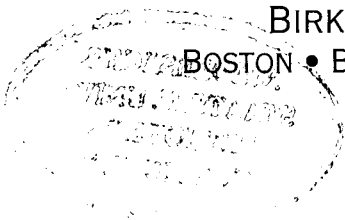


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# DYNAMIC EQUATIONS ON TIME SCALES

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AN INTRODUCTION WITH APPLICATIONS



BIRKHÄUSER  
BOSTON • BASEL • BERLIN

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