Dynamic Fully-Compressed Suffix Trees

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Outline

Suffix Trees are Important

28 min

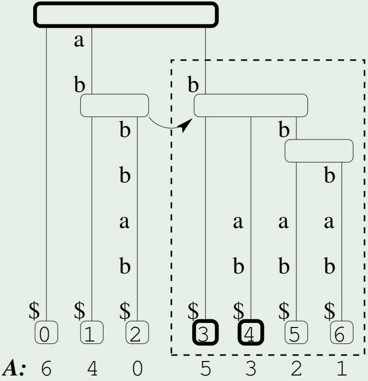
Suffix trees are important for several string problems:

- pattern matching
- longest common substring
- super maximal repeats
- bioinformatics applications
- etc

Suffix Trees are Important

27 min

Example (Suffix Tree for *abbbab*)



Representation Problems

26 min

Problem (Suffix Trees need too much space)

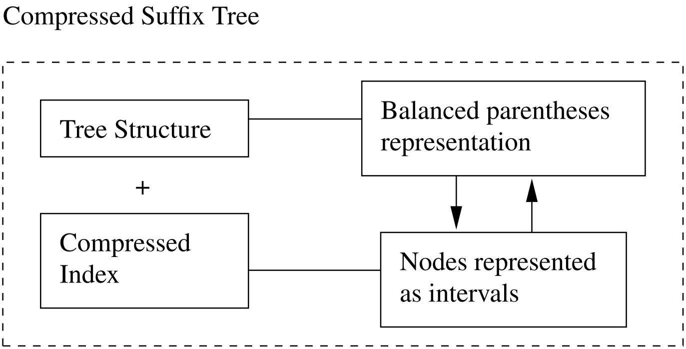
Pointer based representations require $O(n \log n)$ bits.

This is much larger than the indexed string.
State of the art implementations require $[8, 10]n \log \sigma$ bits.

Compressed Representations

25 min

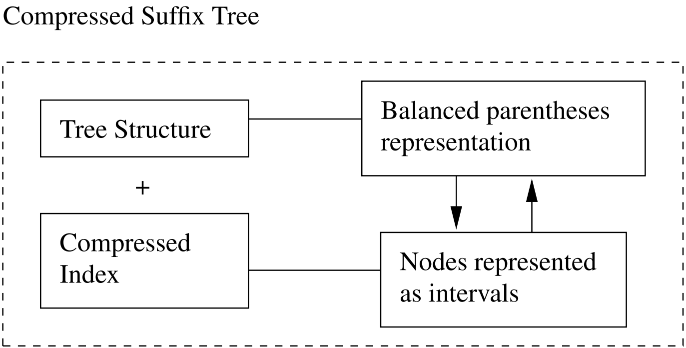
Sadakane proposed a way to represent compressed suffix trees, in $nH_k + 6n + o(n \log \sigma)$ bits.



Compressed Representations

25 min

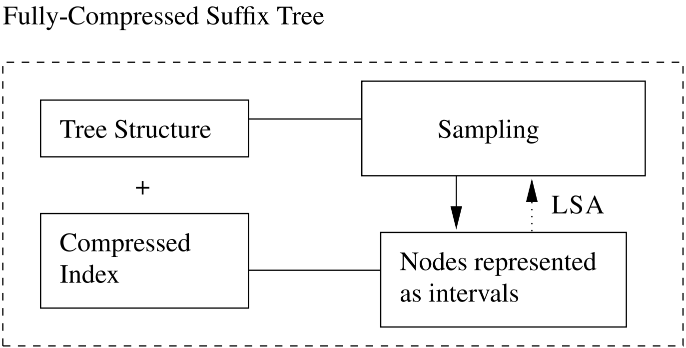
A dynamic representation, by Chan *et al.*, requires $nH_k + \Theta(n) + o(n \log \sigma)$ bits and suffers an $O(\log n)$ slowdown.



Compressed Representations

25 min

- The Fully-Compressed suffix tree representation requires only $nH_k + o(n \log \sigma)$ bits.
- The representation uses the following scheme:

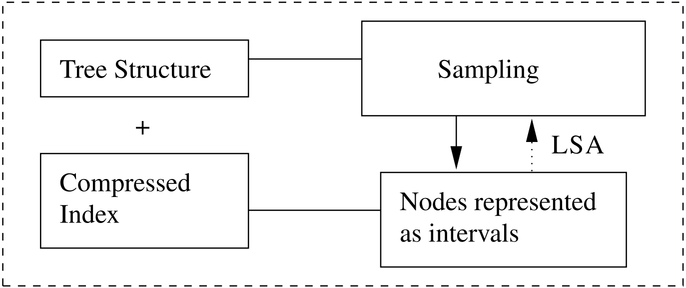


Compressed Representations

25 min

We present dynamic FCST's that require only $nH_k + o(n \log \sigma)$ bits with a $O(\log n)$ slowdown.

Fully-Compressed Suffix Tree



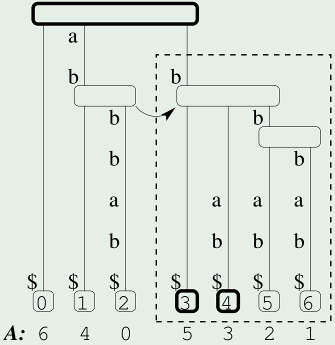
Node Representation

23 min

A node represented as an interval of leaves of a suffix tree.

Example

Interval [3, 6] represents node *b*.



Compressed Indexes

22 min

Compressed indexes are compressed representations of the leaves of a suffix tree.
Their success relies on:

- `Succinct structures`, based on RANK and SELECT.
- `Data compression`, that represent T in $O(uH_k)$ bits.

Examples

FM-index, Compressed Suffix Arrays, LZ-index, etc.

Sadakane used compressed suffix arrays.
We need a compressed index that supports ψ and LF.
For example the Alphabet-Friendly FM-Index.

Suffix Tree self-similarity

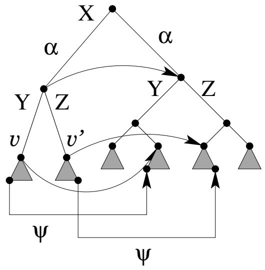
LCA and SLINK

21 min

Lemma

When LCA$(v, v') \neq$ ROOT *we have that:*

$$\text{SLINK}(\text{LCA}(v, v')) = \text{LCA}(\text{SLINK}(v), \text{SLINK}(v'))$$



This self-similarity explains why we can store only some nodes.

Sampling

18 min

FCST's use a sampling such that in any sequence

- v
- SLINK(v)
- SLINK(SLINK(v))
- SLINK(SLINK(SLINK(v)))
- ...

of size δ there is at least one sampled node.

Fundamental lemma

17 min

Lemma

If $\text{SLINK}^r(\text{LCA}(v, v')) = \text{ROOT}$, *and let* $d = \min(\delta, r+1)$.
Then $\text{SDEP}(\text{LCA}(v, v')) =$
$$\max_{0 \le i < d}\{i + \text{SDEP}(\text{LCSA}(\text{SLINK}^i(v), \text{SLINK}^i(v')))\}$$

Proof.

$$\begin{aligned}\text{SDEP}(\text{LCA}(v, v')) &= i + \text{SDEP}(\text{SLINK}^i(\text{LCA}(v, v'))) \\ &= i + \text{SDEP}(\text{LCA}(\text{SLINK}^i(v), \text{SLINK}^i(v'))) \\ &\ge i + \text{SDEP}(\text{LCSA}(\text{SLINK}^i(v), \text{SLINK}^i(v')))\end{aligned}$$

The last inequality is an equality for some $i \le d$. □

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Kernel Operations

12 min

With the previous lemma FCST's compute the following operations:

- $\text{SDEP}(v) = \text{SDEP}(\text{LCA}(v, v)) = \max_{0 \le i < d}\{i + \text{SDEP}(\text{LCSA}(\psi^i(v_l), \psi^i(v_r)))\}$.
- $\text{LCA}(v, v') =$
  $\text{LF}(v[0..i-1],$
  $\text{LCSA}(\psi^i(\min\{v_l, v_l'\}), \psi^i(\max\{v_r, v_r'\}))),$
  for the i in the lemma.
- $\text{SLINK}(v) = \text{LCA}(\psi(v_l), \psi(v_r))$

Dynamic FCST's

11 min

Problem (FCST's are static)

How to insert or remove a text T from a FCST that is indexing a collection $\mathcal{C}$ of texts ?

- Use Weiner's algorithm or delete suffixes from the largest to the biggest.
- Update the CSA and the sampling.

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Dynamic FCST's

10 min

Use a dynamic CSA's.

Theorem (Mäkinen, Navarro)

A dynamic CSA over a collection $\mathcal{C}$ can be stored in $nH_k(\mathcal{C}) + o(n \log \sigma)$ bits, with times $t = \Psi = O(((\log_\sigma \log n)^{-1} + 1) \log n)$, $\Phi = O((\log_\sigma \log n) \log^2 n)$, and inserting/deleting texts T in $O(|T|(t + \Psi))$.

Lets take a closer look at the sampling.

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Lets take a closer look at the sampling.

Reverse tree

9 min

- How do we guarantee the sampling condition, with at most $O(n/\delta)$ nodes?
- We use a purely conceptual reverse tree.

Definition

The **reverse tree** $\mathcal{T}^R$ is the minimal labeled tree that, for every node v of a suffix tree, contains a node v^R denoting the reverse string of the path-label of v.

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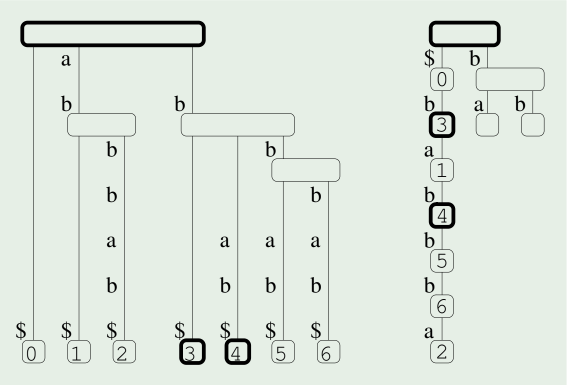
Reverse tree

8 min

Example (Suffix Tree for *abbbab* and its reverse tree)



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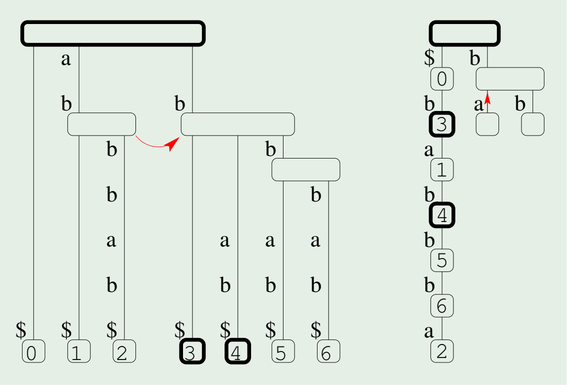

Note that the SLINK's correspond to moving upwards on the reverse tree.

Reverse tree

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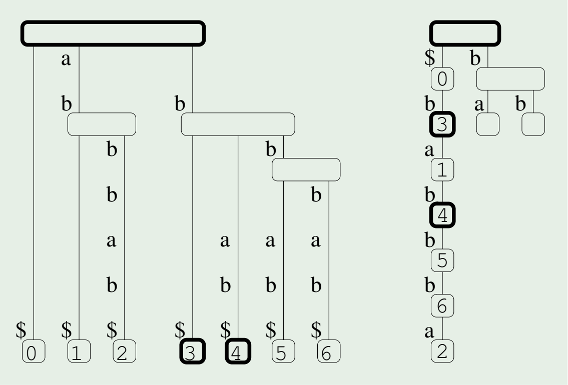

We sample the nodes for which $\text{TDEP}(v^R) \equiv_{\delta/2} 0$ and $\text{HEIGHT}(v^R) \geq \delta/2$.

Reverse tree

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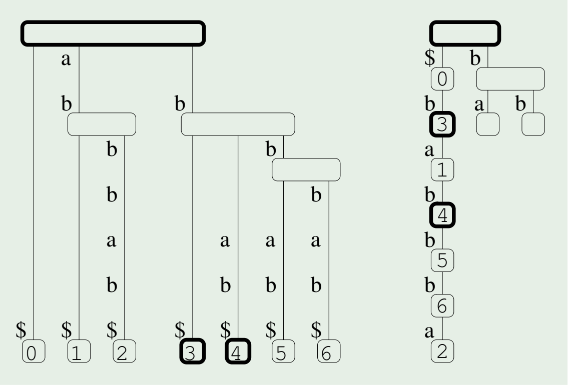

What happens when nodes are inserted or deleted ?

Reverse tree

8 min

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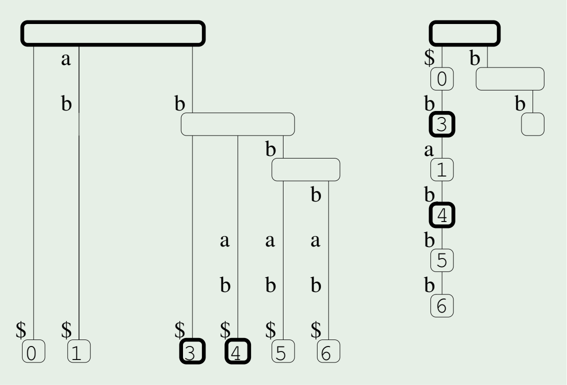

Only the leaves of the reverse tree change.

Reverse tree

8 min

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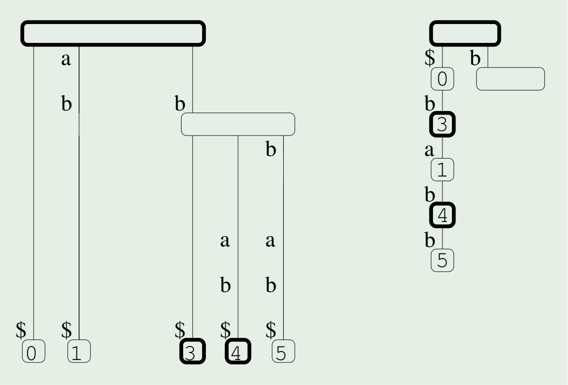

This sampling does not respect the $\text{HEIGHT}(v^R) \geq \delta/2$ condition.

Reverse tree

8 min

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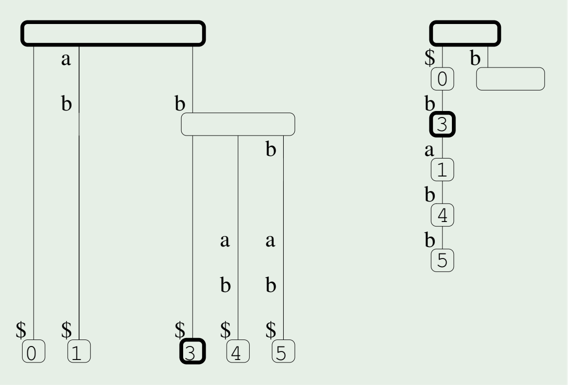

To insert a node we do an upwards scan and sample nodes if necessary.

Reverse tree

8 min

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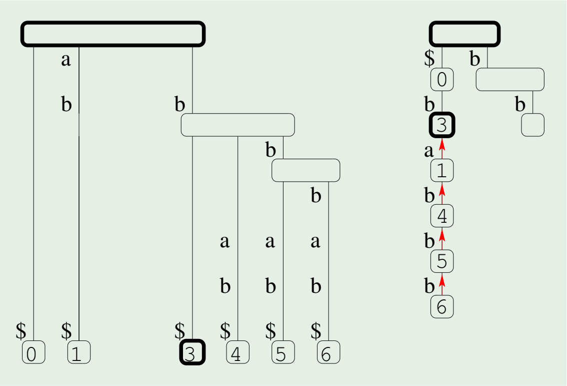

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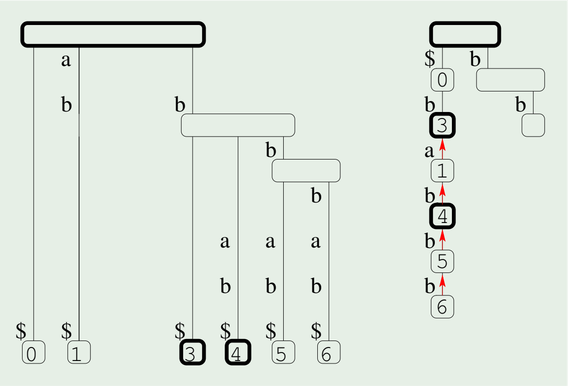

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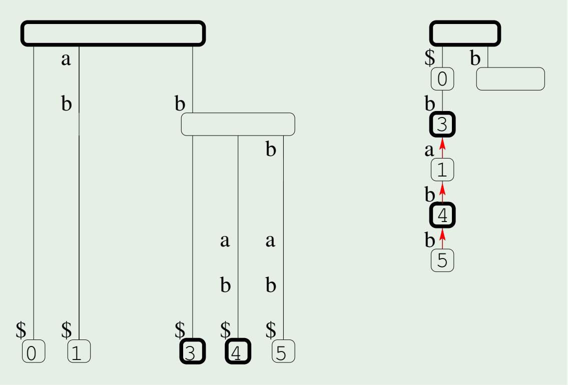

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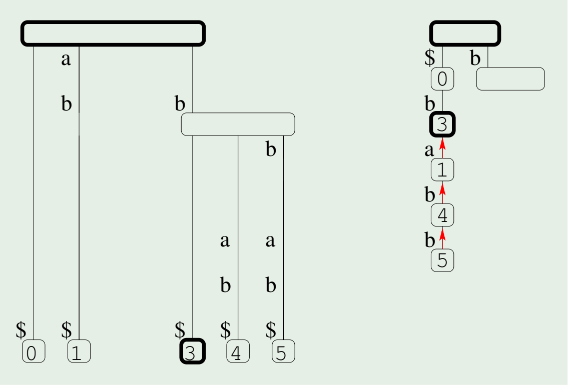

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Other contributions

2 min

- We study the problem of a changing $\lceil log n \rceil$.
- We give a new way to compute LSA.
- We obtain a generalized branching, that determines $v_1.v_2$ for nodes v_1 and v_2 and can be computed directly over CSA's in the sample time as regular branching.

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Summary

1 min

We presented dynamic fully-compressed suffix trees that:

- occupy $uH_k + o(u \log \sigma)$ bits.
- supports usual operations in a reasonable time.

Acknowledgments

0 min

- Veli Mäkinen and Johannes Fisher for pointing out the generalized branching problem.
- FCT grant SFRH/BPD/34373/2006 and project ARN, PTDC/EIA/67722/2006.
- Millennium Institute for Cell Dynamics and Biotechnology, Grant ICM P05-001-F, Mideplan, Chile.

Acknowledgments

0 min

Thanks for listening.