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Dynamic factor analysis models with time-varying parameters offer a valuable tool for evaluating multivariate time series data with time-varying dynamics and/or measurement properties. We use the Dynamic Model of Activation proposed by Zautra and colleagues (Zautra, Potter, & Reich, 1997) as a motivating example to construct a dynamic factor model with vector autoregressive relations and time-varying cross-regression parameters at the factor level. Using techniques drawn from the state-space literature, the model was fitted to a set of daily affect data (over 71 days) from 10 participants who had been diagnosed with Parkinson's disease. Our empirical results lend partial support and some potential refinement to the Dynamic Model of Activation with regard to how the time dependencies between positive and negative affects change over time. A simulation study is conducted to examine the performance of the proposed techniques when (a) changes in the time-varying parameters are represented using the true model of change, (b) supposedly time-invariant parameters are represented as time-varying, and (c) the time-varying parameters show discrete shifts that are approximated using an autoregressive model of differences.

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Many psychological concepts are unobserved and usually represented as latent factors apprehended through multiple observed indicators. With the increased prevalence of longitudinal—in particular, intensive repeated measures—data in psychology (Heron & Smyth, 2010), models that can describe both the structure and time-lagged relationships of latent factors have become essential alternatives to studying intraindividual change. Dynamic factor analysis models fulfill these purposes by combining factor analysis and time series analysis (Browne & Nesselroade, 2005; Browne & Zhang, 2007; Engle & Watson, 1981; Forni, Hallin, Lippi, & Reichlin, 2000, 2005; Geweke, 1977; Geweke & Singleton, 1981; Molenaar, 1985; Nesselroade, McArdle, Aggen, & Meyers, 2002).

Substantively, dynamic factor analysis models have been used to study psychophysiological processes (Molenaar, 1994a), affective processes of couples (Ferrer & Nesselroade, 2003), emotions of patients with Parkinson's disease (S.-M. Chow, Nesselroade, Shifren, & McArdle, 2004), emotion process after a romantic breakup (Sbarra & Ferrer, 2006) and relationship between children's perceived control and school performance (Musher, Nesselroade, & Schmitz, 2002). Methodologically, a number of methods have been proposed over the years for estimating dynamic factor analysis models, including LISREL maximum likelihood (ML) estimation with block Toeplitz matrices (Molenaar, 1985), generalized least squares estimation with block Toeplitz matrices (Molenaar & Nesselroade, 1998), ordinary least squares estimation with lagged correlation matrices (Browne & Zhang, 2007), raw data ML estimation with the Kalman filter (Engle & Watson, 1981; Hamaker, Dolan, & Molenaar, 2005), and the Bayesian approach (Z. Zhang, Hamaker, & Nesselroade, 2008).

In standard dynamic factor analysis models, all modeling parameters are assumed to be constant throughout the entire span of a study. Thus, they are only plausible for stationary time series.[1] Under strict stationarity, the probability distribution of a stochastic process is assumed to be constant over time, whereas weak stationarity only requires the first two moments of a probability distribution to be time invariant (Chatfield, 2004). Unfortunately, nonstationary data are frequently observed and to some extent inevitable in empirical studies (Boker, Xu, Rotondo, & King, 2002; Houtveen & Molenaar, 2001; Kiryu, Saitoh, & Ishioka, 1992; Schack, Bareshova, Grieszbach, & Witte, 1995; Tarvainen, Hiltunen, Ranta-aho, & Karjalainen, 2004; Weber, Molenaar, & Van der Molen, 1992).

Dynamic factor analysis models with time-varying parameters are one way of accommodating a specific kind of nonstationarity. Most time-varying parameter models have been developed for univariate time series. Some examples of popular univariate time-varying parameter models are the local level model

[1]Having time-invariant parameters is a necessary but insufficient condition for stationarity; some processes may have time-invariant parameters and yet are nonstationary.

(Durbin & Koopman, 2001), the local linear trend model (Harvey, 1989), time-varying autoregressive moving average (ARMA) model (Tarvainen, Georgiadis, Ranta–aho, & Karjalainen, 2006; Weiss, 1985), and stochastic regression model (Pagan, 1980). These models are typically formulated within the state-space framework in which the time-varying parameters are formulated as state (i.e., latent) variables. A dynamic model of choice—usually a nonparametric model or related variations that are deemed flexible enough to capture a variety of different change trajectories—is then used to approximate the changes in the time-varying parameters (e.g., Molenaar & Newell, 2003; Tarvainen et al., 2006; Tarvainen et al., 2004).

Relatively few researchers in psychology or psychometrics have considered extending dynamic factor analysis models to include time-varying parameters. The few exceptions include the work of Molenaar (1994b), who considered a one-factor dynamic factor model that conformed to a first-order autoregressive [AR(1)] process at the latent level. Polynomial functions of time were used to represent the dynamics of the time-varying parameters, including AR(1) and factor loading parameters. Several other examples of dynamic factor analysis with time-varying parameters can be found in the statistical and econometrics literature. For instance, Del Negro and Otrok (2008) examined a dynamic factor model with time-varying factor loadings within a Bayesian framework. In addition, Stock and Watson (2008) presented an exploratory analysis aimed at identifying structural breaks (i.e., discrete shifts) in the factor loadings and time series parameters of a dynamic factor analysis model by means of the Chow test (G. C. Chow, 1960).

In this study, we use a theoretical model of affect, namely, the Dynamic Model of Activation proposed by Zautra and colleagues (Reich, Zautra, & Davis, 2003; Zautra, Potter, & Reich, 1997; Zautra, Reich, Davis, Potter, & Nicolson, 2000), to construct a motivating example on which illustrations of the proposed techniques for detecting time-varying parameters are based. Our proposed approach differs from other exploratory applications (e.g., Stock & Watson, 2008) in that a confirmatory model is devised explicitly to test postulates from the Dynamic Model of Activation. Unlike the Bayesian approach taken by Del Negro and Otrok (2008), we perform model inferences within the frequentist framework using techniques drawn from the state-space and Kalman filtering literature. This article also extends the earlier work of Molenaar and colleagues (Molenaar, 1994b; Molenaar, de Gooijer, & Schmitz, 1992) in several ways. First, we consider a bivariate as opposed to a one-factor dynamic factor model. Second, we seek to examine properties of the point as well as standard error (*SE*) estimates obtained from the proposed approach. Molenaar (1994b) used the expectation-maximization (EM) algorithm to estimate other time-invariant parameters in his model. Standard errors of the associated parameters are not available from the EM procedure. The author thus suggested issues pertaining

to *SE* estimation as a future research topic at that point.[2] In the present context, we use a numerical Newton-Raphson procedure to obtain the Gaussian ML estimator of the model parameters. Standard errors can be readily obtained from the observed or expected information matrix of the associated raw data log-likelihood function. Third, in addition to assessing point and *SE* estimates, we also address issues such as coverage rates and Type I error rates (i.e., cases where time-invariant parameters are incorrectly classified as time-varying).

Finally and perhaps most important, whereas Molenaar (1994b) and others (e.g., Del Negro & Otrok, 2008; Stock & Watson, 2008) considered single-subject time series data, we evaluate the performance of the proposed techniques when used with *multiple-subject time series data*. Frequently encountered in ecological momentary assessment studies (Heron & Smyth, 2010), such data are typically characterized by finite time lengths and a small to moderate number of participants. In light of the finite time lengths of such data, we utilize information from all individuals for parameter estimation purposes by constraining all but a subset of time-varying modeling parameters to be invariant across persons. This is in contrast to standard time series approaches that focus on modeling at the individual level. When the assumption of invariance holds across persons (see, e.g., Nesselroade & Molenaar, 1999), such designs offer one way of pooling information from multiple participants for model estimation purposes.

The remainder of the article is organized as follows: We first describe our proposed model and the empirical data set used for illustration purposes. We then outline the estimation procedures used in this article and summarize results from empirical model fitting. This is followed by results from a Monte Carlo simulation study, including some brief illustrations of how the proposed model and techniques can be used to approximate the dynamics of time-varying parameters under different change scenarios. We conclude with some remarks on the strengths and limitations of the proposed techniques.

MOTIVATING EXAMPLE

According to the Dynamic Model of Activation (Reich et al., 2003; Zautra et al., 1997; Zautra et al., 2000), positive affect (PA) and negative affect (NA) are independent under low stress (activation) conditions. However, they tend to collapse into a unidimensional, bipolar structure under high activation. Thus, from a within-person perspective, the concurrent linkage (i.e., correlation) be-

[2]When the EM algorithm is used, bootstrap procedures such as the nonparametric variation presented by Stoffer and Wall (1991) can be used to obtain the associated *SE* estimates. We do not consider this particular approach here. Interested readers are referred elsewhere (Ho, Shumway, & Ombao, 2006; Shumway & Stoffer, 2000) for examples and implementation details.

tween PA and NA is posited to change over time as a function of a time-varying covariate, namely, activation level. When formulated within a dynamic framework, we can further disentangle the directionality of the PA-NA linkage by considering how the lead-lag relationship between PA and NA changes over time. Specifically, we represent the reciprocal PA-NA linkages as cross-regression parameters in a process factor analysis model (Browne & Nesselroade, 2005; McArdle, 1982) with vector autoregressive relations at the factor level. In this regard, our modeling results offer a refinement of Zautra and colleagues' model by providing insights into how the *time dependencies* between PA and NA change as they vary in intensity levels.

In the context of a process factor analysis model, the correlation between two latent factors (i.e., PA and NA) is a function of a number of time series parameters (see, e.g., Browne & Nesselroade, 2005). We focus specifically on capturing changes that arise from over-time fluctuations in the cross-regression parameters. Doing so allows us to clarify whether changes in PA-NA linkage stem more from changes in the PA $\rightarrow$ NA or NA $\rightarrow$ PA influence. Because we do not have a time-varying indicator of stress or activation level in our illustrative data set, we approximate the *deviations* in the PA $\rightarrow$ NA and NA $\rightarrow$ PA cross-regression parameters from their respective baseline values using a first-order autoregressive [AR(1)] model. The proposed process factor analysis model consists of two submodels. The first model, a measurement model, written as

$$\boldsymbol{y}_{i,t} = \boldsymbol{\Lambda}\boldsymbol{\eta}_{i,t} + \boldsymbol{\epsilon}_{i,t}, \tag{1}$$

is used to express $\boldsymbol{y}_{i,t}$, a vector of manifest variables, as a linear function of a vector of latent variables, $\boldsymbol{\eta}_{i,t}$, and unique variables, $\boldsymbol{\epsilon}_{i,t}$, with factor loading matrix, $\boldsymbol{\Lambda}$.

The second model is a dynamic model that expresses the lagged relationships among the latent variables over time. Our illustrative dynamic model consists of four equations, written as

$$\begin{aligned}
PA_{i,t} &= a_P PA_{i,t-1} + b_{PN,i,t-1} NA_{i,t-1} + \zeta_{PA,i,t} \\
NA_{i,t} &= a_N NA_{i,t-1} + b_{NP,i,t-1} PA_{i,t-1} + \zeta_{NA,i,t} \\
(b_{PN,i,t} - \beta_{PN0}) &= \beta_{PN1}(b_{PN,i,t-1} - \beta_{PN0}) + \zeta_{PN,i,t} \\
(b_{NP,i,t} - \beta_{NP0}) &= \beta_{NP1}(b_{NP,i,t-1} - \beta_{NP0}) + \zeta_{NP,i,t},
\end{aligned} \tag{2}$$

where the latent variable vector, $\boldsymbol{\eta}_{i,t} = [PA_{i,t}\ NA_{i,t}\ b_{PN,i,t}\ b_{NP,i,t}]'$, includes the two latent factors corresponding to person i's PA and NA factor score at time t, respectively, the lag-1 NA $\rightarrow$ PA cross-regression weight, $b_{PN,i,t}$, and

the corresponding lag-1 PA → NA cross-regression weight, $b_{NP,i,t}$. In other words, an individual's PA and NA are hypothesized to conform to a vector AR(1) process with autoregression parameter a_P and a_N, respectively, and time-varying cross-regression parameter, $b_{PN,i,t}$ and $b_{NP,i,t}$, respectively. The cross-regression parameters are formulated as latent variables whose changes over time are represented using the third and fourth equations in Equation (2), referred to herein as the *AR(1) of differences* model. In addition, we assume that all the shock variables or process noise components, denoted as $\boldsymbol{\zeta}_{i,t} = [\zeta_{PA,i,t}\ \zeta_{NA,i,t}\ \zeta_{PN,i,t}\ \zeta_{NP,i,t}]'$, are distributed as

$$\boldsymbol{\zeta}_{i,t} \sim \text{MVN}\left(\begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}\sigma^2_{\zeta_{PA}} & \sigma_{\zeta_{PA},\zeta_{NA}} & 0 & 0\\ \sigma_{\zeta_{PA},\zeta_{NA}} & \sigma^2_{\zeta_{NA}} & 0 & 0\\ 0 & 0 & \sigma^2_{\zeta_{PN}} & 0\\ 0 & 0 & 0 & \sigma^2_{\zeta_{NP}}\end{bmatrix}\right), \tag{3}$$

where $\text{MVN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ indicates a multivariate normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.

According to the AR(1) of differences model, within-person, over-time deviations in the PA → NA and NA → PA cross-regression weights from the baseline cross-regression weights (i.e., β_{PN0} and β_{NP0}) are hypothesized to fluctuate over time following an AR(1) processs. The AR parameters, β_{PN1} and β_{NP1}, govern the rates at which such deviations diminish (if $|\beta_{PN1}|$ and $|\beta_{NP1}| < 1.0$) or amplify (if $|\beta_{PN1}|$ and $|\beta_{NP1}| \geq 1.0$) over time, with higher values indicating slower decay or greater amplifications. When $|\beta_{PN1}|$ and $|\beta_{NP1}| < 1.0$, this is consistent with the view that whenever individuals are "perturbed" or are moved away from their affective equilibrium or "set-point," they are able to regulate their emotions by minimizing the discrepancies between their current emotion states and their affective set-points (R. J. Larsen, 2000).

The specification in Equation (2) allows each individual to have his or her own cross-regression weights, $b_{PN,i,t}$ and $b_{NP,i,t}$, that are also allowed to vary *over time* as latent variables. As distinct from conventional single-subject time series analysis, we "borrow strengths" from all individuals' data in estimating the person-specific cross-regression weights by constraining 11 additional time series parameters to be equal across persons. These parameters include (a) the AR(1) parameters for PA and NA, a_P and a_N; (b) the baseline lag-1 NA → PA and PA → NA cross-regression weights, β_{PN0} and β_{NP0}; (c) the AR(1) parameters governing the fluctuations in cross-regression weights, β_{PN1} and β_{NP1}; and (d) the process noise variance and covariance parameters, $\sigma^2_{\zeta_{PA}}$, $\sigma^2_{\zeta_{NA}}$, $\sigma^2_{\zeta_{PN}}$, $\sigma^2_{\zeta_{NP}}$, and $\sigma_{\zeta_{PA},\zeta_{NA}}$. All parameters in the measurement equation in Equation (1) are also constrained to be invariant across persons. Another implication of the specification in Equation (2) is that current latent variable scores in $\boldsymbol{\eta}_{i,t}$ are indirectly affected by previous latent variable scores *beyond*

the immediately preceding time point and previous shock variables through the effects encapsulated in $\boldsymbol{\eta}_{i,t-1}$.

Kitagawa (1981) referred to models with time-varying parameters as self-organizing state-space models because the intrinsic dynamics of the modeling parameters are capable of inducing complex dynamics in a system. The AR(1) of differences model allows individuals' PA and NA to "self-organize" over time through changes in PA, NA, and the associated cross-regression weights. In this way, PA and NA may fluctuate differently at different points in time: they may at times fluctuate in ways that are entirely independent of each other (i.e., with zero coupling or cross-regression influence); at other times, they may show opposing (e.g., when NA is very high and PA is very low) as well as convergent trends (e.g., during "bittersweet" moments when an individual reminisces about some of their happiest times with a diseased spouse; J. T. Larsen, 2001). Examples of these scenarios are depicted in Figure 1.

The modeling choice depicted in Equations (1) and (2) is simply one example of the many time series models that can be used to describe patterns of change in a multivariate time series.[3] Other examples include the shock factor analysis model (Molenaar, 1985) and vector autoregressive moving average (VARMA) models for manifest variables. We chose the AR of differences model as an approximation model because it is flexible enough to approximate a variety of change trajectories, and the parameters in this model offer interesting insights from an affect standpoint. From an estimation standpoint, allowing the cross-regression weights to be estimated as latent variables introduces nonlinearity into the model. For example, from Equation (2), the term $b_{PN,i,t-1}NA_{i,t-1}$ (and by the same token, $b_{NP,i,t-1}PA_{i,t-1}$) involves the multiplication of two latent variables, which constitutes a source of nonlinearity in the model. We account for these nonlinearities using estimation techniques developed within the state-space framework.

ESTIMATION PROCEDURES

Nonlinear State-Space Modeling Framework

The model summarized in Equations (1–2) can be viewed as a special case of a

[3]For instance, if stress is available as a time-varying covariate, the model for $b_{NP,i,t}$ can be modified as $b_{NP,i,t} = \beta_{NP0} + \gamma_{NP}stress_{i,t} + \zeta_{NP,i,t}$. The model for $b_{PN,i,t}$ is modified likewise. Alternatively, if stress is represented as a latent variable (e.g., either as a true score or as a latent factor indicated by multiple observed variables), the dynamic model can be further expanded to represent stress and its associated changes as part of a vector autoregressive moving average [e.g., VARMA(1,1)] process influenced by previous stress, concurrent and previous random shocks as well as the lagged effects of PA and NA. These alternative specifications offer even more direct ways of assessing the Dynamic Model of Activation than the AR(1) of differences model used in this study.

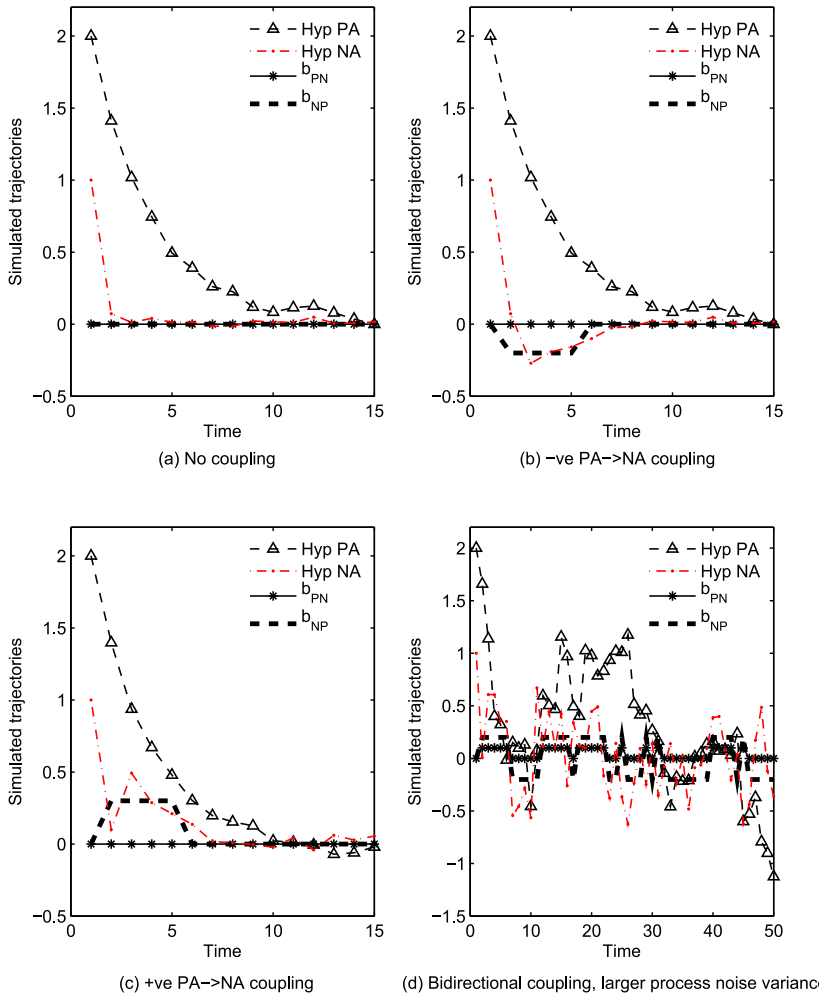

FIGURE 1 Hypothetical trajectories of positive affect (PA) and negative affect (NA) generated using the first two equations in Equation (2) to illustrate the self-organizing properties of models with time-varying parameters. In all scenarios, the equilibrium points of PA and NA are set to be zero. Process noise was added to PA and NA only in scenario (d). (a): $b_{PN,i,t}$ and $b_{NP,i,t}$ are constrained to be zero over time; (b): $b_{NP,i,t}$ becomes negative when either PA or NA exceeds certain threshold values (specifically, lower than $-.5$ or higher than .5). The negative weight of $b_{NP,i,t}$ leads to distinctly low NA levels while PA is high (specifically, for $t \leq 5$); (c): $b_{NP,i,t}$ becomes positive when either PA or NA exceeds the predefined threshold values. The positive weight of $b_{NP,i,t}$ leads to heightened levels of NA while PA is high; (d): Both $b_{NP,i,t}$ and $b_{PN,i,t}$ are allowed to deviate from zero when either PA or NA exceeds the predefined threshold values. PA and NA now show complex, time-varying relationship with respect to each other contingent upon changes in the cross-regression weights. Hyp PA & Hyp NA = hypothetical PA and NA, respectively; $b_{PN} = b_{PN,i,t}$; $b_{NP} = b_{NP,i,t}$.

nonlinear state-space model. Broadly speaking, a state-space model encompasses a dynamic model and a measurement model. Our proposed model of interest is a nonlinear multiple-subject or group-based state-space model wherein certain modeling parameters are held invariant across persons and time. This nonlinear model can be expressed as

$$\boldsymbol{\eta}_{i,t} = g(\boldsymbol{\eta}_{i,t-1}, \boldsymbol{\theta}, \boldsymbol{x}_{i,t}) + \boldsymbol{\zeta}_{i,t}, \quad \boldsymbol{\zeta}_{i,t} \overset{\text{i.i.d}}{\sim} \text{MVN}(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\zeta}}) \tag{4}$$

$$\boldsymbol{y}_{i,t} = h(\boldsymbol{\eta}_{i,t}, \boldsymbol{\theta}, \boldsymbol{x}_{i,t}) + \boldsymbol{\epsilon}_{i,t}, \quad \boldsymbol{\epsilon}_{i,t} \overset{\text{i.i.d}}{\sim} \text{MVN}(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}) \tag{5}$$

$$\boldsymbol{\eta}_{i,1} \sim \text{N}_w(\boldsymbol{a}, \boldsymbol{\Sigma}_{\mathbf{0}}), \quad t = 1, \ldots, T; i = 1, \ldots n,$$

where $g(\boldsymbol{\eta}_{i,t}, \boldsymbol{\theta}, \boldsymbol{x}_{i,t})$ is a set of differentiable linear or nonlinear regression functions, $\boldsymbol{\eta}_{i,t}$ is a $w \times 1$ vector of latent variables (also called "state variables"), $\boldsymbol{\theta}$ is a vector of time-invariant parameters, $\boldsymbol{x}_{i,t}$ is a vector of time-varying covariates, and $\boldsymbol{\zeta}_{i,t}$ is a vector of process noise components. $h(\boldsymbol{\eta}_{i,t}, \boldsymbol{\theta}, \boldsymbol{x}_{i,t})$ is a set of differentiable linear or nonlinear regression functions dictating the measurement relations between the latent variables and the $p \times 1$ vector of manifest variables $\boldsymbol{y}_{it}$; $\boldsymbol{\zeta}_{i,t}$ and $\boldsymbol{\epsilon}_{i,t}$ are $w \times 1$ and $p \times 1$ vectors of process noise and unique components, respectively. In the state-space context, initial condition (specifically, the mean, $\boldsymbol{a}$, and covariance matrix, $\boldsymbol{\Sigma}_{\mathbf{0}}$) of the latent variables, $\boldsymbol{\eta}_{i,1}$, has to be specified. Such information reflects the distributional properties of the latent variables prior to the first observed time point. In our empirical and simulation studies, we adopted a diffuse prior, namely, $\boldsymbol{a}$ was set to be a vector of zeroes and $\boldsymbol{\Sigma}_{\mathbf{0}}$ was set to be a diagonal matrix with large values in its diagonal entries.

Whereas some similarities have been noted between the structural equation modeling tradition and the linear counterpart of the state-space model depicted in Equations (4–5) (S.-M. Chow & Zhang, 2008; MacCallum & Ashby, 1986; Otter, 1986; Oud, van den Bercken, & Essers, 1990), there have been very few empirical applications involving nonlinear state-space models. Because different measurement occasions of the same variables are treated as different manifest variables within the structural equation modeling (SEM) framework, numerical difficulties could arise in the structural equation framework in instances involving large T, especially when $T > N$ (for details see S.-M. Chow & Zhang, 2008; Hamaker, Dolan, & Molenaar, 2003). In contrast, the state-space framework is structured around a set of one-step-ahead difference equations (see, e.g., Equation (4)); the data likelihood function and the associated estimation tools are well suited for handling data sets with larger T, even when $T > N$. Nonlinear state-space models share the same merits as linear state-space models in terms of their flexibility in handling data with longer time lengths. Because of the

nonlinearities in Equations (4–5), however, standard linear estimation procedures can no longer be used. Of all the nonlinear Kalman filter approaches, the extended Kalman filter and the corresponding extended Kalman smoother is by far the most commonly endorsed estimation approach (Gelb, 1974; Molenaar & Newell, 2003). These are the estimation procedures adopted in the present article.

The Extended Kalman Filter, the Extended Kalman Smoother, and the Prediction Error Decomposition Function

Once a model has been expressed in state-space form, the extended Kalman filter and the related extended Kalman smoother can be used to derive longitudinal factor or latent variable scores at each time point (i.e., estimates of $\boldsymbol{\eta}_{i,t}$ conditional on the data). Specifically, suppose the data set $\mathbf{Y}_{i,t} = [\boldsymbol{Y}_{1,i,t}, \boldsymbol{Y}_{2,i,t}, \ldots, \boldsymbol{Y}_{p,i,t}]'$ from person i is available, where $\boldsymbol{Y}_{k,i,t}$ is itself a $t \times 1$ time series that contains the values of the kth manifest variable from time $t = 1$ to t. The extended Kalman filter can be used to derive conditional state estimates based on manifest observations up to time t (i.e., $E(\boldsymbol{\eta}_{i,t}|\mathbf{Y}_{i,t})$), denoted herein as $\boldsymbol{\eta}_{i,t|t}$. Procedures for implementing the extended Kalman filter are summarized in Equations (7–12) in the Appendix.

In one possible Kalman smoother, the fixed interval smoother (Anderson & Moore, 1979), the aim is to derive conditional state estimates using all available data (Anderson & Moore, 1979), yielding $E(\boldsymbol{\eta}_{i,t}|\mathbf{Y}_{i,T})$, denoted herein as $\boldsymbol{\eta}_{i,t|T}$. In practice, this can be accomplished by first performing the extended Kalman filter procedure forward in time (i.e., from $t = 1, \ldots, T$; using Equations (7–12)), followed by backward smoothing from $t = T, \ldots, 1$ (see Equation (13) in the Appendix) to yield smoothed estimates of the latent variables.

To estimate the person- (and time-) invariant parameters in $\boldsymbol{\theta}$, a log-likelihood function, referred to herein as the multiple-subject prediction error decomposition function (Caines & Rissanen, 1974; Schweppe, 1965), can be constructed using output from the extended Kalman filter. The specific nonlinear state-space model considered in our empirical application has several features that simplify estimation of the time-invariant parameters. In particular, all the nonlinearities reside in the deterministic portion (i.e., not in the process noise components) of the dynamic model (see Equation (2)) and the process noise components enter the model in an additive fashion. Thus, provided that $\boldsymbol{\eta}_{i,1}$ and $\boldsymbol{\zeta}_{i,t}$ are multivariate normally distributed (or alternatively, $\boldsymbol{\eta}_{i,1}$ is fixed), the conditional distribution $\boldsymbol{\eta}_{i,t} \mid \boldsymbol{\eta}_{i,t-1}$ is also multivariate normally distributed. Subsequently, because $h(.)$ consists only of linear functions, the distribution of $\boldsymbol{y}_{i,t}|\mathbf{Y}_{i,t-1}$ is also multivariate normally distributed if the uniquenesses in $\boldsymbol{\epsilon}_{i,t}$ are multivariate normally distributed. Thus, a *raw data log-likelihood function* can be constructed based on the distribution of $\boldsymbol{y}_{i,t}|\mathbf{Y}_{i,t-1}$, which is simply a multivariate Gaussian

distribution.[4] We refer to the corresponding log-likelihood function (see Equation (14) in the Appendix) as the multiple-subject version of the prediction error decomposition function. This function can then be optimized with respect to the modeling parameters in $\boldsymbol{\theta}$ to yield ML estimates of these parameters. Smoothed latent variable scores can then be computed using the extended Kalman smoother by fixing the parameter values at their ML estimates.

In sum, our proposed approach utilizes the extended Kalman filter, the extended Kalman smoother, and the prediction error decomposition function. This results in ML point estimates for all the time-invariant parameters and smoothed estimates of all the latent variables, including the cross-regression weights, $b_{PN,i,t}$ and $b_{NP,i,t}$. Standard errors of all the time-invariant parameters can be obtained by taking the square root of the diagonal elements of $\mathbf{I}^{-1}$, where $\mathbf{I}$ is the observed information (i.e., negative numerical Hessian) matrix of the prediction error decomposition function.

EMPIRICAL ILLUSTRATION

A set of previously published data (for details, see S.-M. Chow et al., 2004; Shifren, Hooker, Wood, & Nesselroade, 1997) was used for empirical illustration purposes. The sample consisted of 12 individuals (7 women, 5 men) who had been diagnosed with Parkinson's disease. The participants ages ranged from 59 to 81 years ($M = 68.75$, $SD = 7.24$). They were instructed to give daily self-reports of their feelings on a 5-point scale (with 1 representing *not at all* and 5 representing *all the time*) with respect to 10 PA and 10 NA items for 71 consecutive days. The PA items were enthusiastic, interested, determined, excited, alert, active, strong, proud, inspired, and attentive. The NA items included the following terms: scared, afraid, upset, nervous, ashamed, guilty, distressed, jittery, irritable, and hostile.

Following the data-processing steps used in S.-M. Chow et al. (2004), we used item parceling (Cattell, 1956, 1974; Kishton & Widaman, 1994) to derive three composite indicators for PA and NA, respectively, using the raw item

[4]Note that standard structural equation modeling approaches of fitting dynamic factor models to manifest variable lagged correlation or covariance matrices (e.g., Browne & Zhang, 2007; S.-M. Chow et al., 2004; Molenaar, 1985; Molenaar & Nesselroade, 1998) cannot be used in the present context because such approaches inherently assume stationarity whereas time-varying parameters by definition produce nonstationarity. It is therefore necessary to perform model formulation and estimation at the raw data level and the proposed approach provides one of the most computationally efficient ways of doing so when the nonlinearity, normality, and additive noise assumptions mentioned herein are met. The raw data approach endorsed here also provides a straightforward way to incorporate additional trends into the dynamic model in Equation (4) by specifying time as one of the time-varying covariates in $\boldsymbol{x}_{i,t}$ if deemed necessary.

scores. Linear trends were removed separately from each individual's parcel scores before further analysis. Inspection of the residuals revealed no evidence for higher order trends. Based on the results reported in S.-M. Chow et al. (2004), 10 out of the 12 participants were identified as showing homogeneous lag-0 and lag-1 correlational structures using the homogeneity test proposed by Nesselroade and Molenaar (1999). We thus used the detrended item parcel scores from these 10 participants for model fitting purposes. Summary statistics, lag-0, and lag-1 correlation matrices of the 10 participants' pooled data are shown in Table 1. Approximately 4% of all the measurement occasions were missing, with missingness scattered randomly throughout the 71-day study span. Assuming that the data were missing at random, the missingness can be handled directly by the raw data likelihood form of the prediction error decomposition function.

In S.-M. Chow et al. (2004), a time-invariant dynamic factor analysis model with first-order vector autoregressive [VAR(1)] relations at the latent level was fitted to these data. In the present context, the nonlinear state-space model summarized in Equations (1) and (2) was fitted to the empirical data. Consistent with the results reported in S.-M. Chow et al. (2004), the concurrent covariance between the process noises of PA and NA was not statistically

TABLE 1
Summary Statistics, Lag-0, and Lag-1 Correlations Among the 6 Item Parcels of the 10 Participants in the Empirical Data Set

		$P_{1,t}$	$P_{2,t}$	$P_{3,t}$	$N_{1,t}$	$N_{2,t}$	$N_{3,t}$
SD		*0.53*	*0.56*	*0.47*	*0.50*	*0.46*	*0.39*
Lag-0	$P_{1,t}$	1.00					
	$P_{2,t}$	0.34	1.00				
	$P_{3,t}$	0.46	0.61	1.00			
	$N_{1,t}$	−0.30	−0.37	−0.37	1.00		
	$N_{2,t}$	−0.14	−0.31	−0.30	0.50	1.00	
	$N_{3,t}$	−0.21	−0.40	−0.32	0.67	0.52	1.00
Lag-1	$P_{1,t-1}$	0.40	0.20	0.31	−0.19	−0.06	−0.17
	$P_{2,t-1}$	0.25	0.45	0.39	−0.30	−0.23	−0.29
	$P_{3,t-1}$	0.37	0.39	0.52	−0.29	−0.18	−0.26
	$N_{1,t-1}$	−0.25	−0.32	−0.27	0.42	0.29	0.37
	$N_{2,t-1}$	−0.10	−0.25	−0.18	0.24	0.27	0.22
	$N_{3,t-1}$	−0.16	−0.29	−0.24	0.36	0.29	0.42

Note. $P_{1,j}$, $P_{2,j}$, and $P_{3,j}$ are the three indicators for positive affect (PA) at time j ($j = t$ and $t - 1$); $N_{1,j}$, $N_{1,j}$, and $N_{1,j}$ are the three indicators for negative affect (NA) at time j. Linear trends were removed from the parcel scores separately for each participant after these scores were formed and the corresponding residuals were used for model fitting purposes. Thus, the means of all parcels were, by definition, close to zero and are not reported here.

significant. In addition, the process noise variance for the NA → PA cross-regression parameter, $\sigma^2_{\zeta_{PN}}$, was also not statistically significantly different from zero. This, in conjunction with the finding that β_{PN1} was also not statistically different from zero, indicated that the cross-regression effect from NA to PA was statistically invariant over time. All other parameters were found to be statistically different from zero. Parameter estimates from the final model in which only the statistically significant parameters were retained are summarized in Table 2. All subsequent discussions are based on this particular model.[5]

Compared with the fluctuations in NA, the participants' fluctuations in PA were characterized by greater variability ($\sigma^2_{\zeta_{PA}} = .09$, compared with $\sigma^2_{\zeta_{NA}} = .03$) and slower decay ($a_P = .54$, compared with $a_N = .01$). The smaller process noise variance and near-zero autoregression weight of NA suggested that individuals' NA tended to fluctuate in small magnitudes and did not show strong continuity beyond a particular day. Allowing the PA → NA cross-regression weight to vary over time led to two notable differences in our current results compared with those reported in S.-M. Chow et al. (2004). First, the autoregressive coefficient for NA, a_N, was greatly attenuated (i.e., closer to zero) once the lagged influence from PA to NA was allowed to vary over time. Second, whereas the lagged influence between PA and NA, as a whole, was bidirectional in nature (i.e., both β_{PN0} and β_{NP0} were statistically different from zero), the coupling effect from PA to NA was more volatile. Namely, the PA → NA coupling effect was found to show significant variations over time whereas the coupling effect from NA to PA was time-invariant.

Smoothed estimates of all the latent variables, including the PA → NA and NA → PA cross-regression weights, are shown in Figures 2–3 for the 10 participants.[6] For 9 of the 10 participants, the coupling from PA → NA became more negative around the days when PA and NA showed opposite trends (i.e., when PA was high and NA was low, or vice versa). This can be seen, for instance, in the estimates for Participant 1 around Day 10, Participant 2 around Day 10, Participant 3 around Day 40, Participant 4 around Day 50, Participant 5 around

[5]It may be useful to mention that we also performed some additional exploratory analysis before selecting this final model as our best-fitting model. For example, exploratory tests had been performed elsewhere (S.-M. Chow et al., 2004) to determine the appropriate lag order of the VAR process for this particular data set. We also performed a preliminary simulation study to evaluate the empirical identifiability of the parameters in Equations (1) and (2) and a few other models prior to choosing Equation (2) as our model for the time-varying cross-regression weights. Such exploratory results are omitted here due to space constraints.

[6]Even though the NA → PA coupling was found to be statistically invariant over time, the associated trajectory of $b_{PN,i,t}$, obtained by fixing β_{PN1} and $\sigma^2_{\zeta_{PN}}$ to be zero, constituted a special case of the full model in Equation (2). We thus estimated the trajectory of $b_{PN,i,t} = \beta_{PN0}$ by means of the extended Kalman smoother as opposed to specifying this parameter as a time-invariant parameter to be estimated by means of ML.

TABLE 2
Parameter Estimates From the Best-Fitting Model

Parameters	*Estimates*	*SE*
λ_{P1}	= 1.00	—
λ_{P2}	0.75	0.06
λ_{P3}	0.84	0.05
λ_{N1}	= 1.00	—
λ_{N2}	1.49	0.14
λ_{N3}	1.80	0.17
a_P	0.54	0.05
a_N	0.01	0.004
β_{PN0}	−0.18	0.05
β_{NP0}	−0.26	0.10
β_{NP1}	0.91	0.04
$\sigma^2_{\zeta_{PA}}$	0.09	0.01
$\sigma^2_{\zeta_{NA}}$	0.03	0.01
$\sigma^2_{\zeta_{NP}}$	0.05	0.02
$\sigma^2_{\epsilon_{P1}}$	0.11	0.01
$\sigma^2_{\epsilon_{P2}}$	0.15	0.01
$\sigma^2_{\epsilon_{P3}}$	0.06	0.01
$\sigma^2_{\epsilon_{N1}}$	0.22	0.01
$\sigma^2_{\epsilon_{N2}}$	0.18	0.01
$\sigma^2_{\epsilon_{N3}}$	0.04	0.01

Note. Parameters that were set to zero in the final model included $\sigma^2_{\zeta_{PN}}$, β_{PN1}, and covariances between the process noise components of different latent variables. $\lambda_{P1} - \lambda_{P3}$ = factor loadings of positive affect (PA) on the three observed PA parcels; $\lambda_{N1} - \lambda_{N3}$ = factor loadings of negative affect (NA) on the three observed NA parcels; a_P & a_N = autoregressive weight for PA and NA, respectively; β_{NP0} & β_{PN0} = baseline PA → NA and NA → PA cross-regression weight, respectively; β_{NP1} & β_{PN1} = AR(1) parameter for the deviations in PA → NA and NA → PA cross-regression weight, respectively; $\sigma^2_{\zeta_{PA}}$, $\sigma^2_{\zeta_{NA}}$, $\sigma^2_{\zeta_{NP}}$ and $\sigma^2_{\zeta_{PN}}$ = process noise variance for PA, NA, and the PA → NA and NA → PA cross-regression weights, respectively; $\sigma^2_{\epsilon_{P1}} - \sigma^2_{\epsilon_{N1}}$ unique variances of the observed parcel scores.

Day 35, Participant 6 around Day 10, Participant 8 after Day 50, Participant 9 between Days 45 and 55, and Participant 10 between Days 40 and 55.

For some participants, the PA → NA coupling coefficient became notably more positive when PA and NA showed converging trends (see, e.g., Participant 4 between Days 5 and 20 and Participant 9 between Days 2 and 18). Such incidents where PA and NA ebbed and flowed concurrently with changes in the PA → NA linkage help provide some insights into the dynamic changes that underlie the "bittersweet" feelings discussed by affect researchers (J. T. Larsen, 2001). That is, the increased positive coupling from PA to NA appears to be the driving force in sustaining individuals' bittersweet feelings over a longer time scale.

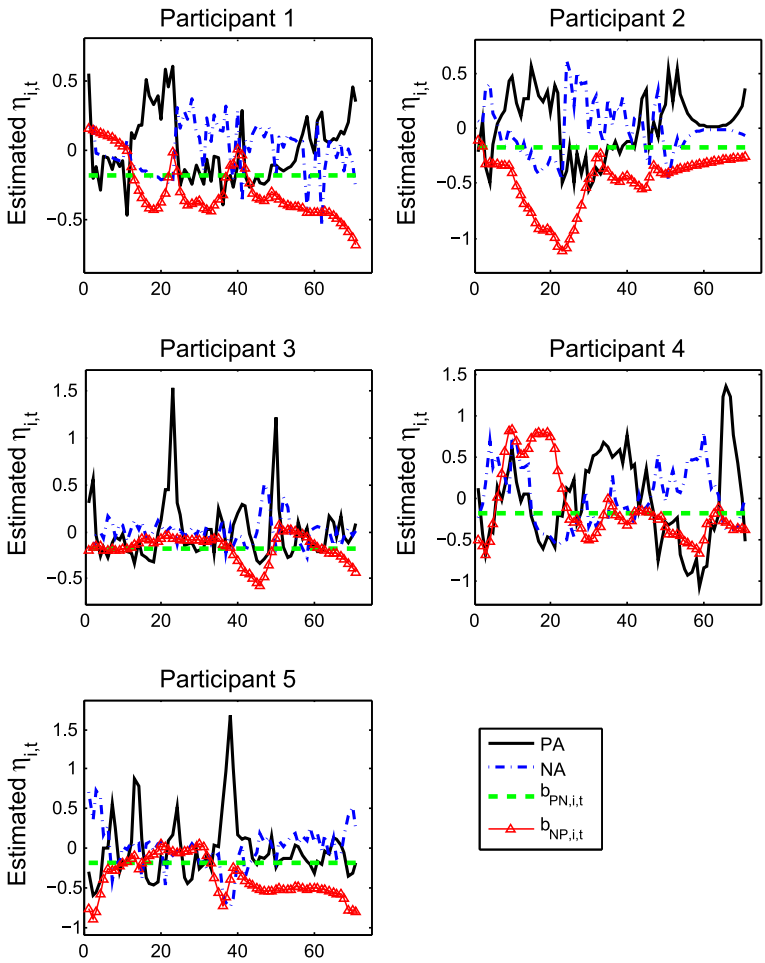

FIGURE 2 Smoothed latent variable estimates for Participants 1 to 5.

Even though no causal statement can be made regarding the changes in PA, NA, and $b_{PN,i,t}$, our findings did lend partial support, and some potential refinement to Zautra and colleagues' (1997; Reich et al., 2003; Zautra et al., 2000) Dynamic Model of Activation. Our empirical results suggest that one way in which the correlation between PA and NA might change around the more emotional days (e.g., the days on which both PA and NA are high or when they show opposite trends) is through changes in the lagged influence of PA on NA. That is, the changes in PA → NA as opposed to NA → PA coupling appear to be the key driving force in altering the dynamic interrelationship between PA

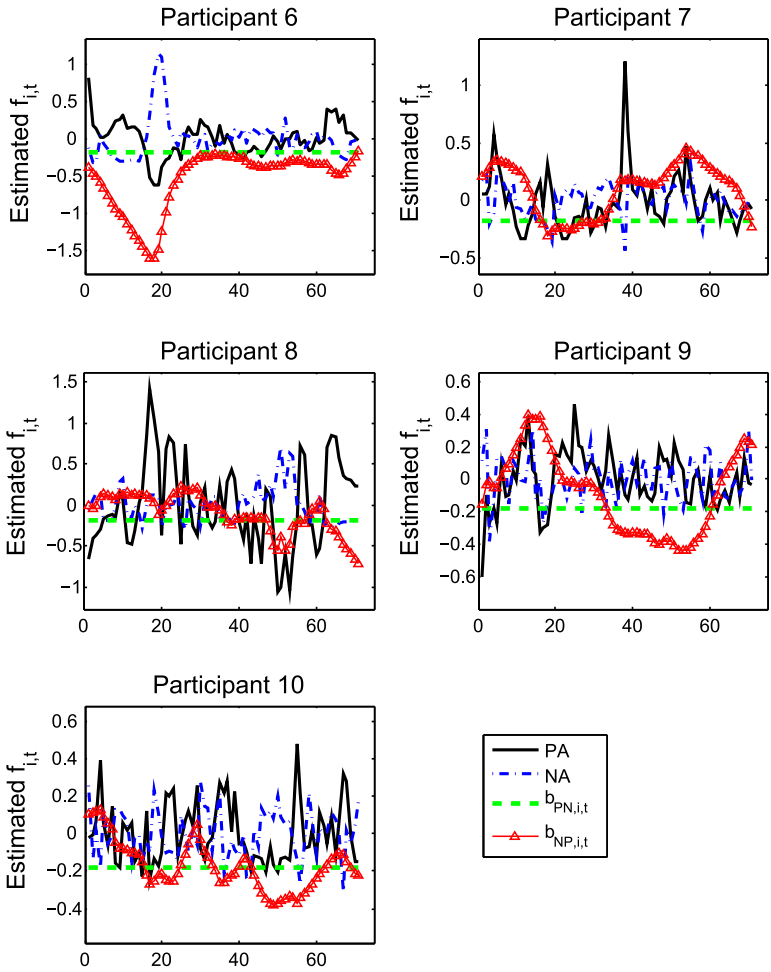

FIGURE 3 Smoothed latent variable estimates for Participants 6 to 10.

and NA. This may suggest, for instance, that it would be more difficult for an individual to alter his or her ability to use yesterday's low NA to increase today's PA (as the NA → PA coupling tends to stay invariant over time) than learning to draw on yesterday's high PA to lower today's NA.

The greater time-related volatility of the PA → NA coupling may stem from two important features of self-report PA and NA data, namely, (a) the generally low variability of and lack of continuity in the NA reported by the participants and (b) the relatively stable, high-amplitude fluctuations evidenced

in self-report PA.[7] These features are not uncommon in retrospective self-report affect data, however (see, e.g., Ram et al., 2005). They are regarded as part of the participants' subject experiences as opposed to mere data artifacts (Gilbert, 2007). In fact, another study that used a slightly different nonlinear dynamic factor model to evaluate PA-NA linkage among a group of college students (S.-M. Chow, Tang, Yuan, Song, & Zhu, 2011) also reported a similar finding, namely, the PA $\rightarrow$ NA, but not the NA $\rightarrow$ PA coupling, was found to be reliably different from zero. Collectively, these findings provide new insights into the important role of PA in emotion regulation.

SIMULATION STUDY

Simulation Designs

The primary purpose of the simulation study is to demonstrate the performance of the proposed methodological approach in recovering the true parameter values; their associated *SE*s; and the true latent variable scores, including values of the time-varying parameters. Two combinations of sample sizes and time series lengths were considered, namely, with $T = 70$, $n = 10$, and $T = 28$, $n = 25$. The first condition was specifically selected to mirror the sample size/time series length of our empirical data. The second condition was chosen to yield the same total number of $n \times T$ observations and to provide a more realistic representation of time series data available from the social and behavioral sciences (with T being much smaller than 100). Missing data were not the focus of this study. We included 4% of missingness in the simulated data following a missing completely at random mechanism to yield comparable total sample size to our empirical data.

The best-fitting model from empirical model fitting was used as a basis to construct our simulation models. To mirror the parameter space of our empirical illustration, the population values of all the time-invariant parameters were chosen to closely approximate those obtained from empirical model fitting. We set

$$\mathbf{\Lambda} = \begin{bmatrix} 1 & .75 & .84 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1.49 & 1.80 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'.$$

The uniquenesses, $\boldsymbol{\epsilon}_{i,t}$, were specified to be normally distributed with zero means and covariance matrix $\mathbf{\Sigma}_{\epsilon} = \text{diag}[.11\ .15\ .06\ .22\ .18\ .04]$. The process noise

[7]Our results did not stem from difference in means between PA and NA, however, because all model fitting was performed using the residual scores.

covariance matrix was set to be $\mathbf{\Sigma}_\zeta = \text{diag}[.09\ .03\ .00\ .05]$. Thus, consistent with the empirical finding that the cross-regression weight from NA to PA, $b_{PN,i,t}$, did not show statistically significant fluctuations over time, the third diagonal element of $\mathbf{\Sigma}_\zeta$, $\sigma^2_{\zeta_{NP}}$, was fixed at zero. In contrast, the cross-regression weight from PA to NA, $b_{NP,i,t}$, was specified to fluctuate over time according to three hypothesized models, denoted here as (1) the true model, (2) a time-invariant model, and (3) a discrete-change model. This yielded a total of 2 (sample size/time series length combinations) $\times$ 3 (change scenarios) $=$ 6 conditions in our simulation study. We conducted N = 8,000 Monte Carlo replications for each condition. The same dynamic model (i.e., Equation (2)) was fitted to the data from all conditions.

(1) True model. In the first condition, the true cross-regression weights were specified to change according to the AR(1) of differences model summarized in Equation (2). The parameter values were set to be the same as the estimates from empirical model fitting (see Table 2), with $a_P = 0.54$, $a_N = 0.01$, $\beta_{PN0} =$ -0.18, $\beta_{NP0} = -0.26$, $\beta_{PN1} = 0.00$, and $b_{PN1} = 0.91$. With β_{PN1} and $\sigma^2_{\zeta_{NP}}$ set to zero, the coupling effect from NA to PA reduced to a time-invariant value, namely, $b_{PN,i,t} = \beta_{PN0} = -0.18$. The coupling effect from PA to NA, in contrast, were specified to show variations over time. Because the fitted model was the true model on which data generation was based, this condition served as a baseline condition against which other conditions were compared.

(2) Time-invariant model. In this condition, both of the cross-regression weights were specified to be invariant over time. This is a special case of the full AR(1) of differences model, with the last diagonal element in $\mathbf{\Sigma}_{\boldsymbol{\zeta}}$ (i.e., $\sigma^2_{\zeta_{NP}}$) as well as β_{NP1} set to zero. This condition provided a basis for testing Type I error rate, namely, the probability that a time-invariant parameter was incorrectly concluded to be time-varying. All other parameter values remained the same as in Condition 1.

(3) Discrete-change model. In this condition, the true PA $\rightarrow$ NA cross-regression weight was specified to show discrete changes as

$$b_{NP,i,t} = \begin{cases} -.26 & \text{if } t \leq \text{ integer}(T/3) \\ .26 & \text{if integer}(T/3) < t \leq 2 * \text{integer}(T/3) \\ -.26 & \text{if } t > 2 * \text{integer}(T/3), \end{cases} \tag{6}$$

whereas $b_{NP,i,t}$ was still specified to be invariant over time as in Conditions 1 and 2. All other parameter values remained the same as in Condition 1. In this condition, the fitted model in Equation (2), which posits that $b_{NP,i,t}$ changes

over time in a continuous manner, was used to approximate the discrete shifts in $b_{NP,i,t}$. This condition thus provided one way of testing the extent to which the point and *SE* estimates obtained from the proposed procedures were affected by the presence of model misspecification.

Simulation Results

Latent variable estimation and recovery of the time-varying parameter. To provide a brief demonstration of the performance of the extended Kalman smoother in recovering latent variable scores, the latent variable estimates from one randomly selected Monte Carlo replication in Conditions 1–3, with $n = 10$ and $T = 70$, were plotted in Figures 4a–c. It can be seen that in Condition 2, the time-invariant $b_{PN,i,t}$ can in fact be represented as a special case of a time-varying parameter with no change (see Panel b). In Condition 3, despite the presence of mild model misspecification, the proposed model still yielded reasonable approximation to the discrete shifts in the time-varying parameter. However, substantial approximation errors were observed near the turning points, as would be expected.

Results from estimating all the latent variable scores using the extended Kalman filter and the extended Kalman smoother are summarized in Table 3.

TABLE 3
Root Mean Squared Errors (RMSEs) of the Latent Variable Estimates Across All Time Points for All Conditions

		$T = 70, n = 10$		$T = 28, n = 25$	
Model	*Variable*	*EKF*	*EKS*	*EKF*	*EKS*
True model	$PA_{i,t}$	0.178	0.165	0.176	0.164
	$NA_{i,t}$	0.104	0.103	0.102	0.101
	$b_{PN,i,t}$	0.045	0.045	0.051	0.051
	$b_{NP,i,t}$	0.404	0.326	0.405	0.338
Time-invariant model	$PA_{i,t}$	0.184	0.172	0.189	0.176
	$NA_{i,t}$	0.099	0.099	0.103	0.103
	$b_{PN,i,t}$	0.073	0.073	0.075	0.075
	$b_{NP,i,t}$	0.039	0.046	0.042	0.049
Discrete-change model	$PA_{i,t}$	0.178	0.169	0.176	0.168
	$NA_{i,t}$	0.100	0.099	0.098	0.098
	$b_{PN,i,t}$	0.060	0.060	0.062	0.062
	$b_{NP,i,t}$	0.273	0.232	0.265	0.240

Note. EKF = Extended Kalman filter; EKS = Extended Kalman smoother; RMSE in this case is defined to be the square root of the mean squared error between each latent variable's true and estimated scores across all time points and persons.

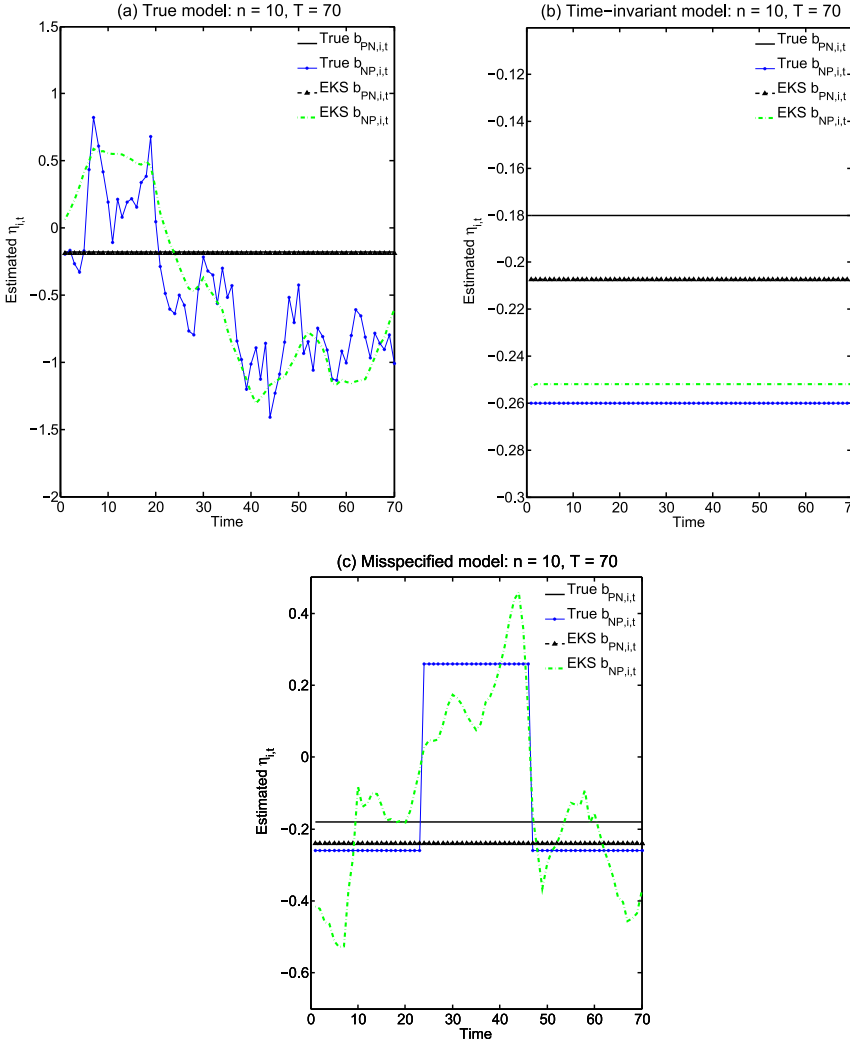

FIGURE 4 The true and latent variable estimates obtained using the extended Kalman smoother when the model in Equation (2) was fitted to data generated during one Monte Carlo replication with $n = 10$ and $T = 70$ using (a) the true model, (b) a time-invariant model that is nested within the true model, and (c) a model dictating discrete changes in $b_{NP,i,t}$ (i.e., a misspecified model).

When the true model was fitted, both the extended Kalman filter and the extended Kalman smoother performed well in estimating factor scores. Greater root mean squared errors (RMSEs) were observed in the estimates of the time-varying parameter, $b_{NP,i,t}$, than all other latent variables. In particular, although $PA_{i,t}$ was associated with a larger process noise or shock variance, which often leads to greater estimation errors, lower RMSEs were observed in the estimates of $PA_{i,t}$ than in the estimates of $b_{NP,i,t}$. This is to be expected, however. It is generally more difficult to track time-varying parameters than latent factors because a time-varying parameter is not indicated directly by an observed variable but rather is identified indirectly through its linkage to other latent factors (i.e., $PA_{i,t}$ and $NA_{i,t}$). Thus, time-varying parameters tend to share less covariance with the observed time series than the latent factors, thus yielding smaller values in the gain matrices, $\mathbf{K}_{i,t} = \text{Cov}(\boldsymbol{\eta}_{i,t}, \boldsymbol{y}_{i,t})\,\text{Var}(\boldsymbol{y}_{i,t})^{-1}$, in the updating stage. Longer time series lengths (i.e., T) are typically needed to track changes in time-varying parameters, especially when a diffuse prior is used. In contrast, the RMSE of $b_{PN,i,t}$ was much smaller because this parameter did not vary over time.

Generally, improvements in estimation accuracy due to smoothing are greater when the process noise variances are large.[8] Thus, improvements from smoothing when the extended Kalman smoother, as opposed to the extended Kalman filter, was used were particularly pronounced in the estimation of the time-varying parameter, $b_{NP,i,t}$. Slight improvements were also observed when the extended Kalman smoother was used to estimate the factor scores of $PA_{i,t}$ but not $NA_{i,t}$. This is not surprising because the former was characterized by a larger process noise variance ($\sigma^2_{\zeta_{PA}} = 0.09$) than the latter ($\sigma^2_{\zeta_{NA}} = 0.03$). No difference was observed between the filtered and smoothed estimates of the time-invariant $b_{PN,i,t}$, whose process noise variance was equal to zero.

Condition 2 constituted a somewhat extreme case in that two latent variables (the two cross-regression weights) had zero process noise variances. It can be seen that the smoothed estimates of $b_{NP,i,t}$ from the extended Kalman smoother were actually characterized by a larger RMSE than the filtered estimates from the extended Kalman filter. In most Monte Carlo runs, estimation errors from the extended Kalman smoother are typically close to or equal to those associated with the filtered estimates. Larger errors were observed among the smoothed estimates, however, in Monte Carlo trials where β_{NP1}, whose true population value was equal to zero, was estimated to have a negative value. Under such a circumstance, incorporating more measurement occasions actually captured

[8]This is related to the smoothability condition of a dynamic system. A latent variable is said to be smoothable if the smoother provides more accurate predictions than those obtained from running the Kalman filter backward in time. Only latent variables that are driven by process noise are smoothable (for details see Gelb, 1974; Thijssen, de Jong, Kateman, & Smit, 1985).

some of the rapid, noisy fluctuations around the baseline cross-regression parameter, β_{NP0}, thus resulting in higher RMSEs for the smoothed estimates. This was not the case for the other time-invariant cross-regression parameter, $b_{PN,i,t}$, because in the fitted model, parameters such as β_{PN0} and $\sigma^2_{\zeta_{PN}}$ were already fixed at the true population value of zero.

In Condition 3 where the fitted model was misspecified, most of the latent variable estimates actually showed lower RMSEs than those observed in Condition 1 in which the true model was fitted. This was because the time-varying parameter $b_{NP,i,t}$ only showed deterministic shifts at selected time points that were easier to track compared with Condition 1 in which an AR(1) of differences process was used as the true model. This result, of course, was specific to the type of misspecification adopted in the present context.

Estimation of time-invariant parameters. Statistical properties of the ML estimators across all conditions are summarized in Tables 4–9. The RMSE and relative bias were used to quantify the performance of the ML point estimator. The empirical *SE* of a parameter (i.e., the standard deviation of the parameter estimates across all Monte Carlo runs) was used as the "true" standard error. We also included the estimated coefficient of variation ($\widehat{CV}$) and average relative deviance of an *SE* estimate (aRDSE) of an estimator. $\widehat{CV}$ is defined to be the ratio between the empirical *SE* to the mean parameter estimates. It measures the relative variation of an estimator. The aRDSE is the difference between an *SE* estimate and the true *SE* over the true *SE*, averaged across Monte Carlo runs. It is a measure of the relative performance of the *SE* estimates.

Ninety-five percent confidence intervals (CIs) were constructed for each of the $N = 8,000$ simulation samples in each condition by adding and subtracting 1.96**SE* estimate in each replication to the parameter estimate from the replication. The coverage performance of a CI was assessed with its empirical coverage rates, namely, the proportion of 95% CIs covering θ across the 8,000 samples. For a parameter whose true value was equal to zero, 1 minus this coverage rate served as a measure of Type I error rate.

Comparing across the two sample size configurations, the design with $T = 70$ and $n = 10$ generally yielded point estimates with lower RMSEs and relative biases compared with the condition where $T = 28$ and $n = 25$ across all change scenarios. When the true model was fitted to the simulated data (Condition 1), the point estimates were generally unbiased, except that the autoregression parameter for NA, a_N, was overestimated. All *SE* estimates were close to the true empirical standard errors, with the exception of the slightly larger aRDSE for a_N and β_{PN0}. Slightly more precise *SE* estimates were observed in the condition with larger T, but the differences were relatively minor. The empirical coverage rates of the CI for a_N, β_{PN0}, β_{NP0}, $\sigma^2_{\zeta_{NA}}$, and $\sigma^2_{\zeta_{NP}}$ were smaller than the 95% nominal rate, whereas those associated with other parameters were close to 95%.

TABLE 4
Summary Statistics Pertaining to the Point and *SE* Estimates for the Time-Invariant Parameters in the True Model Condition (Condition 1) With $T = 70$, $n = 10$

Parameter	θ	$a\hat{\theta}$	*RMSE*	*rBias*	*SE*	$\widehat{CV}$	$a\widehat{SE}$	*aRDSE*	*Coverage*
λ_{P2}	0.75	0.75	0.005	−0.008	0.05	0.07	0.05	0.00	.96
λ_{P3}	0.84	0.84	0.005	−0.005	0.05	0.06	0.05	−0.02	.95
λ_{N2}	1.49	1.51	0.016	0.011	0.12	0.08	0.12	−0.02	.96
λ_{N3}	1.80	1.82	0.024	0.013	0.15	0.08	0.14	−0.06	.95
a_P	0.54	0.52	0.021	−0.039	0.04	0.08	0.04	−0.02	.93
a_N	0.01	0.05	0.037	3.681	0.06	1.26	0.05	−0.23	.80
β_{PN0}	−0.18	−0.18	0.008	0.047	0.06	−0.30	0.04	−0.34	.81
β_{NP0}	−0.26	−0.27	0.009	0.033	0.11	−0.42	0.10	−0.14	.91
β_{NP1}	0.91	0.90	0.011	−0.012	0.04	0.04	0.03	−0.15	.94
$\sigma^2_{\zeta_{PA}}$	0.09	0.09	0.002	0.024	0.01	0.10	0.01	0.02	.96
$\sigma^2_{\zeta_{NA}}$	0.03	0.04	0.005	0.182	0.01	0.17	0.01	−0.03	.89
$\sigma^2_{\zeta_{NP}}$	0.05	0.05	0.001	0.015	0.02	0.42	0.02	−0.12	.89
$\sigma^2_{\epsilon_{P1}}$	0.11	0.11	0.001	−0.010	0.01	0.08	0.01	−0.02	.94
$\sigma^2_{\epsilon_{P2}}$	0.15	0.15	0.000	−0.002	0.01	0.06	0.01	0.00	.95
$\sigma^2_{\epsilon_{P3}}$	0.06	0.06	0.000	−0.006	0.01	0.09	0.01	−0.02	.95
$\sigma^2_{\epsilon_{N1}}$	0.22	0.22	0.000	0.001	0.01	0.06	0.01	0.02	.95
$\sigma^2_{\epsilon_{N2}}$	0.18	0.18	0.000	−0.002	0.01	0.07	0.01	−0.03	.94
$\sigma^2_{\epsilon_{N3}}$	0.04	0.04	0.001	−0.011	0.04	0.28	0.01	−0.06	.95

Note. θ = true value of a parameter; $a\hat{\theta} = \frac{1}{N}\sum_{k=1}^{N}\hat{\theta}_k$, where $\hat{\theta}_k$ = estimate of θ from the kth Monte Carlo replication; root mean squared error (RMSE) = $\sqrt{\frac{1}{N}\sum_{k=1}^{N}(\hat{\theta}_k - \theta)^2}$; rBias = relative bias = $\frac{1}{N}\sum_{k}^{N}(\hat{\theta}_k - \theta)/\theta$; *SE* = standard deviation of estimates across Monte Carlo runs; $\widehat{CV}$ = estimated coefficient of variation = SE/$a\hat{\theta}$; $a\widehat{SE}$ = average standard error estimate across Monte Carlo runs; aRDSE = average relative deviance of $\widehat{SE}$ = $(a\widehat{SE} - SE)/SE$; coverage = number of 95% confidence intervals across the Monte Carlo runs that contain the true θ.

The affected parameters were generally parameters in the dynamic model that had smaller effect sizes (i.e., their true values were very close to zero).

As in Condition 1, parameters whose true population values were close to zero also tended to show larger biases and lower coverage rates in the time-invariant condition (i.e., Condition 2). Some of the $\widehat{CV}$ values (e.g., those for a_N and β_{NP1}) were arbitrarily inflated in this case by the near-zero point estimates even though the associated empirical *SE*s were not very large. Estimates for β_{NP1}, whose true population value was equal to zero, were negatively biased with inflated Type I error rates. When compared with the average *SE* estimates across all Monte Carlo runs (i.e., $a\widehat{SE}$), the mean parameter estimates (i.e., $a\hat{\theta}$ in Tables 6 and 7) did not differ significantly from zero. However, the null hypothesis that $\beta_{NP1} = 0$ in the population was incorrectly rejected in 33% and 30% of the Monte Carlo runs, respectively, for $T = 70$ and $T = 28$, yielding relatively

TABLE 5
Summary Statistics Pertaining to the Point and *SE* Estimates for the Time-Invariant Parameters in the True Model Condition (Condition 1) With $T = 28$, $n = 25$

Parameter	θ	$a\hat{\theta}$	*RMSE*	*rBias*	*SE*	$\widehat{CV}$	$a\widehat{SE}$	*aRDSE*	*Coverage*
λ_{P2}	0.75	0.74	0.015	−0.019	0.05	0.07	0.05	−0.02	.94
λ_{P3}	0.84	0.83	0.014	−0.017	0.05	0.06	0.05	−0.02	.93
λ_{N2}	1.49	1.52	0.033	0.022	0.13	0.09	0.12	−0.07	.95
λ_{N3}	1.80	1.85	0.049	0.027	0.16	0.08	0.14	−0.10	.94
a_P	0.54	0.50	0.043	−0.079	0.04	0.08	0.04	−0.02	.93
a_N	0.01	0.07	0.059	5.908	0.07	1.00	0.04	−0.39	.61
β_{PN0}	−0.18	−0.20	0.024	−0.131	0.06	−0.29	0.04	−0.34	.79
β_{NP0}	−0.26	−0.28	0.021	0.081	0.14	−0.51	0.11	−0.20	.92
β_{NP1}	0.91	0.90	0.010	−0.011	0.04	0.05	0.04	−0.15	.93
$\sigma^2_{\zeta_{PA}}$	0.09	0.10	0.005	0.050	0.01	0.10	0.01	0.02	.95
$\sigma^2_{\zeta_{NA}}$	0.03	0.04	0.006	0.188	0.01	0.18	0.01	−0.05	.89
$\sigma^2_{\zeta_{NP}}$	0.05	0.05	0.000	−0.003	0.02	0.47	0.02	−0.15	.86
$\sigma^2_{\epsilon_{P1}}$	0.11	0.11	0.002	−0.017	0.01	0.08	0.01	−0.02	.94
$\sigma^2_{\epsilon_{P2}}$	0.15	0.15	0.000	−0.002	0.01	0.06	0.01	−0.02	.95
$\sigma^2_{\epsilon_{P3}}$	0.06	0.06	0.000	−0.005	0.01	0.09	0.01	−0.03	.94
$\sigma^2_{\epsilon_{N1}}$	0.22	0.22	0.001	0.002	0.01	0.06	0.01	0.00	.96
$\sigma^2_{\epsilon_{N2}}$	0.18	0.18	0.000	0.002	0.01	0.07	0.01	−0.02	.95
$\sigma^2_{\epsilon_{N3}}$	0.04	0.04	0.001	−0.031	0.01	0.30	0.01	−0.09	.94

Note. θ = true value of a parameter; $a\hat{\theta} = \frac{1}{N}\sum_{k=1}^{N}\hat{\theta}_k$, where $\hat{\theta}_k$ = estimate of θ from the kth Monte Carlo replication; root mean squared error (RMSE) = $\sqrt{\frac{1}{N}\sum_{k=1}^{N}(\hat{\theta}_k - \theta)^2}$; rBias = relative bias = $\frac{1}{N}\sum_{k}^{N}(\hat{\theta}_k - \theta)/\theta$; *SE* = standard deviation of estimates across Monte Carlo runs; $\widehat{CV}$ = estimated coefficient of variation = SE/$a\hat{\theta}$; $a\widehat{SE}$ = average standard error estimate across Monte Carlo runs; aRDSE = average relative deviance of $\widehat{SE}$ = $(a\widehat{SE} - SE)/SE$; coverage = number of 95% confidence intervals across the Monte Carlo runs that contain the true θ.

high Type I error rates. The longer time series length in the $T = 70$ condition actually led to slightly greater negative bias in the estimates of β_{NP1} on average. In both sample size conditions, the now time-invariant $b_{NP,i,t}$ was estimated to show rapid, random fluctuations around β_{NP0}, as reflected in the negative value of β_{NP1}.[9] Other parameters were accurately recovered and the corresponding *SE*s were also close to the true *SE*s despite the elevated Type I error rates.

In the discrete-change condition (i.e., Condition 3), a_N was found to be overestimated. The *SE*s of the time-varying parameter-related parameters (e.g.,

[9]Post hoc evaluations of the β_{NP1} estimates in all Monte Carlo trials revealed that the generally high biases associated with β_{NP1} in both sample size conditions were due largely to the existence of a few extreme outliers. We retained all cases, however, to give a more realistic summary of the simulation results.

TABLE 6
Summary Statistics Pertaining to the Point and *SE* Estimates for the Time-Invariant Parameters in the Time-Invariant Condition (Condition 2) With $T = 70$, $n = 10$

Parameter	θ	$a\hat{\theta}$	*RMSE*	*rBias*	*SE*	$\widehat{CV}$	$a\widehat{SE}$	*aRDSE*	*Coverage*
λ_{P2}	0.75	0.75	0.005	−0.006	0.06	0.08	0.06	−0.01	.93
λ_{P3}	0.84	0.84	0.005	−0.006	0.05	0.06	0.05	−0.01	.94
λ_{N2}	1.49	1.46	0.028	−0.019	0.17	0.12	0.17	0.00	.92
λ_{N3}	1.80	1.75	0.053	−0.029	0.22	0.12	0.22	−0.01	.90
a_P	0.54	0.53	0.013	−0.024	0.05	0.09	0.05	−0.01	.93
a_N	0.01	0.01	0.001	0.121	0.05	4.19	0.05	0.03	.95
β_{PN0}	−0.18	−0.18	0.003	0.014	0.09	−0.50	0.06	−0.35	.80
β_{NP0}	−0.26	−0.27	0.008	0.031	0.04	−0.16	0.04	−0.03	.93
β_{NP1}	0.00	−0.12	0.122	—*	0.50	−4.09	0.51	0.02	.67
$\sigma^2_{\zeta_{PA}}$	0.09	0.09	0.001	0.014	0.01	0.11	0.01	0.01	.94
$\sigma^2_{\zeta_{NA}}$	0.03	0.03	0.001	0.032	0.01	0.21	0.01	0.02	.94
$\sigma^2_{\zeta_{NP}}$	0.00	0.01	0.006	—*	0.01	1.67	0.01	−0.23	.93
$\sigma^2_{\epsilon_{P1}}$	0.11	0.11	0.001	−0.009	0.01	0.08	0.01	−0.01	.94
$\sigma^2_{\epsilon_{P2}}$	0.15	0.15	0.000	−0.001	0.01	0.06	0.01	0.00	.94
$\sigma^2_{\epsilon_{P3}}$	0.06	0.06	0.000	0.001	0.01	0.10	0.01	−0.02	.94
$\sigma^2_{\epsilon_{N1}}$	0.22	0.22	0.001	−0.006	0.01	0.06	0.01	0.00	.93
$\sigma^2_{\epsilon_{N2}}$	0.18	0.18	0.002	−0.009	0.01	0.07	0.01	−0.02	.93
$\sigma^2_{\epsilon_{N3}}$	0.04	0.04	0.001	0.034	0.01	0.29	0.01	−0.03	.93

Note. —* = undefined due to division by zero; θ = true value of a parameter; $a\hat{\theta} = \frac{1}{N}\sum_{k=1}^{N}\hat{\theta}_k$, where $\hat{\theta}_k$ = estimate of θ from the kth Monte Carlo replication; root mean squared error (RMSE) $= \sqrt{\frac{1}{N}\sum_{k=1}^{N}(\hat{\theta}_k - \theta)^2}$; rBias = relative bias $= \frac{1}{N}\sum_{k}^{N}(\hat{\theta}_k - \theta)/\theta$; *SE* = standard deviation of estimates across Monte Carlo runs; $\widehat{CV}$ = estimated coefficient of variation = SE/$a\hat{\theta}$; $a\widehat{SE}$ = average standard error estimate across Monte Carlo runs; aRDSE = average relative deviance of $\widehat{SE}$ = $(a\widehat{SE} - SE)/SE$; coverage = number of 95% confidence intervals across the Monte Carlo runs that contain the true θ.

β_{PN0}, β_{NP0}, β_{NP1}, and $\sigma^2_{\zeta_{NP}}$) were slightly underestimated. In addition, the empirical coverage rates of the CIs for a_N and β_{PN0} were also lower than the 95% nominal rate. All other point and *SE* estimates were essentially unbiased. In particular, point and *SE* estimates for all the parameters in the measurement model were robust to the misspecification of the dynamic model of $b_{NP,i,t}$. In fact, because the time-varying parameter only showed deterministic shifts at selected time points that were easier to track compared with Condition 1 in which the AR(1) of differences process was used as the true model, the biases in the measurement parameters were actually lower in the misspecificied condition.

It is important to note that auto- and cross-regression parameters (and other associated parameters that govern their dynamics, such as β_{PN0}, β_{NP0}, and β_{NP1}) are generally correlated. For instance, the correlations between the estimates

TABLE 7
Summary Statistics Pertaining to the Point and *SE* Estimates for the Time-Invariant Parameters in the Time-Invariant Condition (Condition 2) With $T = 28$, $n = 25$

Parameter	θ	$a\hat{\theta}$	*RMSE*	*rBias*	*SE*	$\widehat{CV}$	$a\widehat{SE}$	*aRDSE*	*Coverage*
λ_{P2}	0.75	0.74	0.013	−0.017	0.06	0.08	0.06	−0.02	.93
λ_{P3}	0.84	0.83	0.013	−0.016	0.05	0.06	0.05	−0.01	.93
λ_{N2}	1.49	1.41	0.082	−0.055	0.16	0.11	0.16	0.03	.88
λ_{N3}	1.80	1.65	0.153	−0.085	0.18	0.11	0.19	0.04	.82
a_P	0.54	0.51	0.034	−0.064	0.05	0.10	0.05	−0.03	.89
a_N	0.01	0.01	0.002	0.232	0.04	3.50	0.05	0.08	.96
β_{PN0}	−0.18	−0.19	0.012	0.064	0.09	−0.49	0.06	−0.35	.80
β_{NP0}	−0.26	−0.28	0.021	0.082	0.04	−0.16	0.04	−0.03	.94
β_{NP1}	0.00	−0.09	0.091	—*	0.47	−5.18	0.45	−0.05	.70
$\sigma^2_{\zeta_{PA}}$	0.09	0.09	0.003	0.032	0.01	0.11	0.01	0.03	.95
$\sigma^2_{\zeta_{NA}}$	0.03	0.03	0.003	0.107	0.01	0.20	0.01	0.02	.95
$\sigma^2_{\zeta_{NP}}$	0.00	0.01	0.007	—*	0.01	1.70	0.01	−0.24	.96
$\sigma^2_{\epsilon_{P1}}$	0.11	0.11	0.002	−0.014	0.01	0.09	0.01	−0.02	.93
$\sigma^2_{\epsilon_{P2}}$	0.15	0.15	0.000	−0.002	0.01	0.07	0.01	−0.02	.94
$\sigma^2_{\epsilon_{P3}}$	0.06	0.06	0.000	0.001	0.01	0.10	0.01	−0.01	.94
$\sigma^2_{\epsilon_{N1}}$	0.22	0.22	0.002	−0.010	0.01	0.06	0.01	0.00	.94
$\sigma^2_{\epsilon_{N2}}$	0.18	0.18	0.003	−0.014	0.01	0.07	0.01	0.00	.93
$\sigma^2_{\epsilon_{N3}}$	0.04	0.04	0.004	0.106	0.01	0.25	0.01	0.02	.93

Note. —* = undefined due to division by zero; θ = true value of a parameter; $a\hat{\theta} = \frac{1}{N}\sum_{k=1}^{N}\hat{\theta}_k$, where $\hat{\theta}_k$ = estimate of θ from the kth Monte Carlo replication; root mean squared error (RMSE) = $\sqrt{\frac{1}{N}\sum_{k=1}^{N}(\hat{\theta}_k - \theta)^2}$; rBias = relative bias = $\frac{1}{N}\sum_{k}^{N}(\hat{\theta}_k - \theta)/\theta$; *SE* = standard deviation of estimates across Monte Carlo runs; $\widehat{CV}$ = estimated coefficient of variation = SE/$a\hat{\theta}$; $a\widehat{SE}$ = average standard error estimate across Monte Carlo runs; aRDSE = average relative deviance of $\widehat{SE}$ = $(a\widehat{SE} - SE)/SE$; coverage = number of 95% confidence intervals across the Monte Carlo runs that contain the true θ.

of a_P and β_{PN0} across all Monte Carlo replications were .43, −.17, and .36, respectively, in Conditions 1–3 when $n = 10$ and $T = 70$. Thus, biases in estimating some of these parameters may "spill over" to influence other related parameters. In our simulation results in Condition 2, we found that a_N, whose true value was close to zero and showed relatively high biases in Conditions 1 and 3, was characterized by very low biases in Condition 2 for both sample size configurations. This is because the time-invariant model in Condition 2 is a simpler model than the simulation models employed in Conditions 1 and 3. When both of the cross-regression parameters were invariant over time, it was easier to recover the autoregression parameters accurately even when one of them (i.e., a_N) was characterized by a very small effect size. By the same token, if supposedly time-varying cross-regression parameters were constrained

TABLE 8
Summary Statistics Pertaining to the Point and *SE* Estimates for the Time-Invariant Parameters in the Discrete-Change Condition (Condition 3) With $T = 70$, $n = 10$

Parameter	θ	$a\hat{\theta}$	*RMSE*	*rBias*	*SE*	$\widehat{CV}$	$a\widehat{SE}$	*aRDSE*	*Coverage*
λ_{P2}	0.75	0.75	0.003	−0.005	0.06	0.08	0.06	−0.01	.96
λ_{P3}	0.84	0.84	0.003	−0.004	0.05	0.06	0.05	−0.01	.95
λ_{N2}	1.49	1.48	0.013	−0.009	0.15	0.10	0.15	−0.01	.95
λ_{N3}	1.80	1.76	0.038	−0.021	0.19	0.11	0.19	−0.03	.93
a_P	0.54	0.52	0.017	−0.033	0.04	0.08	0.04	0.00	.94
a_N	0.01	0.04	0.026	2.589	0.06	1.62	0.05	−0.17	.86
β_{PN0}	−0.18	−0.18	0.003	0.017	0.07	−0.41	0.05	−0.38	.80
β_{NP0}	NA	−0.15	NA	NA	0.12	−0.83	0.07	−0.45	NA
β_{NP1}	NA	−0.12	NA	NA	0.07	0.08	0.05	−0.33	NA
$\sigma^2_{\zeta_{PA}}$	0.09	0.09	0.002	0.017	0.01	0.11	0.01	0.01	.96
$\sigma^2_{\zeta_{NA}}$	0.03	0.03	0.004	0.140	0.01	0.19	0.01	0.00	.94
$\sigma^2_{\zeta_{NP}}$	NA	0.01	NA	NA	0.02	0.72	0.01	−0.37	NA
$\sigma^2_{\epsilon_{P1}}$	0.11	0.11	0.001	−0.009	0.01	0.08	0.01	−0.02	.94
$\sigma^2_{\epsilon_{P2}}$	0.15	0.15	0.000	−0.003	0.01	0.06	0.01	−0.01	.95
$\sigma^2_{\epsilon_{P3}}$	0.06	0.06	0.000	−0.005	0.01	0.09	0.01	0.00	.96
$\sigma^2_{\epsilon_{N1}}$	0.22	0.22	0.001	−0.003	0.01	0.06	0.01	0.03	.95
$\sigma^2_{\epsilon_{N2}}$	0.18	0.18	0.002	−0.008	0.01	0.08	0.01	−0.03	.94
$\sigma^2_{\epsilon_{N3}}$	0.04	0.04	0.001	0.034	0.01	0.30	0.01	−0.03	.95

Note. NA = not applicable because true parameter does not exist; θ = true value of a parameter; $a\hat{\theta} = \frac{1}{N}\sum_{k=1}^{N}\hat{\theta}_k$, where $\hat{\theta}_k$ = estimate of θ from the kth Monte Carlo replication; root mean squared error (RMSE) = $\sqrt{\frac{1}{N}\sum_{k=1}^{N}(\hat{\theta}_k - \theta)^2}$; rBias = relative bias = $\frac{1}{N}\sum_{k}^{N}(\hat{\theta}_k - \theta)/\theta$; *SE* = standard deviation of estimates across Monte Carlo runs; $\widehat{CV}$ = estimated coefficient of variation = SE/a$\hat{\theta}$; $a\widehat{SE}$ = average standard error estimate across Monte Carlo runs; aRDSE = average relative deviance of $\widehat{SE}$ = $(a\widehat{SE} - SE)/SE$; coverage = number of 95% confidence intervals across the Monte Carlo runs that contain the true θ.

and estimated as time-invariant parameters, the misspecification would likely affect not only the cross-regression but also the autoregression parameters. In the present context, we allowed the cross-regression parameters but not the autoregression parameters to be time-varying parameters for theoretical and practical reasons. Better diagnostic tests are needed to help assess the need to incorporate other modeling parameters as time-varying parameters.

Summary of simulation results. Several key conclusions can be drawn from our simulation results. First, improvements in estimation accuracy when the extended Kalman smoother as opposed to the extended Kalman filter was used were more substantial when the process noise variances were large. The advantage of smoothing was especially salient in deriving latent variable estimates for time-varying parameters. Second, the point and *SE* estimates obtained

TABLE 9
Summary Statistics Pertaining to the Point and *SE* Estimates for the Time-Invariant Parameters in the Discrete-Change Condition (Condition 3) With $T = 28$, $n = 25$

Parameter	θ	$a\hat{\theta}$	*RMSE*	*rBias*	*SE*	$\widehat{CV}$	$a\widehat{SE}$	*aRDSE*	*Coverage*
λ_{P2}	0.75	0.74	0.014	−0.019	0.06	0.08	0.06	−0.01	.94
λ_{P3}	0.84	0.83	0.014	−0.016	0.05	0.06	0.05	−0.01	.93
λ_{N2}	1.49	1.48	0.007	−0.004	0.16	0.11	0.14	−0.11	.93
λ_{N3}	1.80	1.77	0.033	−0.018	0.21	0.12	0.17	−0.18	.89
a_P	0.54	0.50	0.038	−0.070	0.05	0.09	0.04	−0.05	.86
a_N	0.01	0.07	0.055	5.511	0.07	1.03	0.04	−0.34	.66
β_{PN0}	−0.18	−0.19	0.014	0.075	0.07	−0.38	0.05	−0.37	.79
β_{NP0}	NA	−0.19	NA	NA	0.16	−0.80	0.08	−0.51	NA
β_{NP1}	NA	−0.09	NA	NA	0.14	0.17	0.08	−0.46	NA
$\sigma^2_{\zeta_{PA}}$	0.09	0.09	0.004	0.045	0.01	0.11	0.01	0.02	.96
$\sigma^2_{\zeta_{NA}}$	0.03	0.04	0.006	0.189	0.01	0.18	0.01	−0.01	.91
$\sigma^2_{\zeta_{NP}}$	NA	0.007	NA	NA	0.02	0.54	0.02	−0.11	NA
$\sigma^2_{\epsilon_{P1}}$	0.11	0.11	0.002	−0.017	0.01	0.08	0.01	−0.02	.94
$\sigma^2_{\epsilon_{P2}}$	0.15	0.15	0.000	−0.002	0.01	0.06	0.01	−0.01	.95
$\sigma^2_{\epsilon_{P3}}$	0.06	0.06	0.000	−0.002	0.01	0.09	0.01	−0.02	.95
$\sigma^2_{\epsilon_{N1}}$	0.22	0.22	0.001	−0.003	0.01	0.06	0.01	−0.01	.95
$\sigma^2_{\epsilon_{N2}}$	0.18	0.18	0.001	−0.008	0.01	0.07	0.01	−0.01	.95
$\sigma^2_{\epsilon_{N3}}$	0.04	0.04	0.001	0.036	0.01	0.30	0.01	−0.07	.93

Note. NA = not applicable because true parameter does not exist; θ = true value of a parameter; $a\hat{\theta} = \frac{1}{N}\sum_{k=1}^{N}\hat{\theta}_k$, where $\hat{\theta}_k$ = estimate of θ from the kth Monte Carlo replication; root mean squared error (RMSE) = $\sqrt{\frac{1}{N}\sum_{k=1}^{N}(\hat{\theta}_k - \theta)^2}$; rBias = relative bias = $\frac{1}{N}\sum_{k}^{N}(\hat{\theta}_k - \theta)/\theta$; *SE* = standard deviation of estimates across Monte Carlo runs; $\widehat{CV}$ = estimated coefficient of variation = SE/$a\hat{\theta}$; $a\widehat{SE}$ = average standard error estimate across Monte Carlo runs; aRDSE = average relative deviance of $\widehat{SE}$ = $(a\widehat{SE} - SE)/SE$; coverage = number of 95% confidence intervals across the Monte Carlo runs that contain the true θ.

from the proposed approach were generally satisfactory, although parameters with very small effect sizes tended to show greater biases and lower coverage rates. Third, when all the modeling assumptions were approximately valid (e.g., all individuals were independent samples from the same population and model misspecification was mild), increasing the number of time points generally led to more accurate results than increasing the number of participants. Fourth, relatively high Type I error rates (i.e., proportions of cases where the time-invariant $b_{NP,it}$ were incorrectly identified as time-varying) were observed in the time-invariant condition. Better diagnostic tests are thus needed to supplement the simple measure of using the statistical significance of the process noise variance of a time-varying parameter to determine whether the parameter indeed varies over time. Fifth, when the dynamic process governing a time-varying parameter was misspecified, some of the dynamic parameters were affected by the model

misspecification but all the measurement parameters were unaffected. The extent of the bias depends on the nature and degree of model misspecification.

DISCUSSION

In this article, we represented time-varying parameters in a dynamic factor analysis model as latent variables and estimated their trajectories jointly with other latent variables (e.g., latent factors). The techniques considered in this article expand the work of many others who have used both exploratory as well as confirmatory methods to represent changes in the statistical properties of a system over time—namely, nonstationarities. Directly relevant is the work of Molenaar and colleagues (Molenaar, 1994b; Molenaar et al., 1992), who presented some of the first applications of dynamic factor analysis model with time-varying parameters in psychology, and Heath (2000), who used the Kalman filter to detect sudden changes in the parameters of AR models.

The AR(1) of differences model used as approximation model in this article is but one possible model of change. This model serves as a reasonable approximation to change processes that are expected to fluctuate around a baseline in a less systematic manner (i.e., stochastically) while still allowing for the special case of a time-invariant process. This is consistent with our conceptualization of how the linkage between PA and NA changes over time. In cases where the changes are expected to unfold in other distinct ways (e.g., showing unbounded fluctuations or fluctuations around a changing mean, increasing/decreasing as a linear function of time, manifesting stepwise monotonic increase/decrease as a function of time), this model may not be appropriate. In instances where preconceived notions of the change processes involved are unclear or unavailable, other nonparametric models (e.g., polynomial and spline-based models; Harvey, 2001; Molenaar, 1994b; Ramsay & Silverman, 2005) may be used.

We took a group-based approach to constructing our illustrative examples. Whether such group-based assumptions are tenable in practice is a separate but important issue that deserves more thorough evaluations (Molenaar, 2004). In this article, we capitalized on a set previously published data in which the assumption of homogeneity across individuals had already been tested (S.-M. Chow et al., 2004) to illustrate the proposed techniques. Along a similar line, the extent to which different individuals show homogeneous factor structure that warrants pooling also deserves careful consideration (Hamaker et al., 2005). In our empirical analysis, we used item parceling to eliminate some of the idiosyncratic interindividual differences in factor structure prior to model fitting. Researchers who undertake this group-based approach to modeling in the future should exercise caution in choosing between modeling at the individual versus group level.

Parameters in the measurement model were found to be unaffected by specification errors in the dynamic model for the time-varying parameter. This finding is potentially attributable to the orthogonality of the measurement parameters with respect to the dynamic parameters that were misspecified. This issue was discussed in the context of structural equation models by Yuan, Marshall, and Bentler (2003). The approaches proposed by these authors for checking the orthogonality of modeling parameters are also applicable to state-space models, but this is beyond the scope of this article. From a practical standpoint, it is encouraging to note that conclusions concerning the structures (e.g., factor loadings and unique variances) of psychological constructs are still trustworthy even though the corresponding dynamic model might be misspecified.

As we have illustrated, dynamic factor analysis models with time-varying parameters provide a natural platform for testing certain psychological processes of interest. Even though some of the theoretical postulates seen in the psychological literature can be readily represented using dynamic models, they are rarely put to the test using the model-fitting approach adopted in this article. More often, carefully construed experiments are designed to detect possible changes in certain modeling parameters of interest. The lack of readily accessible tools for evaluating more complex dynamic models is one reason for the scarcity of modeling work along this line. Another reason stems from many researchers' lack of familiarity with modeling time-varying parameters and more generally, with nonlinear dynamic models. We hope to have provided the reader with a workable example of the potential utility of dynamic factor analysis models with time-varying parameters.

Our consideration of issues pertaining to time-varying parameters is far from exhaustive. In particular, our simulation study was designed to mirror several key features of our empirical data (e.g., in sample size configuration and parameter space). Thus, our simulation results may be limited in generalizability to other conditions and models of change. In addition to considering a broader range of models, sample size, and parameter configurations, a number of other issues warrant further investigation in future studies. For instance, the choice of initial latent variable specification (De Jong, 1991; De Jong & Lin, 2003; Oud, Jansen, van Leeuwe, Aarnoutse, & Voeten, 1999) can often influence the estimation results when time series data of finite lengths are involved. A more thorough investigation of this issue is imperative. Other possible modeling extensions include devising improved ways of diagnosing the presence of time-varying parameters in dynamic models to lower Type I error rates (S.-M. Chow, Hamaker, & Allaire, 2009; De Jong & Penzer, 1998), evaluating the effects of other forms of model misspecification, exploring other point and *SE* estimators in the presence of model misspecification and nonnormal data (e.g., bootstrap approaches; Stoffer & Wall, 1991; G. Zhang & Browne, 2010a, 2010b; Z. Zhang & Nesselroade, 2007), and developing alternative modeling extensions such as

continuous-time dynamic factor analysis models and dynamic factor analysis models with mixed effects (e.g., Jones, 1993; Shumway, 2000; Singer, 1998).

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APPENDIX

The Extended Kalman Filter, the Extended Kalman Smoother, and the Prediction Error Decomposition Function

Suppose the data set $\mathbf{Y}_{i,t} = [\boldsymbol{Y}_{1,i,t}, \boldsymbol{Y}_{2,i,t}, \ldots, \boldsymbol{Y}_{p,i,t}]'$ from person i is available for estimation purposes, where $\boldsymbol{Y}_{k,i,t}$ is itself a $t \times 1$ time series for manifest variable k. The extended Kalman filter (EKF; see Anderson & Moore, 1979) is designed to estimate a subclass of nonlinear state-space models with additive, Gaussian process and measurement noises. Once a model has been expressed in the form of Equations (4–5), the EKF can be used to derive conditional state estimates based on manifest observations up to time t and the associated covariance matrix (i.e., $\boldsymbol{\eta}_{i,t|t} = E[\boldsymbol{\eta}_{i,t}|\mathbf{Y}_{i,t}]$ and $\boldsymbol{P}_{i,t|t} = \mathrm{Cov}[\boldsymbol{\eta}_{i,t}|\mathbf{Y}_{i,t}]$). For each time t, the nonlinear dynamic and measurement functions are linearized locally around the current latent variable estimates, $\boldsymbol{\eta}_{i,t|t-1}$, using first-order Taylor series expansion. The EKF is implemented as follows:

First, conditional on observed information from time t-1 and assuming the the values for the parameters in $\boldsymbol{\theta}$ are known, the latent variable scores are projected from time $t-1$ to t as $\boldsymbol{\eta}_{i,t|t-1} = g(\boldsymbol{\eta}_{i,t-1|t-1}, \boldsymbol{\theta}, \boldsymbol{x}_{i,t})$. Predicted observations are then obtained as $\boldsymbol{y}_{i,t|t-1} = h(\boldsymbol{\eta}_{i,t|t-1}, \boldsymbol{\theta}, \boldsymbol{x}_{it})$. The dynamic and measurement functions are linearized around the latent variable estimates, $\boldsymbol{\eta}_{i,t-1|t-1}$, to yield the Jacobian matrices,

$$\mathbf{G}_{i,t} = \left.\frac{\partial g(\boldsymbol{\eta}_{i,t}, \boldsymbol{\theta}, \boldsymbol{x}_{i,t})}{\partial \boldsymbol{\eta}_{i,t}, t}\right|_{\boldsymbol{\eta}_{i,t-1|t-1}} \text{ and } \mathbf{H}_{i,t} = \left.\frac{\partial h(\boldsymbol{\eta}_{i,t}, \boldsymbol{\theta}, \boldsymbol{x}_{i,t})}{\partial \boldsymbol{\eta}_{i,t}}\right|_{\boldsymbol{\eta}_{i,t|t-1}}, \quad (7)$$

where the jth row and kth column of $\boldsymbol{G}_{i,t}$ carries the partial derivative of the jth dynamic function with respect to the k the latent variable, evaluated at participant i's posterior latent variable estimates from time $t-1$, $\boldsymbol{\eta}_{i,t-1|t-1}$. By the same token, the jth colum and kth row of $\mathbf{H}_{i,t}$ carries the partial derivative of the jth measurement function with respect to the kth latent variable, evaluated at participant i's latent variable estimates at time t, $\boldsymbol{\eta}_{i,t|t-1}$ prior to incorporating update from the manifest observation at time t. The subject index in $\mathbf{G}_{it}$ and $\mathbf{H}_{it}$ is used to indicate that the associated Jacobian matrices have different numerical values because they are evaluated at each person's respective current state estimates, not that the dynamic or measurement functions are subject dependent.

Once information is available at time t, the latent variable estimates and their associated covariance matrix are updated as

$$\boldsymbol{\eta}_{i,t|t} = \boldsymbol{\eta}_{i,t|t-1} + \mathbf{K}_{i,t}\boldsymbol{v}_{i,t|t}, \tag{8}$$

$$\mathbf{P}_{i,t|t} = \mathbf{P}_{i,t|t-1} - \mathbf{K}_{i,t}\mathbf{H}_{i,t}\mathbf{P}_{i,t|t-1}, \tag{9}$$

$$\boldsymbol{v}_{i,t|t} = \boldsymbol{y}_{i,t} - \boldsymbol{y}_{i,t|t-1}, \tag{10}$$

$$\mathbf{F}_{i,t} = \mathbf{H}_{i,t}\mathbf{P}_{i,t|t-1}\mathbf{H}_{i,t}^{'} + \boldsymbol{\Sigma}_{\epsilon}, \tag{11}$$

$$\mathbf{K}_{i,t} = \mathbf{P}_{i,t|t-1}\mathbf{H}_{i,t}^{'}\mathbf{F}_{i,t}^{-1}, \tag{12}$$

where $\mathbf{K}_{i,t}$ is referred to as the Kalman gain matrix, $\boldsymbol{v}_{i,t|t}$ is termed the "innovation vector" as it carries the new information available at time t that is not accounted for by the prediction based on observations up to time $t-1$; $\boldsymbol{\eta}_{i,t|t} = E(\boldsymbol{\eta}_{i,t}|\mathbf{Y}_{i,t})$ and $\boldsymbol{P}_{i,t|t} = \mathrm{Cov}(\boldsymbol{\eta}_{i,t}|\mathbf{Y}_{i,t})$ are the posterior latent variable estimates and the associated covariance matrix. Thus, the EKF retains the first-order (i.e., linear) terms from Taylor series expansions of the associated nonlinear functions.

The fixed interval smoother (Anderson & Moore, 1979) is one possible Kalman smoother, whose function is to derive conditional state estimates using all available data, namely, $E[\boldsymbol{\eta}_{i,t}|\mathbf{Y}_{i,T}] = \boldsymbol{\eta}_{i,t|T}$ and the associated covariance matrix, $\mathbf{P}_{i,t|T}$. One version of the smoother (Shumway & Stoffer, 2000) can be implemented by performing a backward recursion for $t = T, \ldots, 1$ as

$$\begin{aligned}\boldsymbol{\eta}_{i,t|T} &= \boldsymbol{\eta}_{i,t|t} + \mathbf{J}_{i,t}(\boldsymbol{\eta}_{i,t+1|T} - \boldsymbol{\eta}_{i,t+1|t}),\\ \mathbf{P}_{i,t|T} &= \mathbf{P}_{i,t|t} + \mathbf{J}_{i,t}(\mathbf{P}_{i,t+1|T} - \mathbf{P}_{i,t+1|t})\mathbf{J}_{i,t}^{'},\end{aligned} \tag{13}$$

where $\mathbf{J}_{i,t} = \mathbf{P}_{i,t|t}\mathbf{G}_{i,t+1}^{'}\mathbf{P}_{i,t+1|t}^{-1}$.

In the estimation process, several by-products of the EKF are concurrently substituted into a raw data log-likelihood function termed the prediction error decomposition function (Caines & Rissanen, 1974; Harvey, 2001; Schweppe,

1965). If the initial latent variable vector, $\boldsymbol{\eta}_{i,1}$, is regarded as fixed as opposed to random, the log-likelihood function can be written solely as a function of the individual innovation vector, $\boldsymbol{v}_{i,t}$, and its associated covariance matrix, $\mathbf{F}_{i,t}$, yielding

$$\log L(\theta) = \frac{1}{2} \sum_{i=1}^{n} \sum_{t=1}^{T} -p_{it} \log(2\pi) - \log |\mathbf{F}_{i,t}| - \boldsymbol{v}_{i,t}^{'} \mathbf{F}_{i,t}^{-1} \boldsymbol{v}_{i,t}, \qquad (14)$$

where p_{it} is the number of complete manifest variables at time t for person i. This log-likelihood function is then optimized with respect to all the time-invariant parameters in $\boldsymbol{\theta}$ to yield maximum likelihood (ML) estimates of these parameters. Other modified recursions to improve the numerical stability of the optimization process have been proposed elsewhere (Ansley & Kohn, 1985; De Jong, 1991; Koopman, 1997).