Dynamic Elections and Ideological Polarization*

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Abstract

How does political polarization affect the welfare of the electorate? We analyze this question using a framework in which two policy and office motivated parties compete in an infinite sequence of elections. We propose two novel measures to describe the degree of conflict among agents: *antagonism* is the disagreement between parties; *extremism* is the disagreement between each party and the representative voter. We show that forward-looking parties have an incentive to implement policies favored by the representative voter, in an attempt to constraint future challengers. This incentive grows as antagonism increases. On the other hand, extremism decreases the electorate's welfare.

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1 Introduction

A large body of recent research in political science has been devoted to identifying and explaining ideological polarization, especially but not only in the United States. There is strong evidence that the U.S. Congress has grown progressively more polarized since the 1970s (McCarty, Poole and Rosenthal 2006), as well as some evidence of a polarization trend in presidential platforms (Budge, Klingemann, Volkens, Bara and Tanenbaum 2001, Klingemann, Volkens, Bara, Budge and McDonald 2006). Many empirical studies link this increasing ideological divide between the main American parties to legislative gridlock, elite incivility, income inequality, and voter disengagement (Layman and Carsey 2002*b*, Fiorina and Abrams 2008, Hetherington 2001). Across a broader range of countries, polarization is associated with democratic breakdown, corruption, and economic decline (Brown, Touchton and Whitford 2011, Frye 2002, Linz and Stepan 1978). To complement these empirical findings, many existing models of electoral competition and policymaking show that parties with conflictual preferences are socially suboptimal (Persson and Svensson 1989, Alesina and Tabellini 1990, Azzimonti 2011). However, as we hope to show, these conclusions depend crucially on a number of stylized assumptions about the nature of the disagreement among political actors. While we lack a precise notion of ideological polarization when parties care about multiple dimensions, how we measure this concept crucially affects the conclusions on its welfare consequences.

In this article, we develop a model designed to study the policy consequences of ideological conflict in dynamic elections where parties commit to enact the same policy for their entire tenure in office. This *incumbent policy persistence* reflects politicians' inability to credibly promise policies different than those implemented while in office. This constraint is well documented and arises for many concurring reasons: internal party politics which generates organizational hysteresis (Miller and Schofield 2003); voters' focus on parties' records rather than campaign promises (*retrospective voting*, Fiorina 1981); the electoral costs of changing policy position and being perceived as *flip-floppers* (Adams, Clark, Ezrow and Glasgow 2006, DeBacker Forthcoming, Tavits 2007, Tomz and Van Houweling

2008, Tomz and Van Houweling 2012). Not all parties face the same constraints, though. Following an electoral defeat, parties usually replace their leaders. This, together with the fact that their past policy platform has not been enacted and observed by voters, means that challengers are better able to credibly change their policy stance.[1] We assume that incumbents who remain in office find it too costly to implement policies that differ from those of their previous term. In particular, we model two parties who compete in an election in each of an infinite number of periods; the opposition party proposes a policy platform while the incumbent party, if reelected, enacts the same policy from the previous period. A representative voter picks her favorite candidate. The election winner implements the proposed policy and commits to do so for the duration of its tenure. In this sense, the identity of the incumbent party and its policy represent a dynamic linkage across periods.

We highlight that—when parties and voters care about multiple issues—we can use many different measures to describe the degree of conflict—or polarization—among agents' preferences in the political arena, and we propose two such measures. The first measure, which we label *extremism*, is the ideological distance of each party from the decisive voter in the electorate. The second measure, which we call *antagonism*, is the ideological distance that separates the two parties from each other and summarizes the degree of political competition between policymakers. These two measures coincide in a one dimensional policy space, where the ideological distance between the two parties can increase only as they move further away from the representative voter.[2] However, they do not coincide in a two-dimensional setting: here, the two parties can be very close (when they share views on both dimensions) or very different (when they are perfectly opposed in one dimension), without altering their overall distance from the representative voter.

There are two main questions that we wish to address with this simple setting. First, what is the impact of ideological disagreement on implemented policies when

[1] Janda, Harmel, Edens and Goff (1995) and Somer-Topcu (2009) analyze electoral manifestos data and show that this is indeed the case: parties that lost votes in previous elections change their programs more than parties what won votes.

[2] In the classic framework where the two parties' ideal policies are on opposite sides of the ideal policy of the representative voter.

parties are long-lived and care about present as well as future electoral outcomes? Second, how do electoral competition, ideological disagreement and parties' dynamic incentives affect the electorate's welfare?

We fully characterize a Stationary Markov Perfect Equilibrium (SMPE) of the dynamic electoral competition described above and prove it exists for any discount factor, any initial incumbent's policy, and any degree of antagonism and extremism.[3] According to the results of our model, parties alternate in power and long run policies tends to be more moderate, in the sense of being closer to the preferences of the representative voter: (i) the larger is the degree of antagonism between alternating governments; (ii) the smaller is parties' extremism; (iii) the larger is parties' patience. Opposition parties' ability to design a winning policy around the incumbents' commitment drives alternation in power. The key idea behind moderation is that an opposition party—which knows it is tying its hands with the current policy platform and, hence, will be vulnerable to future electoral defeat—has incentives to behave strategically by offering policies that restrict the choice set of future challengers. The more the preferences of the challenger depart from the preferences of the current government, the more the challenger will try to restrict the future opposition's choice set. The degree of conflict between the parties and the representative voter, on the other hand, does not affect the strategic incentives to moderate.

This analysis suggests that the influence voters exert on policies is a function of the degree of antagonism and extremism of the political system. In particular, polarization can have counter-intuitive welfare implications: the electorate is best served by highly antagonist political elites that are perfectly opposed on one dimension.

[3] The only general existence result for dynamic elections applies to settings with countable state spaces (Duggan and Forand 2013). In our model the state space is uncountable and, thus, proving existence is a necessary step of the analysis. Moreover, if we were to consider a model with a countable set of policies, the results in Duggan and Forand (2013) would guarantee existence of an equilibrium but would not provide a characterization of its dynamics or comparative statics with respect to patience, antagonism and extremism.

2 Related Literature

This paper contributes primarily to a growing theoretical literature on dynamic elections with endogenous economic or political state variables (Krusell and Rios-Rull 1999, Bai and Lagunoff 2011, Battaglini 2014).[4] Closely related to this paper, Kramer (1977), Wittman (1977) and Forand (2014) build on static models of partisan competition and study dynamic models of asynchronous policy competition. While Kramer (1977) and Wittman (1977) focus on myopic parties, Forand (2014) considers farsighted parties who take into account future opponents' policy choices and offer policy platforms on a single dimension. To the contrary, the parties and voters in our setup care about multiple dimensions. Expanding the policy space beyond a single dimension allows us to explore the different facets of ideological polarization and to highlight the ambiguous impact of parties' conflict of preferences on the electorates' welfare.[5]

Our paper is also formally related to models of dynamic legislative bargaining with an endogenous status quo and farsighted players (Baron 1996, Kalandrakis 2004, Diermeier and Fong 2011, Dziuda and Loeper 2012, Bowen, Chen and Eraslan Forthcoming). To view our model as a legislative bargaining model, reinterpret the representative voter as the median legislator, the parties as the only legislators that have the power to set the agenda[6], and define the state variable as the status quo policy, that is, the policy implemented by the legislature in the previous period.

Finally, our work is related to models that directly address the relationship between political elites polarization and policy outcomes. In a multidimensional and static framework, Krasa and Polborn (2014) study how ideological polarization on

[4] See Duggan and Martinelli (2014) for a review of the literature on dynamic elections.

[5] With a single dimension, the ideological distance between the two parties coincide with the disagreement between the parties and the median voter. In this framework, the incentive to moderate in order to constraint future incumbents exists but it is unchanged as we increase the ideological distance between the two parties. Nonetheless, our results are related to Forand's. Reducing our model to one dimension, the long-run policies from Proposition 1 coincide with the long-run bound on extremism characterized in Forand (2014).

[6] These two legislators can be interpreted as the median legislators or party leaders of the two parties, as in the procedural cartel theory introduced by Cox and McCubbins (1993, 2005).

one dimension influences the candidates' positions on a second dimension. In most dynamic models, conflictual political preferences are socially suboptimal (Persson and Svensson 1989, Alesina and Tabellini 1990, Azzimonti 2011, Prato 2013). Our paper shares with this literature the idea that forward-looking incumbents have incentives to strategically position current policies to affect future political outcomes, and that these incentives are stronger when the conflict of preferences is starker. On the other hand, in our model disagreement can be over multiple dimensions and the channel to constraint a future incumbent is the demands of the electorate rather than inefficient or misdirected spending. Our novel approach shows that—when the ideological conflict is multidimensional—preference divergence does not necessarily lead to higher inefficiencies and welfare losses for the electorate, but it could have the opposite effect.

More recent studies argued that polarized parties and divergent platforms can be welfare enhancing (Bernhardt, Duggan and Squintani 2009, Bernhardt, Campuzano, Squintani and Camara 2009, Van Weelden 2013, 2014). In these works, polarization coincides with our notion of extremism and helps the electorate through channels different than the one highlighted in our paper. Van Weelden (2013, 2014) shows that, in a more polarized political environment, the incumbent gives up rent extraction for fear of being replaced by a challenger with markedly different policy preferences. Bernhardt, Duggan and Squintani (2009) analyze a static and unidimensional electoral competition where more polarized parties propose more extreme electoral platforms; for moderate levels of polarization, this is welfare enhancing because it provides the median voter with a more varied choice. Bernhardt et al. (2009) study repeated elections where candidates' types are private information and show that the incumbent will compromise more if his potential replacement is drawn from the other side of the political spectrum. In our case, on the other hand, the beneficial effect of antagonism comes from more moderate policies implemented strategically by forward-looking incumbents of known type, and increased extremism is always detrimental to the representative voter.

3 Model and Equilibrium Notion

We describe here a dynamic model of electoral competition between policy and office motivated parties. In each period of an infinite horizon, two parties, 1 and 2, compete in an election decided by a representative voter v.[7] Each period starts with a party as the incumbent office holder and the other party, the opposition, striving to replace it in power. A key feature of our model is *incumbent policy persistence*: incumbents cannot distance themselves from their past policies, while opposition parties can put forward new policies more freely.

The Electoral Process. Elections occur at the beginning of each period. The opposition party contests an election by offering a bi-dimensional policy $\mathbf{p} = (p^1, p^2) \in X \subseteq \mathbb{R}^2$, or stays out of the race.[8] The incumbent party is committed to the policy $\mathbf{q} \in X$ that brought it into office. The representative voter is, thus, confronted with the choice between $\mathbf{p}$, promised by the opposition party, and $\mathbf{q}$, the continuing policy of the incumbent party. The elected party implements its winning policy and becomes the incumbent at the beginning of the next period. The policy implemented in a period becomes the incumbent's policy commitment in the next period and, as such, represents a dynamic linkage between periods.

Stage Utilities. The stage utility player $i \in \{1, 2, v\}$ receives from policy $\mathbf{p} \in X$ is measured by the squared distance of $\mathbf{p}$ from i's *bliss point*, or ideal policy, $\mathbf{b}_i = (b_i^1, b_i^2)$:

$$u_i(\mathbf{p}) = -(p^1 - b_i^1)^2 - (p^2 - b_i^2)^2 \tag{1}$$

[7] A necessary and sufficient condition for the existence of a decisive median voter in a multidimensional policy space is the 'radial symmetry' of voters' ideal policies (see Plott 1967 and Duggan 2012). Radial symmetry obtains, for example, when voters' ideal policies are distributed according to a radially symmetric density, such as a bi-variate normal or uniform distribution on a disk in $\mathbb{R}^2$ (as in Baron, Diermeier and Fong 2012).

[8] The policy space X can be either $\mathbb{R}^2$ or a compact, convex, and proper subset of $\mathbb{R}^2$ that is large enough to include the ideal policies of all the agents, as described below. Restricting attention to such a subset, $X \subset \mathbb{R}^2$, is without loss of generality. Moreover, removing the option of staying out of the race would not alter any of our results: if $X = \mathbb{R}^2$ there always exists a policy that ensures electoral defeat; if $X \subset \mathbb{R}^2$, as we prove below, both parties always want to contest elections in equilibrium.

Denoting by $d(\mathbf{x}, \mathbf{y})$ the usual Euclidean distance between $\mathbf{x} \in X$ and $\mathbf{y} \in X$, we can rewrite equation (1) as $u_i(\mathbf{p}) = -d^2(\mathbf{p}, \mathbf{b}_i)$.[9] We abuse notation slightly and denote by $d(\mathbf{x}) = ||\mathbf{x}||$ the distance from the origin of $\mathbf{x} \in X$. In addition to policy, parties care about being in office. The party in power in a period receives office rents $r \geq \bar{r}$.[10] Office-holding benefits include patronage positions in government and government-owned companies, public financing of party activities and other office perks that are consumed only by the party in government. The utility i derives from a sequence of policies $\mathbf{P} = \{\mathbf{p}_0, \mathbf{p}_1, \ldots\}$ is the discounted sum of payoffs from each period:

$$U_i(\mathbf{P}) = \sum_{t=0}^{\infty} \delta_i^t \left[u_i(\mathbf{p}_t) + \mathbb{I}_{i,t} \cdot r\right] \tag{2}$$

where $\delta_i \in [0, 1)$ is player i's discount factor and $\mathbb{I}_{i,t} = 1$ when party i is in power in period t and zero otherwise. The representative voter receives no utility from r, hence $\mathbb{I}_{v,t} = 0$ for any t. We assume $\delta_1 = \delta_2 \equiv \delta$ and $\delta_v = 0$.[11]

Parties' Antagonism and Extremism. The model is shift and rotation invariant, so, without loss of generality, we define the representative voter's bliss point as the origin of the plane, $\mathbf{b}_v = (0, 0)$, party 1's bliss point as $\mathbf{b}_1 = (b_1, 0)$ and party 2's bliss point as $\mathbf{b}_2 = (b_2^1, b_2^2)$ with $d(\mathbf{b}_2) = b_2$. The distance of the bliss point of party $i \in \{1, 2\}$ from the bliss point of the representative voter is b_i. The parameter b_i, thus, captures the ideological distance between the voter and the party. We call this the degree of *extremism* of party i.

A second, separate, measure of ideological divergence is given by the angle

[9] We use quadratic Euclidean preferences primarily for convenience. In Appendix A2.1 we present a model with general utility functions that are continuous, decreasing and weakly concave in $d(\mathbf{x}, \mathbf{y})$, and prove results analogous to the ones presented below.

[10] The lower bound on office rents, $\bar{r}$, is determined in the equilibrium analysis below. Using the notation introduced in the remainder of this section, the bound is $\bar{r} \equiv \max\{b_1, b_2\}(1 + \delta)$. Assuming $r \geq \bar{r}$ guarantees that parties always prefer to contest an election rather than staying out of the race and leaving the incumbent in power indefinitely. We find such prediction implausible and rule it out by assumption.

[11] Assuming that the representative voter is myopic facilitates the exposition but it is not needed for our results. In Appendix A2.2 we present a model with forward-looking v and prove results similar to the ones presented below.

$\alpha \in [0, \pi]$ formed by the two vectors $\mathbf{b}_1$ and $\mathbf{b}_2$.[12] This parameter captures how different the parties' bliss points are from each other, regardless of their distance from the representative voter's ideal policy. When $\alpha = 0$, the parties' bliss points are on the same ray departing from the origin. When $\alpha \in (0, \pi)$, the two parties diverge on both dimensions, keeping the distance from the origin of the plane constant. When $\alpha = \pi$, the two parties are perfectly opposed on one dimension and share the same ideology on the second dimension. We call α the degree of *antagonism* of the parties.

The equilibrium strategies we characterize below depend solely on the distance between the incumbent's policy and the bliss point of the representative voter, that is, the origin of the plane. We denote with $k_i(\mathbf{x}) = \frac{d(\mathbf{x})}{b_i} \geq 0$ for $i \in \{1, 2\}$ the distance of policy $\mathbf{x} \in X$ from the origin, relative to b_i. With this notation, $k_i(\mathbf{q})\mathbf{b}_i$ is a point on the line connecting $\mathbf{b}_v$ with $\mathbf{b}_i$, at the the same distance from $\mathbf{b}_v$ as the incumbent's policy commitment $\mathbf{q}$. Figure 1 shows the basic model parameters: a set of arbitrary bliss points for the three players ($\mathbf{b}_v$, $\mathbf{b}_1$, and $\mathbf{b}_2$) and the indifference curves generated by their Euclidean preferences over policies; the corresponding degree of antagonism (α) and extremism (b_1, b_2); the incumbent's policy commitment ($\mathbf{q}$) and its distance from the origin ($d(\mathbf{q})$); and a point on the line connecting $\mathbf{b}_2$ with the origin, at the same distance from the origin as the incumbent's policy ($k_2(\mathbf{q})\mathbf{b}_2$). Figure 2 shows examples of parties' bliss points for two different degrees of antagonism ($\alpha' > \alpha$) and two different degrees of extremism ($b'_i > b_i$).

Strategies. We focus on equilibria in pure Markov strategies (Maskin and Tirole 2001). We assume that the decision of the opposition party regarding which policy to run with, should it contest the election, depends solely on the incumbent's identity and the policy it is committed to. Markovian strategies that abstract from the history of play are standard in dynamic models of political economy (Baron 1996, Kalandrakis 2004, 2010, Battaglini, Nunnari and Palfrey 2012, Duggan and Kalandrakis 2012, Forand 2014, Duggan and Forand 2013), capture the simplest form of behavior consistent with rationality, and clearly isolate the underlying

[12] Notice that $\cos\alpha = \frac{\mathbf{b}_1 \cdot \mathbf{b}_2}{b_1 b_2}$ where $\mathbf{b}_1 \cdot \mathbf{b}_2$ is the usual inner product.

Figure 1: Basic Model Parameters

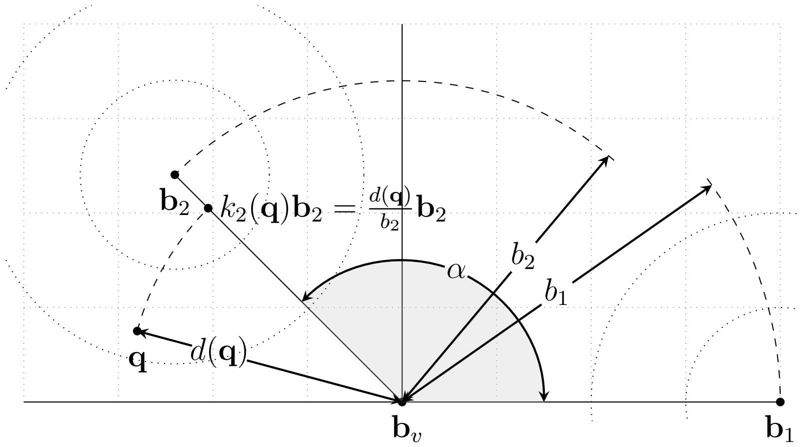
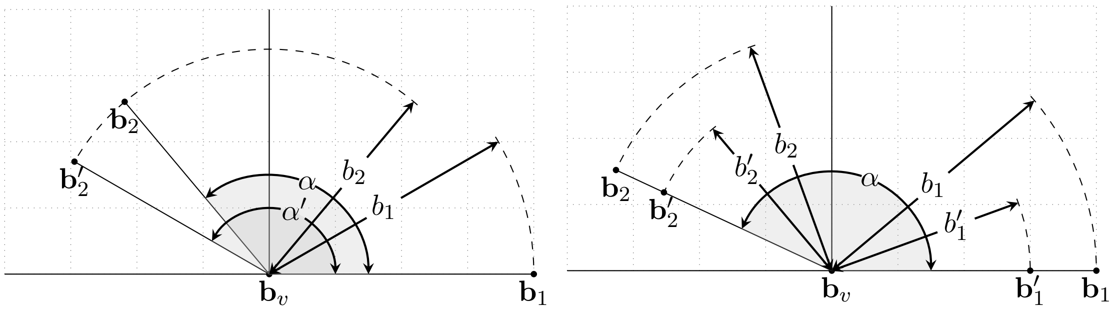


Figure 2: Parties Bliss Points for Different Levels of Antagonism and Extremism

(a) Antagonism (α)

(b) Extremism (b_1, b_2)



strategic motives shaping the competition between the two parties in a dynamic environment, independent of the time horizon. Additionally, the two parties interact over a long time horizon and can be represented by different politicians in different points in time. Therefore, strategies that potentially depend on events from the (distant) past and require coordination might be excessively demanding and inappropriate for the context at hand. Given the policies of the two parties in a contested election, we assume the representative voter elects the party running with the policy she prefers and votes for the opposition when indifferent.[13]

[13] This is a standard assumption in models involving voting over endogenous (proposed) alternatives (see Baron 1996, Bowen, Chen and Eraslan Forthcoming, Diermeier and Fong 2011, Duggan and Kalandrakis 2012, Forand 2014, among others) as it guarantees that the set of

Definition 1. *A Stationary Markov strategy for the opposition party $i \in \{1, 2\}$, given incumbent $j = \{1, 2\} \setminus \{i\}$, is a function $\sigma_i : \{j\} \times X \to X \cup \{Out\}$, mapping j's policy commitment $\mathbf{q} \in X$ into an electoral platform $\mathbf{p} \in X$ or the decision not to contest the election ('Out'). A Stationary Markov strategy for the representative voter v is a function $\sigma_v : (\{1, 2\} \times X) \times X \to \{Yes, No\}$ that maps j's policy commitment $\mathbf{q} \in X$ and the electoral platform of the opposition $\mathbf{p} \in X$ into the decision to elect the opposition.*

Dynamic Utilities. We denote by $V_i^j(\mathbf{q}|\sigma)$ the dynamic utility party $i \in \{1, 2\}$ derives from the infinite sequence of policies generated by the profile of strategies $\sigma = (\sigma_1, \sigma_2, \sigma_v)$, at the beginning of a period with incumbent party $j \in \{1, 2\}$ committed to $\mathbf{q}$. Formally, if $\mathbf{P}(j, \sigma, \mathbf{q}) = \{\mathbf{p}_0, \mathbf{p}_1, \ldots\}$ is a path of policies generated by play according to σ, starting from an incumbent j committed to $\mathbf{q}$, we have:

$$V_i^j(\mathbf{q}|\sigma) = U_i(\mathbf{P}(j, \sigma, \mathbf{q})) = \sum_{t=0}^{\infty} \delta^t \left[u_i(\mathbf{p}_t) + \mathbb{I}_{i,t} \cdot r\right] \tag{3}$$

Equilibrium Notion. We look for a stationary Markov perfect equilibrium.

Definition 2. *A Stationary Markov Perfect Equilibrium (SMPE) is a profile of Stationary Markov strategies $\sigma^* = (\sigma_1^*, \sigma_2^*, \sigma_v^*)$ such that, for any $\mathbf{q} \in X$,*

$$\sigma_i^* \in \arg\max_{\sigma_i} V_i^j(\mathbf{q}|(\sigma_i, \sigma_j^*, \sigma_v^*)) \tag{4}$$

for any $i \in \{1, 2\}$ and $j = \{1, 2\} \setminus \{i\}$, and $\sigma_v^*(j, \mathbf{q}, \mathbf{p}) =$ Yes if and only if

$$u_v(\mathbf{p}) \geq u_v(\mathbf{q}) \tag{5}$$

An equilibrium, as specified in (5), requires that the representative voter supports the opposition if and only if her expected utility from the incumbent's policy is not larger than the expected utility from the opposition's platform. The fact that the opposition optimizes its dynamic utility is ensured by (4).

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policies the representative voter accepts is closed.

4 Equilibrium Analysis

4.1 Simple Strategies

In the remainder, we focus on a class of SMPE of the dynamic electoral competition game, where the two parties use strategies of a simple form, captured by a single parameter $\hat{k}_i$.

Definition 3. *A Simple (Stationary Markov) strategy σ_i for $i \in \{1,2\}$ satisfies*

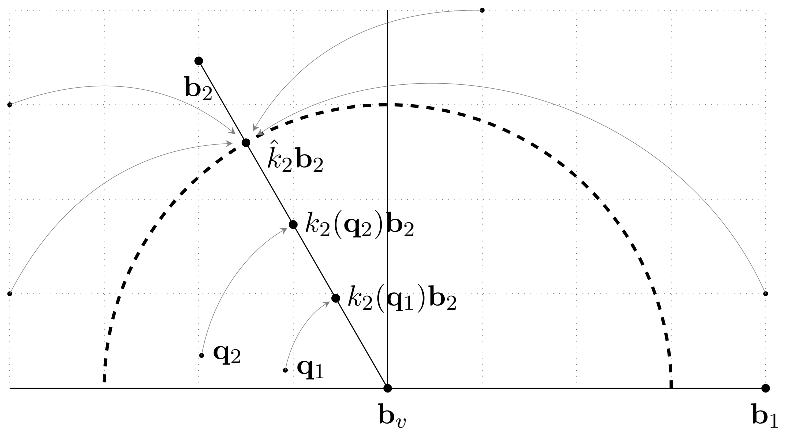
$$\sigma_i(\mathbf{q}) \equiv \mathbf{p}_i(\mathbf{q}) = \begin{cases} k_i(\mathbf{q})\mathbf{b}_i & \textit{for} \;\; k_i(\mathbf{q}) \leq \hat{k}_i \\ \hat{k}_i\mathbf{b}_i & \textit{for} \;\; k_i(\mathbf{q}) \geq \hat{k}_i \end{cases} \tag{6}$$

A Simple Stationary Markov Perfect Equilibrium (SSMPE) is a SMPE where the parties use simple strategies.[14]

As discussed above, $k_i(\mathbf{q})$ measures the distance of the incumbent's policy commitment $\mathbf{q}$ from the origin of the plane. According to these simple strategies, when $\mathbf{q}$ is closer than $\hat{k}_i$ to the origin, the opposition party $i \in \{1,2\}$ contests the election with $k_i(\mathbf{q})\mathbf{b}_i$. This policy is located on the ray that connects $\mathbf{b}_v$ with $\mathbf{b}_i$, and it is at the same distance from $\mathbf{b}_v$ as the incumbent's policy commitment $\mathbf{q}$. When, instead, $\mathbf{q}$ is further than $\hat{k}_i$ from the origin, the opposition party $i \in \{1,2\}$ runs with $\hat{k}_i\mathbf{b}_i$. This policy is on the same ray that connects $\mathbf{b}_v$ with $\mathbf{b}_i$, but it is $\hat{k}_i$ distant from the origin. Figure 3 shows the policies corresponding to these simple strategies: for a number of arbitrary party 1's policy commitments, the arrow indicates the policy platform chosen by party 2. Consider how the policies evolve as $k_2(\mathbf{q}) = \frac{d(\mathbf{q})}{b_2}$ increases, that is, as party 1's policy commitment moves further away from the origin: for low values of $k_2(\mathbf{q})$, the policy $\mathbf{p}_2(\mathbf{q})$ increases linearly—that is, it gets further away from the origin—as $k_2(\mathbf{q})$ increases; once $k_2(\mathbf{q})$ reaches $\hat{k}_2$, $\mathbf{p}_2(\mathbf{q})$ stays constant and it is not affected by a further increase of $k_2(\mathbf{q})$.

[14] In Appendix A2.3 we show that the policy dynamics generated by the unique SSMPE characterized below are identical to the ones generated by the unique subgame perfect equilibrium of the finite horizon version of our model.

Figure 3: Simple Stationary Markov Proposal Strategy



4.2 Results

Proposition 1 below shows that a SSMPE of the dynamic electoral competition game exists and is generally unique, and it fully characterizes it.

Proposition 1 (SSMPE of Dynamic Electoral Competition Game)**.** *Assume, without loss of generality, that* $b_1 \geq b_2$*. Then:*

1. $\hat{k}_1 = 1$ *and* $\hat{k}_2 = \frac{1+\delta \cos \alpha}{1+\delta}$ *characterize a SSMPE;*

2. *if* $b_1 > b_2$*, this SSMPE is unique; if* $b_1 = b_2$*, there exists exactly one additional 'mirror' SSMPE where* $\hat{k}_1$ *and* $\hat{k}_2$ *are reversed;*

3. *in any SSMPE, elections are contested and incumbents always defeated;*

4. *starting from* $\mathbf{q}^0$*, SSMPE policies converge to alternation between*

   (a) $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$ *and* $\hat{k}_2\mathbf{b}_2$*, if* $k_2(\mathbf{q}^0) \geq \hat{k}_2$

   (b) $k_1(\mathbf{q}^0)\mathbf{b}_1$ *and* $k_2(\mathbf{q}^0)\mathbf{b}_2$*, if* $k_2(\mathbf{q}^0) \leq \hat{k}_2$*.*

Proof. See Appendix A1.

In a SSMPE, the representative voter votes for the opposition if it runs with a policy $\mathbf{p}$ that is at least as close to her bliss point as the incumbent's policy commitment $\mathbf{q}$.[15]

The policy platforms chosen by party $i \in \{1, 2\}$ always lie on the ray starting at $\mathbf{b}_v$, the origin of the plane, and passing through $\mathbf{b}_i$, its ideal policy. We call such a ray a $\mathbf{b}_i$-ray. To understand why, consider the dynamic utility i derives from running with a policy $\mathbf{p}$ the representative voter prefers to $\mathbf{q}$: $u_i(\mathbf{p}) + r + \delta V_i^i(\mathbf{p}|\sigma)$. In this expression, the first two terms capture the current utility following an electoral victory. The third term captures the future stream of payoffs, given the policy commitment. This discounted value depends on $d(\mathbf{p})$ but not on the specific location of $\mathbf{p}$: the strategies of all players depend on the distance of the incumbent's policy commitment from the origin, but not on its exact location. As a result, when moving $\mathbf{p}$ along any circle centered at $\mathbf{b}_v$, the dynamic utility of i increases as $\mathbf{p}$ approaches the $\mathbf{b}_i$-ray. This increases the utility accrued in the current period but maintains constant the future utility.

Figure 4: Strategic Incentives in SSMPE

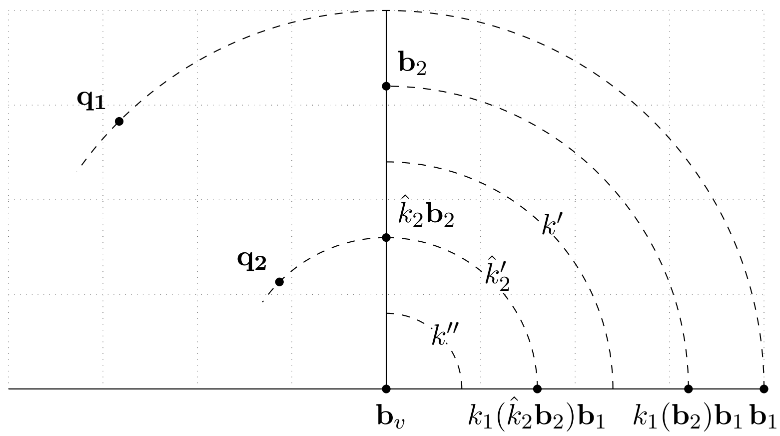


Figure 4 shows many different circles centered at $\mathbf{b}_v$, in dashed lines. The simple strategies from Proposition 1 prescribe that party 2 runs with $\hat{k}_2\mathbf{b}_2$ for any

[15] Figure 4 outlines two such acceptance sets (circles) for two illustrative incumbent's policy commitments $\mathbf{q}_1$ and $\mathbf{q}_2$. For a given incumbent's policy, the representative voter elects the challenger if it proposes any policy on the boundary or strictly inside the corresponding circle.

incumbent's policy commitment $\mathbf{q}$ with $d(\mathbf{q}) \geq d(\mathbf{q}_2)$ and runs with $k_2(\mathbf{q})\mathbf{b}_2$ for any $\mathbf{q}$ with $d(\mathbf{q}) \leq d(\mathbf{q}_2)$. Similarly, party 1 runs with $\hat{k}_1\mathbf{b}_1 = \mathbf{b}_1$ or $k_1(\mathbf{q})\mathbf{b}_1$ depending on whether $d(\mathbf{q}) \gtrless d(\mathbf{q}_1)$.

We first discuss the intuition underlying the strategy of party 2. When $d(\mathbf{q}) \leq d(\mathbf{q}_2)$, running with $k_2(\mathbf{q})\mathbf{b}_2$ means running with a policy on the $\mathbf{b}_2$-ray with the same distance from the origin as $\mathbf{q}$. For incumbent's policy commitments close to the origin, party 2 is constrained by the demand of the representative voter and runs with a winning platform as close as possible to its bliss point $\mathbf{b}_2$. On the other hand, when $d(\mathbf{q}) \geq d(\mathbf{q}_2)$, party 2 runs with $\hat{k}_2\mathbf{b}_2$, a policy strictly inside the voter's acceptance set. Consider, for example, an incumbent party 1 committed to $\mathbf{q}_1$ from Figure 4. In this case, the voter is willing to elect a challenger party 2 that runs with its bliss point, $\mathbf{b}_2$, because this policy is closer to the voters's bliss point than $\mathbf{q}_1$. A perfectly myopic opposition would propose this policy platform. However, a forward-looking opposition finds it optimal to run with policy $\hat{k}_2\mathbf{b}_2$, which is closer to $\mathbf{b}_v$ than $\mathbf{b}_2$. We call this behavior *moderation*.

The incentive to moderate is purely strategic and arises from the dynamic nature of the game. Running with $\hat{k}_2\mathbf{b}_2$, rather than $\mathbf{b}_2$, harms party 2 in the short run, since the policy it implements when in office is further away from its bliss point. At the same time, winning the current election with a more moderate policy helps in the future election: party 2 will be committed to a more moderate platform and this makes the representative voter more demanding. If party 1 wants to re-gain power, it is forced to propose $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$, a policy that both the representative voter and party 2 prefer to $k_1(\mathbf{b}_2)\mathbf{b}_1$ (what party 1 would propose with an incumbent committed to $\mathbf{b}_2$). The extent of moderation, $\hat{k}_2$, is then determined by the strength of two forces. The first force pushes the opposition's policy in the direction of its bliss point, in an attempt to increase its current utility. The second, strategic force, pushes the opposition's policy in the direction of the representative voters's bliss point, in an attempt to constraint the future behavior of the defeated incumbent. As we discuss in the following section, the magnitude of this latter force depends on the parties' patience and on the intensity of their ideological disagreement. The equilibrium extent of moderation is the value of $\hat{k}_2$ that balances the marginal static cost with the marginal dynamic benefit.

We now discuss the equilibrium strategy of party 1. Contrary to party 2, party 1 does *not* moderate: it proposes its ideal policy $\mathbf{b}_1$ whenever the representative voter prefers it to the incumbent's commitment. Why is this the case? While the two parties have similar strategic incentives to moderate, moderation is a *strategic substitute*. Assume party 1 moderates, that is, it proposes a policy closer to the origin than $\mathbf{b}_1$. All moderate policies reduce its current utility from holding office. On the other hand, since party 2 already moderates, not all moderate policies help its electoral prospects in the future. Party 1 will affect the electoral strategy of its challenger only if it moderates to $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$ or to a policy even closer to the origin, for example, referring again to Figure 4, to $k''\mathbf{b}_1$. Moderating less—that is, running with $k'\mathbf{b}_1$ or any other policy between $\mathbf{b}_1$ and $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$—will not make any difference in the following election, as party 2 will run with $\hat{k}_2\mathbf{b}_2$ as prescribed by its moderating strategy. However, moderating to $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$ or more is too costly in terms of foregone current utility and party 1 abandons the idea of moderation altogether. This explains why an equilibrium in simple strategies must be asymmetric, with one party moderating and the other one sticking to its guns. The reason why party 2 is the one who moderates lies in the fact that party 1 has a weaker incentive to moderate: its ideal policy is more extreme and, thus, the future utility gain it gets from constraining its opponent is smaller.[16]

Long-Run Policies and Convergence Dynamics. Proposition 1 also specifies the long run policies that we converge to as a consequence of equilibrium strategies. We have three cases to consider. First, assume the initial incumbent's policy commitment, $\mathbf{q}^0$, is at least as close to the origin as the strategically induced bliss point of party 2, $\hat{k}_2\mathbf{b}_2$. In this case, the representative voter's acceptance set is binding in all elections and all policies will lie at the same distance from the origin as the initial incumbent's policy commitment. The policies implemented will alternate between $k_2(\mathbf{q}^0)\mathbf{b}_2$ and $k_1(\mathbf{q}^0)\mathbf{b}_1$, depending on the identity of the

[16] Notice that an equilibrium in simple strategies has to be asymmetric also when the degree of extremism of the two parties is the same, that is, when $b_1 = b_2$. In this case, the two parties have the same incentive to moderate, but moderation is still a strategic substitute. Provided one party moderates, the opponent has no incentive to moderate at all. As specified by Proposition 1, we have two asymmetric equilibria, one with party 1 moderating and another one with party 2 moderating. These equilibria are equivalent from the point of view of the representative voter.

incumbent.

Second, assume the initial incumbent's policy commitment is further away from the origin than $\mathbf{b}_1$. If party 1 is at the opposition in the first period, it wins the election with its ideal policy $\mathbf{b}_1$. In the second period, party 2 wins the election with $\hat{k}_2\mathbf{b}_2$, a policy the representative voter strictly prefers to the incumbent's policy commitment $\mathbf{b}_1$. In all future periods, the policy implemented will be at the same distance from the origin, alternating between $\hat{k}_2\mathbf{b}_2$, when 2 is in power, and $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$, when 1 is in power. If party 2 is at the opposition in the first period, it wins the election with $\hat{k}_2\mathbf{b}_2$ and the policy dynamic immediately reaches alternation between $\hat{k}_2\mathbf{b}_2$ and $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$.

Finally, assume $\mathbf{q}^0$ is further away from the origin than $\hat{k}_2\mathbf{b}_2$ but closer to the origin than $\mathbf{b}_1$. This case is similar to the second one, except that, if party 1 is at the opposition in the initial period, it is constrained to offer $k_1(\mathbf{q}^0)\mathbf{b}_1$. From the second period, we have the same alternation between $\hat{k}_2\mathbf{b}_2$ and $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$.

5 Representative Voter's Welfare

The welfare of the representative voter from an ex-ante perspective, that is, at the beginning of the game, depends on the identity of the incumbent and on its policy commitment in the first period. Instead of making arbitrary assumptions, we assume that party 1 is the first-period incumbent with probability $\rho \in (0,1)$ and that its policy commitment $\mathbf{q}^0$ is distributed according to a continuous cumulative distribution function $F(\mathbf{q}^0)$ with strictly positive density on X.

Definition 4. *In the SSMPE from Proposition 1, the ex-ante welfare of v is:*

$$W(\hat{k}_1, \hat{k}_2, b_1, b_2) = \int_X \rho u_v(\mathbf{p}_2(\mathbf{z})) + (1-\rho) u_v(\mathbf{p}_1(\mathbf{z}))\, d(F(\mathbf{z})). \tag{7}$$

Lemma 1 shows that the representative voter is strictly worse off as the degree of policy moderation observed in equilibrium decreases and as the parties bliss points diverge from the representative voter's bliss point.

Lemma 1. *The ex-ante welfare of the representative voter is decreasing in $\hat{k}_1$, $\hat{k}_2$, b_1, and b_2 when $\hat{k}_1 > 0$ and $\hat{k}_2 > 0$.*

Proof. See Appendix A1.

6 Comparative Statics

The following proposition formalizes the marginal impact of the model parameters on the strategic force pushing the parties towards moderation:

Proposition 2 (SSMPE Comparative Static)**.** *In the SSMPE from Proposition 1:*

$$\frac{\partial \hat{k}_2}{\partial \alpha} \leq 0 \qquad \frac{\partial \hat{k}_2}{\partial \delta} \leq 0 \tag{8}$$

Proof. Immediate.

Both properties are intuitive. The strategic force pushing the more moderate party towards moderation gains strength as the ideological conflict between the parties becomes more pronounced (higher α) and as the future becomes more important (higher δ). Combining the comparative static results from Proposition 2 with Lemma 1, we have the following corollary about the marginal impact of antagonism, α, and extremism, b_i, on the representative voter's welfare.

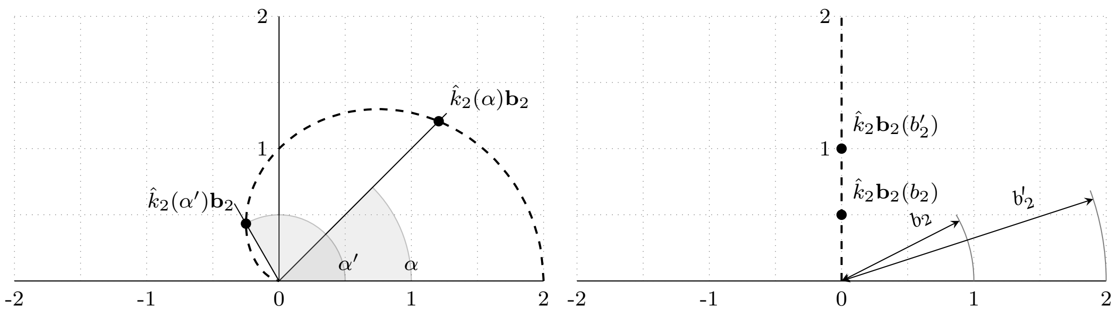
Corollary 1 (SSMPE Representative Voter's Welfare Comparative Static)**.** *In the SSMPE from Proposition 1, the representative voter's ex-ante welfare, W,*

1. *is non-decreasing in α; increasing in α if $\delta > 0$;*
2. *is decreasing in b_i for $i \in \{1, 2\}$.*

Figure 5 shows how the equilibrium strategies of the two parties change when we increase the value of α (Figure 5a) and b_2 (Figure 5b), in the limit as $\delta \to 1$. We focus here on the case where the initial incumbent's policy commitment is sufficiently far from the origin to generate interesting policy dynamics.[17] We know from Proposition 1 and the discussion above that, in this case, the equilibrium policies converge, in at most two periods, to alternation between $\hat{k}_2\mathbf{b}_2$ and $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$. Figure 5 shows the policy implemented whenever party 2 is in power, $\hat{k}_2\mathbf{b}_2$.[18] While this neglects a component of the representative voter's ex-ante welfare—which also depends on the policy implemented by party 1—it captures the main intuition.[19] The dashed line in both figures traces $\hat{k}_2\mathbf{b}_2$ as α increases from 0 to π, and as b_2 increases from 0 to 4.

Figure 5: $\hat{k}_2\mathbf{b}_2$ as a function of α and b_2
$\delta \to 1$ and $b_1 > b_2$

(a) Effect of α for $b_2 = 2$ (b) Effect of b_2 for $\alpha = \frac{1}{2}\pi$



When α increases, the extent of ideological conflict between the two parties increases. This strengthens the strategic force to moderate and the equilibrium policy of party 2, $\hat{k}_2\mathbf{b}_2$, moves closer to the bliss point of the representative voter (Figure 5a). This clearly benefits the representative voter. Increasing the extremism of party 2, b_2, on the other hand, has the opposite effect (Figure 5b). The strength of the moderating force does not change with b_2 and $\hat{k}_2\mathbf{b}_2$ moves away from the origin. Since the representative voter's utility depends on the distance

[17] When $\mathbf{q}^0$ is closer to the origin than $\hat{k}_2\mathbf{b}_2$ neither α or b_2 matter for the moderation of implemented policies. The size of the set of initial incumbent's policy commitments for which this is true increases in b_2 and decreases in α.

[18] To keep the figures simple, we do not show the long-run policy when party 1 is in power, $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$. This policy lies on the horizontal axis, at the same distance from the origin as $\hat{k}_2\mathbf{b}_2$.

[19] The only other policy that can be observed in equilibrium is $\mathbf{b}_1$, the policy platform party 1 chooses when at the opposition in the first period. While α has no impact on this transient policy, the larger is b_1, the further away is this transient policy from the origin and, in turn, the worse off is the representative voter.

of the equilibrium policies from her bliss point, this has a negative effect on her welfare. Why does $\hat{k}_2$ depend on α but not on b_2 (or b_1)? Increasing extremism increases both the marginal benefit and the marginal cost of moderation, *leaving their ratio constant.* On the other hand, increasing antagonism has no influence on the marginal costs of moderation but it increases its marginal benefits.

The next result presents the marginal impact of antagonism and extremism on the variance of the long-run policy outcomes. This is an interesting prediction as it allows us to link preferences' disagreement to gridlock and policy uncertainty. Denote with $d_p = d(\hat{k}_2\mathbf{b}_2, k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1)$ the distance between the two long-run policies in the unique SSMPE from Proposition 1. This measure determines the variance of the long-run equilibrium policies characterized above.

Proposition 3. *Assume that $b_1 > b_2$. d_p is:*

1. *non-decreasing in b_2; increasing in b_2 if $\alpha > 0$; constant in b_1;*
2. *increasing in α when $\alpha \in [0, \alpha']$ and decreasing in α when $\alpha \in [\alpha', \pi]$, where $\alpha' \in [0, \pi]$ and $\alpha' < \pi \Leftrightarrow \delta > \frac{1}{5}$.*

Proof. See Appendix A1.

Figure 6: d_p as a function of α and b_2
$\delta \to 1$ and $b_1 > b_2$

(a) Effect of α for $b_2 = 2$ (b) Effect of b_2 for $\alpha = \frac{1}{2}\pi$

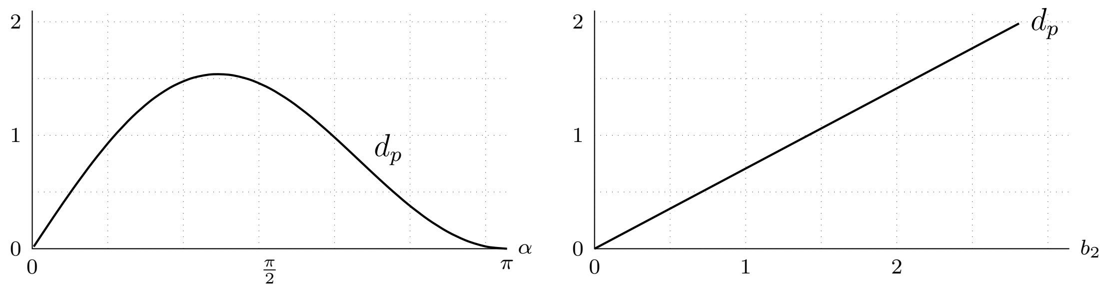


Figure 6 shows how the long-run policies variance change when we increase α (Figure 6a) and b_2 (Figure 6b), in the limit as $\delta \to 1$. Increasing the extremism of party 2 has a straightforward effect on policy volatility, as it pushes the implemented policies away from the representative voter's ideal point in different directions. Increasing the extremism of party 1, on other hand, has no effect because the long-run equilibrium policies depend only on the degree of moderation of party 2.[20] Finally, increasing antagonism has two effects: it distances the long-run policies away from each other but it also moves them closer to the ideal point of the representative voter. The former effects dominates for low α, while the latter dominates for high α.

7 Conclusions

While many commentators and scholars diagnose a sharp and increasing ideological divide between the main American parties, both the popular press and the existing literature are somewhat unclear about what exactly constitutes polarization and how one can measure this concept. In this paper, we study an environment where two ideological and forward-looking parties compete for office in a sequence of elections. We assume that incumbents who are reelected find it too costly to implement policies that differ from those of their first term. On the other hand, challengers are free to offer to the representative voter any bi-dimensional policy platform. We use two different measures to describe the political environment and the degree of conflict among agents' preferences. The first measure, which we label *extremism*, is the ideological distance of each party from the representative voter. The second measure, which we call *antagonism*, is the ideological distance that separates the two parties from each other and summarizes the degree of political competition between policymakers. These two measures coincide in a one dimensional policy space, where the ideological distance between the two parties can increase only as they move further away from a moderate representative voter. However, they do not coincide in a two-dimensional setting: here, the two parties can be very close—when they share views on both dimensions—or very different—when they are perfectly opposed in one dimension—without altering their overall

[20] Proposition 3 assumes $b_1 > b_2$. If $b_1 = b_2$, part two holds with no change. In this case, any increase in b_2 means that party 1 moderates in equilibrium. If we define d_p using the equilibrium extent of moderation by party 1, part one holds except for switching b_1 and b_2.

distance from the representative voter. We show that a stationary Markov perfect equilibrium of this game exists and we fully characterize it for any discount factor, initial incumbent's policy, and degree of extremism and antagonism. In this equilibrium, increasing the degree of extremism reduces the welfare of the representative voter. On the other hand, increasing antagonism increases the influence exerted by the electorate on long run policy outcomes.

In the remainder, we discuss the empirical implications of our analysis. A first empirical prediction delivered by the model is that, even when parties' ideologies are symmetric, moderation is a strategic substitute and only one party moderates, with the other party sticking to its guns. This prediction is in line with the asymmetric evolution of observed polarization in the American Congress, where the Republican representatives are relatively more extreme than the Democratic ones (McCarty, Poole and Rosenthal 1997, 2006). Second, as pointed by Baker, Bloom and Davis (2013) and Fernandez-Villaverde, Guerron-Quintana, Kuester and Rubio-Ramirez (2011), the last decades have been associated with greater-than-historical economic policy uncertainty and volatility in the United States. The theoretical predictions of our model suggest that this could be linked to ideological conflict, through two different channels: both an increase in extremism and an increase in antagonism (starting from a low level) can lead to higher volatility of policy outcomes. Interestingly, the period between the 1970s and 2011 has also been a period of higher-than-historical political polarization, as highlighted by McCarty, Poole and Rosenthal (2006) on the basis of legislators' ideal points estimated from roll call votes.

Finally, does our model suggest that the American electorate is better off with more polarized parties? The answer is likely to depend on the dimensionality of the political conflict. If the political competition is on one dimension—as argued by the theory of "conflict displacement" (Sundquist 1983, Carmines and Stimson 1989, Miller and Schofield 2003) and suggested by the roll call analysis of McCarty, Poole and Rosenthal (2006) for the current historical period—then increased elite polarization coincides with increased extremism, using the language of our model, and this is likely bad news for the moderate voters: as polarization increases, both the distance between parties and the distance of both parties from the moderate voters increase, and the observed policies will be more extreme (even when parties are forward looking and policy motivated). If instead the political competition and the increased ideological conflict is on multiple, correlated dimension—as argued by the theory of "conflict extension" (Layman and Carsey 2002*a*)—this might be good news for moderate voters: the heightened competition between parties will not favor the civility of the political discourse and might lead to policy gridlock, but forward looking and policy motivated parties will propose more moderate policies.

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A1 Proofs

Proof of Proposition 1

Let $\hat{k}_1$ and $\hat{k}_2$ be the parameters associated with simple strategies and let $\hat{k}_1^*$ and $\hat{k}_2^*$ be the parameters associated with the simple strategies of a SSMPE (as described in Definition 3). When talking about both parameters jointly, we use $\hat{k} = (\hat{k}_1, \hat{k}_2)$ and $\hat{k}^* = (\hat{k}_1^*, \hat{k}_2^*)$. Because a simple strategy of $i \in \{1, 2\}$ from Definition 3 is fully determined by $\hat{k}_i$, with a slight abuse of notation, we call $\hat{k}_i$ the strategy of i and $\hat{k} = (\hat{k}_1, \hat{k}_2)$ the profile of strategies.

We start by noting that any SSMPE strategy of v satisfies, for an incumbent $j \in \{1, 2\}$, its policy commitment $\mathbf{q} \in X$ and a platform of the opposition $\mathbf{p} \in X$, $\sigma_v^*(j, \mathbf{q}, \mathbf{p}) = Yes$ if and only if $d(\mathbf{p}) \leq d(\mathbf{q})$. From Definition 2, $\sigma_v^*(j, \mathbf{q}, \mathbf{p}) = Yes$ if and only if $u_v(\mathbf{p}) \geq u_v(\mathbf{q})$, which, from $u_v(\mathbf{x}) = -d^2(\mathbf{x})$ for any $\mathbf{x} \in X$, is equivalent to $d(\mathbf{p}) \leq d(\mathbf{q})$. Because of the simplicity of the representative voter's behavior in any SSMPE, we suppress σ_v from the notation below.

We now claim that, for any SSMPE strategy of v and any simple strategy profile of the two parties $\hat{k}$, the opposition party contests elections and wins. The former follows from $\mathbf{p}_i(\mathbf{q}) \neq Out$ for any $i \in \{1, 2\}$, $\hat{k}_i \in \mathbb{R}$ and $\mathbf{q} \in X$. To prove the latter claim, we need to show that $d(\mathbf{p}_i(\mathbf{q})) \leq d(\mathbf{q})$ for any $i \in \{1, 2\}$, $\hat{k}_i \in \mathbb{R}$ and $\mathbf{q} \in X$. From Definition 3, if $k_i(\mathbf{q}) \leq \hat{k}_i$, then $d(\mathbf{p}_i(\mathbf{q})) = d(k_i(\mathbf{q})\mathbf{b}_i) = k_i(\mathbf{q})b_i = \frac{d(\mathbf{q})}{b_i}b_i$ and if $k_i(\mathbf{q}) \geq \hat{k}_i$, then $d(\mathbf{p}_i(\mathbf{q})) = d(\hat{k}_i\mathbf{b}_i) = \hat{k}_i b_i \leq k_i(\mathbf{q})b_i = d(\mathbf{q})$. Because the opposition party contests elections and wins for any SSMPE strategy of v and any simple strategy profile $\hat{k}$, elections are always contested and incumbents always defeated in any SSMPE. This proves part 3 of the proposition.

We now prove parts 1 (characterization) and 2 (uniqueness or duplicity) of the proposition. By the one-stage-deviation principle, a profile of strategies $\hat{k}^*$ constitutes a SSMPE if, for any $i \in \{1, 2\}$ and $\mathbf{q} \in X$, a deviation by the opposition party i to contest elections with $\mathbf{p} \neq \mathbf{p}_i(\mathbf{q})$ or to stay out is not profitable. We momentarily assume and later verify that both parties, when at the opposition, want to contest elections with platforms that guarantee their victory.

For any $\hat{k}$, $i \in \{1,2\}$ and $\mathbf{q} \in X$, the dynamic utility i receives from running with $\mathbf{p}$ such that $d(\mathbf{p}) \leq d(\mathbf{q})$ is $u_i(\mathbf{p}) + r + \delta V_i^i(\mathbf{p}|\hat{k})$. An opposition party i thus runs with $\mathbf{p}^* \in \arg\max_{d(\mathbf{x}) \leq d(\mathbf{q})} u_i(\mathbf{x}) + r + \delta V_i^i(\mathbf{x}|\hat{k})$. For any $\hat{k}$ and any two policies $\mathbf{p} \in X$ and $\mathbf{p}' \in X$ with $d(\mathbf{p}) = d(\mathbf{p}')$, we have $V_i^i(\mathbf{p}|\hat{k}) = V_i^i(\mathbf{p}'|\hat{k})$. This means that the optimal $\mathbf{p}^*$ has to lie on the ray starting at $\mathbf{b}_v = (0,0)$ and passing through $\mathbf{b}_i$. It hence can be written as $\mathbf{p}^* = k\mathbf{b}_i$ for some $k \geq 0$. Denote by $U_i(k|\hat{k}) = u_i(k\mathbf{b}_i) + r + \delta V_i^i(k\mathbf{b}_i|\hat{k})$ the dynamic utility of party $i \in \{1,2\}$ from running with policy $k\mathbf{b}_i$ when the parties use simple strategies characterized by $\hat{k}$. The key properties of $U_i(k|\hat{k})$ are summarized in the lemma below. To facilitate its proof, we first state several identities for the policy utility of the two parties.

The policy utility $i \in \{1,2\}$ derives from $k\mathbf{b}_j$ where $k \geq 0$ and $j \in \{1,2\}$ is $u_i(k\mathbf{b}_j) = -d^2(k\mathbf{b}_j, \mathbf{b}_i)$. This is a continuous and differentiable function of k. Using $-i = \{1,2\} \setminus \{i\}$, we have

$$
\begin{aligned}
\frac{\partial(-d^2(k\mathbf{b}_{-i}, \mathbf{b}_i))}{\partial k} &= -2(kd^2(\mathbf{b}_{-i}) - \mathbf{b}_{-i} \cdot \mathbf{b}_i) = -2b_i b_{-i}\left(k\tfrac{b_{-i}}{b_i} - \cos\alpha\right) \\
\frac{\partial(-d^2(k\mathbf{b}_i, \mathbf{b}_i))}{\partial k} &= -2(kd^2(\mathbf{b}_i) - \mathbf{b}_i \cdot \mathbf{b}_i) = -2b_i^2(k-1) \\
\frac{\partial^2(-d^2(k\mathbf{b}_j, \mathbf{b}_i))}{\partial^2 k} &= -2d^2(\mathbf{b}_j).
\end{aligned}
\tag{A1}
$$

This proves that $u_i(k\mathbf{b}_j)$ is a concave function of k.

Lemma A1. *For any $i \in \{1,2\}$, fix $\hat{k}_i \in \mathbb{R}$ and $\hat{k}_{-i} \in \mathbb{R}$. Let $\mathbf{q} \in X$ be the incumbent's policy commitment. $U_i(k|\hat{k})$, as a function of $k \in [0, k_i(\mathbf{q})]$, is continuous, differentiable except when $k = \hat{k}_i$ or $k = \hat{k}_{-i}\frac{b_{-i}}{b_i}$, strictly concave on each interval on which it is differentiable and $\frac{\partial U_i(k|\hat{k})}{\partial k} > 0$ for $k < c$, $\frac{\partial U_i(k|\hat{k})}{\partial k} = 0$ for $k = c$ and $\frac{\partial U_i(k|\hat{k})}{\partial k} < 0$ for $k > c$ where*

$$
\begin{aligned}
c &= \tfrac{1+\delta\cos\alpha}{1+\delta} && \text{if } \; k < \min\{\hat{k}_i, \hat{k}_{-i}\tfrac{b_{-i}}{b_i}\} \\
c &= \tfrac{1+\delta\cos\alpha}{1+\delta} && \text{if } \; \hat{k}_i < k < \hat{k}_{-i}\tfrac{b_{-i}}{b_i} \\
c &= 1 && \text{if } \; \hat{k}_{-i}\tfrac{b_{-i}}{b_i} < k < \hat{k}_i \\
c &= 1 && \text{if } \; \max\{\hat{k}_i, \hat{k}_{-i}\tfrac{b_{-i}}{b_i}\} < k.
\end{aligned}
\tag{A2}
$$

Proof. Throughout the proof fix $i \in \{1,2\}$, $-i = \{1,2\} \setminus \{i\}$, $\hat{k}_i \in \mathbb{R}$, $\hat{k}_{-i} \in \mathbb{R}$

and incumbent's policy commitment $\mathbf{q} \in X$. The continuity is easy to see as the simple strategies characterized by $\hat{k}$ give rise to a value function, $V_i^i(\mathbf{x}|\hat{k})$, that is continuous in $\mathbf{x} \in X$ for any $d(\mathbf{x}) \leq d(\mathbf{q})$. For the remaining properties, we have to derive $V_i^i(\mathbf{x}|\hat{k})$ explicitly. Because the opposition party always contests and wins elections for any $\hat{k}$, we have, for any $\mathbf{x} \in X$ with $d(\mathbf{x}) \leq d(\mathbf{q})$,

$$\begin{aligned} V_i^{-i}(\mathbf{x}|\hat{k}) &= u_i(\mathbf{p}_i(\mathbf{x})) + r + \delta V_i^i(\mathbf{p}_i(\mathbf{x})|\hat{k}) \\ V_i^i(\mathbf{x}|\hat{k}) &= u_i(\mathbf{p}_{-i}(\mathbf{x})) + \delta V_i^{-i}(\mathbf{p}_{-i}(\mathbf{x})|\hat{k}). \end{aligned} \tag{A3}$$

Combining the two equations

$$V_i^i(\mathbf{x}|\hat{k}) = u_i(\mathbf{p}_{-i}(\mathbf{x})) + \delta \left[u_i(\mathbf{p}_i(\mathbf{p}_{-i}(\mathbf{x}))) + r + \delta V_i^i(\mathbf{p}_i(\mathbf{p}_{-i}(\mathbf{x}))|\hat{k}) \right] \tag{A4}$$

where

$$\begin{array}{lll} \mathbf{p}_{-i}(\mathbf{x}) = k_i(\mathbf{x})\frac{b_i}{b_{-i}}\mathbf{b}_{-i} & \mathbf{p}_i(\mathbf{p}_{-i}(\mathbf{x})) = k_i(\mathbf{x})\mathbf{b}_i & \text{if } k_i(x) \leq \min\{\hat{k}_i, \hat{k}_{-i}\frac{b_{-i}}{b_i}\} \\ \mathbf{p}_{-i}(\mathbf{x}) = k_i(\mathbf{x})\frac{b_i}{b_{-i}}\mathbf{b}_{-i} & \mathbf{p}_i(\mathbf{p}_{-i}(\mathbf{x})) = \hat{k}_i\mathbf{b}_i & \text{if } \hat{k}_i < k_i(x) < \hat{k}_{-i}\frac{b_{-i}}{b_i} \\ \mathbf{p}_{-i}(\mathbf{x}) = \hat{k}_{-i}\mathbf{b}_{-i} & \mathbf{p}_i(\mathbf{p}_{-i}(\mathbf{x})) = \hat{k}_{-i}\frac{b_{-i}}{b_i}\mathbf{b}_i & \text{if } \hat{k}_{-i}\frac{b_{-i}}{b_i} < k_i(x) < \hat{k}_i \\ \mathbf{p}_{-i}(\mathbf{x}) = \hat{k}_{-i}\mathbf{b}_{-i} & \mathbf{p}_i(\mathbf{p}_{-i}(\mathbf{x})) = \min\{\hat{k}_{-i}\frac{b_{-i}}{b_i}, \hat{k}_i\}\mathbf{b}_i & \text{if } \max\{\hat{k}_i, \hat{k}_{-i}\frac{b_{-i}}{b_i}\} \leq k_i(x) \end{array} \tag{A5}$$

is easy to confirm using properties of the simple strategies along with $k_i(\mathbf{x})b_i = k_{-i}(\mathbf{x})b_{-i}$ for any $\mathbf{x} \in X$.

We sbstitute $\mathbf{x} = k\mathbf{b}_i$ into (A4), using (A5), $k_i(k\mathbf{b}_i) = k$, $u_i(\mathbf{x}) = -d^2(\mathbf{x}, \mathbf{b}_i)$ and $V_i^i(\mathbf{x}|\hat{k}) = V_i^i(k_i(\mathbf{x})\mathbf{b}_i|\hat{k})$, which follows from $d(k_i(\mathbf{x})\mathbf{b}_i) = \frac{d(\mathbf{x})}{b_i}d(\mathbf{b}_i) = d(\mathbf{x})$. After some straightforward algebra, summarizing with χ_t all the terms constant in k, we have:

$$\begin{array}{ll} U_i(k|\hat{k}) = \dfrac{-d^2(k\mathbf{b}_i, \mathbf{b}_i) - d^2(k\frac{b_i}{b_{-i}}\mathbf{b}_{-i}, \mathbf{b}_i)\delta + r}{1-\delta^2} & \text{if } k \leq \min\{\hat{k}_i, \hat{k}_{-i}\frac{b_{-i}}{b_i}\} \\ U_i(k|\hat{k}) = -d^2(k\mathbf{b}_i, \mathbf{b}_i) - d^2(k\frac{b_i}{b_{-i}}\mathbf{b}_{-i}, \mathbf{b}_i)\delta + \chi_1 & \text{if } \hat{k}_i < k < \hat{k}_{-i}\frac{b_{-i}}{b_i} \\ U_i(k|\hat{k}) = -d^2(k\mathbf{b}_i, \mathbf{b}_i) + \chi_2 & \text{if } \hat{k}_{-i}\frac{b_{-i}}{b_i} < k < \hat{k}_i \\ U_i(k|\hat{k}) = -d^2(k\mathbf{b}_i, \mathbf{b}_i) + \chi_3 & \text{if } \max\{\hat{k}_i, \hat{k}_{-i}\frac{b_{-i}}{b_i}\} \leq k \end{array} \tag{A6}$$

for any $k \in [0, k_i(\mathbf{q})]$. Direct verification then shows all the remaining properties of $U_i(k|\hat{k})$.[21] $\square$

Using $U_i(k|\hat{k})$ we can rewrite the optimization problem of opposition party $i \in \{1, 2\}$ regarding which policy to contest elections with, for any incumbent's policy commitment $\mathbf{q} \in X$, as $\max_{0 \leq k \leq k_i(\mathbf{q})} U_i(k|\hat{k})$. To find a SSMPE, we need to find a $\hat{k}^*$ such that the solution to this optimization problem under $\hat{k}^*$, for any $\mathbf{q} \in X$, can be described by $\hat{k}^*$.

We first claim that in any SSMPE, $\hat{k}_i^* \in \{\frac{1+\delta\cos\alpha}{1+\delta}, 1\}$ for $i \in \{1, 2\}$. Notice $\frac{1+\delta\cos\alpha}{1+\delta} \leq 1$ for any $\delta \in [0, 1)$ and $\alpha \in [0, \pi]$. To show the claim, suppose, towards a first contradiction, that $\hat{k}_i^* < \frac{1+\delta\cos\alpha}{1+\delta}$. Suppose $\hat{k}_{-i}^* \frac{b_{-i}}{b_i} \leq \hat{k}_i^*$. Then from Lemma A1, there exists $\bar{\epsilon} > 0$ such that for all $\epsilon \in (0, \bar{\epsilon})$, $U_i'(\hat{k}_i^* + \epsilon|\hat{k}^*) > 0$, which implies that $U_i(\hat{k}_i^*|\hat{k}^*) < U_i(\hat{k}_i^* + \epsilon|\hat{k}^*)$, $\mathbf{p}_i((\hat{k}_i^* + \epsilon)\mathbf{b}_i) = \hat{k}_i^*\mathbf{b}_i$ and $d((\hat{k}_i^* + \epsilon)\mathbf{b}_i) < d((\hat{k}_i^* + \bar{\epsilon})\mathbf{b}_i)$. In words, for incumbent's policy commitment $(\hat{k}_i^* + \bar{\epsilon})\mathbf{b}_i$, i contests elections with $\hat{k}_i^*\mathbf{b}_i$, despite the fact that running with $(\hat{k}_i^* + \epsilon)\mathbf{b}_i$ would ensure its victory and higher dynamic utility, a contradiction. An identical argument leads to a contradiction when $\hat{k}_{-i}^* \frac{b_{-i}}{b_i} > \hat{k}_i^*$. Now suppose, towards a second contradiction, that $\hat{k}_i^* \in (\frac{1+\delta\cos\alpha}{1+\delta}, 1)$. If $\hat{k}_{-i}^* \frac{b_{-i}}{b_i} \leq \hat{k}_i^*$, an argument identical to the one above leads to a contradiction. Suppose $\hat{k}_{-i}^* \frac{b_{-i}}{b_i} > \hat{k}_i^*$. Then from Lemma A1, there exists $\bar{\epsilon} > 0$ such that for all $\epsilon \in (0, \bar{\epsilon})$, $U_i'(\frac{1+\delta\cos\alpha}{1+\delta} + \epsilon|\hat{k}^*) < 0$, which implies that $U_i(\frac{1+\delta\cos\alpha}{1+\delta}|\hat{k}^*) > U_i(\frac{1+\delta\cos\alpha}{1+\delta} + \epsilon|\hat{k}^*)$, $\mathbf{p}_i((\frac{1+\delta\cos\alpha}{1+\delta} + \epsilon)\mathbf{b}_i) = (\frac{1+\delta\cos\alpha}{1+\delta} + \epsilon)\mathbf{b}_i$ and $d(\frac{1+\delta\cos\alpha}{1+\delta}\mathbf{b}_i) < d((\frac{1+\delta\cos\alpha}{1+\delta} + \bar{\epsilon})\mathbf{b}_i)$, a contradiction. Finally suppose, towards a third contradiction, that $\hat{k}_i^* > 1$. Then irrespective of $\hat{k}_{-i}^*$, Lemma A1 implies that there exists $\bar{\epsilon} > 0$ such that, for all $\epsilon \in (0, \bar{\epsilon})$, $U_i'(1 + \epsilon|\hat{k}^*) < 0$, which implies that $U_i(1|\hat{k}^*) > U_i(1 + \epsilon|\hat{k}^*)$, $\mathbf{p}_i((1 + \epsilon)\mathbf{b}_i) = (1 + \epsilon)\mathbf{b}_i$ and $d(\mathbf{b}_i) < d((1 + \bar{\epsilon})\mathbf{b}_i)$, a contradiction.

Having shown that $\hat{k}_i^* \in \{\frac{1+\delta\cos\alpha}{1+\delta}, 1\}$ for $i \in \{1, 2\}$, we now argue that $\hat{k}_i^* =$

[21] For completeness:

$$\chi_1 = u_i(\hat{k}_i\mathbf{b}_i)\delta^2 + r(1 + \delta^2) + \delta^2 V_i^i(\hat{k}_i\mathbf{b}_i|\hat{k})$$
$$\chi_2 = u_i(\hat{k}_{-i}\mathbf{b}_{-i})\delta + u_i(\hat{k}_{-i}\tfrac{b_{-i}}{b_i}\mathbf{b}_i)\delta^2 + r(1 + \delta^2) + \delta^2 V_i^i(\hat{k}_{-i}\tfrac{b_{-i}}{b_i}\mathbf{b}_i|\hat{k})$$
$$\chi_3 = u_i(\hat{k}_{-i}\mathbf{b}_{-i})\delta + u_i(\min\{\hat{k}_{-i}\tfrac{b_{-i}}{b_i}, \hat{k}_i\}\mathbf{b}_i)\delta^2 + r(1 + \delta^2) + \delta^2 V_i^i(\min\{\hat{k}_{-i}\tfrac{b_{-i}}{b_i}, \hat{k}_i\}\mathbf{b}_i|\hat{k})$$

$\frac{1+\delta\cos\alpha}{1+\delta}$ for all $i \in \{1,2\}$ cannot constitute an SSMPE unless $\frac{1+\delta\cos\alpha}{1+\delta} = 1$, that is unless $\delta = 0$ or $\alpha = 0$. Suppose, towards a contradiction, that $\delta > 0$, $\alpha > 0$ and $\hat{k}_i^* = \frac{1+\delta\cos\alpha}{1+\delta}$. This implies that $\hat{k}_i^* < 1$ for all $i \in \{1,2\}$. Suppose, without loss of generality, that $b_1 \geq b_2$. Then Lemma A1 implies that there exists $\bar{\epsilon} > 0$ such that for all $\epsilon \in (0, \bar{\epsilon})$, $U_1'(\hat{k}_1^* + \epsilon|\hat{k}^*) > 0$, which implies that $U_1(\hat{k}_1^*|\hat{k}^*) < U_1(\hat{k}_1^* + \epsilon|\hat{k}^*)$, $\mathbf{p}_1((\hat{k}_1^* + \epsilon)\mathbf{b}_1) = \hat{k}_1^*\mathbf{b}_1$ and $d((\hat{k}_1^* + \epsilon)\mathbf{b}_1) < d((\hat{k}_1^* + \bar{\epsilon})\mathbf{b}_1)$, which means party 1 is not maximizing its dynamic utility when at the opposition. Furthermore, an argument similar to the one used in the second contradiction above implies that $\hat{k}_1^* = \hat{k}_2^* = 1$ cannot constitute an SSMPE unless $\delta = 0$ or $\alpha = 0$.

This leaves three possible cases. *Case 1:* $\delta = 0$ or $\alpha = 0$ and $\hat{k}_1^* = \hat{k}_2^* = 1$. When $\delta = 0$ or $\alpha = 0$, clearly $\hat{k}^* = (1,1)$ constitutes a SSMPE and, because $\hat{k}_i^* \in \{\frac{1+\delta\cos\alpha}{1+\delta}, 1\}$ for $i \in \{1,2\}$, this SSMPE is unique. *Case 2:* $\delta > 0$ and $\alpha > 0$ and $\hat{k}_1^* = 1$ with $\hat{k}_2^* = \frac{1+\delta\cos\alpha}{1+\delta}$. *Case 3:* $\delta > 0$ and $\alpha > 0$ and $\hat{k}_1^* = \frac{1+\delta\cos\alpha}{1+\delta}$ with $\hat{k}_2^* = 1$.

We first claim that when $b_1 > b_2$, Case 3 cannot constitute a SSMPE. When $\delta > 0$ and $\alpha > 0$, $\frac{1+\delta\cos\alpha}{1+\delta} < 1$. From $\hat{k}_1^* = \frac{1+\delta\cos\alpha}{1+\delta}$, $\hat{k}_2^* = 1$ and $b_1 > b_2$, it follows $\frac{1+\delta\cos\alpha}{1+\delta}b_2 < b_2 = \hat{k}_2^* b_2$ and $\frac{1+\delta\cos\alpha}{1+\delta}b_2 < \frac{1+\delta\cos\alpha}{1+\delta}b_1 = \hat{k}_1^* b_1$, or $\frac{1+\delta\cos\alpha}{1+\delta} < \min\{\hat{k}_1^*\frac{b_1}{b_2}, \hat{k}_2^*\}$. We can now use argument similar to the one used in the second contradiction above to establish contradiction with party 2 maximizing its dynamic utility when in opposition.

It remains to be shown that, when $b_1 > b_2$, Case 2 constitutes a unique SSMPE and that, when $b_1 = b_2$, both Cases 2 and 3 constitute an SSMPE. We cover Case 2 irrespective of whether $b_1 > b_2$ or $b_1 = b_2$. When $b_1 = b_2$, Case 3 is similar to Case 2 and is omitted.

Suppose $\delta > 0$, $\alpha > 0$ and $b_1 \geq b_2$. We need to show that $\hat{k}_1 = 1$ and $\hat{k}_2 = \frac{1+\delta\cos\alpha}{1+\delta}$ constitute an SSMPE. Take any incumbent's policy commitment $\mathbf{q} \in X$. The optimization problem of the opposition party regarding the policy to contest elections with is $\max_{k\in[0,k_i(\mathbf{q})]} U_i(k|\hat{k})$. For any $\mathbf{q} \in X$ such that $k_2(\mathbf{q}) > \hat{k}_2$, from Lemma A1 we have $\lim_{k\to\hat{k}_2^-} U_2'(k|\hat{k}) \geq 0$ and $\lim_{k\to\hat{k}_2^+} U_2'(k|\hat{k}) \leq 0$. By piece-wise strict concavity of U_2 established in the same lemma, $U_2(k|\hat{k})$ is, for any $\mathbf{q} \in X$, increasing in k on $[0, \min\{\hat{k}_2, k_2(\mathbf{q})\}]$ and decreasing in k on $[\hat{k}_2, \max\{\hat{k}_2, k_2(\mathbf{q})\}]$.

When $k_2(\mathbf{q}) \leq \hat{k}_2$, it is optimal for party 2 to contest elections with $k_2(\mathbf{q})\mathbf{b}_2$ and when $k_2(\mathbf{q}) > \hat{k}_2$, it is optimal to run with $\hat{k}_2\mathbf{b}_2$. The simple strategy with $\hat{k}_2$ is thus optimal for party 2. A similar argument can be used to show optimality of the simple strategy with $\hat{k}_1$ for party 1. The key to this claim is Lemma A1 along with $\hat{k}_2 b_2 < \hat{k}_1 b_1$ and $\hat{k}_2 b_2 = \frac{1+\delta\cos\alpha}{1+\delta} b_2 < \frac{1+\delta\cos\alpha}{1+\delta} b_1$.

To finish the proof of parts 1 and 2 of the proposition, we have to show that none of the parties, when at the opposition, has a profitable deviation from staying out of the election. Take any $\hat{k}^*$ characterized in the three cases above and any incumbent's policy commitment $\mathbf{q} \in X$. The dynamic utility of opposition party $i \in \{1, 2\}$ on the equilibrium path is $u_i(\mathbf{p}_i(\mathbf{q})) + r + \delta V_i^i(\mathbf{p}_i(\mathbf{q})|\hat{k}^*)$ whereas the dynamic utility from staying out is $u_i(\mathbf{q}) + \delta V_i^{-i}(\mathbf{q}|\hat{k}^*)$. Because on the equilibrium path i contests elections, $V_i^{-i}(\mathbf{q}|\hat{k}^*) = u_i(\mathbf{p}_i(\mathbf{q})) + r + \delta V_i^i(\mathbf{p}_i(\mathbf{q})|\hat{k}^*)$. We need to ensure that the on-path dynamic utility is larger than the off-path one, or $u_i(\mathbf{p}_i(\mathbf{q})) + r + \delta V_i^i(\mathbf{p}_i(\mathbf{q})|\hat{k}^*) \geq \frac{u_i(\mathbf{q})}{1-\delta}$. We derive an upper bound on the right hand side of the inequality and a lower bound on the left hand side of the inequality and show that the upper bound is smaller than the lower bound.

The upper bound is clearly $0 \geq \frac{u_i(\mathbf{q})}{1-\delta}$. We construct the lower bound as follows. $u_i(\mathbf{p}_i(\mathbf{q})) + r + \delta V_i^i(\mathbf{p}_i(\mathbf{q})|\hat{k}^*)$ is the dynamic utility of opposition party i from running with the optimal policy, given incumbent's policy commitment $\mathbf{q} \in X$. Because $\mathbf{p}_i$ is a simple strategy, $U_i(\min\{k_i(\mathbf{q}), \hat{k}_i^*\}|\hat{k}^*) = u_i(\mathbf{p}_i(\mathbf{q})) + r + \delta V_i^i(\mathbf{p}_i(\mathbf{q})|\hat{k}^*)$. From Lemma A1 and the discussion that followed, $U_i(k|\hat{k}^*)$ is increasing in k on $[0, \hat{k}_i^*]$ in any SSMPE. Hence $U_i(0|\hat{k}^*) \leq u_i(\mathbf{p}_i(\mathbf{q})) + r + \delta V_i^i(\mathbf{p}_i(\mathbf{q})|\hat{k}^*)$ for any $\mathbf{q} \in X$. From the proof of Lemma A1, $U_i(0|\hat{k}^*) = \frac{-d^2(0,\mathbf{b}_i)(1+\delta)+r}{1-\delta^2}$. $\frac{-d^2(0,\mathbf{b}_i)(1+\delta)+r}{1-\delta^2} \geq 0$ or equivalently $r \geq d^2(0, \mathbf{b}_i)(1+\delta)$ then ensures that none of the parties, when at the opposition, wants to stay out of the elections. This concludes the proof of parts 1 and 2.

To prove part 4 of the proposition, take the SSMPE $\hat{k}_1^* = 1$ and $\hat{k}_2^* = \frac{1+\delta\cos\alpha}{1+\delta}$ and $b_1 \geq b_2$, so that $\hat{k}_1^* b_1 \geq \hat{k}_2^* b_2$. Starting from $\mathbf{q} \in X$ such that $k_2(\mathbf{q}) \leq \hat{k}_2^*$, $k_2(\mathbf{q})b_2 = k_1(\mathbf{q})b_1 \leq \hat{k}_2^* b_2 \leq \hat{k}_1^* b_1$, both parties run with $k_i(\mathbf{q})\mathbf{b}_i$ when at the opposition in the first period. Since $d(k_i(\mathbf{q})\mathbf{b}_i) = d(\mathbf{q})$, the incumbent's policy commitment at the beginning of the second period is a policy at the same distance from the origin as $\mathbf{q}$. Thus both parties run with $k_i(\mathbf{q})\mathbf{b}_i$ when at the opposition in the second period. The same holds in any future period. Starting from $\mathbf{q} \in X$ such that $k_2(\mathbf{q}) \geq \hat{k}_2^*$, if party 2 is at the opposition in the first period it runs with $\hat{k}_2^*\mathbf{b}_2$ and if party 1 is at the opposition it runs with $\min\{k_1(\mathbf{q}), \hat{k}_1^*\}\mathbf{b}_1$. In the former case, the incumbent's policy commitment at the beginning of the second period satisfies $k_2(\hat{k}_2^*\mathbf{b}_2) = \hat{k}_2^*$, so that the policies alternate on $k_2(\hat{k}_2^*\mathbf{b}_2)\mathbf{b}_2 = \hat{k}_2^*\mathbf{b}_2$ and $k_1(\hat{k}_2^*\mathbf{b}_2)\mathbf{b}_1$ starting from period 2. In the latter case, because $k_2(\min\{k_1(\mathbf{q}), \hat{k}_1^*\}\mathbf{b}_1) = \min\{k_1(\mathbf{q}), \hat{k}_1^*\}\frac{b_1}{b_2} = \min\{k_2(\mathbf{q}), \hat{k}_1^*\frac{b_1}{b_2}\} \geq \hat{k}_2^*$, party 2 in the second period runs with $\hat{k}_2^*\mathbf{b}_2$ and the same alternation obtains from period 3 onwards. $\square$

Proof of Lemma 1

From the proof of Proposition 1 we know that when v uses her SSMPE voting strategy, then, for any $\hat{k}_1$ and $\hat{k}_2$, the opposition party $i \in \{1, 2\}$ contests elections and wins with policy $\mathbf{p}_i(\mathbf{q})$, for any incumbent's policy commitment $\mathbf{q} \in X$. Because $u_v(\mathbf{x}) = -d^2(\mathbf{x})$ is decreasing in $d(\mathbf{x})$ for any $\mathbf{x} \in X$ and $d(\mathbf{p}_i(\mathbf{q})) = \min\{k_i(\mathbf{q}), \hat{k}_i\}b_i$ for any $\mathbf{q} \in X$, all we have to show is that $\min\{k_i(\mathbf{q}), \hat{k}_i\}b_i$ for $i \in \{1, 2\}$ is non-decreasing in $\hat{k}_i$ and b_i for any $\mathbf{q} \in X$, and increasing in the same parameters for some $\mathbf{q}$. The non-decreasing part is immediate and the increasing part is easy to see from $\min\{k_i(\mathbf{q}), \hat{k}_i\}b_i = \hat{k}_i b_i > 0$ when $\hat{k}_i > 0$ and $k_i(\mathbf{q}) > \hat{k}_i$. $\square$

Proof of Proposition 3

By Proposition 1, $\hat{k}_1 = 1$ and $\hat{k}_2 = \frac{1+\delta\cos\alpha}{1+\delta}$. Notice that $\hat{k}_2 \in (0, 1]$ for any $\delta \in [0, 1)$ and $\alpha \in [0, \pi]$. To show part 1, using the transformation of polar coordinated to Cartesian ones — $\hat{k}_2\mathbf{b}_2 = (\hat{k}_2 b_2 \cos\alpha, \hat{k}_2 b_2 \sin\alpha)$ — d_p can be expressed as $d_p = 2b_2\hat{k}_2 \sin\frac{\alpha}{2}$. d_p is clearly non-decreasing in b_2, increasing in b_2 when $\alpha > 0$

and constant in b_1. To show part 2,

$$\frac{\partial\left[2b_2\frac{1+\delta\cos\alpha}{1+\delta}\sin\frac{\alpha}{2}\right]}{\partial\alpha}=\frac{b_2\cos\frac{\alpha}{2}}{1+\delta}\left[1-2\delta+3\delta\cos\alpha\right] \tag{A7}$$

where the first term is positive unless $\alpha=\pi$. Solving $1-2\delta+3\delta\cos\alpha=0$ gives $\alpha=\arccos\left[\frac{2\delta-1}{3\delta}\right]$. From $\{x|\arccos x\in\mathbb{R}\}=[-1,1]$, $-1\leq\frac{2\delta-1}{3\delta}\leq 1$ holds when $\delta\geq\frac{1}{5}$. Because $\arccos x\in[0,\pi)$ for $x\in(-1,1]$, $\arccos\left[\frac{2\delta-1}{3\delta}\right]<\pi$ when $\delta>\frac{1}{5}$. Defining $\alpha'=\arccos\left[\frac{2\delta-1}{3\delta}\right]$ when $\delta\geq\frac{1}{5}$ and $\alpha'=\pi$ when $\delta\in[0,\frac{1}{5})$, $\alpha'<\pi$ when $\delta>\frac{1}{5}$. That d_p is increasing in α when $\alpha\in[0,\alpha']$ and decreasing in α when $\alpha\in[\alpha',\pi]$ then follows from the fact that $\cos\alpha$, and hence $1-2\delta+3\delta\cos\alpha$, is decreasing in α. $\square$

A2 Extensions [For Online Publication]

A2.1 Model with General Utility Functions

The model analyzed in this section is identical to the one in the main part of the paper except for the stage utility player $i\in\{1,2,v\}$ derives from policy $\mathbf{p}$ which now is

$$u_i(\mathbf{p})=f(d(\mathbf{p},\mathbf{b}_i)) \tag{A8}$$

where $f:[0,\infty)\to\mathbb{R}$ is a continuous, decreasing and concave function in $d(\mathbf{p},\mathbf{b}_i)$. We also assume that f is twice continuously differentiable on $(0,\infty)$. Notice that these assumptions allow $f'(0)=0$ and $f(x)=-x^2$, so that the model in the paper is a special case of the model analyzed here.

For space considerations, we refrain from repeating the arguments leading to Proposition 1 and stress only those aspects of the analysis that differ considerably. An argument similar to the one from the proof of Proposition 1 shows that, since parties use simple strategies, it is i) optimal for $i\in\{1,2\}$ to contest elections with policies located only on the $\mathbf{b}_i$-ray and ii) optimal for v to vote for the opposition party running with $\mathbf{p}$ when the incumbent's policy commitment is $\mathbf{q}$ if and only if $d(\mathbf{p})\leq d(\mathbf{q})$.

In an SSMPE, the extent of moderation of party $i \in \{1,2\}$ is the k which maximizes $\tilde{U}_i(k)$ where, for $k \geq 0$,

$$\begin{aligned}\tilde{U}_i(k) &= u_i(k\mathbf{b}_i) + u_i(kb_{ir}\mathbf{b}_{-i})\delta \\ &= f(d(k\mathbf{b}_i, \mathbf{b}_i)) + f(d(kb_{ir}\mathbf{b}_{-i}, \mathbf{b}_i))\delta.\end{aligned} \tag{A9}$$

$\tilde{U}_i(k)$ is the dynamic utility party i receives, devoid of any constant terms, when it runs with policy $k\mathbf{b}_i$ and party $-i$, not moderating to a larger extent, runs with $kb_{ir}\mathbf{b}_{-i}$, where we are using the shorthand $b_{ir} = \frac{b_i}{b_{-i}}$. The derivation of $\tilde{U}_i(k)$ is similar to the derivation of (A6) and is not repeated here.

Condition A1 (Non trivial model)**.** *$\delta \in (0,1)$ and $\alpha \neq 0$.*

Condition A2 (Preserving concavity)**.** *If $\alpha = \pi$, then $f'' < 0$.*[22]

Condition A3 (Concavity at origin)**.** *$f'(0) > f'(b_i\sqrt{2(1-\cos\alpha)})\delta \sin\frac{\alpha}{2}$ for $i \in \{1,2\}$.*

Lemma A2. *$\tilde{U}_i(k)$ is*

1. *continuous and twice continuously differentiable for $k \neq 1$*
2. *increasing for $k \in [0, \cos\alpha]$*
3. *decreasing for $k \geq 1$*
4. *if Conditions A1 and A2 hold, strictly concave for $k \in (\max\{0, \cos\alpha\}, 1)$*
5. *if Condition A1 fails, increasing for $k \in (\max\{0, \cos\alpha\}, 1)$*
6. *if Condition A2 fails, increasing for $k \in (0,1)$*

Proof. The continuity in part 1 follows from the continuity of f, $d(k\mathbf{b}_i, \mathbf{b}_i)$ and $d(kb_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)$. The differentiability follows from the derivatives below, which can

[22] With a slight abuse of terminology, when Condition A2 fails we mean $\alpha = \pi$ and $f'' = 0$, that is, f is linear. Formally, the failure of $f'' < 0$ permits an f that is linear in some parts of its domain and strictly concave in others. Characterizing these intermediate cases provides little additional insights.

be easily checked

$$d(k\mathbf{b}_i,\mathbf{b}_i) = b_i|1-k| \qquad d(kb_{ir}\mathbf{b}_{-i},\mathbf{b}_i) = b_i\sqrt{(k-\cos\alpha)^2+\sin^2\alpha}$$

$$d'(k\mathbf{b}_i,\mathbf{b}_i)\begin{cases} <0 & \text{if } k\in[0,1) \\ \nexists & \text{if } k=1 \\ >0 & \text{if } k\in(1,\infty) \end{cases} \qquad d'(kb_{ir}\mathbf{b}_{-i},\mathbf{b}_i)\begin{cases} <0 & \text{if } k<\cos\alpha \\ =0 & \text{if } k=\cos\alpha\wedge\alpha\neq 0 \\ \nexists & \text{if } k=\cos\alpha\wedge\alpha=0 \\ >0 & \text{if } k>\cos\alpha \end{cases}$$

$$d''(k\mathbf{b}_i,\mathbf{b}_i)\begin{cases} =0 & \text{if } k\neq 1 \\ \nexists & \text{if } k=1 \end{cases} \qquad d''(kb_{ir}\mathbf{b}_{-i},\mathbf{b}_i)\begin{cases} >0 & \text{if } \alpha\notin\{0,\pi\} \\ 0 & \text{if } \begin{array}{l}\alpha=\pi\vee\\(k\neq 1\wedge\alpha=0)\end{array} \\ \nexists & \text{if } k=1\wedge\alpha=0. \end{cases}$$

For parts 2 and 3, we have

$$\tilde{U}_i'(k) = f'(d(k\mathbf{b}_i,\mathbf{b}_i))d'(k\mathbf{b}_i,\mathbf{b}_i) + f'(d(kb_{ir}\mathbf{b}_{-i},\mathbf{b}_i))d'(kb_{ir}\mathbf{b}_{-i},\mathbf{b}_i)\delta \tag{A10}$$

and direct verification shows that $\tilde{U}_i'(k)>0$ for $k<\cos\alpha$, $\lim_{k\to\cos\alpha^-}\tilde{U}_i'(k)\geq 0$, $\tilde{U}_i'(k)<0$ for $k>1$ and $\lim_{k\to 1^+}\tilde{U}_i'(k)\leq 0$.

For the strict concavity for $k\in(\max\{0,\cos\alpha\},1)$ in part 4, we have

$$\begin{aligned}\tilde{U}_i''(k) = \quad & f''(d(k\mathbf{b}_i,\mathbf{b}_i))\left[d'(k\mathbf{b}_i,\mathbf{b}_i)\right]^2 + f'(d(k\mathbf{b}_i,\mathbf{b}_i))d''(k\mathbf{b}_i,\mathbf{b}_i)+ \\ & \delta f''(d(kb_{ir}\mathbf{b}_{-i},\mathbf{b}_i))\left[d'(kb_{ir}\mathbf{b}_{-i},\mathbf{b}_i)\right]^2 + f'(d(kb_{ir}\mathbf{b}_{-i},\mathbf{b}_i))d''(kb_{ir}\mathbf{b}_{-i},\mathbf{b}_i)\delta.\end{aligned} \tag{A11}$$

$\tilde{U}_i''(k)\leq 0$ is a consequence of the fact that all the summands of the expression are non-positive. To see that $\tilde{U}_i''(k)<0$ under Conditions A1 and A2, note that the last summand is either zero (when $d''(kb_{ir}\mathbf{b}_{-i},\mathbf{b}_i)=0\Rightarrow\alpha=\pi$), which implies that the next to last summand is negative ($\alpha=\pi\Rightarrow f''<0$ by Condition A2), or negative.

For part 5, which claims that $\tilde{U}_i(k)$ is increasing on $(\max\{0,\cos\alpha\},1)$ when Condition A1 fails, notice that the failure of the condition implies either $\delta=0$ or

$\alpha = 0$. In the latter case the $(\max\{0, \cos\alpha\}, 1)$ interval is empty so assume $\delta = 0$ and $\alpha > 0$. Substituting $\delta = 0$ into (A9) gives $\tilde{U}_i(k) = f(d(k\mathbf{b}_i, \mathbf{b}_i))$ so that $\tilde{U}_i(k)$ is increasing on $(\max\{0, \cos\alpha\}, 1)$. Finally, when Condition A2 in part 6 fails, we have $\alpha = \pi$ and $f'' = 0$ so that $d'(k\mathbf{b}_i, \mathbf{b}_i) = -d'(kb_{ir}\mathbf{b}_{-i}, \mathbf{b}_i) = -b_i$ for $k \in (0, 1)$. This implies that $\tilde{U}_i'(k) = (1 - \delta)c$ where $c > 0$ so that $\tilde{U}_i(k)$ is increasing on $(0, 1)$. ☐

Proposition A1 (SSMPE with General Utility). *When the stage utility of $i \in \{1, 2, v\}$ is $u_i(\mathbf{p}) = f(d(\mathbf{p}, \mathbf{b}_i))$ where f is a twice continuously differentiable, decreasing and concave function, then:*

1. *if Condition A1 fails, $\hat{k}_1 = \hat{k}_2 = 1$ characterize the unique SSMPE;*
2. *if Condition A2 fails, $\hat{k}_1 = \hat{k}_2 = 1$ characterize the unique SSMPE;*
3. *if Conditions A1 and A2 hold, there exists a unique κ_i for $i \in \{1, 2\}$ given by $\kappa_i = \arg\max_{k\geq 0} \tilde{U}_i(k) \in [\max\{0, \cos\alpha\}, 1]$ and either $\kappa_i b_i > \kappa_{-i} b_{-i}$, in which case $\hat{k}_i = 1$ and $\hat{k}_{-i} = \kappa_{-i}$ characterize the unique SSMPE, or $\kappa_1 b_1 = \kappa_2 b_2$, in which case there exist exactly two SSMPE characterized by $\hat{k}_1 = 1$, $\hat{k}_2 = \kappa_2$ and $\hat{k}_1 = \kappa_1$, $\hat{k}_2 = 1$.*

Proof. When Condition A1 fails, either $\delta = 0$ or $\alpha = 0$, so that none of the players has any incentive to moderate. By Lemma A2 there exists a unique maximizer of $\tilde{U}_i(k)$ for $i \in \{1, 2\}$, $k = 1$, and using arguments similar to the proof of Proposition 1, there exists a unique SSMPE characterized in part 1 of the proposition. When Condition A2 fails, then by Lemma A2 the maximizer of $\tilde{U}_i(k)$ is $k = 1$ for $i \in \{1, 2\}$. Repeating the same argument just made, there exists a unique SSMPE characterized in part 2 of the proposition.

When both Conditions A1 and A2 hold in part 3, the uniqueness of $\kappa_i = \arg\max_{k\geq 0} \tilde{U}_i(k)$ and $\kappa_i \in [\max\{0, \cos\alpha\}, 1]$ follows from Lemma A2. Recalling again the proof of Proposition 1, uniqueness/multiplicity of a SSMPE and its characterization follows.[23] ☐

[23] With general utility it is not necessarily true that $b_1 > b_2$ implies $\kappa_1 b_1 > \kappa_2 b_2$, even though counter-examples are difficult to produce. Using similar argument as in the proof of Proposition

Proposition A2 (SSMPE Comparative Static with General Utility)**.**
Assume Conditions A1, A2 and A3 hold. Then $\kappa_i \in (\max\{0, \cos\alpha\}, 1)$ *for* $i \in \{1,2\}$ *in Proposition A1 and (assuming* $\alpha \neq \pi$ *for the first relation)*

$$\frac{\partial \kappa_i}{\partial \alpha} < 0 \qquad \frac{\partial \kappa_i}{\partial \delta} < 0 \tag{A12}$$

Proof. From Proposition A1, if Conditions A1 and A2 hold, $\kappa_i = \arg\max_{k \geq 0} \tilde{U}_i(k) \in [\max\{0, \cos\alpha\}, 1]$ for $i \in \{1,2\}$. We need to show that Condition A3 implies $\kappa_i \in (\max\{0, \cos\alpha\}, 1)$ for $i \in \{1,2\}$. We show that Condition A3 is required only for $\kappa_i < 1$ and that $\kappa_i > \max\{0, \cos\alpha\}$ holds in general. Fix $i \in \{1,2\}$. By Lemma A2 part 4, $\tilde{U}_i(k)$ is strictly concave for $k \in (\max\{0, \cos\alpha\}, 1)$. To show that $\kappa_i < 1$, it thus suffices to show that $\lim_{k\to 1^-} \tilde{U}_i'(k) < 0$. Substituting

$$\begin{aligned}
\lim_{k\to 1^-} d(k\mathbf{b}_i, \mathbf{b}_i)) &= 0 & \lim_{k\to 1^-} d'(k\mathbf{b}_i, \mathbf{b}_i)) &= -b_i \\
\lim_{k\to 1^-} d(kb_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)) &= b_i\sqrt{2(1-\cos\alpha)} & \lim_{k\to 1^-} d'(kb_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)) &= b_i \sin\tfrac{\alpha}{2}
\end{aligned} \tag{A13}$$

into (A10) gives $\lim_{k\to 1^-} \tilde{U}_i'(k) = -b_i[f'(0) - f'(b_i\sqrt{2(1-\cos\alpha)})\delta \sin\frac{\alpha}{2}] < 0$, where the inequality follows from Condition A3. To show that $\kappa_i > \max\{0, \cos\alpha\}$, it suffices to show that $\lim_{k\to 0^+} \tilde{U}_i'(k) > 0$ when $\alpha \geq \frac{\pi}{2}$ and $\lim_{k\to \cos\alpha^+} \tilde{U}_i'(k) > 0$ when $\alpha < \frac{\pi}{2}$. When $\alpha \geq \frac{\pi}{2}$, substituting

$$\begin{aligned}
\lim_{k\to 0^+} d(k\mathbf{b}_i, \mathbf{b}_i)) &= b_i & \lim_{k\to 0^+} d'(k\mathbf{b}_i, \mathbf{b}_i)) &= -b_i \\
\lim_{k\to 0^+} d(kb_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)) &= b_i & \lim_{k\to 0^+} d'(kb_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)) &= -b_i\cos\alpha
\end{aligned} \tag{A14}$$

into (A10) gives $\lim_{k\to 0^+} \tilde{U}_i'(k) = -b_i f'(b_i)(1 + \delta\cos\alpha) > 0$. When $\alpha < \frac{\pi}{2}$, substituting

$$\begin{aligned}
\lim_{k\to \cos\alpha^+} d(k\mathbf{b}_i, \mathbf{b}_i)) &= b_i(1-\cos\alpha) & \lim_{k\to \cos\alpha^+} d'(k\mathbf{b}_i, \mathbf{b}_i)) &= -b_i \\
\lim_{k\to \cos\alpha^+} d(kb_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)) &= b_i\sin\alpha & \lim_{k\to \cos\alpha^+} d'(kb_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)) &= 0
\end{aligned} \tag{A15}$$

A2 below, a sufficient condition for $b_1 > b_2 \Rightarrow \kappa_1 b_1 > \kappa_2 b_2$ is $\frac{\partial}{\partial b_i}\tilde{U}_i'(\kappa_i) \geq 0$. This condition rewrites as $\kappa_2 \leq \frac{z+\delta\cos\alpha}{z+\delta}$ where $z = \frac{f''(b_i(1-\kappa_i))}{f''(b_i\sqrt{(\kappa_i-\cos\alpha)^2+\sin^2\alpha})} > 0$ and holds for high z or low δ and α. The counter-example we were able to produce uses high δ and α along with $f(x) = -\exp x^2$.

into (A10) gives $\lim_{k \to \cos\alpha^+} \tilde{U}_i'(k) = -b_i f'(b_i(1-\cos\alpha)) > 0$.

Since $\kappa_i \in (\max\{0, \cos\alpha\}, 1)$ under the conditions of the proposition, it is implicitly defined by $\tilde{U}_i'(\kappa_i) = 0$. From the implicit function theorem $\frac{\partial \kappa_i}{\partial x} = \frac{\frac{\partial}{\partial x}\tilde{U}_i'(\kappa_i)}{-\tilde{U}_i''(\kappa_i)}$. The denominator of this expression is positive by strict concavity of $\tilde{U}_i(k)$. The numerator of this expression for $x \in \{\alpha, \delta\}$ is

$$\begin{aligned}\tfrac{\partial}{\partial\alpha}\tilde{U}_i'(\kappa_i) &= f''(d(\kappa_i b_{ir}\mathbf{b}_{-i}, \mathbf{b}_i))d'(\kappa_i b_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)\frac{\partial d(\kappa_i b_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)}{\partial\alpha}\delta \\ &\quad + f'(d(\kappa_i b_{ir}\mathbf{b}_{-i}, \mathbf{b}_i))\frac{\partial d'(\kappa_i b_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)}{\partial\alpha}\delta \\ \tfrac{\partial}{\partial\delta}\tilde{U}_i'(\kappa_i) &= f'(d(\kappa_i b_{ir}\mathbf{b}_{-i}, \mathbf{b}_i))d'(\kappa_i b_{ir}\mathbf{b}_{-i}, \mathbf{b}_i)\end{aligned} \tag{A16}$$

where both expressions are negative since $\frac{\partial}{\partial\alpha}d(\kappa_i b_{ir}\mathbf{b}_{-i}, \mathbf{b}_i) = \frac{b_i\kappa_i\sin\alpha}{\sqrt{(\kappa_i-\cos\alpha)^2+\sin^2\alpha}} > 0$ and $\frac{\partial}{\partial\alpha}d'(\kappa_i b_{ir}\mathbf{b}_{-i}, \mathbf{b}_i) = \frac{b_i\sin\alpha(1-\kappa_i\cos\alpha)}{\left(\sqrt{(\kappa_i-\cos\alpha)^2+\sin^2\alpha}\right)^3} > 0$. □

A2.2 Forward Looking Representative Voter

In this section we study a version of the model from the main part of the paper in which the representative voter is forward-looking. We will show that for any SSMPE identified in Proposition 1, there exists an equilibrium with forward-looking v and that this equilibrium generates a comparative static on the representative voter's welfare with respect to antagonism and extremism which is identical to the one stated in Corollary 1.

Throughout this section assume $\delta_v \in [0, 1)$. The model from the main part requires the following changes. The utility v derives from a sequence of policies $\mathbf{P} = \{\mathbf{p}_0, \mathbf{p}_1, \ldots\}$ is the discounted sum of payoffs from each period

$$U_v(\mathbf{P}) = \sum_{t=0}^{\infty}\delta_v^t u_v(\mathbf{p}_t). \tag{A17}$$

Given a path of policies generated by play according to $\sigma = (\sigma_1, \sigma_2, \sigma_v)$ starting from an incumbent $j \in \{1, 2\}$ committed to $\mathbf{q}$, $\mathbf{P}(j, \sigma, \mathbf{q}) = \{\mathbf{p}_0, \mathbf{p}_1, \ldots\}$, the dynamic utility of v is

$$V_v^j(\mathbf{q}|\sigma) = U_v(\mathbf{P}(j,\sigma,\mathbf{q})) = \sum_{t=0}^{\infty} \delta_v^t u_v(\mathbf{p}_t). \tag{A18}$$

In Definition 2 of SMPE, we now require $\sigma_v^*(j,\mathbf{q},\mathbf{p}) = Yes$ if and only if

$$u_v(\mathbf{p}) + \delta_v V_v^i(\mathbf{p}|\sigma^*) \geq u_v(\mathbf{q}) + \delta_v V_v^j(\mathbf{q}|\sigma^*) \tag{A19}$$

for any incumbent $j \in \{1,2\}$ committed to $\mathbf{q} \in X$ and the opposition $i = \{1,2\} \setminus \{j\}$ contesting elections with $\mathbf{p} \in X$. Given a profile of strategies σ, assuming the initial incumbent's policy commitment $\mathbf{q}^0$ is distributed according to $F(\mathbf{q}^0)$ with strictly positive density on X, and denoting with $\rho \in (0,1)$ the probability that party 1 is the first-period incumbent, the representative voter's ex-ante welfare from the dynamic electoral competition game with parties' extremism b_1 and b_2 and antagonism α is

$$W(b_1,b_2,\alpha|\sigma) = \int_X \rho V_v^1(\mathbf{z}|\sigma) + (1-\rho)V_v^2(\mathbf{z}|\sigma)d(F(\mathbf{z})). \tag{A20}$$

Proposition A3. *For any SSMPE σ' in the model with $\delta_v = 0$ identified in Proposition 1, there exists a SMPE σ^* in the model with $\delta_v \in (0,1)$ such that:*

1. *in σ^*, elections are always contested and incumbents always defeated;*
2. *starting from $\mathbf{q} \in X$, policies converge to identical alternation under σ' and σ^*;*
3. *$W(b_1,b_2,\alpha|\sigma^*)$ is non-decreasing in α; increasing in α if $\delta > 0$; decreasing in b_i for $i \in \{1,2\}$.*

Proof. Fix the SSMPE $\sigma' = (\sigma_1', \sigma_2', \sigma_v')$ from Proposition 1. We construct an SMPE $\sigma^* = (\sigma_1^*, \sigma_2^*, \sigma_v^*)$ for the model with $\delta_v \in [0,1)$ and then show that it possesses the properties from parts 1 through 3. Assume that σ' is such that party 2 moderates, that is $\sigma_1'(\mathbf{q}) = \mathbf{p}_1(\mathbf{q})$ with $\hat{k}_1 = 1$ and $\sigma_2'(\mathbf{q}) = \mathbf{p}_2(\mathbf{q})$ with

$\hat{k}_2 = \frac{1+\delta\cos\alpha}{1+\delta}$ for any $\mathbf{q} \in X$. For the 'mirror' SSMPE identified in Proposition 1 part 2, the proof is similar and omitted.[24]

Denote $\tilde{k}_2 = \sqrt{\left(\frac{b_1}{b_2}\right)^2 (1-\delta_v) + \delta_v \hat{k}_2^2}$ and note that, because $\hat{k}_2 \leq 1$ and $\frac{b_1}{b_2} \geq 1$, $\hat{k}_2 \leq \tilde{k}_2 \leq \frac{b_1}{b_2}$. For any $\mathbf{q} \in X$ define

$$\tilde{p}_1(\mathbf{q}) = \begin{cases} k_1(\mathbf{q}) & \text{for} \quad k_2(\mathbf{q}) \leq \hat{k}_2 \\ k_1(\mathbf{q})\sqrt{\frac{1-\delta_v\left(\frac{\hat{k}_2 b_2}{d(\mathbf{q})}\right)^2}{1-\delta_v}} & \text{for} \quad k_2(\mathbf{q}) \in (\hat{k}_2, \tilde{k}_2) \\ 1 & \text{for} \quad k_2(\mathbf{q}) \geq \tilde{k}_2. \end{cases} \tag{A21}$$

Standard arguments show that $\tilde{p}_1(\mathbf{x}) = \tilde{p}_1(\mathbf{y})$ for any $\mathbf{x} \in X$ and $\mathbf{y} \in X$ if $d(\mathbf{x}) = d(\mathbf{y})$ and that $\tilde{p}_1(\mathbf{x})$ is continuous in $d(\mathbf{x})$ and increasing in $d(\mathbf{x})$ if $k_2(\mathbf{x}) \in [0, \tilde{k}_2]$.

We now construct σ^*. For party 2 set $\sigma_2^* = \sigma_2'$, that is σ_2^* is a simple strategy $\mathbf{p}_2$ from Definition 3 with $\hat{k}_2 = \frac{1+\delta\cos\alpha}{1+\delta}$. For party 1 and any $\mathbf{q} \in X$, set $\sigma_1^*(\mathbf{q}) = \tilde{p}_1(\mathbf{q})\mathbf{b}_1$ and note that $\sigma_1^* = \sigma_1'$ when $\delta_v = 0$. For v set σ_v^* such that $\sigma_v^*(j, \mathbf{q}, \mathbf{p}) = Yes$ if and only if, when $j = 1$

$$\begin{aligned} & d(\mathbf{p}) \leq d(\mathbf{q}) && \text{for} \quad k_2(\mathbf{q}) \leq \hat{k}_2 \\ & d(\mathbf{p}) \leq \sqrt{d^2(\mathbf{q})(1-\delta_v) + \delta_v(\hat{k}_2 b_2)^2} && \text{for} \quad k_2(\mathbf{q}) \in (\hat{k}_2, \tfrac{b_1}{b_2}) \\ & d(\mathbf{p}) \leq \sqrt{d^2(\mathbf{q}) - \delta_v b_1^2 + \delta_v(\hat{k}_2 b_2)^2} && \text{for} \quad k_2(\mathbf{q}) \geq \tfrac{b_1}{b_2} \end{aligned} \tag{A22}$$

[24] The reason why σ' is not an SMPE when $\delta_v > 0$ is the fact that rejecting $\mathbf{p}$ of the opposition party and voting for incumbent's policy commitment $\mathbf{q}$ whenever $d(\mathbf{p}) > d(\mathbf{q})$ need not be optimal for v. To see this suppose the moderating party 2 is committed to $\mathbf{q} \in X$ with $d(\mathbf{q}) > \hat{k}_2 b_2$. According to σ_1', party 1 contests elections with $\mathbf{p} \in X$ such that $d(\mathbf{p}) = d(\mathbf{q})$. Suppose party 1 deviates and runs with $\mathbf{p}' \in X$ such that $d(\mathbf{p}') = d(\mathbf{q}) + \epsilon$ for some small $\epsilon > 0$. The dynamic utility of v from rejecting $\mathbf{p}'$ is $u_v(\mathbf{q}) + \delta_v u_v(\mathbf{p}) + \delta_v^2 \frac{u_v(\hat{k}_2\mathbf{b}_2)}{1-\delta_v}$ while the dynamic utility from accepting $\mathbf{p}'$ is $u_v(\mathbf{p}') + \delta_v \frac{u_v(\hat{k}_2\mathbf{b}_2)}{1-\delta_v}$. The condition for rejection to be optimal rewrites as $u_v(\mathbf{p}) - u_v(\mathbf{p}') > \delta_v(u_v(\hat{k}_2\mathbf{b}_2) - u_v(\mathbf{p}))$. The left hand side of the condition is positive for any $\epsilon > 0$ and tends to zero as $\epsilon \to 0$. Because the right hand side of the condition is positive whenever $\delta_v > 0$, there exists ϵ small enough such that the condition fails. Intuitively, when the moderating party 2 is committed to $\mathbf{q}$ such that $d(\mathbf{q}) > \hat{k}_2 b_2$, party 1, by contesting elections, releases party 2 from its policy commitment and starts the process of convergence to policies at distance $\hat{k}_2 b_2$ from the origin. Because $d(\mathbf{q}) > \hat{k}_2 b_2$, this provides v with a discrete increase in her future utility so that v is willing to accept a moderate decrease in her current utility.

and when $j = 2$

$$\begin{aligned}
&d(\mathbf{p}) \leq d(\mathbf{q}) && \text{for} \quad k_2(\mathbf{q}) \leq \hat{k}_2 \\
&d(\mathbf{p}) \leq \tilde{p}_1(\mathbf{q}) b_1 && \text{for} \quad k_2(\mathbf{q}) \in (\hat{k}_2, \tilde{k}_2) \\
&d(\mathbf{p}) \leq \sqrt{d^2(\mathbf{q}) + \delta_v b_1^2 - \delta_v (\hat{k}_2 b_2)^2} && \text{for} \quad k_2(\mathbf{q}) \geq \tilde{k}_2
\end{aligned} \tag{A23}$$

and note that $\sigma_v^* = \sigma_v'$ when $\delta_v = 0$.

We now argue that $\sigma^* = (\sigma_1^*, \sigma_2^*, \sigma_v^*)$ constitutes an SMPE. From (A22) and $\sigma_2^*(\mathbf{q}) = \min\{k_2(\mathbf{q}), \hat{k}_2\}\mathbf{b}_2$, party 2 contests and wins elections for any policy commitment $\mathbf{q} \in X$ of party 1. From (A23) and $\sigma_1^*(\mathbf{q}) = \tilde{p}_1(\mathbf{q})\mathbf{b}_1$, party 1 contests and wins elections for any policy commitment $\mathbf{q} \in X$ of party 2. Because elections are always contested and incumbents always defeated in σ' as well as in σ^*, because $\sigma_2^*(\mathbf{q}) = \sigma_2'(\mathbf{q})$ and $k_2(\sigma_2^*(\mathbf{q})) \leq \hat{k}_2$ for any $\mathbf{q} \in X$ and because $\sigma_1^*(\mathbf{q}) = \sigma_1'(\mathbf{q})$ for any $\mathbf{q} \in X$ such that $k_2(\mathbf{q}) \leq \hat{k}_2$, it follows that $V_1^1(\mathbf{q}|\sigma^*) = V_1^1(\mathbf{q}|\sigma')$ for any $\mathbf{q} \in X$ and $V_2^2(\mathbf{q}|\sigma^*) = V_2^2(\mathbf{q}|\sigma')$ for any $\mathbf{q} \in X$ such that $k_2(\mathbf{q}) \leq \hat{k}_2$. In addition, because $\tilde{p}_1(\mathbf{q}) \geq k_1(\mathbf{q})$ for any $\mathbf{q} \in X$ such that $k_2(\mathbf{q}) \leq \frac{b_1}{b_2}$ and because $\tilde{p}_1(\mathbf{q}) = \hat{k}_1 = 1$ for any $\mathbf{q} \in X$ such that $k_2(\mathbf{q}) \geq \frac{b_1}{b_2}$, it follows that $V_2^2(\mathbf{q}|\sigma^*) \leq V_2^2(\mathbf{q}|\sigma')$ for any $\mathbf{q} \in X$ such that $k_2(\mathbf{q}) > \hat{k}_2$.

From the proof of Proposition 1 we know that $u_1(k\mathbf{b}_1) + \delta V_1^1(k\mathbf{b}_1|\sigma')$ is increasing in k for $k \in [0, 1]$ and decreasing in k for $k \geq 1$. Since $V_1^1(k\mathbf{b}_1|\sigma^*) = V_1^1(k\mathbf{b}_1|\sigma')$ for any $k \in \mathbb{R}_{\geq 0}$, σ_1^* is optimal for party 1. For a policy commitment $\mathbf{q} \in X$ of party 2 such that $k_2(\mathbf{q}) \leq \tilde{k}_2$, the largest policy on the $\mathbf{b}_1$-ray that generates a victory of party 1 is $\tilde{p}_1(\mathbf{q})\mathbf{b}_1$, and the same policy maximizes the dynamic utility of party 1. For a policy commitment $\mathbf{q} \in X$ of party 2 such that $k_2(\mathbf{q}) > \tilde{k}_2$, running with policy $\mathbf{b}_1$ guarantees the electoral victory for party 1, and the same policy maximizes the dynamic utility of party 1.

From the proof of Proposition 1 we know that $u_2(k\mathbf{b}_2) + \delta V_2^2(k\mathbf{b}_2|\sigma')$ is increasing in k for $k \in [0, \hat{k}_2]$ and decreasing in k for $k \geq \hat{k}_2$. Since $V_2^2(k\mathbf{b}_2|\sigma^*) = V_2^2(k\mathbf{b}_2|\sigma')$ for $k \in [0, \hat{k}_2]$ and $V_2^2(k\mathbf{b}_2|\sigma^*) \leq V_2^2(k\mathbf{b}_2|\sigma')$ for $k > \hat{k}_2$, σ_2^* is optimal for party 2. For a policy commitment $\mathbf{q} \in X$ of party 1 such that $k_2(\mathbf{q}) \leq \hat{k}_2$, the largest policy on the $\mathbf{b}_2$-ray that generates the victory of party 2 is $k_2(\mathbf{q})\mathbf{b}_2$, and the same policy maximizes the dynamic utility of party 2. For a policy commitment $\mathbf{q} \in X$ of party 1 such that $k_2(\mathbf{q}) > \hat{k}_2$, running with policy $\hat{k}_2\mathbf{b}_2$ guarantees the electoral victory for party 2, and the same policy maximizes the dynamic utility of party 2.

It remains is to be shown that σ_v^* satisfies the definition of SMPE. From the definition, for any incumbent $j \in \{1,2\}$ committed to $\mathbf{q} \in X$ and the opposition $i = \{1,2\} \setminus \{j\}$ contesting elections with $\mathbf{p} \in X$, $\sigma_v^*(j, \mathbf{q}, \mathbf{p}) = Yes$ if and only if

$$u_v(\mathbf{p}) + \delta_v V_v^i(\mathbf{p}|\sigma^*) \geq u_v(\mathbf{q}) + \delta_v V_v^j(\mathbf{q}|\sigma^*). \tag{A24}$$

Using σ_1^*, σ_2^* and $u_v(\mathbf{q}) = -d^2(\mathbf{q})$ for any $\mathbf{q} \in X$, straightforward algebra gives

$$\begin{aligned} V_v^1(\mathbf{q}|\sigma^*) &= \begin{cases} -\frac{d^2(\mathbf{q})}{1-\delta_v} & \text{for} \quad k_2(\mathbf{q}) \leq \hat{k}_2 \\ -\frac{(\hat{k}_2 b_2)^2}{1-\delta_v} & \text{for} \quad k_2(\mathbf{q}) > \hat{k}_2 \end{cases} \\ V_v^2(\mathbf{q}|\sigma^*) &= \begin{cases} -\frac{d^2(\mathbf{q})}{1-\delta_v} & \text{for} \quad k_2(\mathbf{q}) \leq \tilde{k}_2 \\ -b_1^2 - \delta_v \frac{(\hat{k}_2 b_2)^2}{1-\delta_v} & \text{for} \quad k_2(\mathbf{q}) > \tilde{k}_2 \end{cases} \end{aligned} \tag{A25}$$

which can be used to derive σ_v^* as stated in (A22) and (A23). This concludes the proof that σ^* constitutes an SMPE.

Part 1 of the proposition, that elections are always contested and incumbents always defeated under σ^*, has already been noted. Part 2 claims that policies converge to identical alternation under σ' and σ^*. From Proposition 1, this means alternation between $k_1(\mathbf{q})\mathbf{b}_1$ and $k_2(\mathbf{q})\mathbf{b}_2$ when $k_2(\mathbf{q}) \leq \hat{k}_2$ and alternation between $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$ and $\hat{k}_2\mathbf{b}_2$ when $k_2(\mathbf{q}) > \hat{k}_2$. When $k_2(\mathbf{q}) \leq \hat{k}_2$, σ^* clearly generates the identical alternation since $\sigma_2^* = \sigma_2'$ and $\sigma_1^*(\mathbf{q}) = k_1(\mathbf{q})\mathbf{b}_1$. When $k_2(\mathbf{q}) > \hat{k}_2$, the first time party 2 contests elections it does so with $\sigma_2^*(\mathbf{q}) = \hat{k}_2\mathbf{b}_2$ and from then on the equilibrium policies alternate between $k_1(\hat{k}_2\mathbf{b}_2)\mathbf{b}_1$ and $\hat{k}_2\mathbf{b}_2$. To show part 3, taking derivatives with respect to α and b_i for $i \in \{1,2\}$ of the expressions in (A25) shows that $\frac{\partial}{\partial \alpha} V_v^j(\mathbf{q}|\sigma^*) \geq 0$ and $\frac{\partial}{\partial b_i} V_v^j(\mathbf{q}|\sigma^*) \leq 0$ for any $i \in \{1,2\}$, any $j \in \{1,2\}$ and any $\mathbf{q} \in X$. At the same time $\frac{\partial}{\partial b_1} V_v^2(\mathbf{q}|\sigma^*) < 0$ when $k_2(\mathbf{q}) > \tilde{k}_2$, $\frac{\partial}{\partial b_2} V_v^1(\mathbf{q}|\sigma^*) < 0$ when $k_2(\mathbf{q}) > \hat{k}_2$ and $\frac{\partial}{\partial \alpha} V_v^1(\mathbf{q}|\sigma^*) > 0$ when $k_2(\mathbf{q}) > \hat{k}_2$ and $\delta > 0$. $\square$

A2.3 Unique SPE with Finite Horizon

In this section, we analyze a finite horizon version of the model studied so far. We show that there exists a unique subgame perfect equilibrium (SPE) and describe the policy dynamics generated by this equilibrium strategy profile.

Assume the parties compete in T consecutive elections, where $T < \infty$. For simplicity, we assume that $b_1 = b_2$ and that the two parties have lexicographic preferences over office rents and policy outcomes. This implies that the opposition party, for any incumbent's policy commitment $\mathbf{q} \in X$, contests the election with a winning policy, if such a policy exists. Denote $\kappa = \frac{1+\delta\cos\alpha}{1+\delta}$ and note that $\kappa \in (0, 1]$ for any $\delta \in [0, 1)$ and $\alpha \in [0, \pi]$. We use the notation $\mathbf{p}_i(\mathbf{q}|\hat{k}_i)$ for the simple strategies from Definition 3, in order to make explicit their dependence on $\hat{k}_i$.

In any SPE, the voting strategy of the representative voter in any period has to satisfy, for any incumbent party $j \in \{1, 2\}$, any incumbent's policy commitment $\mathbf{q} \in X$ and any electoral platform of the opposition party $\mathbf{p} \in X$, $\sigma_v(j, \mathbf{q}, \mathbf{p}) = Yes$ if and only if $d(\mathbf{p}) \leq d(\mathbf{q})$.

We proceed by backward induction. Consider the last period T. For any incumbent's policy commitment $\mathbf{q} \in X$, the opposition party i contests elections with policy $\mathbf{p}^* \in \arg\max_{\mathbf{p}\in\{\mathbf{x}\in X|d(\mathbf{x})\leq d(\mathbf{q})\}} -d^2(\mathbf{p}, \mathbf{b}_i)$. Clearly, $\mathbf{p}^*$ has to lie on the $\mathbf{b}_i$-ray and, hence, can be written as $k\mathbf{b}_i$ for some $k \geq 0$. Denote $\tilde{U}_{i,T}(k) = -d^2(k\mathbf{b}_i, \mathbf{b}_i)$. Then the optimization problem of party i can be written as $\max_{k\in[0,k_i(\mathbf{q})]} \tilde{U}_{i,T}(k)$. This problem has a unique solution $k = 1$ if $k_i(\mathbf{q}) \geq 1$ and $k = k_i(\mathbf{q})$ if $k_i(\mathbf{q}) \leq 1$. In other words, $\mathbf{p}_i(\mathbf{q}|1)$ is the uniquely optimal strategy of the opposition party i for all $i \in \{1, 2\}$ and $\mathbf{q} \in X$. Denote $\mathbf{p}_{i,T}(\mathbf{q}) = \mathbf{p}_i(\mathbf{q}|1)$ for $i \in \{1, 2\}$.

Consider the period $T - 1$. For any incumbent's policy commitment $\mathbf{q} \in X$, the opposition party i contests elections with a policy on the $\mathbf{b}_i$-ray because both parties use simple strategies in period T that depend on $d(\mathbf{q})$ but not on the exact location of $\mathbf{q}$. The optimization problem of the opposition party i is $\max_{k\in[0,k_i(\mathbf{q})]} \tilde{U}_{i,T-1}(k)$ where $\tilde{U}_{i,T-1}(k) = -d^2(k\mathbf{b}_i, \mathbf{b}_i) - d^2(\mathbf{p}_{-i,T}(k\mathbf{b}_i), \mathbf{b}_i)\delta$. Because $\mathbf{p}_{-i,T}(k\mathbf{b}_i) = k\mathbf{b}_{-i}$ for $k \in [0,1]$ and $\mathbf{p}_{-i,T}(k\mathbf{b}_i) = \mathbf{b}_{-i}$ for $k \geq 1$,

$$
\begin{aligned}
\tilde{U}_{i,T-1}(k) &= -d^2(k\mathbf{b}_i, \mathbf{b}_i) - d^2(k\mathbf{b}_{-i}, \mathbf{b}_i)\delta \\
\tilde{U}_{i,T-1}(k) &= -d^2(k\mathbf{b}_i, \mathbf{b}_i) - d^2(\mathbf{b}_{-i}, \mathbf{b}_i)\delta
\end{aligned}
\tag{A26}
$$

when $k \in [0,1]$ and $k \geq 1$ respectively. An identical argument to the one used to prove Lemma A1 then shows that $\tilde{U}_{i,T-1}(k)$ has a unique maximum at $k = \kappa$, is increasing on $[0,\kappa]$ and decreasing on $[\kappa,\infty)$. Thus, the optimization problem of the opposition party i has a unique solution $k = \kappa$ if $k_i(\mathbf{q}) \geq \kappa$ and $k = k_i(\mathbf{q})$ if $k_i(\mathbf{q}) \leq \kappa$. In other words, $\mathbf{p}_i(\mathbf{q}|\kappa)$ is the uniquely optimal strategy of the opposition party i for all $i \in \{1,2\}$ and $\mathbf{q} \in X$. Denote $\mathbf{p}_{i,T-1}(\mathbf{q}) = \mathbf{p}_i(\mathbf{q}|\kappa)$ for $i \in \{1,2\}$.

Consider now the period $T-2$. Without repeating the obvious details, for any incumbent's policy commitment $\mathbf{q} \in X$, the optimization problem of the opposition party i writes $\max_{k\in[0,k_i(\mathbf{q})]} \tilde{U}_{i,T-2}(k)$ where

$$
\tilde{U}_{i,T-2}(k) = -d^2(k\mathbf{b}_i, \mathbf{b}_i) - d^2(\mathbf{p}_{-i,T-1}(k\mathbf{b}_i), \mathbf{b}_i)\delta - d^2(\mathbf{p}_{i,T}(\mathbf{p}_{-i,T-1}(k\mathbf{b}_i)), \mathbf{b}_i)\delta^2. \tag{A27}
$$

Because $\mathbf{p}_{-i,T-1}(k\mathbf{b}_i) = k\mathbf{b}_{-i}$ for $k \in [0,\kappa]$ and $\mathbf{p}_{-i,T-1}(k\mathbf{b}_i) = \kappa\mathbf{b}_{-i}$ for $k \geq \kappa$, we have $\mathbf{p}_{i,T}(\mathbf{p}_{-i,T-1}(k\mathbf{b}_i)) = k\mathbf{b}_i$ if $k \in [0,\kappa]$ and $\mathbf{p}_{i,T}(\mathbf{p}_{-i,T-1}(k\mathbf{b}_i)) = \kappa\mathbf{b}_i$ if $k \geq \kappa$. Hence

$$
\begin{aligned}
\tilde{U}_{i,T-2}(k) &= -d^2(k\mathbf{b}_i, \mathbf{b}_i) - d^2(k\mathbf{b}_{-i}, \mathbf{b}_i)\delta - d^2(k\mathbf{b}_i, \mathbf{b}_i)\delta^2 \\
\tilde{U}_{i,T-2}(k) &= -d^2(k\mathbf{b}_i, \mathbf{b}_i) - d^2(\kappa\mathbf{b}_{-i}, \mathbf{b}_i)\delta - d^2(\kappa\mathbf{b}_i, \mathbf{b}_i)\delta^2
\end{aligned}
\tag{A28}
$$

when $k \in [0,\kappa]$ and $k \geq \kappa$ respectively. We first note that $\tilde{U}_{i,T-2}(k)$ is increasing on $[0,\kappa]$, which follows from $\tilde{U}'_{i,T-2}(k) = \tilde{U}'_{i,T-1}(k) - 2b_i^2\delta^2(k-1) > \tilde{U}'_{i,T-1}(k) > 0$ for any $k \in (0,\kappa)$. Furthermore, $\tilde{U}_{i,T-2}(k)$ is clearly increasing on $[\kappa,1]$ and decreasing on $[1,\infty)$. Thus the optimization problem of the opposition party i has a unique solution $k = 1$ if $k_i(\mathbf{q}) \geq 1$ and $k = k_i(\mathbf{q})$ if $k_i(\mathbf{q}) \leq 1$. In other words, $\mathbf{p}_i(\mathbf{q}|1)$ is the uniquely optimal strategy of the opposition party i for all $i \in \{1,2\}$ and $\mathbf{q} \in X$. Denote $\mathbf{p}_{i,T-2}(\mathbf{q}) = \mathbf{p}_i(\mathbf{q}|1)$ for $i \in \{1,2\}$.

Consider the period $T-3$. For any $\mathbf{q} \in X$, the optimization problem of the opposition party i writes $\max_{k\in[0,k_i(\mathbf{q})]} \tilde{U}_{i,T-3}(k)$, where similar arguments as in the previous period show

$$\begin{aligned}\tilde{U}_{i,T-3}(k) &= -d^2(k\mathbf{b}_i,\mathbf{b}_i) - d^2(k\mathbf{b}_{-i},\mathbf{b}_i)\delta - d^2(k\mathbf{b}_i,\mathbf{b}_i)\delta^2 - d^2(k\mathbf{b}_{-i},\mathbf{b}_i)\delta^3 \\ \tilde{U}_{i,T-3}(k) &= -d^2(k\mathbf{b}_i,\mathbf{b}_i) - d^2(k\mathbf{b}_{-i},\mathbf{b}_i)\delta - d^2(\kappa\mathbf{b}_i,\mathbf{b}_i)\delta^2 - d^2(\kappa\mathbf{b}_{-i},\mathbf{b}_i)\delta^3\end{aligned} \tag{A29}$$

when $k \in [0,\kappa]$ and $k \geq \kappa$ respectively. Denoting by $\upsilon_i(k) = -d^2(k\mathbf{b}_i,\mathbf{b}_i) - d^2(k\mathbf{b}_{-i},\mathbf{b}_i)\delta$, $\tilde{U}_{i,T-3}(k) = (1+\delta^2)\upsilon_i(k)$ if $k \in [0,\kappa]$ and $\tilde{U}_{i,T-3}(k) = \upsilon_i(k) + c$ if $k \geq \kappa$, where c is constant in k. Because $\upsilon_i(k)$ is increasing in k on $[0,\kappa]$ and decreasing in k on $[\kappa,\infty)$, $\tilde{U}_{i,T-3}(k)$ has a unique maximum at $k=\kappa$. By the now familiar arguments, $\mathbf{p}_i(\mathbf{q}|\kappa)$ is the uniquely optimal strategy of the opposition party i for all $i \in \{1,2\}$ and $\mathbf{q} \in X$. Denote $\mathbf{p}_{i,T-3}(\mathbf{q}) = \mathbf{p}_i(\mathbf{q}|\kappa)$ for $i \in \{1,2\}$.

So far, we have shown that it is uniquely optimal for the opposition party i to contest elections with $\mathbf{p}_i(\mathbf{q}|\hat{k}_i)$, where $\hat{k}_i = 1$ for T and $T-2$ and $\hat{k}_i = \kappa$ for $T-1$ and $T-3$. Suppose that this patterns repeats for periods up to $T-s+2$ and $T-s+1$ where s is even.

Consider period $T-s$. We need to show that it is uniquely optimal for the opposition party i to contest elections with $\mathbf{p}_i(\mathbf{q}|1)$ for all $i \in \{1,2\}$ and $\mathbf{q} \in X$. Because in $T-s+1$ the opposition party contests elections with $\mathbf{p}_i(\mathbf{q}|\kappa)$,

$$\begin{aligned}\tilde{U}_{i,T-s}(k) &= \upsilon_i(k)\sum_{t=0}^{\frac{s-2}{2}}\delta^{2t} - d^2(k\mathbf{b}_i,\mathbf{b}_i)\delta^s \\ \tilde{U}_{i,T-s}(k) &= -d^2(k\mathbf{b}_i,\mathbf{b}_i) + c\end{aligned} \tag{A30}$$

when $k \in [0,\kappa]$ and $k \geq \kappa$ respectively. Because $\upsilon_i(k)$ is increasing in k on $[0,\kappa]$ and decreasing in k on $[\kappa,\infty)$, and because $-d^2(k\mathbf{b}_i,\mathbf{b}_i)$ is increasing in k on $[0,1]$ and decreasing in k on $[1,\infty)$, $\tilde{U}_{T-s}(k)$ has a unique maximum $k=1$. Hence, it is uniquely optimal for the opposition party i to contest elections with $\mathbf{p}_i(\mathbf{q}|1)$ for all $i \in \{1,2\}$ and $\mathbf{q} \in X$.

Consider $T-s-1$. We need to show that it is uniquely optimal for the opposition party i to contest elections with $\mathbf{p}_i(\mathbf{q}|\kappa)$ for all $i \in \{1,2\}$ and $\mathbf{q} \in X$. Because in $T-s$ the opposition party contests elections with $\mathbf{p}_i(\mathbf{q}|1)$ and in $T-s+1$

with $\mathbf{p}_i(\mathbf{q}|\kappa)$,

$$\begin{aligned}\tilde{U}_{i,T-s-1}(k) &= \upsilon_i(k)\sum\nolimits_{t=0}^{\frac{s}{2}}\delta^{2t}\\ \tilde{U}_{i,T-s-1}(k) &= \upsilon_i(k)+c\end{aligned}\tag{A31}$$

when $k \in [0,\kappa]$ and $k \geq \kappa$ respectively. Because $\upsilon_i(k)$ is increasing in k on $[0,\kappa]$ and decreasing in k on $[\kappa,\infty)$, $\tilde{U}_{T-s-1}(k)$ has a unique maximum $k=\kappa$. Hence, it is uniquely optimal for the opposition party i to contest elections with $\mathbf{p}_i(\mathbf{q}|\kappa)$ for all $i \in \{1,2\}$ and $\mathbf{q} \in X$.

To summarize, there exists a unique SPE in which the opposition party i contests elections with $\mathbf{p}_i(\mathbf{q}|\hat{k}_i)$ for all $i \in \{1,2\}$ and $\mathbf{q} \in X$. $\hat{k}_i = 1$ for all periods $T-s$ with $s \in \mathbb{N}_{\geq 0}$ and s even and $\hat{k}_i = \kappa$ for all periods $T-s$ with $s \in \mathbb{N}_{\geq 1}$ and s odd. With an appropriately chosen initial-period incumbent party, the SPE just described generates identical policy dynamics to the one generated by the SSMPE from Proposition 1, for any incumbent's policy commitment $\mathbf{q} \in X$. When party 2 is at the opposition in the initial period of the infinite horizon model, both parties moderate, if at the opposition, in the initial period of the finite horizon model with T even. When party 1 is at the opposition in the initial period of the infinite horizon model, none of the parties moderates, if at the opposition, in the initial period of the finite horizon model with T odd.