Dynamic Game Model for Deregulated Electricity Markets Considering the Ramp Rate Constraints

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***Abstract*—In this paper, we investigate how generators' ramp rate constraints may influence generators' equilibrium strategy formulation. In the market model, generators' ramp rate constraints are explicitly represented. In order to fully characterize the inter-temporal nature of the ramp rate constraints, a dynamic game model is presented. The subgame perfect Nash equilibrium is adopted as the solution of the game and the backward induction procedure is designed. Due to the inter-temporal nature of the ramp rate constraints, the subgame perfect Nash equilibrium strategy should be a Markov strategy. This, in turn, suggests that the subgame perfect Nash equilibrium of the proposed game should be characterized as the Markov perfect equilibrium. Finally, two examples including a simple discrete strategy example and a numerical illustration of applying the proposed approach are presented.**

***Index Terms*—Electricity markets, ramp rate constraints, dynamic game theory, subgame perfect Nash equilibrium, Markov perfect equilibrium.**

I. Introduction

RECENTLY, the electricity industry is being restructured around the world. As restructuring continues, many studies have been performed considering the unique characteristics of electricity as well as electricity market economics.

Among many physical characteristics of electricity, much research has focused on understanding the roles of transmission networks in a deregulated electricity industry. Borenstein et al. studied the competitive effects of a transmission line that connects two electricity markets [1]. They showed that there may be no direct relationship between the competitive effect of a transmission line and the actual line flow on the line. Moreover, with a sufficiently large capacity line, the full benefits of competition can be achieved even in cases where the equilibrium line flow is zero. For sufficiently large line capacity, the market outcome is equivalent to the case where the markets are merged; that is, where there is unlimited capacity between the markets. Their work also included an empirical analysis of the California electricity market modeled as a duopoly. Willems studied a very similar market model to that of Borenstein et al. and investigated the role of the network operator for promoting competition among the generators [2]. Quick and Carey applied the "dominant firm price-leadership model" to assess market power in Colorado's electricity industry and showed that strategies exist to reduce market power [3]. Leautier studied regulatory contracts for the operators of transmission networks and proposed a regulatory contract that induces network operators to "optimally" expand the grid [4]. Stoft investigated market power issues when the generators serve a demand with capacity constrained transmission lines [5]. He considered the effect on market power of financial transmission rights (FTRs) and the resulting distribution of the congestion rent. Cho investigated the competitive equilibrium in electricity markets over a network with finite capacity [6]. He suggested a tool to check whether an equilibrium is efficient. He also examined markets for firm transmission rights in a market with a specific structure.

Considering many studies on the transmission network constraints, generation unit's physical constraints have been less studied in the context of markets. Baldick and Hogan applied a supply function equilibrium model to analyze electricity markets with capacity constrained generators [7]. Arroyo and Conejo described a market clearing tool which considers generator's minimum up and down time constraints [8].

One of the important constraints that considerably affect generators' economic production is the ramp rate constraint. Wang and Shahidehpour proposed an algorithm to solve unit commitment problems considering the ramp rate constraint in the vertically integrated industry environment [9]. Lee et al. presented a price-based ramp rate model [10]. However, they did not consider generators' strategic interaction in the market. Shrestha et al. studied the ramp rate constraint in deregulated markets [12]. Even though they addressed generators' strategic dispatch decision, they did not consider strategic interaction.

In this paper, a dynamic game model is proposed in order to consider generators' strategic interaction with the ramp rate constraints. The solution of the game is obtained based on the subgame perfect Nash equilibrium concept [11, 13]. Backward induction approach is adopted for characterizing the subgame

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perfect Nash equilibrium of the game. After equilibrium analysis, the inter-temporal nature of the ramp rate constraints shows that the equilibrium strategy should be a Markov strategy. That is, the subgame perfect Nash equilibrium is characterized by the Markov perfect equilibrium [13]. As an illustration, a simple numerical example is presented.

This paper is organized as follows. Section II describes the electricity market model with explicit representation of the ramp rate constraints. In section III, a dynamic game model is presented and the market equilibrium is analyzed. Section IV presents a simple numerical illustration of the approach. Finally, the conclusion is provided in Section V.

II. Electricity Market Model

We consider a series of electricity spot markets of which time index is denoted by $t \in \{1,2,\cdots,T\}$. There are N generators in the market and the production cost function $C_i : \Re_+ \to \Re_+$ of generator i, where $i \in \{1,2,\cdots,N\}$, is of a quadratic form:

$$C_i(q_i) = \frac{1}{2} a_i q_i^2 + b_i q_i + c_i , \quad (1)$$

where q_i is generator i's production, and a_i, b_i, and c_i are parameters. Demand in the market at time t is assumed to be characterized by an inverse-demand function denoted by P_t : $\Re_+ \to \Re_+$ and is represented by an affine curve with a negative slope:

$$P_t(q) = -\alpha_t q + \beta_t , \text{ where } \alpha_t, \beta_t \in \Re_+. \quad (2)$$

Let $q_{i,0}$ denote the initial production quantity of generator i. Let, also, Δq_{iu} and Δq_{id} denote generator i's ramp up rate and ramp down rate for the interval between two consecutive time indices, respectively. Then, generator i's ramp rate constraints are written as:

$$\forall t \in \{1,2,\cdots,T\}, \quad q_{i,t-1} - \Delta q_{id} \le q_{i,t} \le q_{i,t-1} + \Delta q_{iu} , \quad (3)$$

where $q_{i,t}$ is generator i's production quantity at time t.

III. Market Equilibrium Analysis

A. Game Model

In the game model, generators compete against each other by choosing their production quantities (Cournot assumption). Generators' ramp rate constraints described in (3) are inter-temporal constraints in nature and, therefore, we apply a dynamic game theory in order to fully characterize generators' strategic interaction in the markets. Fig. 1 shows the extensive form representation of the dynamic game for the electricity market considered in this paper.

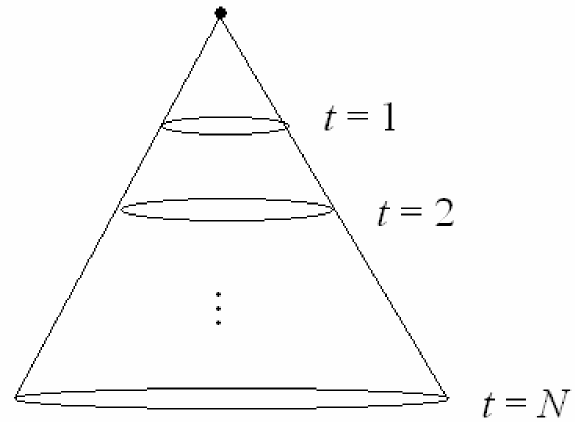

Fig. 1. Extensive form for the electricity market

In this game model, there is a static game embedded in the whole dynamic game at each time t. That is, at each t, generators compete against each other to serve demand at that time by 'simultaneously' choosing their production quantities.

One of the popular solution concepts in dynamic game theory is the subgame perfect Nash equilibrium. In this paper, the market equilibrium is defined by the subgame perfect Nash equilibrium. In order for every subgame characterization, we define generator i's payoff function $\Pi_{i,t}$ for the subgame from time t to T:

$$\Pi_{i,t} = \sum_{\tau=t}^{T} \pi_{i,\tau} , \quad (4)$$

where $\pi_{i,t}$ denotes generator i's profit from the spot market at time t.

B. Equilibrium Analysis

One way to characterize a subgame perfect Nash equilibrium is backward induction. Backward induction is applied for equilibrium analysis in this paper. The first step of this approach is analyzing the last node subgames at time T.

At time T, generator i's subgame payoff function is its profit $\pi_{i,T}$ at time T and the profit is defined as:

$$\pi_{i,T} = \left(-\alpha_T \sum_{j=1}^{N} q_{j,T} + \beta_T \right) q_{i,T} - C_i(q_{i,T}) . \quad (5)$$

Due to the ramp rate constraint, the possible production choice $q_{i,T}$ is restricted according to generator i's production quantity $q_{i,T-1}$ at the previous time T-1. More generally, the ramp rate constraint at time $t \in \{1,2,\cdots,T\}$ is expressed as:

$$\underline{q}_{i,t} \le q_{i,t} \le \overline{q}_{i,t} , \quad (6)$$

where $\underline{q}_{i,t} = \max(0, q_{i,t-1} - \Delta q_{id})$, and $\overline{q}_{i,t} = q_{i,t-1} + \Delta q_{iu}$.

Due to the ramp rate constraint, there are three cases for characterizing the best response $q_{i,T}^{BR}$ of generator i for the subgame at time T. Since $\pi_{i,T}$ is a concave function and

$\frac{\partial \pi_{i,T}}{\partial q_{i,T}}$ is an increasing function with respect to $q_{i,T}$, the best response $q_{i,T}^{BR}$ is expressed as:

$$q_{i,T}^{BR} = \begin{cases} \underline{q}_{i,T}, & if\ \frac{\partial \pi_{i,T}}{\partial q_{i,T}}\left(q_{1,T},\cdots,\underline{q}_{i,T},\cdots,q_{N,T}\right) < 0 \\ \bar{q}_{i,T}, & if\ \frac{\partial \pi_{i,t}}{\partial q_{i,t}}\left(q_{1,T},\cdots,\bar{q}_{i,T},\cdots,q_{N,T}\right) > 0 \\ -\frac{\alpha_T}{2\alpha_T + a_i}\sum_{j=1,j\neq i}^{N} q_{j,T} + \frac{\beta_T - b_i}{2\alpha_T + a_i}, & otherwise \end{cases} . \tag{7}$$

The third row of (7) is derived by the first-order necessary condition for optimality:

$$\frac{\partial \pi_{i,T}}{\partial q_{i,T}} = -\alpha_T \sum_{j=1}^{N} q_{j,T} + \beta_T - \alpha_T q_{i,T} - a_i q_{i,T} - b_i = 0 . \tag{8}$$

The Nash equilibrium strategy profile $q_T^{Nash} = \left\lfloor q_{1,T}^{Nash},\cdots,q_{N,T}^{Nash} \right\rfloor$ for the spot market at time T is determined by simultaneously solving (7) for all generators. Moreover, the Nash equilibrium payoff profile $\Pi_T^{Nash} = \left\lfloor \Pi_{1,T}^{Nash},\cdots,\Pi_{N,T}^{Nash} \right\rfloor$ for the subgame at time T is obtained using the determined equilibrium strategy profile q_T^{Nash}. An important observation is that the equilibrium strategy profile q_T^{Nash} for the subgame at time T would be a function of the production quantity profile $q_{T-1} = \left\lfloor q_{1,T-1},\cdots,q_{N,T-1} \right\rfloor$ at the previous time T-1. This holds in general for the subgame from time t to T. That is, the equilibrium strategy profile q_t^{Nash} for the subgame from time t to T would be a function of the production quantity profile $q_{t-1} = \left\lfloor q_{1,t-1},\cdots,q_{N,t-1} \right\rfloor$ at time t-1.

This explicitly shows the nature of the inter-temporal dynamics of the ramp rate constraints. Since the ramp rate constraints restrict generators' production quantities at only two consecutive times, the subgame perfect equilibrium strategy should be a Markov strategy. That is, the subgame equilibrium strategy profile q_t^{Nash} is only dependent on the previous production profile q_{t-1}, but not on the production profiles before time t-1, $q_1,\cdots,q_{t-2}$:

$$q_t^{Nash}\left(q_1,\cdots,q_{t-1}\right) = q_t^{Nash}\left(q_{t-1}\right). \tag{9}$$

This, in turn, suggests that the subgame perfect equilibrium of the proposed model should be Markov perfect equilibrium [13].

Now, consider we have the solution for the subgame from time t to T. Then, the subgame equilibrium strategy profile q_{t-1}^{Nash} is obtained by simultaneously solving:

$$\forall i \in \{1,\cdots,N\},$$
$$q_{i,t-1}^{Nash} = \arg\max_{q_{i,t-1}} \Pi_{t-1}\left(q_{1,t-1}^{Nash},\cdots,q_{i,t-1},\cdots,q_{N,t-1}^{Nash}\right), \tag{10}$$

where $\Pi_{i,t-1} = \pi_{i,t-1} + \Pi_{i,t-1}\left(q_{1,t}^{Nash},\cdots,q_{N,t}^{Nash}\right)$. Since we have determined the equilibrium for the subgame at time T, by using the backward induction procedure, the Markov perfect Nash equilibrium strategy profile q^{Nash} can be obtained as:

$$q^{Nash} = \left\lfloor \left(q_{1,1}^{Nash},\cdots,q_{1,T}^{Nash}\right),\cdots,\left(q_{N,1}^{Nash},\cdots,q_{N,T}^{Nash}\right) \right\rfloor. \tag{11}$$

The Markov perfect equilibrium path can also be obtained by determining the actual equilibrium outputs sequentially from the initial production profile $q_0 = \left\lfloor q_{1,0},\cdots,q_{N,0} \right\rfloor$.

IV. Examples

In order to show the validity of the proposed approach, two examples are shown. The first example is a discrete strategy game example and mainly aimed to provide an illustration. The second example is a numerical example using a simple market model is considered.

A. Discrete Strategy Game Example

In order to illustrate how the equilibrium strategy can be specified in a simpler way due to the Markov property, we present a discrete strategy game example. Suppose that there are two strategic generators competing to each other. Three consecutive spot markets at t = 1, 2, 3 are considered with a given initial result $\left(q_{1,0}, q_{2,0}\right)$ the market at t = 0. At each spot market, generator i, i = 1, 2, determines one of two choices: ramping up R_i^{up} and down R_i^{down}. We assume that, at each spot market, the ramping up and down rate of each generator i are the same and denoted by Δq_i. Following the Cournot game framework, the strategy of generator i at market t is defined as generator i's output quantity $q_{i,t}$. Fig. 2 shows the extensive form representation of the game.

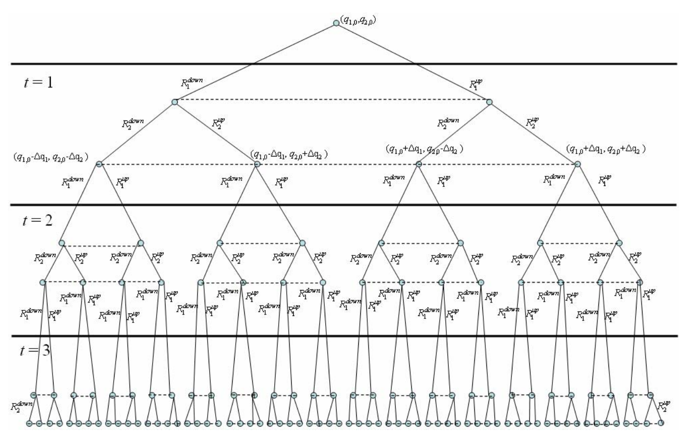

Fig. 2. Extensive form representation of the game

Let us consider generator 1's Nash equilibrium strategy $q_{1,3}^{Nash}$ in the market at t = 3. Without considering the Markov property, there will be 16 equilibrium strategies for $q_{1,3}^{Nash}$, $q_{1,3}^{Nash,1},\cdots,q_{1,3}^{Nash,16}$, as in the final stage game shown in Fig. 2, and $q_{1,3}^{Nash}$ can be represented as (12):

$$q_{1,3}^{Nash} = \begin{cases} q_{1,3}^{Nash,1} & if\ q_{1,1} = q_{1,0} - \Delta q_1, q_{2,1} = q_{2,0} - \Delta q_2, \\ & q_{1,2} = q_{1,0} - 2\Delta q_1, q_{2,2} = q_{2,0} - 2\Delta q_2, \\ q_{1,3}^{Nash,2} & if\ q_{1,1} = q_{1,0} - \Delta q_1, q_{2,1} = q_{2,0} - \Delta q_2, \\ & q_{1,2} = q_{1,0} - 2\Delta q_1, q_{2,2} = q_{2,0}, \\ \vdots & \\ q_{1,3}^{Nahs,16} & if\ q_{1,1} = q_{1,0} + \Delta q_1, q_{2,1} = q_{2,0} + \Delta q_2, \\ & q_{1,2} = q_{1,0} + 2\Delta q_1, q_{2,2} = q_{2,0} + 2\Delta q_2. \end{cases} \quad (12)$$

On the other hand, if we utilize the Markov property, then there will be only 9 equilibrium strategies for $q_{1,3}^{Nash}$, $q_{1,3}^{Mar,1}, \cdots, q_{1,3}^{Mar,9}$, which are Markov perfect equilibrium strategies. Using the Markov perfect equilibrium concept, $q_{1,3}^{Nash}$ is represented as (13):

$$q_{1,3}^{Nash} = \begin{cases} q_{1,3}^{Mar,1} & if\ q_{1,2} = q_{1,0} - 2\Delta q_1, q_{2,2} = q_{2,0} - 2\Delta q_2, \\ q_{1,3}^{Mar,2} & if\ q_{1,2} = q_{1,0} - 2\Delta q_1, q_{2,2} = q_{2,0}, \\ \vdots & \\ q_{1,3}^{Mar,3} & if\ q_{1,2} = q_{1,0} + 2\Delta q_1, q_{2,2} = q_{2,0} + 2\Delta q_2. \end{cases} \quad (13)$$

As shown in this example, even though we considered only 3 markets and 2 discrete strategies for each player at each market, the number of equilibrium strategies in the final stage game reduced considerably. For more realistic cases with large number of stages and with continuous strategy spaces, consideration of the Marko property would significantly simplify equilibrium strategy representation.

B. Numerical Example

We consider two stage game with $t = 1,2$, and the market at $t = 0$ is assumed to be previously cleared. The market clearing result at $t = 0$ is given as an initial condition as the intial production quantity profile $q_0 = [0.25, 0.25]$.

There are two generators with the same cost function:

$$C_i(q_i) = \frac{1}{2} q_i^2 + q_i, \quad i = 1,2. \quad (14)$$

The ramp rates for two generators are:

$$\Delta q_{1u} = \Delta q_{1d} = 0.1, \quad \Delta q_{2u} = \Delta q_{2d} = 0.05. \quad (15)$$

The considered time horizon is set to 2, that is, $t \in \{1,2\}$. The inverse demand functions are:

$$P_1(q) = -2q + 4, \quad P_2(q) = -q + 2. \quad (16)$$

Following the backward induction procedures, first consider the subgame at $t = 2$. The equilibrium strategies for the subgame are determined as:

$$q_{1,2}^{Nash} = \begin{cases} q_{1,1} - 0.1 & if\ q_{1,1} > \frac{1.3 - q_{2,2}^{Nash}}{3} \\ q_{1,1} + 0.1 & if\ q_{1,1} < \frac{0.7 - q_{2,2}^{Nash}}{3}, \\ \frac{1 - q_{2,2}^{Nash}}{3} & otherwise \end{cases} \quad (17)$$

$$q_{2,2}^{Nash} = \begin{cases} q_{2,1} - 0.05 & if\ q_{2,1} > \frac{1.15 - q_{1,2}^{Nash}}{3} \\ q_{2,1} + 0.05 & if\ q_{2,1} < \frac{0.85 - q_{1,2}^{Nash}}{3}. \\ \frac{1 - q_{1,2}^{Nash}}{3} & otherwise \end{cases} \quad (18)$$

Considering all the feasible values of $q_1 = [q_{1,1}, q_{2,1}]$, the equilibrium strategy profile for the subgame is obtained as:

$$q_2^{Nash} = [0.25, 0.25]. \quad (19)$$

Note that (17) is not a value but a function with a constant value. The equilibrium payoff profile for the subgame is also determined as:

$$\Pi_2^{Nash} = [0.0938, 0.0938]. \quad (20)$$

Now, consider the subgame from $t = 1$ to $t = 2$. The payoff profile for this subgame is:

$$\Pi_1 = [\Pi_{1,1}, \Pi_{2,1}] = [\pi_{1,1} + 0.0938, \pi_{2,1} + 0.0938]. \quad (21)$$

By finding Nash equilibrium for this subgame considering the initial production quantity profile, the Markov perfect Nash equilibrium strategy profile is obtained as:

$$q^{Nash} = [(q_{1,1}^{Nash}, q_{1,2}^{Nash}), (q_{2,1}^{Nash}, \cdots, q_{2,2}^{Nash})] = [(0.35, 0.25), (0.3, 0.25)]. \quad (22)$$

V. Conclusions

This paper proposed a game theoretic approach for studying generators' strategic interaction in the deregulated electricity markets considering generators' ramp rate constraints. In this paper, a dynamic game model has been proposed in order to consider generators' strategic interaction with the ramp rate constraints. The subgame perfect Nash equilibrium has been adopted as the solution of the game. Backward induction approach has been applied to determining the subgame perfect Nash equilibrium of the game. The inter-temporal nature of the ramp rate restricts the subgame perfect equilibrium strategy to be a Markov strategy. This, in turn, characterizes the subgame perfect Nash equilibrium of the proposed game as the Markov perfect equilibrium. Finally, as an illustration, we presented two examples including a simple discrete strategy example and a numerical example.

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