \ \sigma = 10\%, \ \alpha = 60\%. \qquad (10)$$

The creditors have a discount rate $\rho = 5\%$. The firm asset generates a constant stream of cash flow at a rate of 10% per annum, which is paid out to the creditors as interest payments. The interest payments are attractive since the interest rate r is much higher than the creditors' discount rate ρ. We choose the firm's rollover frequency δ to be 10, which implies an average debt maturity of about 37 days $(365/\delta)$. This implied maturity matches the average maturity of outstanding asset-backed commercial paper in February 2009 (Federal Reserve Release). $\phi = 0.2$ implies that the firm asset on average lasts for 5 years $(1/\phi)$, which is much longer than the debt maturity and resembles the typical duration of a mortgage bond. $\theta = 2$ means that conditional on every maturing creditor choosing to run, the firm can survive on average for 18 days $(1/\theta\delta)$.[22] The firm fundamental y_t has a growth rate of $\mu = 5\%$ per annum and a volatility of $\sigma = 10\%$ per annum. Finally, when the firm liquidates its asset prematurely, it only recovers $\alpha = 60\%$ of the asset's fundamental value. This implies that $L = 0.24$ and $l = 0.6$ in equation (3). Under these baseline parameters, the equilibrium rollover threshold is $y_* = 1.19$.

5.1 Liquidation Recovery Rate

We first illustrate the key threshold rat race mechanism using a simple thought experiment. Suppose that initially the liquidation recovery rate of the firm asset is α_h, and, correspondingly, every creditor uses an equilibrium threshold level $y_{*,0}$. Unexpectedly, at a certain time, all creditors find out that the recovery rate drops to a lower level $\alpha_l < \alpha_h$. What would the new equilibrium threshold be? Let's start with an individual creditor's threshold choice, which depends on others' choice. Suppose that all the other creditors still use the original threshold $y_{*,0}$. Then, by solving the HJB equation in (9), we can derive the creditor's optimal threshold $y_{*,1}$, which is higher than $y_{*,0}$ because the lower liquidation value generates

[22] This θ value is rather modest relative to the recent experience of Bear Stearns, which lasted for 3 days under the runs of its creditors and clients before a forced sale to JP Morgan in March 2008, e.g., Cox (2008).

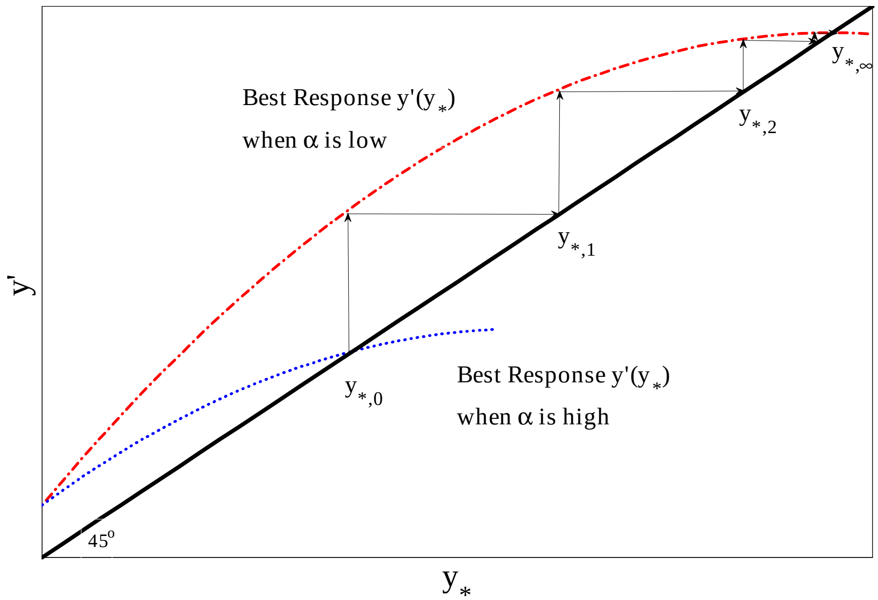

Figure 2: An illustration of the rat race between creditors in choosing rollover thresholds.

a greater expected loss to the creditor in the event that the firm is forced into a premature liquidation during his contract period. Of course, each creditor will go through this same calculation and choose a new threshold. If all creditors choose the threshold $y_{*,1}$, then an individual creditor's optimal threshold would be $y_{*,2}$, another level even higher than $y_{*,1}$. If all creditors choose $y_{*,2}$, then each creditor would go through another round of updating, and so on and so forth. Figure 2 illustrates this updating process until it eventually converges to a fixed point $y_{*,\infty}$, the new equilibrium threshold.

The difference between the threshold levels $y_{*,1}$ and $y_{*,0}$ represents the necessary safety margin a creditor would demand in response to the reduced asset liquidation value if other creditors' rollover strategies stay the same. This increase in threshold is eventually amplified to a much larger increase $y_{*,\infty} - y_{*,0}$ through the rat race between creditors. This amplification mechanism, which is absent from the single-creditor benchmark, plays a key role in driving the debt runs in our model.

To illustrate the magnitude of this amplification effect, we examine the change in the equilibrium rollover threshold as we vary α from its baseline value of 0.6. We measure the threshold by the fundamental value of the firm asset at y_*, $F(y_*) = \frac{r}{\rho+\phi} + \frac{\phi}{\rho+\phi-\mu} y_*$, which is directly comparable to the firm's outstanding liability, 1.

In Figure 3, the flat thin solid line represents the equilibrium threshold $F(y_{*,0}) = 1.59$ when α takes the baseline value 0.6. The thick solid line shows that as α deviates from its baseline value of 0.6 and decreases from 0.7 to 0.3, $F(y_{*,\infty})$ rises monotonically from

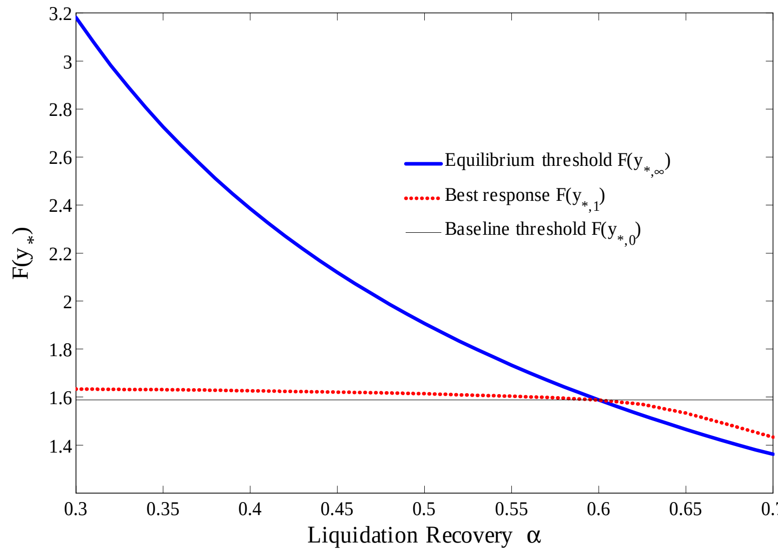

Figure 3: The equilibrium rollover threshold vs the liquidation recovery rate α. This figure uses the following baseline parameters: $\rho = 5\%$, $r = 0.10$, $\delta = 10$, $\phi = 0.2$, $\theta = 2$, $\mu = 5\%$, $\sigma = 10\%$, $\alpha = 60\%$. The threshold is measured in the firm's fundamental value $F(y_*)$. The thin solid line is the baseline threshold level, $F(y_{*,0})$, under the baseline parameters. The thick solid line plots the equilibrium threshold $F(y_{*,\infty})$. The dashed line plots a creditor's best response $F(y_{*,1})$ to the change in α from its baseline value while fixing the other creditors' threshold at $F(y_{*,0})$.

1.36 to 3.18. Note that $F(y_{*,\infty})$ is always above 1. As each maturing creditor only holds a partial stake in the firm, it makes sense for him to run and get his money back before the firm's fundamental value drops below the outstanding liability. This is because he does not internalize the cost imposed by his run on the whole firm.

Moreover, the equilibrium threshold decreases with α because a lower liquidation value increases the expected loss to each creditor in the event of a forced liquidation. We formally prove this result in the following proposition:

Proposition 7 *The equilibrium rollover threshold y_* decreases with the firm's premature liquidation recovery rate α.*

We further decompose $F(y_{*,\infty}) - F(y_{*,0})$, the effect of an α change on $F(y_*)$, into two components. The dashed line in Figure 3 plots the best response of a creditor in the absence of the rat race between creditors. Suppose α drops unexpectedly from its baseline level 0.6 to 0.4. After the drop in α, by solving the HJB equation in (9) numerically, we find that an individual creditor will choose an optimal threshold $F(y_{*,1}) = 1.63$ (on the dashed line) if the other creditors' rollover threshold is fixed at the baseline level $F(y_{*,0}) = 1.59$ (the thin solid line). The difference $F(y_{*,1}) - F(y_{*,0}) = 0.04$ represents the safety margin necessary to compensate the creditor for the increased expected bankruptcy loss in the absence of the rat race. Of course, once we take into account the rat race, each creditor ends up choosing a higher equilibrium threshold of $F(y_{*,\infty}) = 2.38$ (on the thick solid line). The difference $F(y_{*,\infty}) - F(y_{*,1})$ represents the amplification effect of the rat race, which is about 20 times the effect without the rat race. Overall, this decomposition shows that in the absence of the rat race between creditors, a change in α only has a rather modest effect on each creditor's threshold choice. However, the rat race dramatically amplifies this effect on the equilibrium rollover threshold.

5.2 Fundamental Volatility

Fundamental volatility σ affects an individual creditor's optimal rollover threshold through several channels. We can intuitively discuss these channels through various terms in the creditor's value function in equation (8). First, when the firm's fundamental volatility increases, its insolvency risk, which is reflected by the term $\min(1, y_\tau)\,\mathbf{1}_{\{\tau=\tau_\phi\}}$, rises because it becomes more likely that the firm's asset value at the asset maturity could be insufficient to pay off its liability. The increased insolvency risk prompts each creditor to use a higher rollover threshold. Second, a higher volatility also increases the firm's rollover risk through the term $\min(1, L + ly_\tau)\,\mathbf{1}_{\{\tau=\tau_\theta\}}$ (i.e., other creditors might choose to run and cause the firm to fail before the creditor's debt matures.) More precisely, through a rat race similar to the one described in the previous subsection, imperfect coordination between creditors causes each creditor to choose an even higher threshold to protect himself against other creditors' runs in the future. Third, once the creditor's debt matures, he has the option to roll over his debt and take advantage of the debt's high interest payments if the firm fundamental is sufficiently strong. Through this embedded option, which is reflected by the term $\max_{\text{rollover or run}}\{V(y_\tau; y_*), 1\}\,\mathbf{1}_{\{\tau=\tau_\delta\}}$, a higher fundamental volatility motivates the creditor to choose a lower rollover threshold. The effect of the embedded option works in an opposite direction to those of the insolvency risk and rollover risk.

Figure 4 illustrates the net effect of these three channels. As σ deviates from its baseline value of 10% and increases from 5% to 20%, the creditors' equilibrium rollover threshold $F(y_*)$ (the thick line) increases from 1.51 to 1.63. We can formally prove that the equilibrium threshold increases with σ if the firm's credit lines are sufficiently unreliable, i.e., θ is sufficiently high. Under this condition, the firm would easily fail under a run, and consequently the embedded-option channel becomes dominated by the other two channels. In fact, our numerical exercises show that this result also holds when θ takes a modest value.

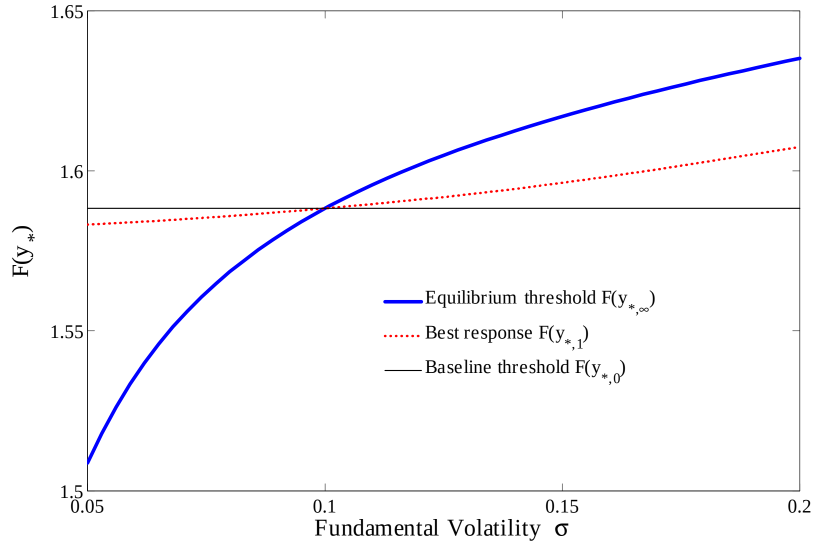

Figure 4: The equilibrium rollover threshold vs the fundamental volatility σ. This figure uses the following baseline parameters: $\rho = 5\%$, $r = 0.10$, $\delta = 10$, $\phi = 0.2$, $\theta = 2$, $\mu = 5\%$, $\sigma = 10\%$, $\alpha = 60\%$. The threshold is measured in the firm's fundamental value $F(y_*)$. The thin solid line is the baseline threshold level $F(y_{*,0})$ under the baseline parameters. The thick solid line plots the equilibrium threshold $F(y_{*,\infty})$. The dashed line plots a creditor's best response $F(y_{*,1})$ to the change in α from its baseline value while fixing the other creditors' threshold at $F(y_{*,0})$.

Proposition 8 *Suppose that θ is sufficiently high. Then, the equilibrium rollover threshold y_* increases with the firm's fundamental volatility σ.*

To highlight the effect of the rat race, we also plot an individual creditor's best response $F(y_{*,1})$ to the change in σ (the dashed line) while fixing the other creditors' threshold at the baseline level $F(y_{*,0}) = 1.59$ when σ takes its baseline level 10%. When σ rises above its baseline level, the increase $F(y_{*,1}) - F(y_{*,0})$ represents the safety margin that the creditor would demand to protect himself against the increased rollover risk in the absence of the rat race between creditors. Note that $F(y_{*,0})$ already accounts for the increase in the firm's insolvency risk and the increase in the creditor's embedded-option value.

As σ varies from 5% to 20%, $F(y_{*,1})$ increases from 1.51 to 1.63. Relative to the dashed line, the thick solid line shows that the range of the equilibrium threshold $F(y_{*,\infty})$ is wider. For instance, when we increase σ from 10% to 15%, an individual creditor will only raise his threshold by 0.01, from $F(y_{*,0}) = 1.59$ to $F(y_{*,1}) = 1.60$, if the other creditors' threshold is fixed at 1.59. However, after taking into account the rat race between creditors, each would use a new equilibrium threshold of 1.62, which implies that the rat race amplifies the effect of the volatility increase by 200%.

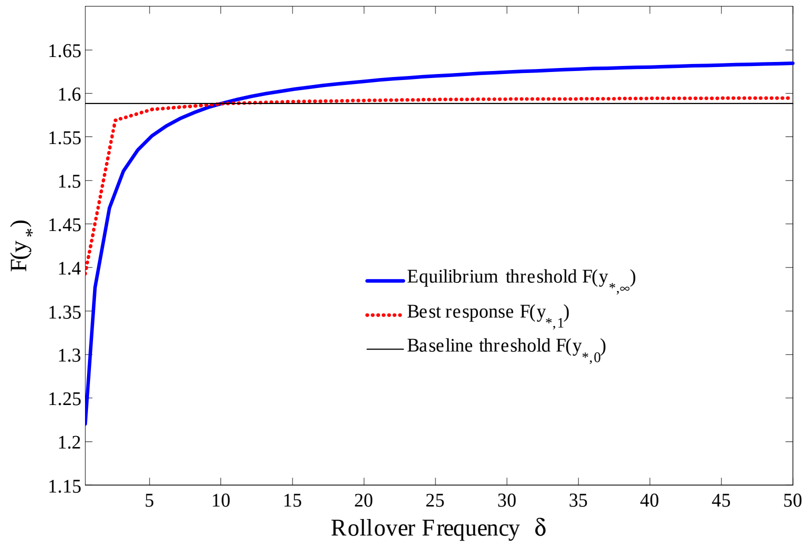

Figure 5: The equilibrium rollover threshold vs the rollover frequency δ. This figure uses the following baseline parameters: $\rho = 5\%$, $r = 0.10$, $\delta = 10$, $\phi = 0.2$, $\theta = 2$, $\mu = 5\%$, $\sigma = 10\%$, $\alpha = 60\%$. The threshold is measured in the firm's fundamental value $F(y_*)$. The thin solid line is the baseline threshold level $F(y_{*,0})$ under the baseline parameters. The thick solid line plots the equilibrium threshold $F(y_{*,\infty})$. The dashed line plots a creditor's best response $F(y_{*,1})$ to the change in α from its baseline value while fixing the other creditors' threshold at $F(y_{*,0})$.

5.3 Rollover Frequency

The firm's rollover frequency δ is another key determinant of its rollover risk. As δ increases, each creditor's contract period, which has an expected duration of $1/\delta$, gets shorter. This generates two opposing effects on the equilibrium. First, each individual creditor is locked in for a shorter period. As a result, his embedded option on the firm is more valuable as he has more flexibility to pull out if the firm fundamental deteriorates. The increased embedded-option value makes the creditor more willing to roll over his debt, i.e., to choose a lower rollover threshold. On the other hand, a higher δ also means that the other creditors are locked in for a shorter period. As a result, during the creditor's contract period, the firm is more susceptible to the rollover risk created by the other creditors. The increased rollover risk therefore motivates him to choose a higher rollover threshold. The equilibrium threshold y_* trades off the embedded-option effect and the rollover-risk effect.

Figure 5 plots the equilibrium rollover threshold (the thick solid line) as we vary δ from its baseline value of 10 to a range between 0.2 to 50, along with an individual creditor's best response (the dashed line) to the δ change while fixing the other creditors' rollover threshold at the baseline level of 1.59. As δ increases from 0.2 to 50, the equilibrium rollover threshold $F(y_*)$ increases from 1.15 to 1.64. This monotonically increasing pattern in $F(y_*)$ suggests that the rollover risk effect dominates the embedded-option effect in this illustration. In unreported numerical analysis, we also find that this holds true over a wide range of parameter values. The embedded-option effect becomes dominant only when θ is low, i.e., the firm's credit lines are sufficiently reliable and the firm's rollover risk is modest.

We again observe a dramatic amplification effect caused by the rat race among the creditors in choosing higher and higher thresholds. For instance, consider raising δ from the baseline level 10 to 50, which implies an average debt maturity of about 1 week. An individual creditor would slightly increase his rollover threshold by 0.005 in the absence of the rat race, while the new equilibrium threshold is higher by 0.05, implying that the rat race amplifies the effect of the δ increase by about 10 times.

6 Implications of Dynamic Runs

This section discusses implications of our model for various issues related to dynamic runs.

6.1 Panic-driven vs. Fundamental-driven Runs

There are two different and long-standing views about runs on banks. The first view is advocated by Friedman and Schwartz (1963) and Kindleberger (1978). They attribute many historical banking crises to unwarranted panics and assert that the banks that were forced to liquidate in such episodes were illiquid rather than insolvent. The alternative view, proposed by Mitchell (1941) and others, suggests that runs happen when depositors have fundamental concerns about the health of banks. Each of these views has motivated a body of theoretical models of bank runs. Diamond and Dybvig (1983), Postlewaite and Vives (1987), Peck and Shell (2003), and Caballero and Krishnamurthy (2008) offer models of panic-driven runs, while Bryant (1980), Gorton (1988), Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), and Allen and Gale (1998) focus on the fundamental risk of bank loans and the depositors' signal extraction problem in driving runs. Gorton and Winton (2003) and Allen and Gale (2007) offer two recent reviews of the history of financial crises and different theories of runs.

Both of these views are relevant for understanding the recent runs on the non-bank financial institutions. Brunnermeier (2009), Krishnamurthy (2009), and Shin (2009) show that these institutions suffered large losses from real-estate-related exposures in the period preceding the credit crisis in 2008, although the losses at the time were insufficient to justify the financial distress faced by these institutions. Federal Reserve Chairman Ben Bernanke

(2008), former SEC Chairman Christopher Cox (2008), and US Treasury Secretary Timothy Geithner (2008) all emphasized in their respective public speeches that Bear Stearns and other financial firms experienced panic runs by their creditors, counterparties and clients. Covitz, Liang, and Suarez (2009) provide a recent study of the runs in the asset-backed commercial paper (ABCP) market in 2007, and find that both deteriorating fundamentals and market panics contributed to the observed runs. During the first few weeks of the turmoil, all types of ABCP programs experienced difficulty in issuing new commercial paper, indicating widespread market panics. However, runs eventually centered on fundamentally impaired programs.

Our model integrates deteriorating fundamentals and the coordination problem between creditors as joint drivers of debt runs in a dynamic framework. It is the possibility of the firm's future fundamental deterioration that generates each creditor's fear of the firm's future rollover risk—if the fundamental declines later, the firm might fail and he might end up bearing the cost of liquidating the firm asset at a fire-sale price. The coordination problem between creditors further amplifies this fear through the rat race mechanism described in Section 5.1 and causes each maturing creditor to run at a threshold substantially higher than he would in the absence of the coordination problem. This intricate interaction between the firm's rollover risk and fundamental risk explains why it is often difficult to identify a financial crisis as a fundamental crisis or liquidity crisis. Our analysis in the previous section shows that under a wide range of model parameters, firms with declining fundamentals, higher fundamental volatility, greater asset illiquidity, or shorter debt maturities are more exposed to runs.

Our model shares some spirit of the bank run models of Rochet and Vives (2004) and Goldstein and Pauzner (2005), which build on the global games approach discussed in Section 3.3.3. These models are static and extend the Diamond-Dybvig bank-run setting by allowing the bank fundamental to be unobservable and depositors to possess noisy private signals about the fundamental. Depositors use their private signals to coordinate their withdrawal decisions and a unique equilibrium emerges. In this equilibrium, uncertainty about the bank fundamental interacts with the coordination problem between depositors and leads to inefficient runs on the bank. In contrast to these models, our dynamic model incorporates time-varying fundamentals and staggered debt structures, which allow us to analyze the effects of fundamental volatility and debt maturity in driving runs.

The wave of bank failures during the Great Depression motivated the government to provide deposit insurance to protect bank depositors, and thus mitigate their incentive to run on banks. The credit crisis of 2007-2008 shows that non-bank financial institutions are exposed to similar runs by their creditors, but they are not protected by any government program. As discussed by Cecchetti (2009), during the credit crisis the Fed failed to restore the financial system back to normalcy by using its standard monetary tools, such as cutting the cost of discount borrowing, increasing the term of the loans, and cutting the target federal funds rate. Instead, the Fed had to invent several unconventional tools.[23] Given the potential systemic risk created by runs on "illiquid but solvent" financial institutions, which we discuss in Section 6.4, many issues remain for regulators and researchers regarding how to establish new safeguards for the non-bank financial system. Our model provides useful insights for these issues.

6.2 Capital Adequacy Standards

How much capital does a financial firm need to prevent inefficient panic runs? It is well known that a well capitalized firm—i.e., one that has more assets than liabilities—can have funding problems because of the illiquidity of its assets. As a result, the firm needs to maintain an adequate capital cushion to offset its asset illiquidity. A common-sense criterion is that adequate capital to counterbalance the illiquidity discount of a firm's assets in the current market is sufficient to prevent runs. This criterion is effective in the standard bank-run models, such as Diamond and Dybvig (1983), Rochet and Vives (2004), and Goldstein and Pauzner (2005). The intuition is simple. If the firm's capital is sufficient to pay back its liability after a forced liquidation, there is no need for any creditor to worry about runs by other creditors.

However, our model shows that this criterion is insufficient. Figure 6 plots the creditors' equilibrium rollover threshold with respect to the firm's fundamental volatility σ over a wider range than Figure 4. Once σ rises above 30%, the creditors' equilibrium rollover threshold $F(y_{*,\infty})$ (the thick solid line) surpasses $1/\alpha$ (the thin solid line), the level at which the firm's asset value is sufficient to cover its liability even after the liquidation discount. That is, even though the firm is so well capitalized that it can pay back its liability even after a forced liquidation, creditors are still not assured and may choose to run. This type of frantic run, derived as the third case in Theorem 1, is consistent with the run on Bear Stearns, e.g., Cox (2008).

[23] For example, the Primary Dealer Credit Facility, created in March 2008 after the collapse of Bear Stearns, allows major investment banks to access the discount window and borrow from the Fed. Following a prominent money-market fund's "breaking the buck" (i.e., a decline of its net assets below par) in September 2008, the Fed also created the Asset-Backed Commercial Paper (ABCP) Liquidity Facility to assist money-market funds that hold such paper in meeting demands for redemptions by investors. Through these facilities, the Fed acts as the lender of last resort and provides credit lines and backstop liquidity to investment banks and money-market funds.

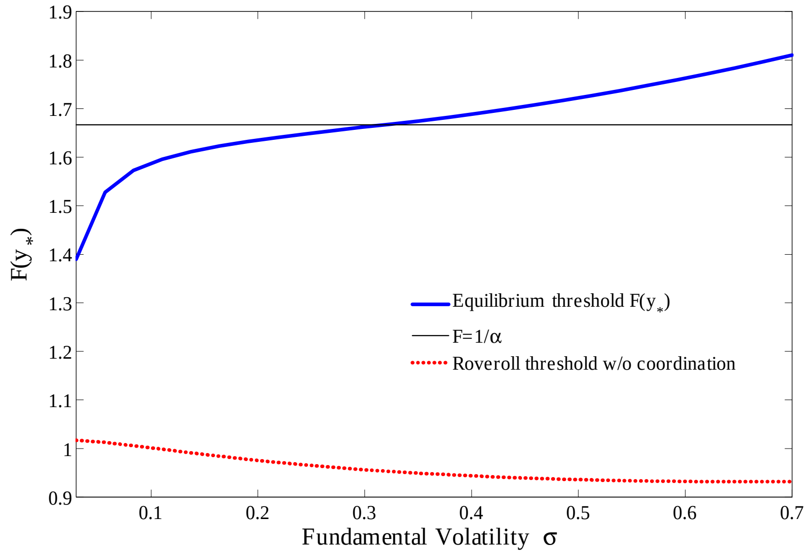

Figure 6: The equilibrium rollover threshold vs the fundamental volatility σ. This figure uses the following baseline parameters: $\rho = 5\%$, $r = 0.10$, $\delta = 10$, $\phi = 0.2$, $\theta = 2$, $\mu = 5\%$, $\sigma = 10\%$, $\alpha = 60\%$. The threshold is measured in the firm's fundamental value $F(y_*)$. The thick solid line plots the equilibrium threshold $F(y_*)$, while the thin solid line gives the level $\frac{1}{\alpha}$, at which the firm's capital is sufficient to cover its liability even after the liquidation discount. The dashed line plots the rollover threshold $F(y_s)$ in the absence of any coordination problem.

What drives this type of frantic run? The driving force is exactly the firm's rollover risk, the central theme of our model. The capacity for the firm's liquidation value to pay back its liability now is not a guarantee for future periods when the firm fundamental may deteriorate (as the liquidation value falls with the fundamental.) In particular, a maturing creditor is worried that during his next contract period, the fundamental might fall and other creditors would choose to run and cause the firm to fail. The firm's liquidation value at that time may not be sufficient to pay off its liability. When this concern becomes sufficiently strong, he chooses to run ahead of future maturing creditors despite the firm's ample capital cushion now. Figure 6 shows that this occurs when the firm's fundamental is sufficiently volatile.

As we discussed in Section 5.2, an increase in fundamental volatility affects the equilibrium rollover threshold through three channels: the insolvency-risk, rollover-risk and embedded-option channels. To further highlight rollover risk as the driving force of frantic runs, we also consider a firm financed by a single large creditor, based on the setting described in Section 4. Suppose that a small creditor also holds a negligible fraction of the firm's debt. Since the large creditor will always roll over his debt (Proposition 6), the small creditor's rollover threshold choice is only affected by the firm's insolvency risk and his embedded option in the firm.[24] As shown by the dashed line in Figure 6, his rollover threshold decreases with the firm's fundamental volatility, suggesting that the embedded-option effect dominates the insolvency-risk effect. The contrast between this line and the thick solid line confirms that frantic runs are driven by the creditors' fear of the firm's rollover risk.

We can formally prove the following proposition:

Proposition 9 *When the firm's fundamental volatility is sufficiently large, creditors start to run on the firm even when its current liquidation value is sufficient to pay off its liability, i.e.,* $F(y_*) > 1/\alpha$.

This proposition highlights the severity of the creditors' preemptive motive to run ahead of each other in a dynamic environment. It also demonstrates that once we account for the dynamic coordination problem between creditors, higher capital adequacy standards are required to prevent inefficient panic runs.

6.3 Shortening of Debt Maturity

Krishnamurthy (2009) documents a dramatic shortening in the maturity structure of commercial paper issuance around mid-September 2008. The issuance of commercial paper with maturities less than 9 days increased by more than 50%, replacing maturities over 9 days. Anecdotally, much of the shortening is in fact to overnight paper.

What explains the dramatic maturity shortening? Our discussion in Section 5.3 suggests that each creditor would prefer a shorter debt maturity so that he has more flexibility to pull out of a firm before others if the firm runs into trouble later. We formally derive this in the following proposition:

Proposition 10 *Fixing the other creditors' rollover frequency, each creditor's value function increases with his own rollover frequency.*

This proposition suggests that in the absence of any commitment device like debt covenants or regulatory requirement, the firm could use shortening debt maturity as a survival tool when the creditors refuse to roll over their maturing debt. According to our earlier analysis, this would happen when the firm fundamental falls, when the fundamental volatility rises, or when the secondary market of the firm asset becomes more illiquid. However, our analysis in Section 5.3 also shows that as other creditors' debt maturity becomes shorter, each creditor has a greater incentive to run because he anticipates the firm's increasing rollover risk in the

[24] The value of his debt is given by equation (8) with the rollover risk term $\min(1, L + ly_\tau)\,\mathbf{1}_{\{\tau=\tau_\theta\}}$ removed.

future. Thus, the maturity shortening acts as a self-enforcing tightening mechanism to push the firm closer and closer to bankruptcy. This also suggests that maturity shortening of a firm's debt issuance tends to precede creditors' runs. Formally analyzing this issue requires making debt maturity a choice variable, which is beyond our current framework.[25]

6.4 Spillover and Systemic Risk

Federal Reserve Chairman Ben Bernanke (2008) describes the potential systemic risk following the collapse of Bear Stearns as the key reason that led the Fed to open the discount window to every major investment bank. We can readily extend our model with multiple firms holding similar assets to analyze this type of systemic risk triggered by creditors' panic runs on one firm. As these firms face the same downward sloping demand curve for their assets in an illiquid secondary market, the liquidation recovery rate α of each firm depends on other firms' liquidation, e.g., Shleifer and Vishny (1992). Suppose that one firm, say Bear Stearns, suffers idiosyncratic negative shocks to its fundamentals. As a result, when this firm experiences runs by its creditors and needs to liquidate its asset, the liquidation would potentially push down the liquidation values of other firms. This in turn increases the losses of other firms' creditors in the event that their firms are forced into liquidation. Thus, through this liquidation-value channel, panic runs spill over to these firms as their creditors now have greater incentives to run, even though there is no fundamental deterioration in these firms.[26] The possibility of other firms experiencing runs also feeds back to the creditors of the initial firm in distress, creating even greater incentives to run. In this way, a rat race to exit risky debt is underway not just between creditors of one firm, but also between creditors of all firms holding similar assets.[27] Thus, market liquidity evaporates and systemic risk becomes imminent.

6.5 Credit Risk

The standard credit modeling approach, following the classic structural model of Merton (1974), focuses on insolvency risk (i.e., the risk that a firm's asset value could fall below its liability) as the only source of credit risk (i.e., the risk that a firm defaults on its debt). The

[25] Brunnermeier and Oehmke (2009) provide such an analysis in a different and simplified setting. They show that the conflict of interest between short-term and long-term creditors leads to a maturity rat race between creditors, through which the firm ends up with excessive short-term debt in equilibrium.

[26] This spillover mechanism is complementary to the existing ones proposed by Allen and Gale (2000) through the interbank lending channel and by Kyle and Xiong (2001) through the wealth effect of financial intermediaries.

[27] This mechanism is closely related to market runs analyzed by Bernardo and Welch (2004) and Morris and Shin (2004), who treat each firm's credit constraint as exogenously imposed.

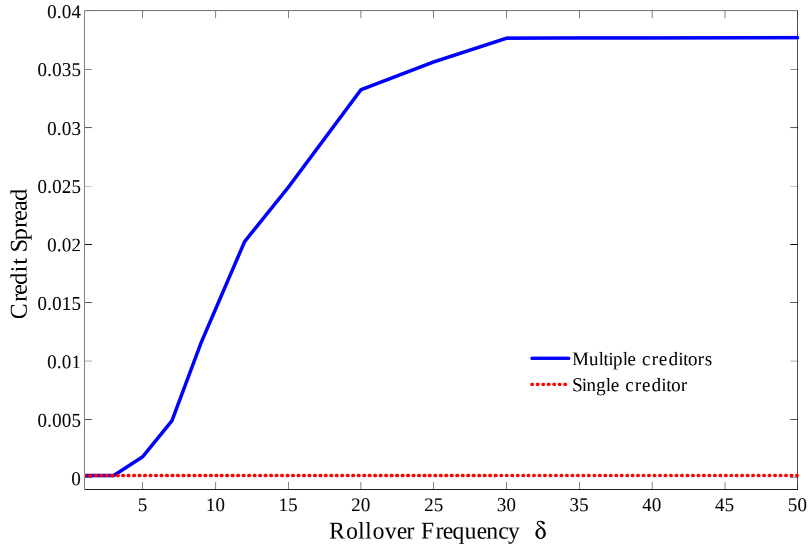

Figure 7: Credit spread vs debt rollover frequency. This figure uses the following baseline parameters: $\rho = 5\%$, $r = 0.10$, $\delta = 10$, $\phi = 0.2$, $\theta = 2$, $\mu = 5\%$, $\sigma = 10\%$, $\alpha = 60\%$, $y_0 = 1.25$, and $T = 0.25$. The solid line plots the credit spread of a firm financed by a continuum of small creditors. The dashed line plots the credit spread of a firm financed by a single creditor.

credit crisis of 2007-2008 vividly highlights that liquidity risk, the risk that a firm might not be able to raise capital when it needs to, plays a key role in driving credit risk of firms, such as Bear Stearns and Lehman Brothers.

In our model, rollover risk is an important source of credit risk. To illustrate this effect, we examine the credit spread of a hypothetical infinitesimal zero-coupon bond issued by the firm analyzed in our model, which has a face value of 1 and a fixed maturity T. This bond provides the following payoff depending on three scenarios: 1) if the firm's asset matures before T and before any forced liquidation, the bond pays $\min\left(y_{\tau_\phi}, 1\right)$; 2) if a forced liquidation occurs before T and before the asset matures, the bond pays $\min\left(L + l y_{\tau_\theta}, 1\right)$, the liquidation value of the firm asset; 3) otherwise, the bond pays 1. This payoff effectively captures the firm's credit risk before time T. The credit spread is the difference between the bond yield and the yield of a risk-free bond with the same maturity.[28] For comparison, we also introduce another firm identical in all other dimensions except that it is financed by a single creditor with deep pockets. As the single creditor will always roll over his debt (Proposition 6), this firm has no rollover risk.

[28] Our risky bond receives a payoff at a random time before the bond maturity T. For a fair comparison, we also impose the same random maturity on the risk-free bond, which has a value of $\frac{\phi}{\rho+\phi} + \frac{\rho}{\rho+\phi} e^{-(\rho+\phi)T}$. Then we calculate the yield earned by the risk-free bond as $\beta_{riskfree} = -\frac{1}{T} \ln\left(\frac{\phi}{\rho+\phi} + \frac{\rho}{\rho+\phi} e^{-(\rho+\phi)T}\right)$. The credit spread is measured relative to this yield.

We numerically calculate the value of this hypothetical bond with $y_0 = 1.25$, $T = 0.25$ (3 months) for both firms, and plot the credit spreads with respect to their debt rollover frequency δ in Figure 7. The difference between these two credit spreads measures the contribution of rollover risk to the credit risk of the firm with multiple creditors. The credit spread of the firm with a single creditor is independent of δ. However, the credit spread of the firm with multiple creditors increases sharply from less than 2 basis points to over 380 basis points as δ increases from 1 to 50 (i.e., from once every one year to once every week). This illustration shows that rollover risk could be a substantial part of a firm's credit risk.

In summary, the firm's credit risk depends on the interaction between its fundamental insolvency risk and the coordination problem between its creditors. Fear of the firm's future rollover risk when its fundamental deteriorates causes each maturing creditor to preemptively run on the firm and thus causes the firm to fail even when its fundamental value is substantially higher than its liability. As a result, the firm's corporate bond spreads depend not only on the common measures of credit risk, such as fundamental risk and leverage, but also on its debt maturity structure, which has a significant effect on the firm's rollover risk as we show in Section 5.3.[29]

7 Conclusion

In this paper, we develop a dynamic model of panic runs by creditors on a firm, which invests in an illiquid asset by rolling over staggered short-term debt contracts. In particular, fear of runs by future maturing creditors motivates each creditor to preemptively run ahead of others. Our model highlights that the firm's rollover risk is intertwined with its fundamental risk, thus making it difficult to separate the firm's liquidity problems from fundamental problems. Our model provides a set of implications on the roles played by volatility, illiquidity and debt maturity in driving debt runs, as well as on issues related to firms' capital adequacy standards and credit risk.

Runs are a natural self-protection mechanism for individual agents in many other realistic settings where intrinsic conflicts of interest between agents are present. For example, fear of an asset market becoming illiquid could motivate individual investors to exit the market, which in turn makes the market even more illiquid, e.g., Bernardo and Welch (2004) and Morris and Shin (2004). An individual worker's fear of other workers not exerting effort, which reduces his marginal productivity, also motivates him to reduce his effort, e.g., Cooper

[29] Morris and Shin (2004, 2009) also model the effect of the creditors' coordination problem on firms' credit risk. They adopt the global games approach in two-period settings. In contrast, our continuous-time setting has a potential advantage in calibrating the effect.

and John (1988). Since interactions between fundamental fluctuations and the running incentives of individual agents (exit a market or not exert effort) are also present in these settings, our model framework can be extended to study potentially interesting dynamics in these contexts.

A Appendix

A.1 Proof of Theorem 1

Using the HJB equation in (9), we first construct an individual creditor's value function by utilizing the fact that in any symmetric equilibrium all creditors (including this individual creditor) use the same monotone strategy with threshold y_*. The equilibrium threshold must then be the solution to the equation $V(y_*; y_*) = 1$. Of course, individual optimality requires that $V(y; y_*) > 1$ for $y > y_*$ and $V(y; y_*) < 1$ for $y < y_*$, a condition that we will verify in Lemma 13. Later in Lemma 14 we also show that there does not exist any asymmetric threshold equilibrium.

When all creditors use the same threshold y_*, the HJB equation (9) becomes

- If $y < y_*$,

$$0 = \frac{\sigma^2}{2} y^2 V_{yy} + \mu y V_y - [\rho + \phi + (\theta + 1)\delta] V(y; y_*) + \phi \min(1, y) + \theta\delta \min(L + ly, 1) + r + \delta; \quad (11)$$

- If $y \geq y_*$,

$$0 = \frac{\sigma^2}{2} y^2 V_{yy} + \mu y V_y - (\rho + \phi) V(y; y_*) + \phi \min(1, y) + r. \quad (12)$$

The value function has to satisfy these two differential equations and be continuous and differentiable at the boundary point y_*. In solving these differential equations, we need to introduce the two roots to the first fundamental equation for (11):

$$\frac{1}{2}\sigma^2 x(x-1) + \mu x - [\rho + \phi + (1+\theta)\delta] = 0, \quad (13)$$

which are

$$-\gamma_1 = -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 [\rho + \phi + (1+\theta)\delta]}}{\sigma^2} < 0, \quad (14)$$

and

$$\eta_1 = -\frac{\mu - \frac{1}{2}\sigma^2 - \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 [\rho + \phi + (1+\theta)\delta]}}{\sigma^2} > 1; \quad (15)$$

and the two roots to the second fundamental equation for (12):

$$\frac{1}{2}\sigma^2 x(x-1) + \mu x - (\rho + \phi) = 0, \quad (16)$$

which are

$$-\gamma_2 = -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 (\rho + \phi)}}{\sigma^2} < 0, \quad (17)$$

and

$$\eta_2 = -\frac{\mu - \frac{1}{2}\sigma^2 - \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\sigma^2 (\rho + \phi)}}{\sigma^2} > 1. \quad (18)$$

We summarize the constructed value function below.

Lemma 11 *Given the equilibrium rollover threshold y_*, the value function of an individual creditor is given by the following three cases:*

1. *If $y_* < 1$,*

$$V(y;y_*)=\begin{cases}\frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}+\frac{\phi+\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu}y+A_1y^{\eta_1} & \text{when } 0<y\le y_*\\ \frac{r}{\rho+\phi}+\frac{\phi}{\rho+\phi-\mu}y+A_2y^{-\gamma_2}+A_3y^{\eta_2} & \text{when } y_*<y\le 1\\ \frac{r+\phi}{\rho+\phi}+A_4y^{-\gamma_2} & \text{when } 1<y\end{cases}. \tag{19}$$

The four coefficients A_1, A_2, A_3, and A_4 are given by

$$\begin{aligned}A_1 &= \frac{[H_3\gamma_2+H_1]-y_*^{-\eta_2}(\gamma_2H_4+H_2y_*)}{(\eta_1+\gamma_2)\,y_*^{\eta_1-\eta_2}},\\ A_2 &= \frac{y_*^{\gamma_2}}{\eta_2+\gamma_2}\left[\eta_2H_4-H_2y_*+A_1(\eta_2-\eta_1)\,y_*^{\eta_1}\right],\\ A_3 &= \frac{y_*^{-\eta_2}}{\eta_2+\gamma_2}\left[\gamma_2H_4+H_2y_*+A_1(\eta_1+\gamma_2)\,y_*^{\eta_1}\right],\\ &= \frac{1}{\eta_2+\gamma_2}\left[H_3\gamma_2+H_1\right],\\ A_4 &= A_2-\frac{1}{\eta_2+\gamma_2}\left[H_3\eta_2-H_1\right],\end{aligned}$$

where

$$\begin{aligned}H_1 &= -\frac{\phi}{\rho+\phi-\mu},\\ H_2 &= \frac{\theta\delta l(\rho+\phi-\mu)-\phi(1+\theta)\delta}{(\rho+\phi+(1+\theta)\delta-\mu)(\rho+\phi-\mu)},\\ H_3 &= -\frac{\phi\mu}{(\rho+\phi)(\rho+\phi-\mu)},\\ H_4 &= \frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}-\frac{r}{\rho+\phi}+H_2y_*.\end{aligned}$$

2. *If $1 < y_* \le \frac{1-L}{l}$,*

$$V(y;y_*)=\begin{cases}\frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}+\frac{\phi+\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu}y+B_1y^{\eta_1} & \text{when } y\le 1,\\ \frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}+\frac{\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu}y+B_2y^{-\gamma_1}+B_3y^{\eta_1} & \text{when } 1<y\le y_*,\\ \frac{r+\phi}{\rho+\phi}+B_4y^{-\gamma_2} & \text{when } y_*<y.\end{cases} \tag{20}$$

The four coefficients B_1, B_2, B_3, and B_4 are given by

$$\begin{aligned}B_1 &= B_3-\frac{M_2\gamma_1+M_1}{\eta_1+\gamma_1},\\ B_2 &= \frac{M_2\eta_1-M_1}{\eta_1+\gamma_1}<0,\\ B_3 &= \frac{(\gamma_1-\gamma_2)\,B_2\,(y_*)^{-\gamma_1}+\gamma_2M_3-\frac{\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu}y_*}{(\eta_1+\gamma_2)\,y_*^{\eta_1}},\\ B_4 &= \frac{\eta_1+\gamma_1}{\eta_1+\gamma_2}B_2y_*^{\gamma_2-\gamma_1}+\frac{\eta_1-1}{\eta_1+\gamma_2}\frac{\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu}y_*^{\gamma_2+1},\\ &\quad -\frac{\eta_1}{\eta_1+\gamma_2}\left[\frac{r+\phi}{\rho+\phi}-\frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}\right]y_*^{\gamma_2},\\ &= \frac{(\eta_1+\gamma_1)\,B_2y_*^{-\gamma_1}-\eta_1M_3-\frac{\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu}y_*}{(\eta_1+\gamma_2)\,y_*^{-\gamma_2}},\end{aligned}$$

where

$$\begin{aligned} M_1 &= \frac{\phi}{\rho+\phi+(1+\theta)\,\delta-\mu}, \\ M_2 &= \frac{\phi\mu}{(\rho+\phi+(1+\theta)\,\delta)\,(\rho+\phi+(1+\theta)\,\delta-\mu)}, \\ M_3 &= \frac{r+\phi}{\rho+\phi}-\frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\,\delta}-\frac{\theta\delta l}{\rho+\phi+(1+\theta)\,\delta-\mu}y_*. \end{aligned}$$

3. *If* $y_* > \frac{1-L}{l}$,

$$V(y;y_*) = \begin{cases} \frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}+\frac{\phi+\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu}y+C_1y^{\eta_1} & \text{when } y\le 1, \\ \frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}+\frac{\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu}y+C_2y^{-\gamma_1}+C_3y^{\eta_1} & \text{when } 1<y\le\frac{1-L}{l}, \\ \frac{r+\phi+\theta\delta+\delta}{\rho+\phi+(1+\theta)\delta}+C_4y^{-\gamma_1}+C_5y^{\eta_1} & \text{when } \frac{1-L}{l}<y\le y_*, \\ \frac{r+\phi}{\rho+\phi}+C_6y^{-\gamma_2} & \text{when } y>y_*. \end{cases} \quad (21)$$

The six coefficients C_1, C_2, C_3, C_4, C_5 *and* C_6 *are given by*

$$\begin{aligned} C_1 &= C_3-\frac{K_4\gamma_1+K_5}{\eta_1+\gamma_1}, \\ C_2 &= \frac{K_4\eta_1-K_5}{\eta_1+\gamma_1}, \\ C_3 &= C_5+\frac{K_2\gamma_1-K_3\frac{1-L}{l}}{(\eta_1+\gamma_1)\left(\frac{1-L}{l}\right)^{\eta_1}}, \\ C_4 &= C_2-\frac{K_2\eta_1+K_3\frac{1-L}{l}}{(\eta_1+\gamma_1)\left(\frac{1-L}{l}\right)^{-\gamma_1}}, \\ C_5 &= \frac{(\gamma_1-\gamma_2)\,C_4y_*^{-\gamma_1}-\gamma_2K_1}{(\eta_1+\gamma_2)\,y_*^{\eta_1}}, \\ C_6 &= \frac{(\eta_1+\gamma_1)\,C_4y_*^{-\gamma_1}+\eta_1K_1}{(\eta_1+\gamma_2)\,y_*^{-\gamma_2}}, \end{aligned}$$

where

$$\begin{aligned} K_1 &= \frac{r+\phi+\theta\delta+\delta}{\rho+\phi+(1+\theta)\,\delta}-\frac{r+\phi}{\rho+\phi}, \\ K_2 &= \frac{\theta\delta\,(1-L)}{\rho+\phi+(1+\theta)\,\delta}-\frac{\theta\delta\,(1-L)}{\rho+\phi+(1+\theta)\,\delta-\mu}, \\ K_3 &= \frac{\theta\delta l}{\rho+\phi+(1+\theta)\,\delta-\mu}, \\ K_4 &= \frac{\phi}{\rho+\phi+(1+\theta)\,\delta-\mu}-\frac{\phi}{\rho+\phi+(1+\theta)\,\delta}, \\ K_5 &= \frac{\phi}{\rho+\phi+(1+\theta)\,\delta-\mu}. \end{aligned}$$

Proof. We can derive the three cases listed above using the same method. For illustration we just solve the first case with $y_* < 1$. Depending on the value of y, we have the following three scenarios.

- If $0 < y \le y_*$:

$$\frac{\sigma^2}{2}y^2V_{yy}+\mu yV_y-[\rho+\phi+(1+\theta)\,\delta]\,V(y)+(\phi+\theta\delta l)\,y+r+\theta\delta L+\delta=0.$$

The general solution of this differential equation is given in the first line of equation (19) with the coefficient A_1 to be determined by the boundary conditions. Note that to ensure the value of V is finite as y approaches zero, we have ruled out another power solution $y^{-\gamma_1}$ of the equation .

- If $y_* < y \leq 1$:
$$\frac{\sigma^2}{2}y^2V_{yy} + \mu y V_y - (\rho + \phi) V(y) + \phi y + r = 0.$$
The general solution of this differential equation is given in the second line of equation (19) with the coefficients A_2 and A_3 to be determined by the boundary conditions.

- If $y > 1$:
$$\frac{\sigma^2}{2}y^2V_{yy} + \mu y V_y - (\rho + \phi) V(y) + r + \phi = 0.$$
The general solution of this differential equation is given in the third line of equation (19) with the coefficient A_4 to be determined by the boundary conditions. Note that to ensure the value of V is finite as y approaches infinity, we have ruled out another power solution y^{η_2} of the equation.

To determine the four coefficients A_1, A_2, A_3, and A_4, we have four boundary conditions at $y = y_*$ and 1, i.e., the value function $V(y)$ must be continuous (value-matching) and differentiable (smooth-pasting) at these two points. Solving these boundary conditions leads to the coefficients given in Lemma 11. ■

Based on the value function derived in Lemma 11, we now show that there exists a unique threshold y_* for the equilibrium condition to hold.

Lemma 12 *There exists a unique y_* such that*

$$V(y_*; y_*) = 1.$$

Proof. Define

$$W(y) \equiv V(y; y).$$

We need to show that there is a unique y_* such that $W(y_*) = 1$.

We first show that $W(y)$ is monotonically increasing when $y < 1$. In this case, we can directly extract the value of $W(y)$ from equation (19), which, by neglecting terms independent of y, is

$$\begin{aligned} W(y) &= \left[\frac{\phi + \theta\delta l}{\rho + \phi + (1+\theta)\delta - \mu} - \frac{1+\gamma_2}{\eta_1 + \gamma_2}H_2\right] y + \frac{[H_3\gamma_2 + H_1]}{\eta_1 + \gamma_2} y^{\eta_2} \\ &= \left[-H_1 + \frac{\eta_1 - 1}{\eta_1 + \gamma_2}H_2\right] y + \frac{[H_3\gamma_2 + H_1]}{\eta_1 + \gamma_2} y^{\eta_2}. \end{aligned}$$

Note that

$$\begin{aligned} \frac{dW(y)}{dy} &= -H_1 + \frac{\eta_1 - 1}{\eta_1 + \gamma_2}H_2 + \frac{[H_3\gamma_2 + H_1]}{\eta_1 + \gamma_2}\eta_2 y^{\eta_2 - 1} \\ &> -H_1 + \frac{\eta_1 - 1}{\eta_1 + \gamma_2}H_2 + \frac{[H_3\gamma_2 + H_1]}{\eta_1 + \gamma_2}\eta_2 \\ &= -H_1 + \frac{\eta_1 - 1}{\eta_1 + \gamma_2}H_2 + \frac{\eta_2}{\eta_1 + \gamma_2}H_1 + \frac{\gamma_2\eta_2}{\eta_1 + \gamma_2}H_3 \\ &= \frac{\eta_1 - 1}{\eta_1 + \gamma_2}(H_2 - H_1) + \frac{\eta_2 - \gamma_2 - 1}{\eta_1 + \gamma_2}H_1 + \frac{\gamma_2\eta_2}{\eta_1 + \gamma_2}H_3, \end{aligned}$$

where the second inequality is due to the fact that $H_3 < 0$ and $H_1 < 0$ (defined in Lemma 11).

In the first term above,

$$H_2 - H_1 = \frac{\theta\delta l + \phi}{\rho + \phi + (1+\theta)\delta - \mu}$$

is positive according to the parameter restriction in (5). For the second term, note that $\eta_2 - \gamma_2 - 1 = -2\frac{\mu}{\sigma^2}$. Then after some algebraic substitutions (note that $\gamma_2\eta_2 = \frac{2(\rho+\phi)}{\sigma^2}$), the sum of the second and third terms is

$$-2\frac{\mu}{\sigma^2}\frac{1}{\eta_1+\gamma_2}H_1 + \frac{\gamma_2\eta_2}{\eta_1+\gamma_2}H_3 = 0.$$

Thus, $\frac{dW(y)}{dy} > 0$.

We now show that $W(y)$ is monotonically increasing when $1 < y \le \frac{1-L}{l}$. Equation (20) implies that

$$\begin{aligned} W(y) &= \frac{r+\phi}{\rho+\phi} + B_4 y^{-\gamma_2} \\ &= \frac{M_2\eta_1 - M_1}{\eta_1+\gamma_2} y^{-\gamma_1} + \frac{\eta_1 - 1}{\eta_1+\gamma_2}\frac{\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu} y \\ &\quad + \frac{\gamma_2}{\eta_1+\gamma_2}\frac{r+\phi}{\rho+\phi} + \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}. \end{aligned}$$

We now show $\eta_1 < \frac{M_1}{M_2} = \frac{\rho+\phi+(1+\theta)\delta}{\mu}$. Plugging $x = \frac{\rho+\phi+(1+\theta)\delta}{\mu}$ into the first fundamental equation (13), we find that the value is positive, which implies that $\eta_1 < \frac{M_1}{M_2}$. Therefore $M_2\eta_1 - M_1 < 0$, and the first term is increasing in y. Because $\eta_1 > 1$, the second term is increasing in y. As a result, $W(y)$ is increasing in y.

Similarly we can show that $W(y)$ is increasing in y for $y > \frac{1-L}{l}$. Equation (21) implies that

$$W(y) = \frac{r+\phi}{\rho+\phi} + C_6 y^{-\gamma_2} = \frac{\gamma_2}{\eta_1+\gamma_2}\frac{r+\phi}{\rho+\phi} + \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} + \frac{(\eta_1+\gamma_1)C_4 y^{-\gamma_1}}{\eta_1+\gamma_2}.$$

Since $\frac{K_5}{K_4} = \frac{K_3\frac{1-L}{l}}{-K_2} = \frac{\rho+\phi+(1+\theta)\delta}{\mu} = M_1/M_2$, we have

$$\begin{aligned} \frac{(\eta_1+\gamma_1)C_4 y^{-\gamma_1}}{\eta_1+\gamma_2} &= \frac{K_4\eta_1 - K_5 + \frac{-K_2\eta_1 - K_3\frac{1-L}{l}}{\left(\frac{1-L}{l}\right)^{-\gamma_1}}}{\eta_1+\gamma_2} y^{-\gamma_1} \\ &= \frac{\eta_1 - M_1/M_2}{\eta_1+\gamma_2}\left(K_4 y^{-\gamma_1} + (-K_2)\left(\frac{ly}{1-L}\right)^{-\gamma_1}\right). \end{aligned}$$

Therefore, because $\eta_1 - M_1/M_2 < 0$ as shown in the case of $1 < y \le \frac{1-L}{l}$, and we can check that $K_4 > 0$ and $-K_2 > 0$, $W(y)$ is strictly increasing.

Next, we need to ensure that $W(0) < 1$. Equation (19) implies that

$$W(0) = \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} + \frac{\gamma_2}{\eta_1+\gamma_2}\frac{r}{\rho+\phi}.$$

The parameter restriction in (4) ensures that

$$\frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} < 1 \quad \text{and} \quad \frac{r}{\rho+\phi} < 1,$$

thus, $W(0) < 1$.

Finally note that under our parameter restrictions in (4) and (6) we have

$$W(\infty) = \frac{\gamma_2}{\eta_1+\gamma_2}\frac{r+\phi}{\rho+\phi} + \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} > 1.$$

Because $W(y)$ is continuous and monotonically increasing, and because $W(0) < 1$ and $W(\infty) > 1$, there exists a unique y_* such that $W(y_*) = 1$. ■

Lemma 12 implies that there can be at most one symmetric monotone equilibrium. Next, we verify that a monotone strategy with the threshold level determined in Lemma 12 is indeed optimal for an individual creditor if every other creditor uses this threshold.

Lemma 13 *If every other creditor uses a monotone strategy with a threshold y_* identified in Lemma 12, then the same strategy is also optimal for an individual creditor.*

Proof. To show that the value function constructed in Lemma 11 is indeed optimal for an individual creditor, i.e., the value function solves the HJB equation (9), we need to verify that $V(y; y_*) > 1$ for $y > y_*$ and $V(y; y_*) < 1$ for $y < y_*$. By construction in Lemma 11, $V(0; y_*) = \frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} < 1$ and $V(\infty; y_*) = \frac{r+\phi}{\rho+\phi} > 1$. We just need to show that $V(y; y_*)$, as a function of y, only crosses 1 once at y_*. Later in this proof we simply write $V(y; y_*)$ as $V(y)$.

We first consider the case where $y_* < 1$.

We prove by contradiction. Suppose that $V(y)$ also crosses 1 at another point below y_*. Then, there exists $y_1 < y_* < 1$ such that

$$V(y_1) > V(y_*) = 1,\ V'(y_1) = 0, \text{ and } V''(y_1) < 0.$$

Using the differential equation (11), we have

$$\begin{aligned} V(y_1) &= \frac{\frac{1}{2}\sigma^2 y_1^2 V_{yy}(y_1) + \phi \min(1, y_1) + \theta\delta(L + l y_1) + r + \delta}{\rho + \phi + (\theta + 1)\delta} \\ &< \frac{(\phi + \theta\delta l)\, y_1 + \theta\delta L + r + \delta}{\rho + \phi + (\theta + 1)\delta} < \frac{\phi + \theta\delta l + \theta\delta L + r + \delta}{\rho + \phi + (\theta + 1)\delta} < 1. \end{aligned}$$

The last inequality is implied by the parameter restrictions in (4) and (7). This is a contradiction with $V(y_1) > 1$. Thus, $V(y)$ cannot cross 1 at any y below y_*.

Next, we show that $V(y)$ is monotonic in the region $y \geq y_*$. Suppose that $V(y)$ is non-monotone, then there exist two points $y_* \leq y_1 < y_2$ such that

$$V(y_1) > V(y_2),\ V'(y_1) = V'(y_2) = 0, \text{ and } V''(y_1) < 0 < V''(y_2).$$

(If, say, y_1 happens to be on the break point 1 where the second derivative is not necessary continuous, then take the point as $1+$ as $V''(1+)$ has to be negative. The same caveat applies to the case where $y_1 = y_*$.) According to the differential equation (12), we have

$$\begin{aligned} V(y_1) &= \frac{\frac{1}{2}\sigma^2 y_1^2 V_{yy}(y_1) + r + \phi \min(1, y_1)}{\rho + \phi} \\ &> \frac{\frac{1}{2}\sigma^2 y_2^2 V_{yy}(y_2) + r + \phi \min(1, y_2)}{\rho + \phi} = V(y_2), \end{aligned}$$

which is a contradiction.

We next consider the case where $y_* \geq 1$. We do not separate the two cases of $1 < y_* \leq \frac{1-L}{l}$ and $y_* > \frac{1-L}{l}$, as the following proof applies to both.

The expression in equation (20) or (21) implies that $V(y)$ has to approach $\frac{r+\phi}{\rho+\phi}$ from below (because $\frac{r+\phi}{\rho+\phi}$ is the debt holder's highest possible payoff), thus B_4 or C_6 is strictly negative. This implies that $V(y)$ is increasing on $[y_*, \infty)$, and

$$V'(y_*) > 0.$$

Now consider the region $[0, y_*)$, it is easy to check that $V'(0) > 0$. Therefore, if $V(y)$ is not monotonic on $[0, y_*)$, there must exist two points $y_1 < y_2$ such that

$$V(y_1) > V(y_2), \; V'(y_1) = V'(y_2) = 0, \text{ and } V''(y_1) < 0 < V''(y_2).$$

According to the HJB equation, we have

$$\begin{aligned} V(y_1) &= \frac{\frac{1}{2}\sigma^2 y_1^2 V_{yy}(y_1) + r + \phi \min(1, y_1) + \delta [1 + \theta \min(L + l y_1, 1)]}{\rho + \phi + (1+\theta)\delta} \\ &< \frac{\frac{1}{2}\sigma^2 y_2^2 V_{yy}(y_2) + r + \phi \min(1, y_2) + \delta [1 + \theta \min(L + l y_2, 1)]}{\rho + \phi + (1+\theta)\delta} = V(y_2), \end{aligned}$$

which is a contradiction. Thus, $V(y)$ is also monotonically increasing on $[0, y_*)$.

To summarize, we have shown that $V(y)$ only crosses 1 once at y_*. Thus, it is optimal for an individual creditor to roll over his debt if $y > y_*$ and to run if $y < y_*$. ■

Finally, we prove that there does not exist any asymmetric monotone equilibrium.

Lemma 14 *There does not exist any asymmetric monotone equilibrium in which creditors choose different rollover thresholds.*

Proof. We prove by contradiction. Suppose that there exists an asymmetric monotone equilibrium. Then, there exist at least two groups of creditors who use two different monotone strategies with thresholds $y_{*,1} < y_{*,2}$. For creditors who use the threshold $y_{i,*}$, we denote their value function as $V^i(y)$. At the corresponding thresholds, we must have

$$V^1(y_{1,*}) = V^2(y_{2,*}) = 1.$$

Moreover, we must have

$$V^1(y_{2,*}) = V^2(y_{1,*}) = 1,$$

because each creditor is free to switch between these two strategies. Then for all $y \in [y_{1,*}, y_{2,*}]$, we must have $V^1(y) = V^2(y) = 1$. Otherwise the threshold strategies cannot be optimal. This implies that each creditor is indifferent between choosing any threshold in $[y_{1,*}, y_{2,*}]$. Denote by $\zeta(y)$ the measure of creditors who use a threshold lower than $y \in [y_{1,*}, y_{2,*}]$. Then, V^i has to satisfy the HJB equation in this region:

$$\begin{aligned} \rho V^i(y) &= \mu y V_y + \frac{\sigma^2}{2} y^2 V_{yy} + r + \phi \left[\min(1, y) - V^i(y)\right] \\ &\quad + \theta\delta\zeta(y) \left[\min(L + l y, 1) - V^i(y)\right] + \delta \max\left\{1 - V^i(y), 0\right\}. \end{aligned}$$

Since $V^i(y) = 1$ for any $y \in [y_{1,*}, y_{2,*}]$, we have

$$\rho = r + \phi [\min(1, y) - 1] + \theta\delta\zeta(y) [\min(L + l y, 1) - 1].$$

Note that $\zeta(y)$ is non-decreasing in y because it is a distribution function. Since both $\min(1, y)$ and $\min(L + l y, 1)$ are also non-decreasing in y, the only possibility that the above equation holds is that $L + l y > 1$ and $y > 1$ for $y \in [y_{1,*}, y_{2,*}]$. Then, $\rho = r$ has to hold. This contradicts the parameter restriction that $\rho < r$ in (4). ■

A.2 Proof of Proposition 2

As mentioned in the main text, in this modified synchronous setting the firm's debt contracts all expire at time 0. At this time, each creditor decides whether to run or to roll over into a perpetual debt contract lasting until the firm asset matures at τ_ϕ. If all creditors choose to run, we assume that there is a probability $\theta_s \in (0,1)$ that the firm cannot find new creditors to replace the outgoing ones and is forced into a premature liquidation.[30] The current firm fundamental is y_0.

We first derive an individual creditor's value function $U(y)$ if the bank survives the creditors' rollover decisions at time 0 and thus will be able to stay until the asset maturity at τ_ϕ. $U(y)$ satisfies the following differential equation:

$$\rho U = \mu y U_y + \frac{1}{2}\sigma^2 y^2 U_{yy} + \phi\left[\min(1,y) - U\right] + r.$$

It is direct to solve this differential equation:

$$U(y) = \begin{cases} \frac{r}{\rho+\phi} + \frac{\phi}{\rho+\phi-\mu} y + D_1 y^{\eta_2} & \text{if } 0 < y < 1 \\ \frac{r+\phi}{\rho+\phi} + D_2 y^{-\gamma_2} & \text{if } y > 1 \end{cases}, \tag{22}$$

where

$$\begin{aligned} D_1 &= -\frac{\frac{\phi}{\rho+\phi-\mu} + \gamma_2 \frac{\phi\mu}{(\rho+\phi-\mu)(\rho+\phi)}}{\eta_2+\gamma_2} \\ D_2 &= \frac{-\frac{\phi}{\rho+\phi-\mu} + \eta_2 \frac{\phi\mu}{(\rho+\phi-\mu)(\rho+\phi)}}{\eta_2+\gamma_2}. \end{aligned}$$

D_1 and D_2 are constant and independent of the liquidation recovery parameter α. Because $U(y)$ is dominated by the fundamental value of the bank asset, $U(y) < \frac{r}{\rho+\phi} + \frac{\phi}{\rho+\phi-\mu} y$. This implies that $D_1 < 0$. In addition, since $U(\infty) = \frac{r+\phi}{\rho+\phi}$, $D_2 < 0$ and $U(y)$ approaches $\frac{r+\phi}{\rho+\phi}$ from below. Therefore $U(y)$ is a monotonically increasing function with

$$U(0) = \frac{r}{r+\phi} < 1 \quad \text{and} \quad U(\infty) = \frac{r+\phi}{\rho+\phi} > 1.$$

Then the intermediate value theorem implies that there exists $y_l > 0$ such that $U(y_l) = 1$.

Define $y_h \equiv \frac{1-L}{l}$. According to the parameter restriction (6), $y_h > 1$. We impose the following condition so that a premature liquidation is sufficiently costly, i.e., α is sufficiently small:

$$\alpha < \frac{\rho+\phi-\mu}{\phi\left[\left(D_2 \frac{r-\rho}{r+\phi}\right)^{\frac{1}{\gamma_2}} + \frac{r(\rho+\phi-\mu)}{\phi(\rho+\phi)}\right]}. \tag{23}$$

This condition is analogous to the parameter restriction (6) in our main model. Given this condition and that $\frac{1-L}{l} = \frac{\rho+\phi-\mu}{\phi\alpha} - \frac{r(\rho+\phi-\mu)}{\phi(\rho+\phi)}$, we have

$$U\left(\frac{1-L}{l}\right) = \frac{r+\phi}{\rho+\phi} + D_2\left(\frac{1-L}{l}\right)^{-\gamma_2} > 1,$$

which further implies that $y_l < y_h = \frac{1-L}{l}$.

Next, we show that if $y_0 > y_h$, then it is optimal for an individual creditor to roll over, even if all the other creditors choose to run (so that the liquidation probability is θ_s). Note that the liquidation value of the

[30] In this synchronous rollover setting, the liquidation probability parameter θ_s has to be inside $(0,1)$, while the liquidation intensity parameter θ in the main model can be higher than 1 (conditional on creditors' runs the liquidation probability over $(t, t+dt)$ is $\theta\delta dt$.)

bank asset is sufficient to pay off all the creditors because $L + ly_0 > 1$. Thus, the creditor's expected payoff from choosing run is $\theta_s + (1 - \theta_s) = 1$. His expected payoff from choosing rollover is $\theta_s + (1 - \theta_s) U(y_0)$, which is higher than the expected payoff from choosing run.

Next, we show that if $y_0 < y_l$, then it is optimal for an individual creditor to run even if all the other creditors choose rollover. In this case, the bank will always survive no matter what the individual creditor's decision is. If he chooses to run, he gets a payoff of 1, while if he chooses to roll over, his continuation value function is $U(y_0) < 1$. Thus, it is optimal for the creditor to run.

Finally, we consider the case when $y_0 \in [y_l, y_h]$. If all the other creditors choose to roll over, then an individual creditor's payoff from run is 1, while his continuation value function is $U(y_0) > 1$. Thus it is optimal for him to roll over too. If all the other creditors choose to run, then his expected payoff from run is $\theta_s (L + ly_0) + (1 - \theta_s)$. His expected payoff from choosing rollover is $(1 - \theta_s) U(y_0)$, because once the bank is forced into a premature liquidation, the liquidation value of the bank asset is not sufficient to pay off the other outgoing creditors and the creditor who chooses rollover gets zero. Therefore we need to ensure that $\theta_s (L + ly_0) > (1 - \theta_s)(U(y_0) - 1)$. Analogous to the parameter restriction (7) of our main model, we impose a parameter restriction on θ_s so that it is sufficiently large:

$$\frac{\theta_s}{1 - \theta_s} > \frac{1}{L} \frac{r - \rho}{\rho + \phi}.$$

Then, because $U(y_0) - 1 < \frac{r+\phi}{\rho+\phi} - 1 = \frac{r-\rho}{\rho+\phi}$, we have $(1 - \theta_s)(U(y_0) - 1) < (1 - \theta_s) \frac{r-\rho}{\rho+\phi} < \theta_s L < \theta_s (L + ly_0)$. As a result, it is optimal for the creditor to run with other creditors.

A.3 Proof of Proposition 3

To be consistent with our main model, we restrict ourselves to monotone strategies based on the bank fundamental. Since the bank fundamental is constant, an individual creditor's strategy is to choose always rollover or run. Considering more flexible strategies would only make multiple equilibria more likely to emerge.

Suppose that all the other creditors always choose run in the future. When an individual creditor needs to make his rollover decision, his payoff from run is 1, and his value from always choosing rollover, based on the random debt maturity, is $\frac{r+\phi \min(y,1)+\theta\delta \min(L+ly,1)}{\rho+\phi+\theta\delta}$. Define

$$y_h^c \equiv \min \{y : r + \phi \min(y, 1) + \theta\delta \min(L + ly, 1) \geq \rho + \phi + \theta\delta\}.$$

Thus, if the other creditors always choose run in the future, rollover is optimal for the creditor if $y > y_h^c$, and run is optimal if $y \leq y_h^c$.

Now suppose that all the other creditors always choose to roll over in the future. When an individual creditor needs to make his rollover decision, his payoff from run is 1, and his value function from always choosing rollover, based on the random debt maturity, is $\frac{r+\phi \min(y,1)}{\rho+\phi}$. Define

$$y_l^c \equiv \max \{y : r + \phi \min(y, 1) \leq \rho + \phi\}.$$

Thus, if the other creditors always choose to roll over in the future, run is optimal for an individual creditor if $y < y_l^c$, and rollover is optimal if $y \geq y_l^c$.

Next, we show that $y_h^c > y_l^c$. According to the definition of y_l^c, it suffices to show that

$$r + \phi \min(y_h^c, 1) > \rho + \phi.$$

Note that $y_h^c < \frac{1-L}{l}$, because (recall that $\frac{1-L}{l} > 1$ implied by (6))

$$r + \phi \min\left(\frac{1-L}{l}, 1\right) + \theta\delta \min\left(L + l\frac{1-L}{l}, 1\right) = r + \phi + \theta\delta > \rho + \phi + \theta\delta.$$

Therefore according to the definition of y_h^c,

$$\begin{aligned} & r + \phi \min\left(y_h^c, 1\right) + \theta\delta\left(L + l y_h^c\right) = \rho + \phi + \theta\delta \\ \Rightarrow \quad & r + \phi \min\left(y_h^c, 1\right) = \rho + \phi + \theta\delta\left(1 - L - l y_h^c\right) > \rho + \phi, \end{aligned}$$

which implies that $y_h^c > y_l^c$. Therefore when $y \in [y_l^c, y_h^c]$, a creditor finds both rollover and run optimal depending on other creditors' strategies.

A.4 Proof of Proposition 4

1. $\mu > 0$ Case.

When $\mu > 0$, the bank fundamental will eventually travel to the upper dominance region, in which all creditors will always choose to roll over independent of other creditors' strategies. Let us first consider the value function of a creditor who is locked in by his current contract under the assumption that the other creditors in the future will always roll over:

$$V^R(y) \equiv E\left[\int_0^{\tau_\phi} e^{-\rho t} r dt + e^{-\rho\tau_\phi} \min\left(y_{\tau_\phi}, 1\right) \middle| y_0 = y\right]. \tag{24}$$

It is easy to see that $V^R(y)$ is increasing with y and $V^R(1) = \frac{r+\phi}{\rho+\phi} > 1$. Define $y_{\mu+}$ as the unique solution to the equation $V^R(y) = 1$; and it is clear that $y_{\mu+} < 1$. When $y > y_{\mu+}$, $V^R(y) > 1$. Thus, in this region, it is optimal for a maturing creditor to choose rollover knowing that every creditor after him will choose rollover. That is, the equilibrium is uniquely defined in the region $y > y_{\mu+}$, and the value function of an individual creditor who is currently in a debt contract is

$$V^{\mu+}(y) = V^R(y) \quad \text{if } y > y_{\mu+}.$$

However, when $y < y_{\mu+}$, it is optimal for a maturing creditor to run even if the other maturing creditors in the future will always choose rollover. Thus, it is reasonable to conjecture that in the equilibrium each maturing creditor indeed chooses run when $y \le y_{\mu+} < 1$. We verify this conjecture in two steps: first, we construct the value function of a creditor under the assumption that every creditor (including himself) uses a monotone strategy with threshold $y_{\mu+}$; second, we show that $V^{\mu+}(y) < 1$ for $y < y_{\mu+}$.

Note that when $y < y_{\mu+}$, $V^{\mu+}$ satisfies

$$(\rho + \phi + (1+\theta)\delta) V^{\mu+} = \mu y V_y^{\mu+} + r + \phi y + \theta\delta(L + ly) + \delta, \tag{25}$$

with the boundary condition that $V^{\mu+}(y_{\mu+}) = 1$. Solving this equation gives that $V^R(0) = \frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}$. Parameter restrictions (4) and (7) imply that

$$\frac{r + \phi + \theta\delta(L + l) + \delta}{\rho + \phi + (1+\theta)\delta} < 1,$$

which in turn provides that $V^{\mu+}(0) < 1$. Therefore, if $V^{\mu+}(y) > 1$ for some $y < y_{\mu+}$, then we must have some point $\widehat{y}$ such that $V^{\mu+}(\widehat{y}) > 1$ and $V_y^{\mu+}(\widehat{y}) = 0$. But then equation (25) implies that

$$V^{\mu+}(\widehat{y}) < \frac{r + \phi + \theta\delta(L + l) + \delta}{\rho + \phi + (1+\theta)\delta} < 1,$$

which contradicts $V^{\mu+}(\widehat{y}) > 1$. Thus, $V^{\mu+}(\widehat{y}) < 1$ if $y < y_{\mu+}$. That is, it is optimal for a maturing creditor to choose run when $y \leq y_{\mu+}$.

This monotone equilibrium is unique, because there is only one $y_{\mu+}$ that satisfies the equilibrium condition of the threshold: $V^R(y_{\mu+}) = 1$.

2. $\mu < 0$ Case.

When $\mu < 0$, the bank fundamental will eventually travel to the lower dominance region, in which each maturing creditor will choose to run independent of other creditors' strategies. We first consider the value function $V^W(y)$ of a creditor who is locked in by his current contract, under the assumption that the other creditors will all choose run in the future. V^W satisfies

$$(\rho + \phi + (1+\theta)\delta) V^W = \mu y V_y^W + r + \phi \min(1, y) + \theta\delta(L + ly) + \delta, \tag{26}$$

with the boundary condition $V^W(0) = \frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} < 1$. It is easy to show that V^W is increasing with y, therefore there exists a unique $y_{\mu-}$ such that $V^W(y_{\mu-}) = 1$. For $y < y_{\mu-}$, the general solution to equation (26) is

$$V^W(y) = \frac{r + \theta\delta L + \delta}{\rho + \phi + (1+\theta)\delta} + \frac{\phi + \theta\delta l}{\rho + \phi + (1+\theta)\delta - \mu} y + A y^{\frac{\rho+\phi+(1+\theta)\delta}{\mu}},$$

where A is constant. Because $\mu < 0$, A has to be zero because, otherwise, $V^W(0)$ diverges. Therefore, $V^W(1) < \frac{r+\phi+\theta\delta(L+l)+\delta}{\rho+\phi+(1+\theta)\delta} < 1$, which in turn implies that $y_{\mu-} > 1$. Thus, when $y < y_{\mu-}$, the equilibrium is uniquely determined and each maturing creditor chooses run knowing that other maturing creditors afterward will choose run. The value function of an individual creditor who is currently in a debt contract is

$$V^{\mu-}(y) = V^W(y) \quad \text{if } \; y < y_{\mu-}.$$

However, when $y > y_{\mu-}$, it is optimal for a maturing creditor to roll over even if other maturing creditors in the future will always choose run. Thus, it is reasonable to conjecture that in the equilibrium each maturing creditor indeed chooses rollover when $y > y_{\mu-} > 1$. We again verify this in two steps: first, we construct the value function of a creditor under the assumption that every creditor (including himself) uses a monotone strategy with threshold $y_{\mu-}$; second, we show that $V^{\mu-}(y) > 1$ for $y > y_{\mu-}$.

Note that if $y > y_{\mu-}$, $V^{\mu-}(y)$ satisfies

$$(\rho + \phi) V^{\mu-} = \mu y V_y^{\mu-} + r + \phi,$$

with the boundary condition $V^{\mu-}(y_{\mu-}) = 1$. The solution is $\frac{r+\phi}{\rho+\phi} + B y^{\frac{\rho+\phi}{\mu}}$ where the constant $B < 0$. This function is monotonically increasing. Thus, $V^{\mu-}(y) > 1$ if $y > y_{\mu-}$. In other words, rollover is optimal for a maturing creditor in equilibrium if $y > y_{\mu-}$.

This monotone equilibrium is unique, because there is only one $y_{\mu-}$ that satisfies the equilibrium condition of the threshold: $V^W(y_{\mu-}) = 1$.

A.5 Proof of Proposition 5

For any increasing sequence $\{\delta_n\}$ such that $\delta_n \to \infty$, denote the corresponding equilibrium threshold sequence as $\{y_*(\delta_n)\}$ which satisfies $W(y_*(\delta_n); \delta_n) = 1$ with $W(y; \delta)$ defined in Lemma 12. If $\{y_*(\delta_n)\}$ does not converge to $\frac{1-L}{l}$, then for any $\varepsilon > 0$ and any large $\overline{\delta}$ there exists a $\delta_N > \overline{\delta}$ such that $y_*(\delta_N) \notin \left[\frac{1-L}{l} - \varepsilon, \frac{1-L}{l} + \varepsilon\right]$ and $W(y_*(\delta_N); \delta_N) = 1$. We have three cases to consider. In the following derivation, keep in mind that γ_1 and η_1 are in the order of $\sqrt{\delta_N}$, while γ_2 and η_2 are constant.

- Suppose that $y_*(\delta_N) > \frac{1-L}{l} + \varepsilon > 1$. Then

$$W(y_*(\delta_N);\delta_N) = \frac{\gamma_2}{(\eta_1+\gamma_2)}\frac{r+\phi}{\rho+\phi} + \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\phi+\theta\delta_N L+\delta_N}{\rho+\phi+(1+\theta)\delta_N}$$
$$+ [y_*(\delta_N)]^{-\gamma_1}\frac{\eta_1 - M_1/M_2}{\eta_1+\gamma_2}\left(K_4 + (-K_2)\left(\frac{l}{1-L}\right)^{-\gamma_1}\right).$$

The first term goes to zero, and the second term goes to $\frac{\theta L+1}{1+\theta} < 1$. The third term goes to zero, because 1) $[y_*(\delta_N)]^{-\gamma_1}$ goes to zero, and 2) $\frac{\eta_1 - M_1/M_2}{\eta_1+\gamma_2}$ is in the order of $\sqrt{\delta_N}$ (recall $M_1/M_2 = \frac{\rho+\phi+(1+\theta)\delta_N}{\mu}$ while K_4 is in the order of $(\delta_N)^{-2}$ and $-K_2$ is in the order of $(\delta_N)^{-1}$). Thus the sum of these terms contradicts $W(y_*(\delta_N);\delta_N) = 1$.

- Suppose that $1 \le y_*(\delta_N) < \frac{1-L}{l} - \varepsilon$. Then

$$\begin{aligned} W(y_*(\delta_N);\delta_N) &= \frac{M_2\eta_1 - M_1}{\eta_1+\gamma_2}[y_*(\delta_N)]^{-\gamma_1} + \frac{\eta_1 - 1}{\eta_1+\gamma_2}\frac{\theta\delta_N l}{\rho+\phi+(1+\theta)\delta_N-\mu}y_*(\delta_N) \\ &\quad + \frac{\gamma_2}{\eta_1+\gamma_2}\frac{r+\phi}{\rho+\phi} + \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\phi+\theta\delta_N L+\delta_N}{\rho+\phi+(1+\theta)\delta_N}. \end{aligned}$$

The first and third terms go to zero. The sum of second and fourth term converges to

$$\frac{\theta l}{1+\theta}y_*(\delta_N) + \frac{\theta L+1}{1+\theta} < 1 - \frac{\theta l}{1+\theta}\varepsilon,$$

which is again a contradiction to $W(y_*(\delta_N);\delta_N) = 1$.

- Suppose that $y_*(\delta_N) < 1$. Then

$$\begin{aligned} W(y_*) &= \frac{[H_3\gamma_2 + H_1]}{\eta_1+\gamma_2}y_*(\delta_N)^{\eta_2} + \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\theta\delta_N L+\delta_N}{\rho+\phi+(1+\theta)\delta_N} + \frac{\gamma_2}{\eta_1+\gamma_2}\frac{r}{\rho+\phi} \\ &\quad + \left[\frac{\eta_1-1}{\eta_1+\gamma_2}\frac{\theta\delta_N l+\phi}{\rho+\phi+(1+\theta)\delta_N-\mu} + \frac{1+\gamma_2}{\eta_1+\gamma_2}\frac{\phi}{(\rho+\phi-\mu)}\right]y_*(\delta_N) \\ &\to \frac{\theta L+1}{1+\theta} + \frac{\theta l}{1+\theta}y_*(\delta_N) < \frac{1+\theta(L+l)}{1+\theta} < 1. \end{aligned}$$

which is a contradiction. This concludes the proof.

This proof can be slightly modified to show that as $\theta \to \infty$, the equilibrium rollover threshold y_* also converges to $\frac{1-L}{l}$. In this case, simply note that γ_1 and η_1 are in the order of $\sqrt{\theta}$ when θ is sufficiently large.

A.6 Proof of Proposition 6

We use a guess-and-verify approach. We first construct the single creditor's value function if he always chooses to roll over the debt, and then verify that this value function is higher than the payoff $\min(L+ly, 1)$ from run if α is sufficiently low.

Denote the single creditor's value function as $V^s(y_t)$. We can simply modify the HJB equation in (9) to get the following one:

$$\rho V^s = \mu y V_y^s + \frac{\sigma^2}{2}y^2 V_{yy}^s + r + \phi[\min(1,y) - V^s] + \delta \max_{\text{rollover or run}}\{0, \min(L+ly,1) - V^s\}.$$

If the single creditor always chooses to roll over, this equation becomes

$$(\rho+\phi)V^s(y) = \frac{\sigma^2}{2}y^2 V_{yy}^s + \mu y V_y^s + \phi\min(1,y) + r.$$

This equation is identical to the equation for U in Appendix A.2, and therefore admits the same solution expressed in equation (22). The fact that D_1 and D_2 are negative implies that $V^s(y)$ is globally concave. With the same condition (23) so that the liquidation cost is sufficiently large, we have

$$V^s\left(\frac{1-L}{l}\right) > 1.$$

Since $V^s(y)$ is increasing in y, $V^s(y) > \min(L+ly, 1)$ for $y > \frac{1-L}{l}$. For $0 < y < \frac{1-L}{l}$, note that $V^s(y) > L+ly$ hold for both end points, i.e., $V^s(0) > L$ and $V^s(\frac{1-L}{l}) > 1$. Because $V^s(y)$ is concave and $L+ly$ is linear, $V^s(y)$ is always above $L+ly$ in the region $y \in \left(0, \frac{1-L}{l}\right)$. Thus, $V^s(y) > \min(L+ly, 1)$ always holds. That is, the single creditor will always choose to roll over.

A.7 Proof of Proposition 7

Note that y_* is determined by the condition that $W(y_*) = V(y_*; y_*) = 1$. Theorem 1 implies that if $y_* > \frac{1-L}{l}$, it is determined by the following implicit function:

$$\begin{aligned} 1 &= W(y_*) = \frac{\eta_1 - M_1/M_2}{\eta_1+\gamma_2}\left(K_4 y_*^{-\gamma_1} + (-K_2)\left(\frac{l y_*}{1-L}\right)^{-\gamma_1}\right) \\ &\quad + \frac{\gamma_2}{(\eta_1+\gamma_2)}\frac{r+\phi}{\rho+\phi} + \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}, \end{aligned} \tag{27}$$

where $L = \frac{\alpha r}{\rho+\phi}$ and $l = \frac{\alpha\phi}{\rho+\phi-\mu}$ increase with α, and M_1/M_2, and K_4 are independent of α. By the implicit function theorem, $\frac{dy_*}{d\alpha} = -\frac{\partial W/\partial\alpha}{\partial W/\partial y_*}$. Since we have shown that $\partial W/\partial y_* > 0$ in Lemma 12, to prove the claim we need to show that $\partial W/\partial\alpha > 0$. There are two terms in W that involve α: 1) because $-K_2 = \frac{\mu\theta\delta(1-L)}{(\rho+\phi+(1+\theta)\delta)(\rho+\phi+(1+\theta)\delta)}$, the second term in the first bracket is proportional to $-\frac{(1-L)^{1+\gamma_1}}{l^{\gamma_1}}$, which is increasing in α; and 2) the second term $\frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}$ in the second line is increasing in α. Therefore $\partial W/\partial\alpha > 0$, and $\frac{dy_*}{d\alpha} < 0$.

When $1 < y_* \le \frac{1-L}{l}$, it is determined by the following implicit function:

$$\begin{aligned} 1 &= W(y_*) = \frac{M_2\eta_1 - M_1}{\eta_1+\gamma_2} y_*^{-\gamma_1} + \frac{\eta_1 - 1}{\eta_1+\gamma_2}\frac{\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu} y_* \\ &\quad + \frac{\gamma_2}{\eta_1+\gamma_2}\frac{r+\phi}{\rho+\phi} + \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}. \end{aligned} \tag{28}$$

Therefore

$$\partial W/\partial\alpha = \eta_1\frac{\theta\delta\frac{r}{\rho+\phi}}{\rho+\phi+(1+\theta)\delta} + \frac{\eta_1-1}{\eta_1+\gamma_2}\frac{\theta\delta\frac{\phi}{\rho+\phi-\mu}}{\rho+\phi+(1+\theta)\delta-\mu} y_* > 0, \tag{29}$$

which implies $\frac{dy_*}{d\alpha} > 0$.

When $y_* < 1$, it is determined by the following implicit function:

$$\begin{aligned} W(y_*) &= \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} + \frac{\gamma_2}{\eta_1+\gamma_2}\frac{r}{\rho+\phi} + \frac{[H_3\gamma_2+H_1]}{(\eta_1+\gamma_2)} y_*^{\eta_2} \\ &\quad + \left[\frac{\eta_1-1}{\eta_1+\gamma_2}\frac{\theta\delta l+\phi}{\rho+\phi+(1+\theta)\delta-\mu} + \frac{1+\gamma_2}{(\eta_1+\gamma_2)}\frac{\phi}{(\rho+\phi-\mu)}\right] y_* = 1, \end{aligned}$$

where H_3 and H_1 are independent of α. Then

$$\partial W/\partial\alpha = \frac{\eta_1}{\eta_1+\gamma_2}\frac{\theta\delta\frac{r}{\rho+\phi}}{\rho+\phi+(1+\theta)\delta} + \frac{\eta_1-1}{\eta_1+\gamma_2}\frac{\theta\delta\frac{\phi}{\rho+\phi-\mu}}{\rho+\phi+(1+\theta)\delta-\mu} > 0. \tag{30}$$

Taken together, the equilibrium rollover threshold y_* decreases with α.

A.8 Proof of Proposition 8

As we noted at the end of Appendix 5, when $\theta \to \infty$, $y_* \to \frac{1-L}{l} > 1$. Thus, by the continuity of y_* with respect to θ, if θ is sufficiently high, $y_* > 1$. Our numerical exercises also show that this holds true over a wide range of parameter values. Thus, we will focus on showing that y_* increases with σ^2 in the range where $y_* > 1$.

Since y_* is determined by the implicit function $W(y_*) = V(y_*, y_*) = 1$, to show that y_* increases with σ^2, we only need to verify that $\frac{\partial W(y)}{\partial \sigma^2} < 0$.

We first note several inequalities. Directly from condition (5), we have $\frac{\partial \eta_i}{\partial \sigma^2} < 0$ and $\frac{\partial \gamma_i}{\partial \sigma^2} < 0$ for $i = 1, 2$. Moreover, by using the definitions of η_1 in (15) and γ_2 in (17), we can also show that

$$\frac{\partial \left(\frac{\gamma_2}{\eta_1+\gamma_2}\right)}{\partial \sigma^2} < 0. \tag{31}$$

We now consider the case where $1 < y_* \leq \frac{1-L}{l}$. Based on $W(y)$ given in equation (28), we have

$$\begin{aligned}\frac{\partial W(y)}{\partial \sigma^2} &= \frac{\partial \left(\frac{\eta_1 - M_1/M_2}{\eta_1+\gamma_2}\right)}{\partial \sigma^2} M_2 y^{-\gamma_1} + \frac{\partial \left(\frac{-1}{\eta_1+\gamma_2}\right)}{\partial \sigma^2} \frac{\theta\delta l}{\rho+\phi+(1+\theta)\delta-\mu} y \\ &\quad + \frac{\eta_1 - M_1/M_2}{\eta_1+\gamma_2} M_2 y^{-\gamma_1} \ln y \frac{\partial(-\gamma_1)}{\partial \sigma^2} + \frac{\partial \left(\frac{\gamma_2}{\eta_1+\gamma_2}\right)}{\partial \sigma^2} \left(\frac{r+\phi}{\rho+\phi} - \frac{r+\phi+\theta\delta(L+ly)+\delta}{\rho+\phi+(1+\theta)\delta}\right).\end{aligned}$$

As $\frac{r+\phi}{\rho+\phi} - \frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta} > 0$, inequality in (31) implies that the last term is negative. Also, $\frac{\partial\left(\frac{-1}{\eta_1+\gamma_2}\right)}{\partial \sigma^2} < 0$ implies that the second term is negative. Moreover, because $M_2\eta_1 - M_1 < 0$ (shown in the proof of Lemma 12), and $\frac{\partial(-\gamma_1)}{\partial \sigma^2} > 0$, the third term is negative. Finally, note that when θ is sufficiently large, η_1 and γ_2 are in the order of $\theta^{0.5}$. Since $M_1/M_2 = \frac{\rho+\phi+(1+\theta)\delta}{\mu}$, the first part of the second term $\frac{\partial\left(\frac{\eta_1 - M_1/M_2}{\eta_1+\gamma_2}\right)}{\partial \sigma^2}$ is approximately equal to $-\frac{\partial\left(\frac{1}{\eta_1+\gamma_2}\right)}{\partial \sigma^2} M_1/M_2$, which is negative. Taken together, $\frac{\partial W(y)}{\partial \sigma^2} < 0$.

We now consider the case where $y_* > \frac{1-L}{l}$. Based on $W(y)$ in equation (27), we have

$$\begin{aligned}\frac{\partial W(y)}{\partial \sigma^2} &= \frac{\partial \left(\frac{\gamma_2}{\eta_1+\gamma_2}\right)}{\partial \sigma^2} \left(\frac{r+\phi}{\rho+\phi} - \frac{r+\phi+\theta\delta L+\delta}{\rho+\phi+(1+\theta)\delta}\right) + \frac{\partial \left(\frac{\eta_1 - M_1/M_2}{\eta_1+\gamma_2}\right)}{\partial \sigma^2} \left(K_4 y^{-\gamma_1} + (-K_2)\left(\frac{ly}{1-L}\right)^{-\gamma_1}\right) \\ &\quad + \frac{\eta_1 - M_1/M_2}{\eta_1+\gamma_2} \frac{\partial \left(K_4 y^{-\gamma_1} + (-K_2)\left(\frac{ly}{1-L}\right)^{-\gamma_1}\right)}{\partial \sigma^2}.\end{aligned}$$

Using arguments similar to those presented in the previous case, it is easy to show that every term in this expression is negative. Thus, $\frac{\partial W(y)}{\partial \sigma^2} < 0$. This concludes the proof.

A.9 Proof of Proposition 9

We only need to verify that when σ^2 is sufficiently large, $W(y) = V(y, y)$ is below 1 at $y = \frac{1-L}{l}$. This implies $y_* > \frac{1-L}{l}$ because $W'(y) > 0$ and y_* is determined by $W(y_*) = 1$. Note that showing

$$\begin{aligned}W\left(\frac{1-L}{l}\right) &= \frac{M_2\eta_1 - M_1}{\eta_1+\gamma_2}\left(\frac{1-L}{l}\right)^{-\gamma_1} + \frac{-1}{\eta_1+\gamma_2}\frac{\theta\delta(1-L)}{\rho+\phi+(1+\theta)\delta-\mu} \\ &\quad + \frac{\gamma_2}{\eta_1+\gamma_2}\frac{r+\phi}{\rho+\phi} + \frac{\eta_1}{\eta_1+\gamma_2}\frac{r+\phi+(1+\theta)\delta}{\rho+\phi+(1+\theta)\delta} < 1\end{aligned}$$

is equivalent to showing

$$\begin{aligned}
&\frac{M_1 - M_2\eta_1}{\eta_1+\gamma_2}\left(\frac{1-L}{l}\right)^{-\gamma_1} + \frac{1}{\eta_1+\gamma_2}\frac{\theta\delta\left(1-L\right)}{\rho+\phi+\left(1+\theta\right)\delta-\mu} \\
> \quad & \frac{\gamma_2}{\eta_1+\gamma_2}\frac{r-\rho}{\rho+\phi} + \frac{\eta_1}{\eta_1+\gamma_2}\frac{r-\rho}{\rho+\phi+\left(1+\theta\right)\delta}.
\end{aligned}$$

When $\sigma^2 \to \infty$, $\eta_1 \to 1$, $\gamma_1 \to 0$, and $\gamma_2 \to 0$. Together with $M_1 - M_2 = \frac{\phi}{\rho+\phi+(1+\theta)\delta}$, showing the inequality above is equivalent to showing $\frac{\phi}{\rho+\phi+(1+\theta)\delta} + \frac{\theta\delta(1-L)}{\rho+\phi+(1+\theta)\delta-\mu} > \frac{r-\rho}{\rho+\phi+(1+\theta)\delta}$, which holds because $\phi+\rho > r$, condition (4). Thus, when σ^2 is sufficiently large, $y_* > \frac{1-L}{l}$.

A.10 Proof of Proposition 10

We distinguish between an individual creditor i's rollover frequency δ_i and other creditors' rollover frequency δ_{-i}. We can rewrite the individual creditor's HJB equation for his value function V^i:

$$\begin{aligned}
\rho V^i\left(y_t;y_*\right) \quad = \quad & \mu y_t V_y^i + \frac{\sigma^2}{2}y_t^2 V_{yy}^i + r + \phi\left[\min\left(1,y_t\right) - V\left(y_t;y_*\right)\right] \\
& +\theta\delta_{-i}\mathbf{1}_{\{y<y_*\}}\left[\min\left(L+ly_t,1\right) - V\left(y_t;y_*\right)\right] + \delta_i \max_{\text{rollover or run}}\left\{1-V\left(y_t;y_*\right),0\right\}.
\end{aligned} \tag{32}$$

Suppose that we increase δ_i from δ to $\delta' > \delta$. We need to show that the creditor i's value function with parameter δ' is strictly higher than that with parameter δ. To facilitate the comparison, we consider a new problem, in which the creditor's contract expires with rate δ', but he is only allowed to withdraw at his contract expiration if an independent random variable $X = 1$. This variable X can take values of 1 or 0 with probabilities of $\lambda = \delta/\delta' < 1$ and $1-\lambda$, respectively. This random variable effectively reduces the creditor's release rate to δ. Thus, in this constrained problem with parameter δ', the creditor has the same value function as in the unconstrained problem with parameter δ.

Next, consider the creditor's value function in the unconstrained problem (or, $\lambda = 1$ always) with parameter δ', which should be strictly higher than that in the constrained problem. This is because if the creditor is allowed to withdraw when $X = 0$ and $y_t < y_*$, his value function is strictly increased even if he keeps the same threshold. Then, it is obvious that the creditor's value function in the unconstrained problem with parameter δ' is strictly higher than that in the same problem with parameter δ.

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