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DYNAMIC MODELING OF BEHAVIOR CHANGE

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Abstract. We consider a conceptual and quantitative modeling approach for investigating dynamic behavior change. While the approach is applicable to behavior change in eating disorders, smoking, substance abuse and other behavioral disorders, here we present our novel dynamical systems modeling approach to understand the processes governing an individual's behavior in the context of problem drinking. Recent advances in technology have resulted in large intensive longitudinal data sets which are particularly well suited for study within such frameworks. However, the lack of previous work in this area (specifically, on the inter- and intra-personal factors governing the drinking behavior of individuals) renders this a daunting and unique challenge. As a result, issues which are typically routine in mathematical modeling require considerable effort such as the determination of key quantities of interest, and the timescale on which to represent them. We discuss the construction of an initial mathematical model for two starkly distinct individuals and make a case for the potential for such efforts to help in

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understanding the underlying mechanisms responsible for behavior change in problem drinkers.

1. Introduction. In the study of drug and alcohol abuse, there has been extensive information collected on substance use, a participant's willingness to change behavior, and a participant's success in a particular treatment. Field experts have formulated various ideas concerning factors that control a patient's motivations and behavior. However, the relative contributions of these driving factors, and specifically the mechanisms for behavior change, are unclear. These interacting factors, which are inherently complex and nonlinear, change over time. From a causal perspective, it is natural to explore these issues within the framework of a mathematical model, which enables us to describe these processes quantitatively as a dynamical system. Automated data collection systems have enabled the collection of large data sets involving both substantial numbers of patients as well as time points (warranting the term 'intensive longitudinal data sets'). We seek to model drinking behavior and associated behavior change as informed by such a dataset, Project MOTION, to gain insight into the underlying mechanisms.

Certainly, the application of mathematical modeling to understand learning processes and social behavior is not a new field, and there have been fruitful efforts in the past. Briefly, but not exhaustively, some relevant works include those of Frey and Lau [16] in which a system of integral equations was used to describe ways in which governments make decisions. Van Geert for example in [40] and [41], employs technically diverse tools in the form of a logistical model and a cellular automaton model to understand the cognitive development of children. Notably, the work of Stephen Grossberg, summarized nicely in [19] and [20], is a significant contribution to the theory of learning in the context of cognitive mechanisms as well as associated neural network dynamics.

We note that there has been other recent work resulting from the collaborations of field experts in substance abuse and mathematicians. These efforts have been largely at the 'population-level', or, specifically, the characterization of individuals according to their use ('heavy', 'light', 'abstaining', etc.), similar to epidemiological or ecological models. The goal is then to investigate which scenarios result in overall less or more drinking/substance use in the population. Some contributions along these lines are those of Scribner et al. [37] and Ackleh et al. [1], which focus on student populations and exploring campus alcohol policies with the goal of reducing 'wetness'. This work was further extended in Rasul et al. [33], where the model was calibrated (via parameter estimation) to a student population and specific alcohol policy scenarios were explored.

Another series of efforts exploits ideas from theoretical epidemiology. In 2004, Gorman et al. [17] outlined the ways in which studies on alcohol could be improved via dynamic systems modeling and control theory. This note resulted from a meeting on Ecological Modeling of Alcohol-Related Behavior sponsored by the NIAAA and possibly began the collaboration that gave rise to a series of papers. Sanchez's initial paper [35], classifying individuals as susceptible (nondrinkers, S), drinkers (D), and recovered alcohol users (R) has the same structure as a typical SIR model. The difference is that the relapse term (recruitment of R to D) is modeled by an interaction term, giving rise to a backward bifurcation. This implies that recovered alcohol users can easily relapse through even just a few social interactions with other drinkers. This work was extended in [13], to explore the role of nonlinear relapse among networks of drinking communities with varying connectivities. Mubayi (in [31, 30]) extends this work to examine the contagion of drinking (using the same basic 'SDR' principles) among individuals in low- and high-risk communities, and explored the role of *residence times* in each community. Also, it is shown that social contacts or the extent of mixing within communities drastically affects the outcomes. In [31], the deterministic model was extended to consider variability in social interactions of drinkers and increasing/decreasing levels of drinking. Then the distribution of drinking levels under prevention, intervention, and a combination of both was presented and discussed.

Research on substance use over the past several decades has attempted to identify predictors of successful achievement of abstinence and moderation. There are four sets of categories or constructs that have been broadly identified as being related to sustaining the status quo or prompting changes in drinking behavior. These categories are: 1) stable characteristics of the patient such as age, gender, and drinking history, 2) mood and affect, 3) environmental factors, such as social networks and stressful events, and 4) internal process factors, such as motivation to change, commitment to changing, self-efficacy, etc.

The MOTION data set is comprised of information collected via an interactive voice recording (IVR) from drinkers who wanted to reduce their drinking, although not necessarily cease drinking altogether. Ideally, each individual answered this 41-question survey once per day for eight weeks, or 56 days, and gave information about inter- and intra-personal factors that could potentially influence their drinking as well as their drinking behavior in the past 24 hours and their commitment to avoid heavy drinking or drinking at all in the next day.

Within Project MOTION, both the IVR-based daily survey and the fixed assessments were attempts to collect data across these broad categories in order to exhibit the process of change for individuals aiming to moderate their drinking. These data were collected from participants both within the context of a brief treatment and through independent self-monitoring. Data yielded from each of these measures and individual items were utilized to help create an informed picture/guide as to how individuals attempt to change their drinking and how the trajectories of change may differ across groups.

Generally, the use of mathematical modeling to learn about a physical or biological process is done as an iterative process in which an initial model is formulated and then compared with observed or experimental data. This comparison gives insight into any discrepancies between the observed processes and model predictions, suggesting modifications and/or refinements to the model. This process is repeated until the model provides sufficient information to capture key aspects of behavior of the observed system and to answer questions of interest.

However, in most applications, previous models have been developed and/or there is previous knowledge of quantitative behavior of underlying processes, so modeling is typically initiated with an idea of the appropriate basic framework and possible mechanisms. This is not the case for this particular problem, and the modeling here is with the goal of working toward such a framework. As such, model terms representative of possible mechanisms are considered on the basis of how well the model solutions then reproduce the observed trends in data. Only after the success of several such efforts are the use of more precise inverse problem methods, such as parameter estimation, model sensitivity analyses, etc., appropriate, as they rely on the mathematical model being a relatively good description of the observed processes. Currently, there are also some challenges in the observation process, or measurements, that would impede the application of such methods.

While the aim of the current work was simply to attempt to develop a mathematical model that might aid in our understanding of any behavioral mechanisms relevant to drinking behavior, the potential is more far-reaching as we feel that similar approaches would be valuable in understanding the behavior changes in dieting and eating disorders, smoking, substance abuse, etc. A cohesive theory of how and why individuals change as they modify their behavior long-term is lacking, not only in problem drinking but in other areas of behavioral disorder. In such investigations it is not expected that all mechanisms will be at work in all individuals as there is considerable variability, but there are likely groups or cohorts of people who change similarly. Working toward mechanism-based descriptions of behavior change, our and similar efforts could potentially help clinicians identify patients as part of a characterized cohort and thereby potentially improve therapy.

Secondary to this goal was the aim of improving upon current data collection procedures and study designs, which can greatly improve our ability to learn about these processes and make them more amenable to mathematical approaches. As we shall discuss, the development of the first two initial models has been enlightening, and some immediate changes to propose have become clear. Further changes representing somewhat of a paradigm shift are discussed in the closing remarks of this paper.

There is no straightforward way of organizing data on such a large variety of aspects of daily life into a reasonable number of variables to study within a mathematical modeling framework. To begin formulating conceptual variables, we applied linear methods such as calculating coefficients of determination, linear interpolation via least-squares approximation and principal component analysis. However, these methods do not provide useful information on relationships between the responses to questions. It is sensible to construct our model variables on questions based on similar ideas used in the IVR. We shall examine how these categories relate to each other and how to best use this data in formulating a model. Our initial interpretation of this data is based on knowledge and predictions from the field of substance abuse and recovery, visually assessing qualitative trends between similar questions within topical categories, and grouping them into variables. Once variables were decided upon, qualitative trends between them were examined to determine the existence and nature of relationships between them within a single patient.

We constructed a first model based on prior knowledge of hypothesized relationships between the model variables using knowledge of substance abuse therapy and relationships observed in the data from Project MOTION. We discuss the construction of this initial model along with modifications to it which improve the agreement between solutions and data. One of these is supported by the use of a model comparison statistic, an example of one way that inverse problem methods can help to refine these models once their solutions are reasonably close to data. Building upon lessons learned from the initial patient's model, we developed a model for another patient, thereby demonstrating the use of dynamic models as a means of describing and studying these processes within individuals. Finally, we comment on issues with the current data set and suggest improvements to future studies so that these data sets are better able to quantify the underlying processes.

2. Project MOTION: Rationale, procedures, and data. Motivational interviewing (MI) has been demonstrated to be an effective stand-alone intervention for alcohol use disorders (AUD). The consistency, magnitude, and durability of its effects, especially given its brevity, suggest that powerful mechanisms of behavior change (MOBC) are operating to reduce drinking. Thus, gaining a better understanding of the underlying MOBC in MI is important. However, existing MOBC studies of MI have yielded limited and contradictory findings. Project MOTION (described in greater detail in [9] and [10]) aimed to rigorously examine MOBC in MI by improving on prior methods and using an enhanced conceptual framework that considered nonspecific therapy factors and self-change mechanisms. The treatment conditions were labeled Full MI (FMI) and Spirit-Only MI (SOMI).

General advertising online and in local media was used to recruit 89 participants seeking to reduce but not stop drinking. Participants were considered eligible for the study if they were: (1) between the ages of 18 and 65; (2) had an estimated average weekly consumption of greater than 15 or 24 standard drinks per week for women and men, respectively; and (3) had a primary alcohol use disorder (AUD). Participants were excluded from the study if they: (1) presented significant substance use or a current substance use disorder (for any substance other than alcohol, marijuana, nicotine or caffeine), which was defined for our assessment purposes as greater than once weekly use in the past month; (2) presented a serious psychiatric illness or substantial suicide or violence risk; (3) demonstrated clinically severe alcoholism, as evidenced by physical withdrawal symptoms or a history of serious withdrawal symptoms; (4) were legally mandated to complete a substance abuse treatment program; (5) reported social instability (e.g., homeless); (6) expressed a desire to achieve abstinence at baseline; or (7) expressed a desire or intent to obtain additional substance abuse treatment while in the study.

Treatment was delivered in 4 sessions that lasted between 45 minutes to an hour long at weeks 1, 2, 4, and 8. Participants were blind to condition assignment, meaning they were not told which therapy they would be receiving. All participants were followed for a total of 9 weeks and participated in 90-minute assessments at weeks 0, 1, 4 and 8. Participants in MI and SOMI were called for an additional 4 weeks post-treatment (week 12) to collect drinking data via the Timeline Follow Back (TLFB, described further below). Follow up rates for assessments at weeks 1, 4, 8 and 12 were 100%, 96%, 92.1%, and 68% respectively.

All participants were asked to complete a daily telephone survey at the end of each day for the duration of the eight weeks (one week of baseline information and seven weeks of treatment) of the study, for a total of 56 possible days. At the end of their initial screening visit (a week prior to randomization), a research assistant (RA) provided each participant with a 15-minute training session on how to use the Interactive Voice Recording (IVR) system, a system developed using TELESAGE SmartQ 5.2, a software package specifically designed for the administration of automated surveys [39]. Each day, participants responded to a series of questions about potential mediators of drinking behavior such as mood, commitment to not drinking heavily or not at all, and confidence to do so, in addition to the number and types of drinks they consumed in the last 24 hours. Once familiar with the system, the daily IVR session required approximately 5 minutes to complete. Each participant was provided a toll-free phone number and an anonymous participant identification number to ensure confidentiality. The IVR system could be accessed between 4:00 p.m. and 10:00 p.m. This time period was judged to be when participants most likely would be able to reflect on their alcohol use that occurred past the prior day's assessment and before most individuals would be likely to consume large amounts of alcohol. This time window had the advantage of providing consistent report timing and facilitated compliance by creating routines for participants. If participants failed to call into the system by 8:00 p.m., an automated reminder call was made. Participants' data were coded as missing if they were not able to complete a call. This daily questionnaire was considered a form of self-monitoring one's drinking.

3. Interpretation of data to inform modeling. While the wealth of information in this rich data set presents a unique opportunity to examine aspects of drinking behavior change longitudinally, it also presents challenges. In contrast to other areas, such as the physical sciences, in which the state variables representing key players in the processes are more obvious, no such information is known. In addition, the relevant timescales on which the variables are changing and therefore should be observed are unclear. It is possible that some of the measures, if not deemed to be important on the timescale of the current data set, may be driving other aspects of drinking behavior either on a shorter or longer timescale than that on which we choose to focus.

As we initially approached the task of developing dynamical systems models of these processes, we needed to assess the information content in the Project MOTION data. Individuals' drinking behavior, and their response to treatment, is highly variable, and it is likely that one model describing all individuals is not possible. At any rate, to be so ambitious with first attempts at discerning key mechanisms is unreasonable, and focusing on groups of individuals who respond in similar ways is a more feasible approach. With that in mind, we proceeded to organize the data in ways that would best inform modeling efforts, described in much greater detail in [9].

Given the longitudinal nature and frequency of the IVR responses along with the desire to understand change, it is natural to focus on those data for the development of a dynamic model. We also considered which of the fixed assessment battery provides novel or equivalent information to that in the IVR. While these data are not sufficiently informative on a longitudinal basis to be considered as main components of a model at this stage, it is possible that these data may have supplemented possible missing responses from the IVR, or the novel data could be information that should be included in future IVR-type questionnaires. The details of this exploration are given in [9], but ultimately, we did not use any of this information in the models presented here.

The timeline followback responses proved to be useful in that we were able to determine criteria upon which to divide the subjects into ‘responders’, ‘nonresponders’, or neither. We defined responders as those individuals who demonstrated at least a 40% decrease in their drinking by their last treatment session or the week 8 assessment. Nonresponders were identified as demonstrating no change in their drinking or increasing their drinking by the week 8 assessment. Thus, there are several individuals who would be classified neither as a responder nor as a nonresponder. We thought it prudent to focus on individuals with the desired behavior-changing mechanisms in place, or at least those who exhibited the desired behavior changes, for initial modeling efforts. Therefore, we chose to focus on the eight subjects classified as ‘responders’. With these individuals’ data, we determined an initial grouping of IVR data questions (as it is not reasonable to consider each of the 42 questions as independent information, or variables) based on intuition, categories the investigators had in mind when constructing the IVR questionnaire, and previous hypotheses of interactions of key factors.

4. Initial model formulation. In the hopes of identifying meaningful groups of similar IVR items to be used as the basis for model variables, we turned to common statistical techniques that are frequently used in the analysis of such data sets among psychologists. The techniques employed were least-squares regression, principal component analysis, and factor analysis. We note that these test for static relationships, and therefore we would not necessarily expect them to be very informative concerning processes that may change nonlinearly in time. These results and an explanation of the methods are contained in [9] and more succinctly in [10].

The static methods used to search for affine relationships between variables failed to produce strong evidence for any linear patterns. While regression models were informative in that some existing weak relationships between variables, or observed data categories, were revealed, the nature of these relationships were not made clear. Results from principal component analysis did not yield any unique trends in patient data. First, PCA in each individual did not identify possible subsets of responses to group together. Moreover, the groups created did not explain the variance in the data any better than the original variables. Finally the groupings of variables that PCA produced did not follow any patterns when comparing individuals. We also performed factor analysis, a method similar to PCA and discussed by R. J. Rummel in [34], on the data. While factor analysis may be implemented using any one of a variety of methods, we employed the maximum likelihood method. The factor analysis resulted in a so-called Heywood case, an anomaly in communality estimates, suggesting that factor analysis is not an appropriate method to use on this data set. In factor analysis the covariance of the assumed error in the measurements is estimated in an iterative process when using the maximum likelihood method, so if a value on the diagonal of this covariance matrix becomes less than or equal to 0, the maximum likelihood method is converging to an inappropriate result. It is likely that the non-normal distribution of data and the small number of possible values for most observations negatively affected the estimations of covariance components and factor loadings.

Our static linear analysis did not show any clear patterns between IVR questions. Some of the questions included in the Project MOTION IVR asked about redundant or closely related information. Thus, we began with a reasonable grouping of IVR questions to base our model variables on the categorical groupings of IVR questions.

It is reasonable to represent composite scores of the categorical groupings (and hence the variables) as fluctuating on a continuum as opposed to a discrete set of numbers, as a discrete set may not appropriately reflect how an individual is feeling. For example, a person may not be simply happy or sad, and there are many degrees or intensities at which happiness or sadness may be felt that do not correspond to five digits. Discrete scales are more amenable to questionnaires, particularly when administered via the telephone. However, when developing a dynamic model, the use of a continuum of values is preferred, and likely more reasonable, as it can be argued. Similar arguments can be made concerning the true nature of the dynamics of the processes in time, so we will consider the model as a function of continuous time as opposed to discrete time. We thus assume that only the data collection times and observation values are discrete, while the underlying processes of interest change in a continuous manner.

We determined that a 'triweekly' timescale, seen in Figure 1, would be most suitable to observe the processes. Daily data appeared to have too many fluctuations, so that any trends (and hence, mechanisms or dynamics) were not evident, and data averaged by week appeared not to manifest sufficient dynamics, or that the information was essentially 'averaged out'. A triweekly timescale appeared to be a reasonable level of refinement for observing the data. It consists of 3 time points per week: weekend (Friday night through Sunday prior to IVR call), Sunday and Monday, and Tuesday through Thursday.

4.1. *Patient 6029 model.* We initially chose to focus on patient 6029 since, based on the TLFB assessment seen in Figure 2, it appeared that his drinking significantly changed during the observed time period in a seemingly systematic way. That is, there appeared to be some heavy drinking initially that the individual gradually reduced so that there was consistently and noticeably less drinking at the end of the observation period. This patient responded consistently to the IVR system and appeared to generally exhibit characteristics that the clinicians would identify as indicators of successful behavior change.

Causal relationships between potential variables were determined based on visual analysis of the data, and 'categorical models' based upon these relationships were constructed for a select four responding patients, including patient 6029. The categorical model is a schematic representation of model variables and their relationships. In general, whether categories were included depended entirely on whether the category appeared to be directly related to drinking (represented as daily averages over the 2- or 3-day time period). Relationships were considered among the variables that were initially selected as being pertinent to the preliminary categorical model. We did not discuss possible relationships among variables that did not appear to be drinking related, and therefore were not considered in these categorical models. Consequently, it could be that some of the variables are related to each other but that relationship is not represented in the categorical model

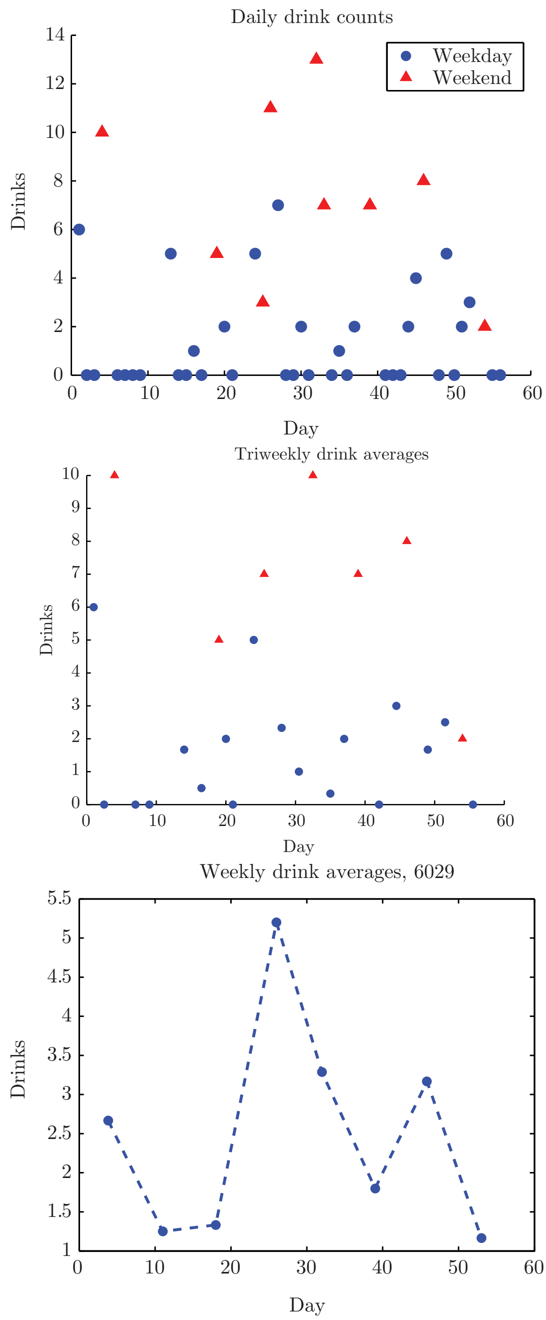

FIG. 1. Daily (top), triweekly (middle) and weekly (bottom) drink counts for patient 6029. Red triangles indicate weekends, and blue dots indicate weekdays.

unless each of them is related to drinking. Solid lines in the schematics for categorical

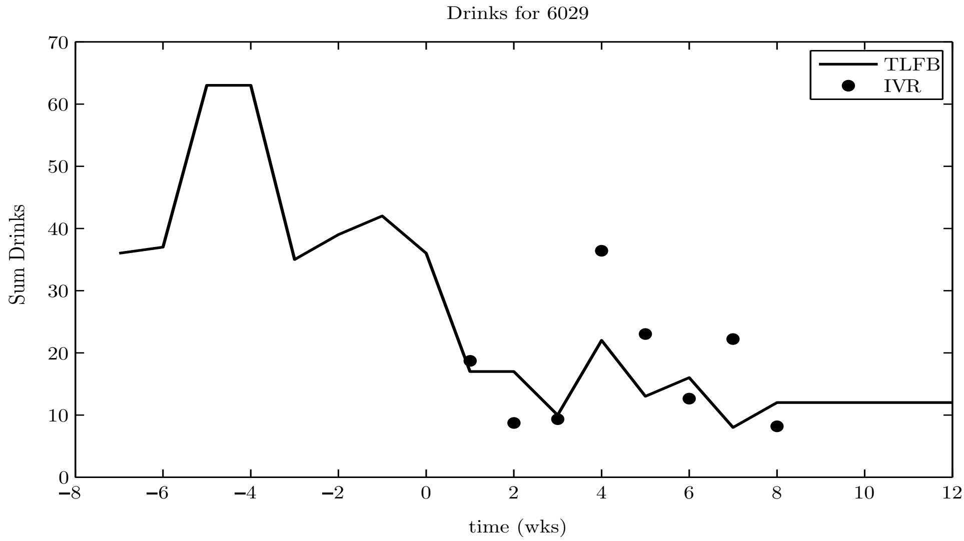

FIG. 2. Weekly totals of drinks as reported by patient 6029 in the Timeline Followback (TLFB) assessment and the IVR system.

models denote 'strong' relationships, while 'weaker' relationships are represented with dotted lines. Arrows indicate causality or direction.

The representation of the categorical model for patient 6029 can be seen in Figure 3. The graphs of the data from which we decided there are noticeable relationships are shown in Figure 4. All relationships for this individual seemed to be relatively strong when present.

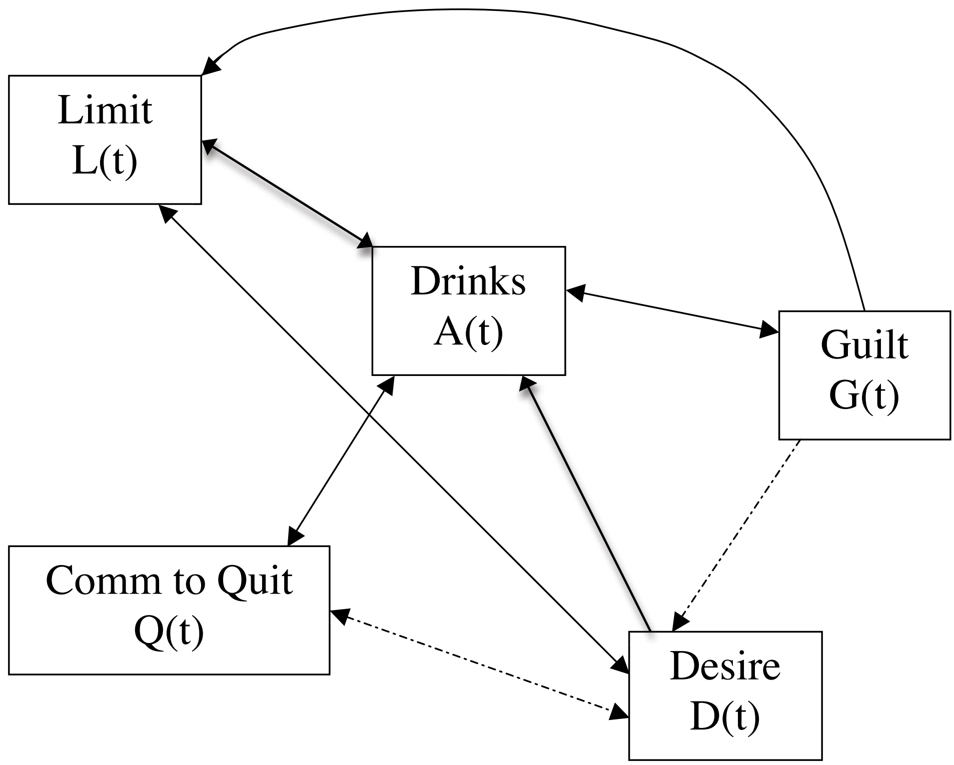

FIG. 3. Categorical model for patient 6029 based on hypothesized relationships in the IVR data. 'Strong' relationships are represented by solid lines, and 'weak' relationships are represented by dashed lines. All relationships are taken to be bidirectional.

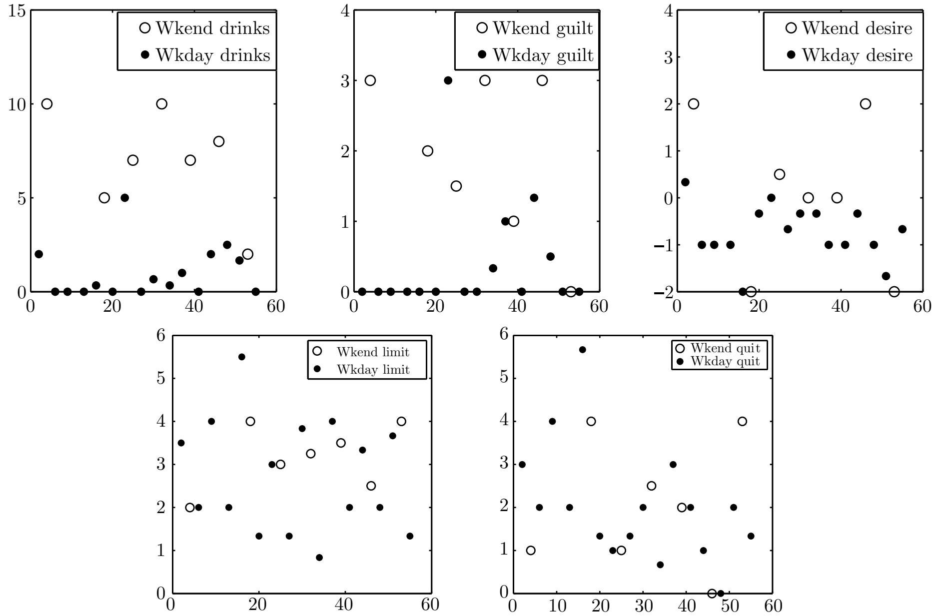

FIG. 4. IVR data for variables deemed important for the development of a mathematical model.

The initial model, given by

$$\frac{d}{dt}A(t) = -a_{12}\chi_{(G>0)}\left(\int_{-\tau_1}^{0} G(t+s)\kappa_1(s)ds\right)^2 + a_{13}\chi_{(D>0)}D(t)$$
$$- a_{14}(Q(t) + \chi_{(Q>0)}Q^2(t)) - a_{15}L(t), \tag{1}$$

$$\frac{d}{dt}G(t) = a_{21}\left(\int_{-\tau_2}^{0} A(t+s)\kappa_2(s)ds - (1 + c_1\chi_{W(t)})A_G^*\right), \tag{2}$$

$$\frac{d}{dt}D(t) = -a_{34}Q(t) - a_{3,5}L(t) - a_{32}\left[\exp\left(\frac{1}{G_{D1}^*}\int_{-\tau_3}^{0} G(t+s)\kappa_3(s)ds\right) - G_{D2}^*\right], \tag{3}$$

$$\frac{d}{dt}Q(t) = -a_{43}\left(1 + \chi_{W(t)}\chi_{(D>0)}\right)D(t)$$
$$- a_{41}\left[\exp\left(A_{Q1}^*\min(0, (A(t-\tau_4) - A_{Q1}^*))\right) - (1 - \chi_{W(t)})A_{Q2}^*\right], \tag{4}$$

$$\frac{d}{dt}L(t) = a_{52}G(t) - a_{53}D(t) - a_{51}\left[\exp\left(\frac{1}{A_{L1}^*}\min(A_{L1}^*, (A(t-\tau_5) - A_{L1}^*))\right) - A_{L2}^*\right], \tag{5}$$

was derived from patterns in the IVR data and confirmed by plausible explanations of drinking behavior processes. It includes all possible mechanisms considered in these investigations, and we did not think it likely that all terms would remain as we proceed in an iterative modeling effort [11, Chapter 1]. However, there are some key features that will likely be necessary to include in even the simplest mathematical representation of individual-level behavior change in general and specifically concerning one's alcohol intake. Namely, these are nonlinearities, delayed and/or cumulative effects, and time-dependent threshold behavior.

To illustrate such model components and their use, we consider equation (2) describing the rate of change of guilt:

$$\frac{d}{dt}G(t) = a_{21}\left(\int_{-\tau_2}^{0} A(t+s)\kappa_2(s)ds - (1 + c_1\chi_{W(t)})A_G^*\right).$$

This equation is the only one that was not modified as the model solutions were compared to data (described in Section 4.2), and exhibits most of the features listed above. It can be interpreted as the individual's 'guilt', or more specifically his feeling that his drinking over the previous day was excessive, increases with the number of drinks consumed since τ_2 days prior, if they exceed the threshold $(1 + \chi_W(t))A_G^*$. We note that since $A(t)$ is the drinking *rate*, the number of drinks consumed from τ_2 days prior to the present time is represented by $\int_{-\tau_2}^{0} A(t+s)ds$. The function $\kappa_2(s)$ is used to effectively weight times at which drinking may have a more or less significant effect on his guilt. For example, if $\kappa_2(s)$ increases with s going from $-\tau_2$ to 0, then this would indicate that his drinking rate at the present time more strongly influences his guilt than that in the past. This particular individual's (and likely most individuals') idea of 'excessive drinking' changes from the weekday to the weekend, and thus the function $\chi_{W(t)}$, which is equal to one during the weekend and zero during the week, changes the individual's level of 'acceptable drinking' or his threshold from A_G^* during the week to $(1 + c_1)A_G^*$ during the weekend.

4.2. *6029 model modification.* Attempting to examine how well solutions agree with data is nontrivial. Numerous parameters are not readily psychologically or emotionally interpreted, thus complicating the estimation of feasible ranges for their values. While we may be able to anticipate the model solution of each individual term, there is no way to anticipate the nature of solutions of the model when all terms are included simultaneously. Further, we do not expect that all terms will be included in a final model which we consider the best description for the system as justified by the current data set. Therefore, we took a reductionist approach. We began with the simplest system of drinking behavior that we anticipated to be capable of producing solutions with reasonably close behavior to that seen in the data. Once we reproduced the data as well as possible with the simple model, we added additional components if necessary to capture features in the data not explained by the current model. This approach corresponds to a general modeling philosophy of including as much complexity in the model as needed, but no more, to capture the key features manifested in the data.

The overall goal of the project is to understand *drinking behavior*, so the simplest model necessarily includes the equation for the drinking rate $\frac{d}{dt}A(t)$, and mechanisms driving and suppressing alcohol consumption. To have one without the other would lead to strictly increasing or decreasing alcohol consumption, and from the IVR data, we know this is not the case. We focused on the desire $D(t)$ and 'guilt' $G(t)$ (which may be more accurately interpreted as a norm violation measure, but still referred to here in shorthand as guilt) variables. These were identified as two variables that appeared to have the strongest relationships with drinks in the IVR data. Therefore, the simple model initially considered is

$$\frac{d}{dt}A(t) = -a_{12}\chi_{G>G^*}(G(t) - G^*) + a_{13}D(t), \tag{6}$$

$$\frac{d}{dt}G(t) = a_{21}\left[\int_{-1}^{0} A(t+s) - (1 + c_1\chi_{W(t)})A^*\right], \tag{7}$$

$$\frac{d}{dt}D(t) = -a_{32}\chi_{G>G^*}(G(t) - G^*). \tag{8}$$

Immediately upon inspection of the equations (6)-(8) there is an issue in that the only driving force in the alcohol equation is the desire to drink. But the equation governing desire, equation (8), results in a non-increasing time course for desire $D(t)$. Thus, as written, desire will not provide a driving force for drinking, as it will not increase beyond its initial value. Also in the original model, the only way for the individual's desire to increase is through a negation of essentially controlling mechanisms. Therefore, the model was missing a mechanism to drive the patient's drinking, and upon further consultation with the data, a notable weekend/weekday pattern in both the desire and drinking data was discerned. That is, the individual's desire to drink and his drinking increased going into the weekend and remained elevated during the weekend, and then decreased as the weekend ended. Thus, we include the term $c_2h(\hat{t})$ in the $\frac{d}{dt}D(t)$ equation, where $\hat{t} = t$

mod 7 and $h(\hat{t})$ is given by

$$h(\hat{t}) = \begin{cases} 2(\hat{t}-1.5) & 1.5 \le \hat{t} < 2 \text{ (Fri. a.m. thru Fri. p.m.)}, \\ -2(\hat{t}-2.5) & 2 \le \hat{t} < 2.5 \text{ (Fri. p.m. thru Sat. a.m.)}, \\ -2(\hat{t}-3.5) & 3.5 \le \hat{t} < 4 \text{ (Sun. a.m. thru Sun. p.m.)}, \\ 2(\hat{t}-4.5) & 4 \le \hat{t} < 4.5 \text{ (Sun. p.m. thru Mon. a.m.)}, \\ 0 & \text{else.} \end{cases}$$

A possible interpretation of this mechanism is discussed briefly in Section 4.4.

Setting $a_{32} = 0$ so that the desired equation is just $\frac{d}{dt}D(t) = c_2 h(\hat{t})$ allows us to see the effect of this weekend-dependent mechanism $h(t)$ on desire, shown in Figure 5. This form of the desire equation appeared to agree well with the desire data as seen in the bottom panel of Figure 6. Therefore, we have good reason to think that this subject's desire is well described by our representation of the weekend/weekday mechanism alone.

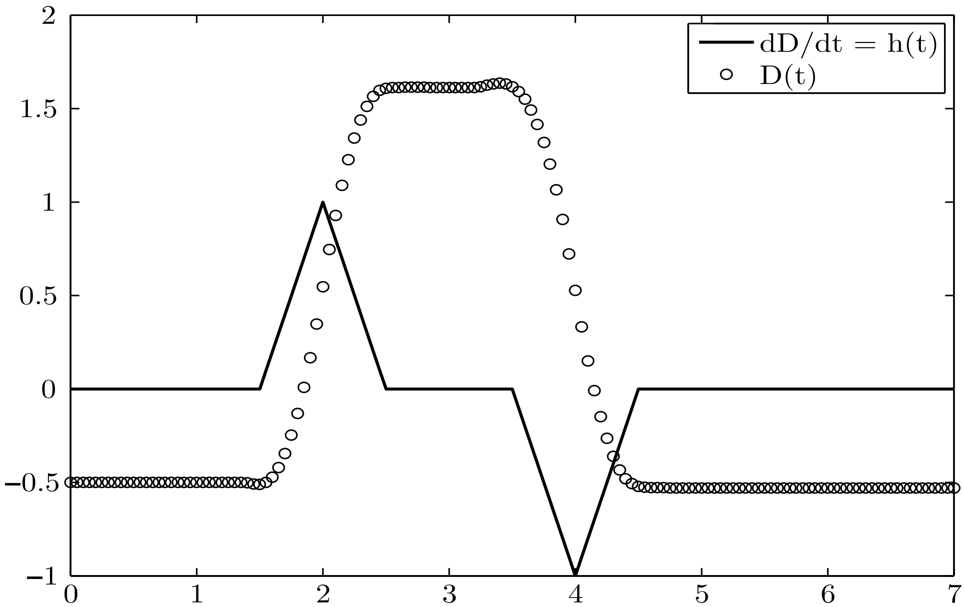

FIG. 5. Solution of $\frac{d}{dt}D(t) = c_2 h(\hat{t})$ and $c_2(\hat{t})$ with $c_2 = 4.25$ and $D(0) = -0.5$.

We note that not all discussions of *data fitting* in this document are actually referring to a least-squares type of fitting, but rather in some cases are the choosing of parameter values and manual adjustment to see the effects on the solution as compared with data, i.e., a type of simulation-based sensitivity analysis for model response with respect to parameter values. The parameter values must be somewhat close, and/or with feasible ranges determined to hope for any optimization routine to be able to minimize the difference between the model and data, thereby resulting in reasonable parameter estimates. Therefore, the manual 'fitting' discussed here is a necessary first step prior to estimating parameters via inverse problem methods.

Once the desire data was well explained, we examined the dependence of the individual's drinking on desire, which we had hypothesized as the driving mechanism. It became apparent that to have that term in the $\frac{d}{dt}A(t)$ equation depend on $D(t)$ was flawed, since any positive value of $D(t)$ will result in an *increase* in the drinking *rate* (i.e., drinking is accelerated). What is more accurate is that a positive but possibly waning desire, for example on a Sunday, would mean that the individual is consuming alcohol albeit at a slower rate, and thus, their rate of drinking is positive $A(t) > 0$, but decreasing (say, as they go into the work week) $\frac{d}{dt}A(t) < 0$. This is directly proportional to the rate of desire, not the value of the desire variable. Intuitively, it does make sense that the rate of change of alcohol consumption should be more directly related to the rate of change of desire, and thus, the number of drinks should be correlated with the individual's desire level. This is a subtle point, but it does require attention to maintain the fidelity of the processes represented in the mathematical model. The simple model is then more reasonably given by

$$\frac{d}{dt}A(t) = -a_{12}\chi_{G>G^*}(G(t) - G^*) + a_{13}\frac{dD}{dt}, \tag{9}$$

$$\frac{d}{dt}G(t) = a_{21}\left[\int_{-1}^{0} A(t+s)ds - (1 + c_1\chi_{W(t)})A^*\right], \tag{10}$$

$$\frac{d}{dt}D(t) = -a_{32}\chi_{G>G^*}(G(t) - G^*) + c_2 h(t). \tag{11}$$

Then the system may be reduced by the substitution of the equation $\frac{d}{dt}D(t)$ into the second term of equation $\frac{d}{dt}A(t)$, resulting in

$$\frac{d}{dt}A(t) = -a_{12}\chi_{G>G^*}(G(t) - G^*) + a_{13}\left(-a_{32}\chi_{G>G^*}(G(t) - G^*) + c_2 h(t)\right),$$
$$\frac{d}{dt}G(t) = a_{21}\left[\int_{-1}^{0} A(t+s)ds - (1 + c_1\chi_{W(t)})A^*\right].$$

If we take $\tilde{a}_{12} = a_{12} + a_{32}$ and $\tilde{a}_{13} = a_{13} + c_2$, the model is further simplified to

$$\frac{d}{dt}A(t) = -\tilde{a}_{12}\chi_{G>G^*}(G(t) - G^*) + \tilde{a}_{13}h(t), \tag{12}$$

$$\frac{d}{dt}G(t) = a_{21}\left[\int_{-1}^{0} A(t+s)ds - (1 + c_1\chi_{W(t)})A^*\right]. \tag{13}$$

The drinking and guilt data are shown along with solutions to the model (12)-(13) with $\tilde{a}_{12} = 0$ in Figure 6, along with the desire data and the solution of $\frac{d}{dt}D(t) = c_2 h(t)$. The parameters and initial conditions are given in Table 1. The agreement between the solution and data is good in the sense that drinking episodes (with the exception of two) are predicted, and suggests that the individual's elevated drinking episodes may be primarily attributed to the weekend/weekday pattern. Further, it appears that his desire and drinking are closely linked, being that the same function $h(t)$ can be used to reproduce both of the trajectories shown in the data. While we suspect that the effect of guilt on drinking is relatively insignificant compared to the more dominant weekend-dependent effect seen in the desire data, as reflected in the parameter values in Table 1,

TABLE 1. Parameter values

$\tilde{a}_{12}$	0	c_1	8.1	$A(0)$	1
$\tilde{a}_{13}$	16	A^*	1	$G(0)$	0
a_{21}	0.8	c_2	4.25	$D(0)$	-0.5

we make use of an inverse problem technique in the next section to determine if it is reasonable to exclude this term in the model.

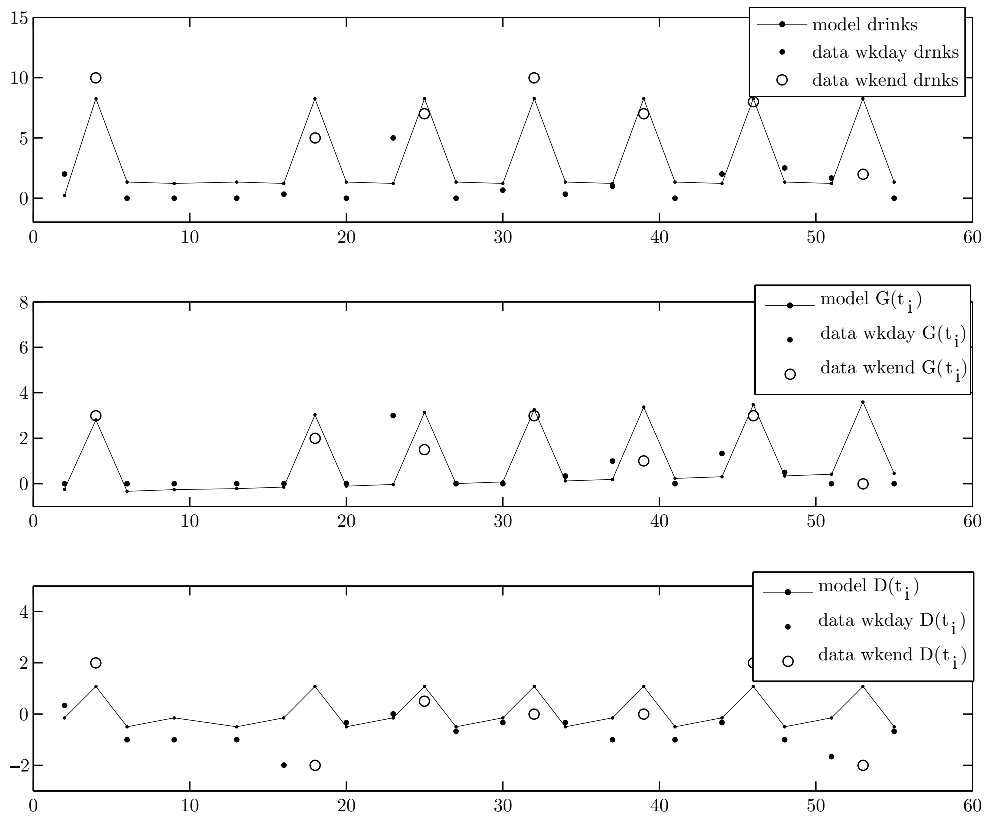

FIG. 6. Agreement between the mathematical model and patient 6029's triweekly IVR data.

4.3. *Model comparison statistic.* The model comparison statistic used here (given in [11, Chapter 3]; more precise and thorough discussions of this test statistic can be found in [4, 6, 7, 8]) can be used to determine if a more complicated version of a simpler model gives a statistically significant improvement in the agreement between model solution and data. That is, it is appropriate when comparing 'nested' models, or when one model can be written as a special case of the other. As such, it is well suited to help determine whether specific terms in models can be reasonably simplified, or even neglected altogether. We employ it here to determine whether the model

$$\frac{d}{dt}A(t) = -a_{12}\chi_{G>G^*}(G(t) - G^*) + a_{13}h(t),$$
$$\frac{d}{dt}G(t) = a_{21}\left[\int_{-1}^{0} A(t+s)ds - (1 + c_1\chi_{W(t)})A^*\right]$$

(where the tilde's above parameters a_{12} and a_{13} are neglected here and throughout the rest of the paper) with $a_{12} \geq 0$ provides a statistically significant improvement, if any, to the model with $a_{12} = 0$ when comparing model solutions with data.

The statistic involves the residual sum of squares (RSS), which is a measure of the distance between the model solution and data and is given by

$$RSS = \sum_{i=1}^{n} \left| f^{(1)}(t_i;\theta) - y_i^{(1)} \right|^2 + \left| f^{(2)}(t_i;\theta) - y_i^{(2)} \right|^2,$$

where model parameters are represented by θ, $\{y^{(1)}\}_{i=1}^{n_1}$ is the drinking data,

$$\{f^{(1)}(t_i;\theta)\}_{i=1}^{n_1} = \{\int_{-1}^{0} A(t_i + s;\theta)ds\}_{i=1}^{n_1}$$

computed with parameters θ, $\{y^{(2)}\}_{i=1}^{n_2}$ is the guilt data, and $\{f^{(2)}(t_i;\theta)\}_{i=1}^{n_2} = \{G(t_i;\theta)\}_{i=1}^{n_2}$ and $n_1 = n_2 = 23$ for the data on the triweekly timescale. The corresponding model values are also converted to the triweekly timescale in the equivalent way (averaging over 2 or 3 days as described immediately before the beginning of Section 4.1).

We estimate model parameters θ by minimizing the objective functional

$$J(\theta) = \sum_{i=1}^{n_1} \left| \int_{-1}^{0} A(t_i+s)ds - y_i^{(1)} \right|^2 + \sum_{i=1}^{n_2} \left| G(t_i) - y_i^{(2)} \right|^2 \tag{14}$$

over a feasible parameter space Θ. In comparison, estimating parameters with the constraint $a_{12} = 0$ is done by minimizing the same functional over a restricted parameter space $\Theta^H = \{\theta \in \Theta | a_{12} = 0\}$ which is a subspace of the original space ($\Theta^H \subset \Theta$). This is a reduction in the degrees of freedom in parameters by one, and in general, the resulting residual sum of squares when allowing a_{12} to vary (minimizing over Θ) will be at least as small or smaller than when minimizing over Θ^H or fixing $a_{12} = 0$. The test statistic can be used to determine whether this improvement, if any, is statistically significant and is defined by

$$U = \frac{n\left(J(\theta^H) - J(\theta)\right)}{J(\theta)}, \tag{15}$$

where $n = n_1 + n_2$.

TABLE 2. Some values for the $\chi^2(1)$ distribution

$\chi^2(1)$	confidence
1.32	75%
2.71	90%
3.84	95%
6.63	99%
10.83	99.9%

We performed this test in two ways: comparing residuals from estimating model parameters $\theta = (a_{12}, a_{21})^T$ with estimating $\theta^H = (0, a_{21})^T$ (where $a_{12} = 0$ and a_{21} is allowed to vary), and $\theta = (a_{12}, a_{13})^T$ compared with $\theta^H = (0, a_{13})^T$. In the first case, the feasible parameter space is $\Theta = \{[0, 10] \times [0, 2]\}$, and in the second case $\Theta = \{[0, 10] \times [8, 20]\}$. The statistic when comparing the residuals for $\theta = (a_{12}, a_{21})^T$ and $\theta^H = (0, a_{21})^T$ is

$$U = \frac{46(114.28 - 113.10)}{113.10} \approx 0.47993.$$

This statistic may then be compared to a χ^2 distribution with one degree of freedom (select values of which can be seen in Table 2) to determine whether we reject the null hypothesis. For this example, the null hypothesis is that the restricted parameter space (with $a_{12} = 0$) is the appropriate one. We note that this procedure does not allow us to accept the null hypothesis, but only to either reject it or not. Therefore, we conclude that allowing $a_{12} > 0$ does not result in a statistically significant improvement in model fit to data.

When beginning the minimization to estimate parameters $\theta = (a_{12}, a_{13})^T$ we initialized the procedure with $a_{12} = 0.1$. However, the parameter value for a_{12} that minimized the objective functional was $a_{12} = 0$, and thus the residuals (RSS) were the same. Therefore, there was no need to compute the statistic, and we can directly interpret that there is no improvement in the fit. Similarly, when estimating all three parameters $\theta = (a_{12}, a_{13}, a_{21})^T$, as compared with estimating $\theta^H = (0, a_{13}, a_{21})^T$, the difference in residuals is approximately 10^{-6}, which results in a model comparison statistic value (also approximately 10^{-6}) that indicates a very insignificant improvement to the fit. Therefore, we cannot reject the null hypothesis with any reasonable level of confidence. Thus, it is reasonable to take $a_{12} = 0$, which amounts to excluding the term $-a_{12}\chi_{G>G^*}(G(t) - G^*)$ in the $\frac{d}{dt}A(t)$ equation for the final model of patient 6029.

4.4. *Final 6029 model.* The model that best describes the dynamics in patient 6029's data is

$$\frac{d}{dt}A(t) = a_{13}h(t), \tag{16}$$

$$\frac{d}{dt}G(t) = a_{21}\left[\int_{-1}^{0} A(t+s)ds - (1 + c_1\chi_{W(t)})A^*\right]. \tag{17}$$

While there is always room for improvement in the development of a model, at this stage there is likely not much to be gained from the inclusion of additional mechanisms. We note that the modeled processes are not as inherently predictable as those in biology, physics, or engineering. Therefore, our goals and standards for agreement between model solutions and data should be adjusted accordingly. We deem the 'fit' seen in Figure 6 to be relatively good since the episodes of drinking are, with the exception of two, accurately predicted by the model. We surmise that it is not necessarily a reasonable goal to predict the number of drinks during the episodes, but rather, whether or not they occur. In future modeling efforts, it may be that the episodes not predicted by the model are especially instructive, as they may be indicative of mechanisms missing from the model.

This individual reduced his drinking substantially at the beginning of the study and not during the IVR observation period. Therefore, it is difficult to speculate on the associated mechanisms accompanying his behavior change. However, the characterization of his drinking after reduction is instructive. The individual appeared not to drink much, and also exhibited a corresponding low desire to drink during the week, but his desire and drinking increased substantially on the weekends. Incorporating the $h(t)$ function accounted for this and allowed for considerable agreement between model solutions and data. One possible explanation and therefore justification of including this term (weekday/weekend) relates to the perceived availability, craving, and consumption literature. Field and Cox ([15]) in their review of this literature note that the intensity of craving following cue exposure has been shown to vary depending on the participants' belief that they would be able to consume the substance. For example, numerous experiments have shown that smokers who are told the opportunity to smoke is imminent exhibit a higher subjective craving than when they are told they will not be able to smoke for several hours. Perceived availability has also been used to explain differences in responses to cues among substance users continuing to use substances and those seeking to remain abstinent [42]. We know from the clinical records that 6029 set a goal of reduced drinking on the weekday, but would allow himself to drink more heavily on the weekend. It may be that by clearly defining this limit, 6029 changed the perceived availability of heavy drinking and that his desire reflected when he felt he could drink. We can speculate from his clinical record and ability to reduce drinking that 6029 possessed strong internal control, and perhaps this strong internal control was manifested in the ability to set limits such that there was a corresponding reduction in desire. By contrast, someone with weaker internal control might set a limit, but still experience strong desire.

One puzzling question is why commitment and self-efficacy, variables hypothesized to be critical to control, do not appear in the equation. It may be that because 6029 expected to drink more on the weekend, he did not rate his commitment or self-efficacy lower during these periods. Alternatively, it may be that these variables are important determinants of change in the decision to reduce drinking, but that once a plan has been set in motion, they recede as operative variables, thus their fluctuation is not a determinant to drink, unless the plan fails. We return to a fuller discussion of these issues in the discussion section.

It is striking that this individual's drinking is described well by the weekend-dependent driving mechanism, and also that his desire mirrors the same behavior. This close link may be an indication that his internal control is very good. That the other measured control variables (such as the confidence and commitment to limit drinking, or the commitment to abstain completely from drinking) do not appear to capture this information may suggest that these questions are not good indicators of internal control for this patient. It may also be that there are other issues with the data collection process that do not allow for those relationships to surface in the data. At any rate, these efforts provide motivation to look at the measures of control further.

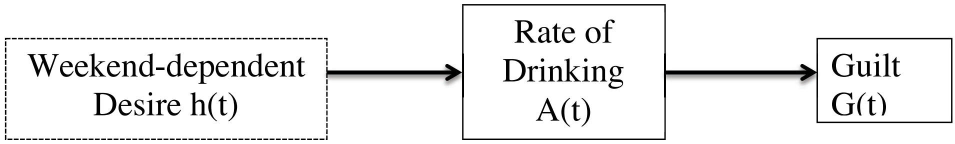

FIG. 7. Updated categorical model for patient 6029

The first categorical model (Figure 3), which was provided as a schematic of hypothesized possible relationships between model variables, can now be updated to reflect the model (16)-(17), and is depicted in Figure 7. While we did not use inverse problem methods to go from the more complex model to the resulting simple one, we did iterate between the model, comparisons between solutions and data, interpretation of data, and using current knowledge and plausible hypotheses to inform the model. Indeed, each arrow in the categorical model, indicative of terms in the mathematical model, represents a hypothesis about the underlying process. The decision to keep, discard, or refine the term is in a sense hypothesis testing, although it is not always done in the usual formal way described at the end of the last section. Typical statistical methods addressing hypothesis testing would be limited in their ability to reveal the dynamic relationships we focus on here, as this type of information is lost as data is aggregated across individuals or responses (means, standard deviations, etc.). Mathematical modeling provides a way for us to precisely formulate hypotheses concerning dynamic variables. If we are able to make use of inverse problem methods and estimate model parameters, then we can use statistical methods in conjunction to test whether model modifications provide a statistically significant difference in the fit to data. As illustrated above, these methods involve the calculation of a statistic based upon the residuals (the sum of the differences between the model solution and data point) from two models that have been fit to the same data set. Other examples of use of such statistically based model comparison techniques are given in [11, Chapter 3] and [8, Chapter V].

It is encouraging to be able to arrive at such a relatively simple explanation of this patient's behavior, but this individual is atypical in a crucial way. Frequently found among responding subjects in Project Motion, the individual experiences some pivotal or transformative period of change after which their alcohol consumption pattern is significantly changed. While patient 6029's drinking pattern is still not the picture of perfection (he continues to violate his norms on weekends), it does not change substantially during the time period of the IVR. It would likely be more instructive to study an individual who experiences this pivotal change during the IVR period so that we would have observations before, during, and after it to better characterize this process of change.

While we should not expect a simple model to describe a typical problem drinker (indeed the close connection between their desire and actual drinking behavior is likely not present), our ability to methodically do so in this case demonstrates mathematical modeling as a useful tool in understanding these data. Notably, it appears to provide a significant advantage over the use of static methods in which the identification of important factors (without attempting to determine precise relationships between them) relevant to the patient's drinking behavior proved difficult.

5. Patient 6009 model. Patient 6009 was selected as the next model subject because his drinking data displayed a clear and consistent downward trend. Therefore, we hoped that information on any pivotal experience that he may have had preceding and during his drinking reduction may be observed in the data. Additionally, this individual was homeless and unemployed when he enrolled in the study and was able to secure stable housing during the IVR. Therefore, we suspected that the supporting mechanisms of his behavior change would be different than those seen in patient 6029. Namely, patient 6029 appeared to exhibit strong internal control, whereas it is possible that patient 6009 was able to reduce his drinking as a result of external factors.

We considered that there may be other mechanisms not considered in the initial categorical model or in the IVR data categories, such as the weekend-dependent desire mechanism that subsequently was key to understanding patient 6029's desire and drinking data. Of the data categories included in the categorical model for patient 6009 (shown in Figure 8), we looked to single question responses for possible relationships with drinks, as the variables in the resulting model for patient 6029 are from single questions, discussed in more detail in Section 7. In the end, the simple model of patient 6029 agreed with the dominating characteristics documented in the supporting notes of the clinical team. As a result, we begin by using the clinical summary of the patient, along with those selected to be in the initial categorical model (Figure 8) to select items in the IVR that may be most relevant to his drinking. We then looked for whether any dynamic relationships appeared to exist between these variables and his drinking data, and thereby selected for inclusion in the model. We note that this modification of our approach - use of single items and clinical summary data - is characteristic of the iterative nature of modeling. That is, it is common that each modeling exercise reveals new information which is then used to inform the next model.

6. Initial model for patient 6009. We determined that guilt $G(t)$, loneliness $L(t)$, *confidence* $C_1(t)$ to resist drinking heavily, *commitment* $C_2(t)$ to resist drinking heavily, and *commitment* $C_3(t)$ to completely resist drinking are the best candidates for state variables for a model that would describe patient 6009's drinking behavior. For the reader's convenience, data from these variables, overlayed with drinking data, is displayed in Figure 9. Not surprisingly as seen in Figure 10, patient 6009 did not exhibit a clear weekday versus weekend pattern as suggested in his case summary where it was noted that he had a lot of unstructured time (that he filled by drinking) due to being unemployed. The timescale was kept consistent with the triweekly timescale of patient 6029, as it is still the desired resolution. With the exception of guilt $G(t)$, these variables were selected based on the effects that they appear to have on drinking. That is, there were notable

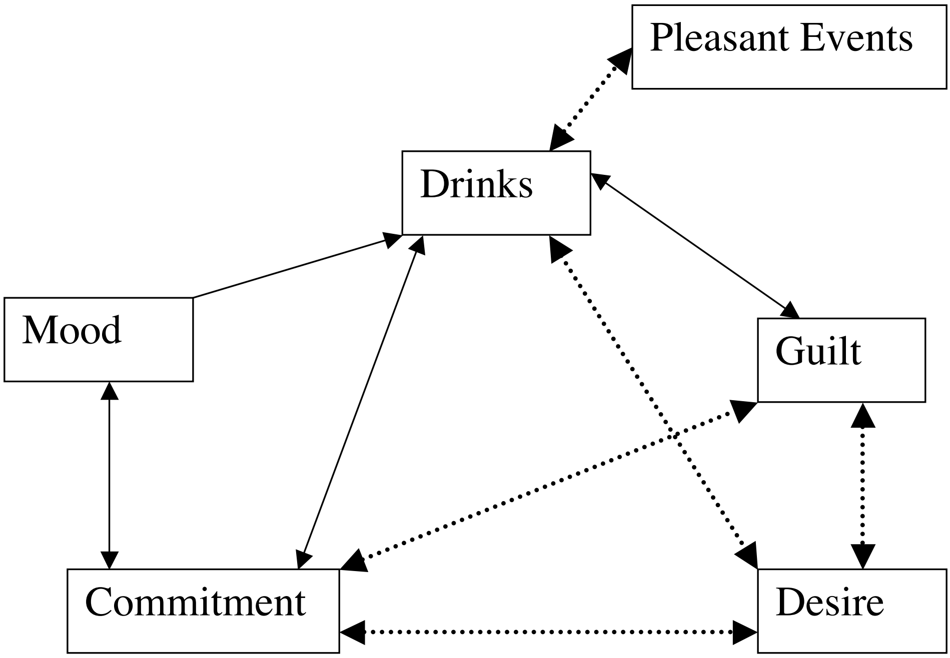

FIG. 8. Categorical model for patient 6009 from initial efforts, discussed further in [9].

features in the evolution of these variables in time that seemed to correlate with a feature in the drinks consumed over time, along with a plausible explanation for such an effect.

For a complete dynamical system, we would like to also be able to derive equations for all state variables $(A(t), G(t), L(t), C_1(t), C_2(t), C_3(t))$, and therefore we would need to be able to attribute changes in these variables to changes in other state variables, or to other time-dependent functions as we did with the $h(t)$ function for patient 6029. The state variables $(L(t), C_1(t), C_2(t), C_3(t))$ may depend on other variables not in the current model (and therefore, not directly related to drinking), but may also be influenced by external events and/or internal processes not captured in the IVR data. For example, we know from 6029's clinical record that he received housing somewhere between day 10 and 21. This event likely contributed to the reduction in his drinking during that period, but was not captured in the IVR script. We return to the issue of missing data in a later section of this paper. Regardless, it does not appear that the fluctuations in these variables are related in any clear way to the other variables in the model as can be seen when comparing the data for the supporting variables directly. Therefore, we deemed it not likely that we would be able to construct model equations for the variables $(L(t), C_1(t), C_2(t), C_3(t))$ depending on other model variables. We constructed functions for these variables that captured key aspects of their dynamics, similar to the function $h(t)$ that reflected the observed weekend-weekday pattern for patient 6029. That is, we constructed 'best fit' lines through these data (piecewise linear functions) and used these functions as inputs into the equations for drinking rate $A(t)$ and guilt $G(t)$. The

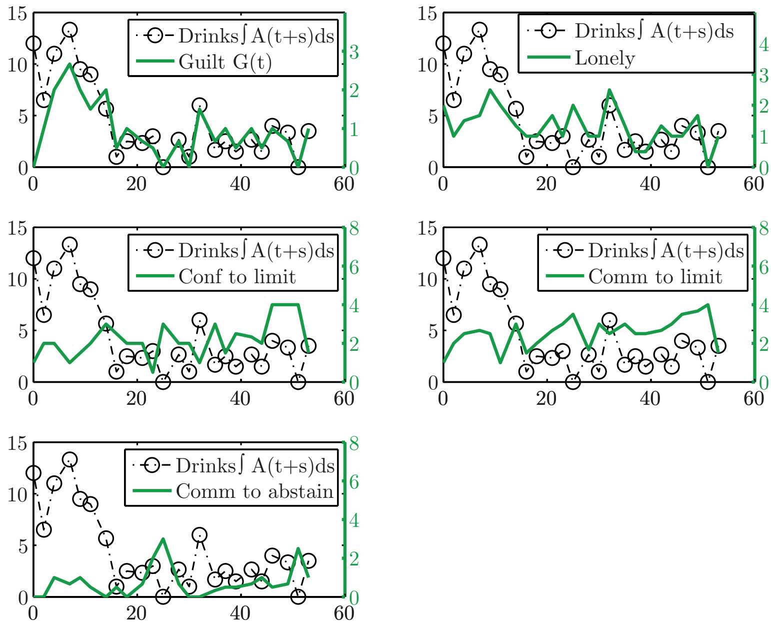

FIG. 9. Data for initial 6009 model variables (left axis) overlayed with drinks (right axis).

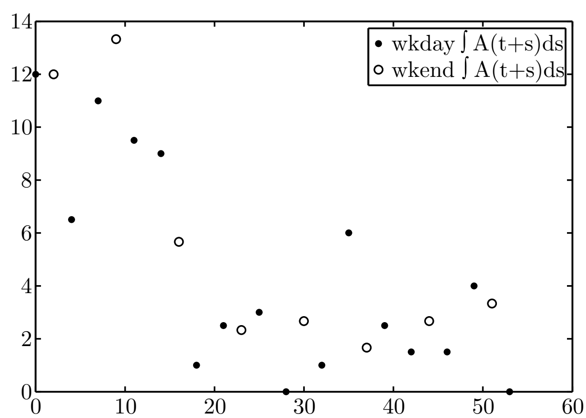

FIG. 10. Weekday and weekend drinks plotted on the triweekly timescale.

use of the observed dynamics of the supporting variables in the model equations for $A(t)$ and $G(t)$ that give a reasonable approximation to the observed data thereby suggest that these data are observations of dynamic processes. Further, the use of these dynamic data to reproduce dynamics in drinking and guilt data provide encouraging support for the use of mathematical models to provide meaningful explanations for these observations.

A revised categorical model, or schematic of the hypothesized relationships between variables is given in Figure 11. Supporting variables, or variables that are used as inputs are only represented by boxes with dotted borders, and causal relationships are again represented by arrows. The appearance of the lack of relationships between support variables in this schematic is not to say that they do not exist, but that we are unable to determine them with our knowledge at this time and from the data set on hand.

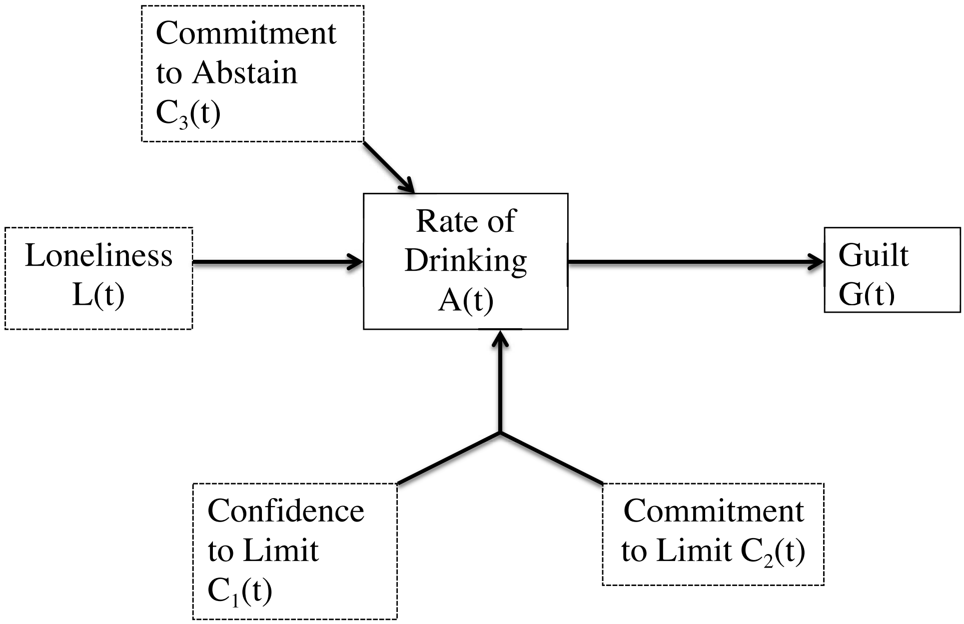

FIG. 11. Updated categorical model, or schematic of hypothesized relationships amongst variables reflected in the initial model.

We constructed an initial model by examining the individual's IVR response data, considering the previous experience modeling patient 6029, and constructing quantitative relationships that represent mechanisms consistent with our knowledge of processes underlying drinking behavior. We initially represented the mechanisms governing the state variables $A(t)$ and $G(t)$ by

$$\begin{aligned}\frac{d}{dt}A(t) &= c_1 + a_{13}\left(L^*(t) - L(t)\right) - a_{17}\chi_{C_3>C_3^*}C_3(t) \\ &\quad - a_{15}\chi_{\{C_1>C_1^*\}}\chi_{\{C_2>C_2^*\}}\left(C_1(t) - C_1^*\right)\left(C_2(t) - C_2^*\right), \qquad (18)\end{aligned}$$

$$\frac{d}{dt}G(t) = a_{21}\left(\int_{-\tau}^{0} A(t+s)ds - A^*(t)\right). \qquad (19)$$

6.1. *Drinking* $\frac{dA}{dt}$ *equation.* The equation for $\frac{dA}{dt}$, which is the change in the individual's rate of drinking with respect to time, is our primary focus since understanding the individual's drinking behavior is our goal. The first constant term, c_1, represents the individual's relatively constant desire to drink, so is an ever-present source driving his drinking up. This is an unknown quantity, but we can determine a range of values that this parameter could possibly take on (a necessary first step in doing computations and also for parameter estimation, if we were to pursue it). The only other variable that causes an increase in drinking is the feeling of loneliness $L(t)$ (perhaps relating to depression). Drinking appears to increase proportionally to loneliness, until it reaches a certain threshold, which may be interpreted as perhaps an *unbearable* level. The first of these switches occurs around $t = 11$, and twice more at $t = 32$ and $t = 49$. It appears that the threshold level of loneliness does not change until $t = 32$, at which point it decreases substantially. So $L^*(t)$ is represented by the piecewise constant function

$$L^*(t) = \begin{cases} L_1^*, & 0 < t \leq 32, \\ L_2^*, & t > 32, \end{cases}$$

and the constants L_1^* and L_2^*, are apparent from the IVR data, and are therefore known values. Loneliness less than the unbearable threshold $L < L^*(t)$ causes an increase in drinking, and the drinking decreases once this threshold is surpassed, thus the term is $a_{13}(L^*(t) - L)$.

A further controlling mechanism for patient 6009's drinking is reflected in his confidence $C_1(t)$ in his ability to resist drinking heavily, and his commitment $C_2(t)$ to resist drinking heavily. These variables induce a decrease in drinking only when both are above a threshold level, C_1^* and C_2^*, which should be at least a response of 'moderately' or in the case above the numerical value 2 (we recall that responses were in the range 0 to 4). This effect is represented by the term $-a_{15}\chi_{\{C_1 - C_1^*\}}\chi_{\{C_2 - C_2^*\}}\,(C_1 - C_1^*)\,(C_2 - C_2^*)$. The product of the characteristic functions $\chi_{\{C_1 > C_1^*\}}$ and $\chi_{\{C_2 > C_2^*\}}$ will only 'turn the term on', i.e., be equal to 1 when both functions are 1. The effect when both commitment and confidence to resist heavy drinking appears to be quite strong and is likely well represented by their product. It is possible that this effect may better be modeled by a saturating effect, in which increases in these variables will have a lesser effect on the control of his drinking at very 'high levels' or intense feelings of commitment and confidence. With the current data set, we will not likely need to include such an effect to reproduce the individual's responses since they are bounded ($C_1, C_2 \in [0, 4]$).

The individual's commitment $C_3(t)$ to resist drinking entirely rarely raises above a level indicating the presence of such a commitment, but on such occasions, it appears to indicate a strong control mechanisms. These occasions occur at times $t = 25$ and $t = 51$, both of which occur simultaneously with the rare occasions where he does abstain entirely from drinking. While the rate a_{17} is unknown, as are most of the others, it is likely much larger than a_{15}.

6.2. *Guilt $\frac{dG}{dt}$ equation.* We begin with the equation

$$\frac{d}{dt}G(t) = a_{21}\left(\int_{-\tau_1}^{0} A(t+s)ds - A^*(t)\right).$$

Patient 6009's responses for guilt, or the perception of the individual's drinking over the previous day as excessive, exhibit the same dynamics as was seen with patient 6029, so is initially modeled similarly. As with patient 6029, it does not appear that 'guilt' $G(t)$ influences his drinking, but rather the other way around. Thus, his guilt is proportional to his drinking over the past day $\int_{-1}^{0} A(t+s)ds$ such that it only increases if his drinking has exceeded his personal standard for an 'acceptable level', or a threshold $A^*(t)$. Patient 6009 differs from patient 6029 in that his acceptable level $A^*(t)$ does not change depending on whether it is a weekend or weekday, but rather is a high constant level A_1^* before his initial pivotal change at $t = \bar{t}_1$ and then afterwards it is a much lower level A_2^* (with $A_1^* > A_2^*$). Thus, the threshold is represented by

$$A^*(t) = \begin{cases} A_1^*, & 0 < t < \bar{t}_1, \\ A_2^*, & \bar{t} \le t \le T_f. \end{cases}$$

While we cannot be sure of the actual day $t = \bar{t}_1$ the individual experienced his change, we can conclude that it was between the narrow window of $\bar{t}_1 \in [11, 14]$ when we see the first significant decrease in drinks. Even more telling is that he reports an increase in feeling that his drinking was excessive at $t = 14$ when he drank roughly 6 drinks in a day, as opposed to around 10 drinks the previous time point at $t = 11$. This clearly indicates a change in personal standards within this time interval.

6.3. *Computational approach and model modifications.* We constructed piecewise linear functions to best estimate and capture important aspects of data in the supporting variables $(L(t), C_1(t), C_2(t), C_3(t))$. These functions, which can be seen in Figure 12, were then used as inputs in the solution of the drinking equation $\frac{d}{dt}A(t)$.

As with the 'fitting' of the model for patient 6029 to his data, the process of working toward a model solution that agreed with patient 6009's drinking data led to model clarifications. The most important of these was that the nature of the dependence of the equation $\frac{d}{dt}A(t)$ on the supporting variables is more accurate on their time-derivatives, as was seen with the relationship between desire and drinking rate with patient 6029. This dependence first became evident when using the loneliness data in the drinking equation. From the data, we hypothesized this dependence as being threshold-dependent (Equation (18)); but as $L(t)$ increases, while being above the threshold L^*, this induces a decrease in drinking rate. If the threshold L^* was taken to be at or close to a peak (local maximum) of the loneliness data, the drinking rate would not decrease for a long enough time period to reproduce the decrease as seen in the drinking data. Rather, the increases and decreases of loneliness occur simultaneously with those of drinks. If the input for $L(t)$ is changed from the piecewise linear function shown in Figure 12 to the slopes of those lines instead, the drinks as predicted by the $\frac{d}{dt}A(t)$ equation agree much better with the fluctuations seen in the data. Through similar reasoning, when considering the inputs of $C_3(t)$, $C_1(t)$, and $C_2(t)$, it became clear that $\frac{d}{dt}A(t)$ should depend on the time derivatives of these variables as well. Therefore, the slopes of the piecewise linear functions were used to approximate the data of the supporting variables and were used as inputs instead of the piecewise linear functions themselves. Not only are these relationships more natural in their interpretation, but they gave a model solution that better agrees with the drinking data. The drinks calculated from the model solution and shown on the triweekly time scale along with the triweekly averaged drinking data is shown in Figure 13.

The final model of the drinking data, from which the solution shown in Figure 13 was computed, is given by

$$\frac{d}{dt}A(t) = a_1(t)\dot{L}(t) - a_2(t)\dot{C}_1(t)\dot{C}_2(t) - a_3(t)\dot{C}_3(t), \tag{20}$$

where $\dot{X}(t)$ denotes the time derivative of the variable X, or $\dot{X}(t) = \frac{d}{dt}X(t)$. The coefficients are time-dependent, so that each piece of the input or supporting variable is weighted to give a better quantitative fit to the drinking data. For example, the estimated loneliness variable increases from time $t = 0$ to $t = 9$, so the positive slope of that line is taken as $\dot{L}(t)$ and the coefficient $a_1(t)$ is constant over the interval $0 \leq 0 \leq 9$. After $t = 9$, loneliness decreases until $t = 16$, so that negative slope is then the input for $\dot{L}(t)$ and the coefficient $a_1(t)$ has a possibly different constant over the time interval $9 < t \leq 16$. The coefficients and other supporting variables are constructed accordingly as piecewise constant functions.

The need to take the coefficients as time-dependent may be indicative of missing information in the model, but also is likely due to the current limitations in precisely quantifying the supporting variables which do indeed appear to represent dynamic processes. Encouragingly, even if constant coefficients are used, the corresponding solution (seen in Figure 14) does give the same fluctuations as seen in the data, although the number of drinks are not as accurately predicted. Namely, the drinks are over-predicted after the initial decrease around time $t = 11$. It is entirely possible that as our ability to quantify the variables improves, there will be less missing information in the data sets and therefore less, if any, need to use time-dependent coefficients.

Although the guilt data exhibited a similar relationship to the drinking data with patient 6009 and patient 6029, solutions to the initial model equation (19) did not reasonably reproduce the dynamics seen in the guilt data. As with the other variables, it appeared that it was due to the *rate* of guilt not being dependent on the drinks over the last day at a given time, but rather, the level of guilt was directly dependent on the drinks over the last day at a given time. As seen in the top right panel of Figure 15, the solution to

$$\frac{d}{dt}G(t) = a_{21}\left(\int_{-\tau}^{0} A(t+s)ds - A^*(t)\right)$$

exhibits the same issues as with the other supporting variables. With drinks above the threshold $A^*(t)$, the guilt increases, although this is not what the data shows. Rather, the data shows a more direct relationship with drinks, and so the bottom two plots are used to illustrate that the relationship seen in the data that guilt is best modeled as being directly proportional to the number of drinks over the past day $G(t) = k\int_{-1}^{0} A(t+s)ds$. We note that while mathematically it is also possible to represent drinking as being completely described as proportional to the guilt variable, it does not agree with our interpretation of this patient and is not as likely true in general of the relationship between guilt and drinking. That is, patient 6009 has not cited guilt from drinking over the previous day as a key contributing reason for drinking. On the other hand, guilt, or one's reflection on his/her previous day's drinking, is by nature dependent on the previous day's drinks. Therefore, for patient 6009, the relationship between guilt and drinking appears to be

$$G(t) = k\int_{-1}^{0} A(t+s)ds. \tag{21}$$

6.4. *Final 6009 model.* Beginning with the case summary as motivated by our modeling experience with patient 6029, we were able to identify some IVR items which contained dynamics that could explain 6009's drinking behavior, albeit incompletely. As we needed to use these dynamics as inputs, much along the same lines as with the weekend/weekday effect displayed in the desire variable data of patient 6029, we note that we are not able to construct a full dynamical system, as these supporting variables do not appear to be influenced by each other. Rather, this is evidence of missing information in the IVR questionnaire, further improvements to which are discussed in the next section. It is likely that if all future modeling efforts begin with case summaries, the insight gained may be limited. That is, it may be that there are insights to be gained from other data not emphasized in the case summary and therefore perhaps excluded from modeling efforts. For that reason, it is important that the data collection be improved to the extent that inverse problem techniques for more complete coupled dynamical systems can be applied to future data sets. In light of that, some suggestions are offered in the next section for data collection, and hopefully the modeling of behavior change will be furthered as a result.

Again, the drinking behavior of patient 6009 appears to be dynamic and has been demonstrated as being influenced by other dynamically changing variables. Patient 6009, as opposed to 6029, appears to change in a much different way, with external events playing a key role. It is noteworthy that the variables (e.g., guilt, mood) that we would have previously expected to reflect his change in drinking behavior, regardless of the pattern of change, did not. This suggests that generally there is something missing from the battery of questions in the survey and that it may be that the inclusion of a free form response question in the survey would help to address that. In addition, it is interesting that the desire variable was not closely related to drinking, which is another variable that we would expect to be closely related to drinking regardless of the individual's motivation. This raises questions concerning which aspects of an individual's behavior and environment we should focus on to understand behavior change.

6.5. *Patient 6029 revisited.* Upon reflecting on our findings with patient 6009, one immediate question is whether patient 6029's guilt data may be better explained by a direct proportionality relationship. A comparison from the final model in Section 4.4 and this direct proportionality can be found in Figure 16. While both solutions predict the drinking episodes with roughly the same accuracy, the solution from the differential equation generally increases with time (as can be seen when focusing on the periods of abstaining from drinking), and that of the direct proportionality relationship does not. This suggests that the simpler form (proportionality) might indeed be a more accurate explanation of the relationship between drinking and guilt, or question in the IVR of whether or not the individual feels their drinking was excessive. This begs the question as to the true quantitative nature of the relationships inquired about in the IVR data, as discussed further in the last part of the next section.

7. Improvements to data. Through the process of developing mathematical models for these two initial patients, some issues in data collection have become clear. Some of these are addressed more easily than others, and we do our best to offer solutions where possible in this section. However, some points we discuss here are with the intention of bringing to light more general and likely persistent challenges that complicate the application of mathematical methods to study questions in the behavioral sciences.

We advise without hesitation that it is better to avoid averaging of information. While it was intuitive that averaging longitudinal trends from different individuals together would result in the loss of information, our initial approach of averaging responses to similar IVR questions for one individual seemed appropriate. However, after constructing the categorical models, it became clear that distinctive longitudinal patterns were only seen in data categories based on few questions, and most variables that were successfully chosen were based on one IVR question. In the case of patient 6029 the desire data, originally the average of three questions, was the only variable based on more than one IVR question. However, when difficulties arose when working toward an agreement between model solutions and these data, we examined the longitudinal patterns of each question response individually compared with the average (shown in Figure 17). There appears to be a loss of information in that the average response does not reflect general trends among the other statements. More troubling is that there does not appear to be a general trend among the statements, and the responses to each statement change in different ways in time. This indicates that these items are not equivalent or indicative of the same idea, as previously thought. In fact, comparing two seemingly opposite statements ('I don't feel like drinking' and 'The idea of drinking is appealing'), one would expect perfectly opposite responses, although that is not the case, as seen in Figure 18. In light of these observations, we recommend in the future that only one survey question (as opposed to several questions) be asked for a given purpose.

While we can use single statements in the future to inquire about a single concept, understanding *why* the responses to the statements shown in Figure 18 do not follow essentially an opposite pattern is more difficult. One possibility is that the patient may have interpreted the meanings of the statements as being slightly different than the intended one. The clinical staff noted that, in some cases, the patients did seem slightly confused as to the intent of the question. This can be addressed by a period in which the participants in the study are trained on the meanings of the questions, analogous to instrument calibration in other sciences. That is, the goal is to fine-tune the instrument, in this case the study participant, so that the measurements are accurate and reflect the intended measured process. The issue may still be a problem simply due to human error, but hopefully could be diminished in its impact on model development.

One of the first aspects of developing any model is to determine the appropriate framework based on the quantitative nature of the state variables and how they change with time. While the data in the IVR is discrete, we think that many of the measured processes are more accurately continuous, as many questions inquire about the intensity of feelings. This could be improved by simply allowing for more responses (ranking on a scale from 0 to 9 if via telephone or even more refined responses via a computer where real numbers in contrast to integers are possible). Another improvement would be to frame the questions in a more quantitative way, asking the patients to qualify how much or to what extent they experience feelings, or agree with statements, etc. This is in contrast to the current IVR items which inquire, for example, if the patient agrees with a statement. The most possible responses were 'definitely false', 'false', 'neither true nor false', 'true', and 'definitely true'.

With such few possible responses, being unable to report intensities more or less than the available options is an obvious limitation, and is known as censoring. For example, an individual could report having no confidence in their ability to resist drinking heavily one day, and could be even less confident the next day, but would not be able to report this decrease. One solution is to include a follow-up question as to whether their true response would be off the possible scale of responses, either above or below. If so, then there are statistical techniques, such as expectation maximization (the EM algorithm), that would allow us to estimate the censored data point. This is a frequently used technique when fitting dynamic models to censored data in using inverse problem methodology (e.g., [2, 3, 14]) with data truncated or filtered due to measurement limitation either at high or low levels of detection.

Another aspect of observation error is consistency over time. This is an issue virtually universal in all experimental setups, typically addressed by the experimenter calibrating the instrument, so that the measurement taken on a given day can be directly compared with those taken on previous days. That is, if something weighs 24.8 grams on one day and 24.6 grams the next day, we wish to have some confidence that the difference in measurements accurately reflects a difference of 0.2 grams in mass. This is complicated in the current application, as it may be difficult for individuals to accurately rank their feelings or experiences through time. It may be possible to reduce this error by informing the individual of their previous responses, thereby allowing them to reflect on their experiences, feelings, etc. at the previous time(s) and relate that to their current state.

With a fixed set of questions, it is entirely possible to miss an important event (either external or internal) that is affecting the person's behavior in a crucial way. We suspect this to be the case with patient 6009, since generally the expected variables (confidence and commitment to limit drinking, guilt, desire, etc.) do not clearly explain his reduction in drinking. It may be due to there being no account for the major external influences in his life in the current data set. For example, all 3 control variables peak at around $t = 25$ as well as loneliness, and the individual refrains from drinking at this point. This is a striking feature of the data and suggests that there may be something significant happening with this individual, but the current data collection process does not capture it. This also happens in the biological and physical sciences (for example, not measuring glucagon in understanding glucose-insulin balance in bloodstream), but it is more challenging for this system as *many* of the key psychological/physiological players are unclear. The inclusion of a free form response, or question in which the individual is asked to reflect on their day and to specifically speak of anything he/she feels or which might be going on that was not included in the questions, could be advantageous in this regard. This would have allowed us to at least understand what was missing in the data, after collection, in this case. In the future, it may even be possible to then tailor the questionnaire to the patient, or to call the patient into a clinic for more intense observation.

The observation that the selected variables with patient 6009 and the desire variable with patient 6029 were actually related to the drinking rate $A(t)$ directly, and therefore, their time derivatives $\frac{d}{dt}$ related, brings to light another challenge in our ability to select for model variables. Longitudinal trends in each variable are difficult enough to determine given the issues in data collection mentioned earlier in this section. The task of identifying possible relationships among these variables is complicated by these not easily discernible trends. As a result, we have selected variables that fluctuate directly together and not those that affect their time derivatives, which are more difficult to identify from data. This indicates not only a need for improvement in data collection, so that quantitative longitudinal trends will be clearer, but also a need for improvement in the approach for constructing mathematical models. In the context of data collection, it may be beneficial to design questions to directly inquire about the increase or decrease of a variable, thereby collecting data directly on their rates of change. Doing so will likely reduce the amount of censored responses that may occur, or possibly abolish it completely. Future studies could investigate the benefit of including such questions in patient surveys.

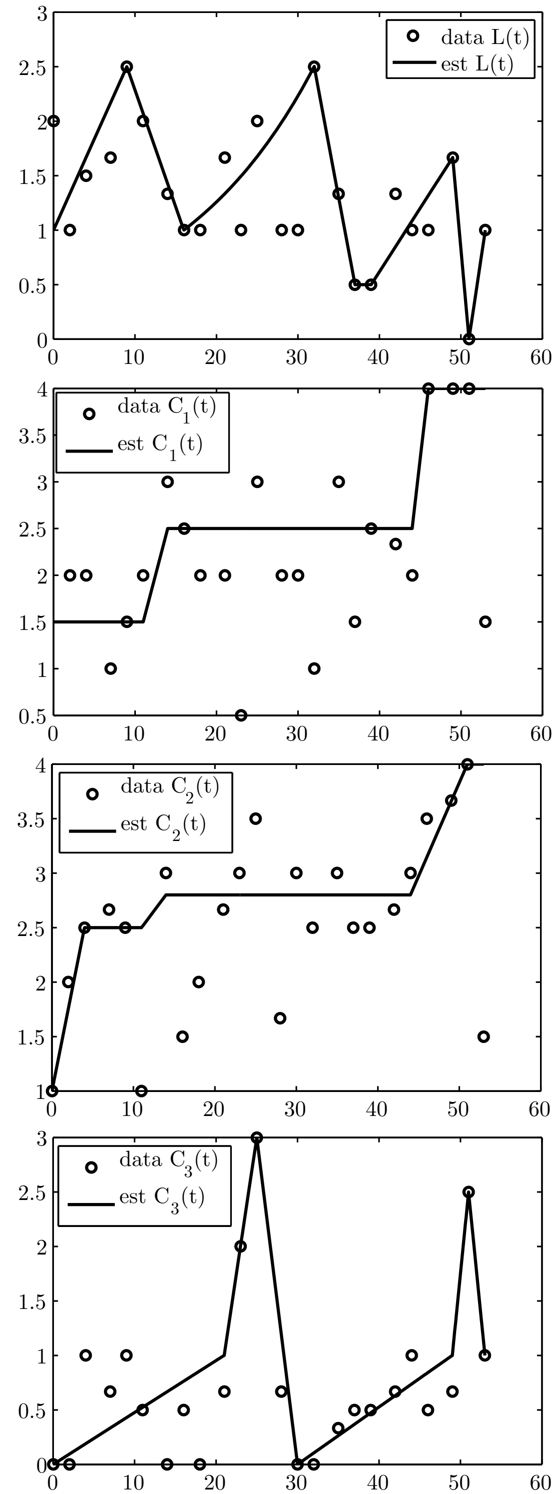

FIG. 12. Constructed functions for supporting variables $(L(t), C_1(t), C_2(t), C_3(t))$, used as inputs in the $\frac{d}{dt}A(t)$ equation for patient 6009.

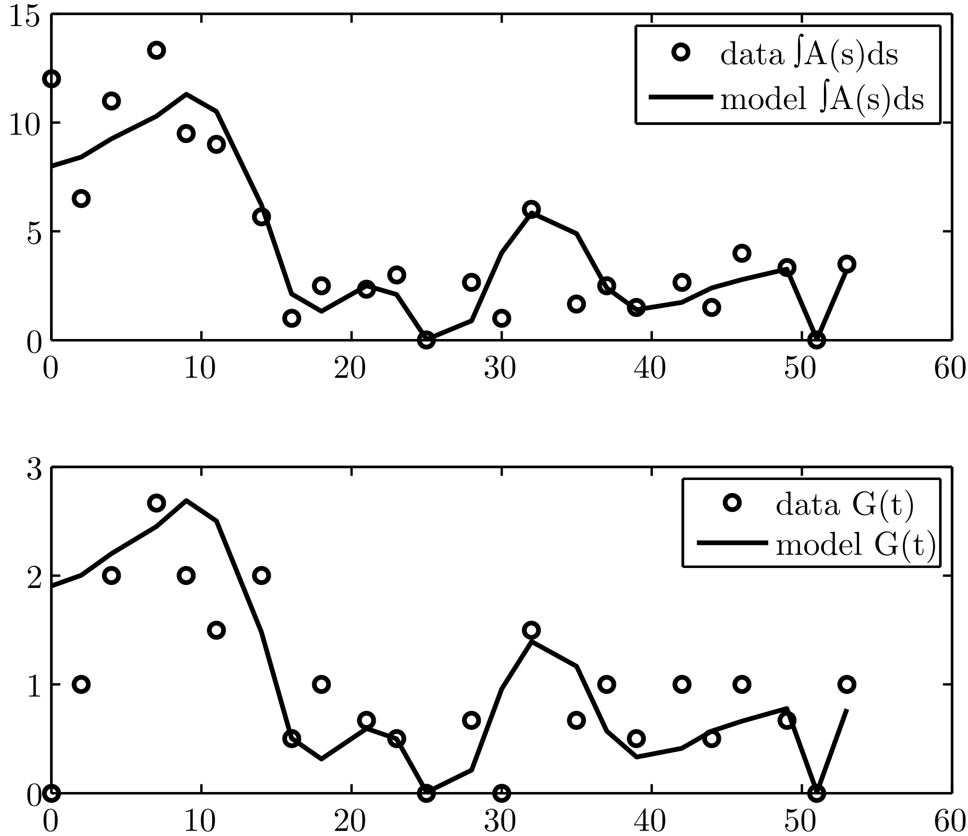

FIG. 13. The 'best fit' model solution shown on the triweekly timescale along with drinking (top) and guilt (bottom) data for patient 6009.

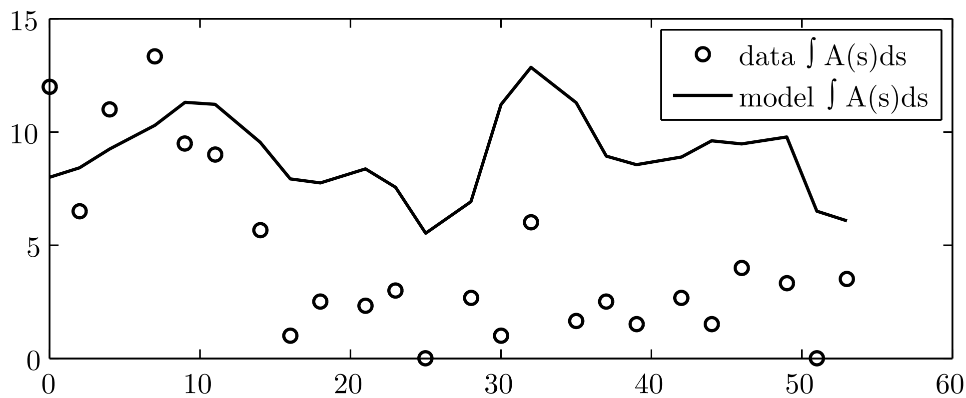

FIG. 14. The 'best fit' model solution, where coefficients a_1, a_2, a_3 are constant, shown on the triweekly timescale along with drinking data for patient 6009.

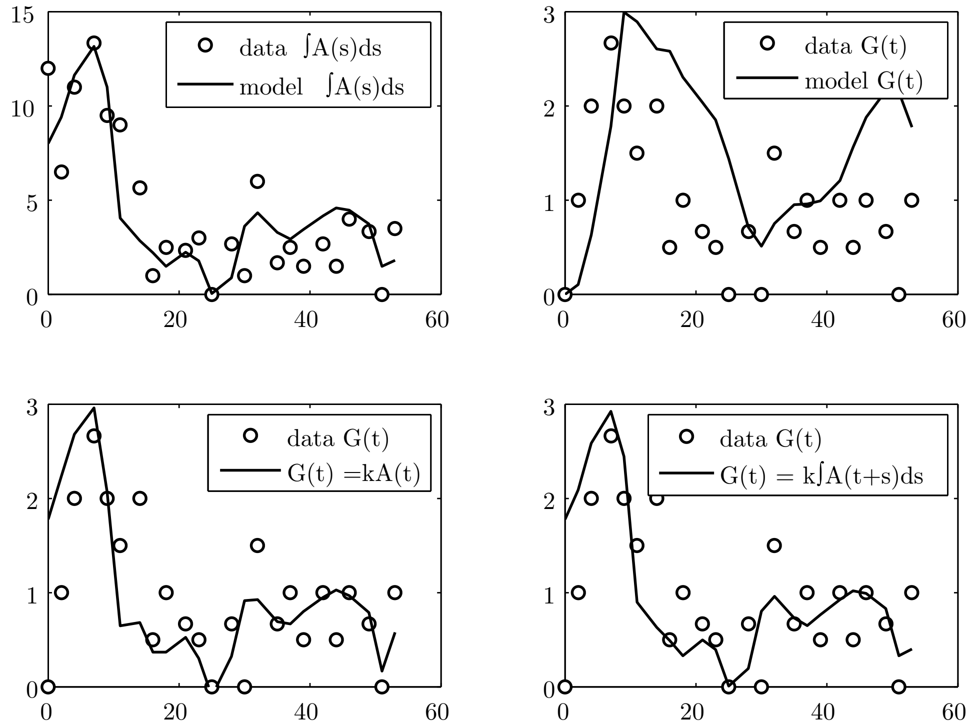

FIG. 15. The top left shows the model solution for the number of drinks $\int_{-1}^{0} A(t+s)ds$ used to calculate guilt $G(t)$. The top right panel shows guilt $G(t)$ plotted on the triweekly scale as calculated from the model solution (19), along with data. The bottom left panel shows guilt if it is directly proportional to the drinking rate $G(t) = kA(t)$, and the bottom right shows guilt if directly proportional to the number of drinks $G(t) = k\int_{-1}^{0} A(t+s)ds$.

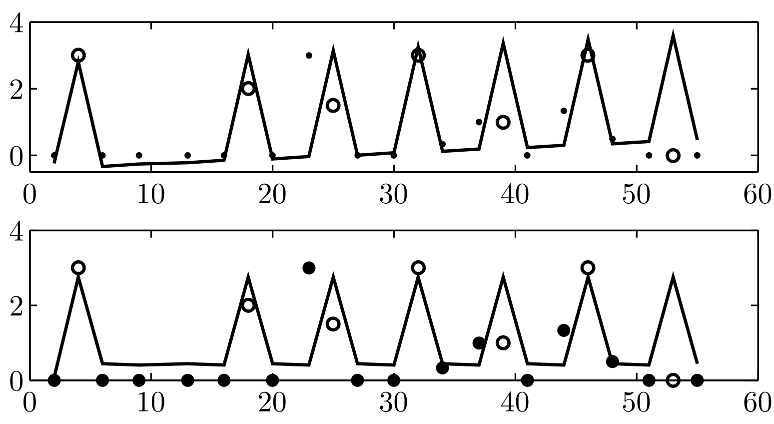

FIG. 16. Guilt as modeled by the differential equation (17) discussed in Section 4.4 (top), compared with guilt modeled by a direct proportionality to drinks (bottom).

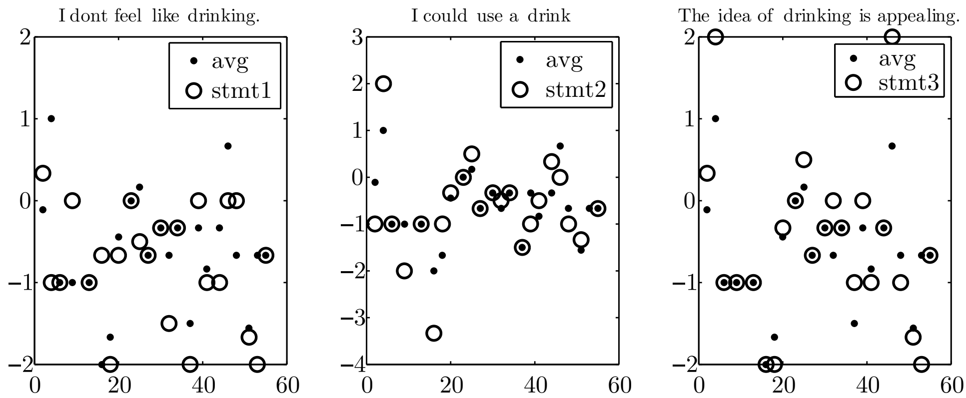

FIG. 17. Drinks and model variables of patient 6009's drinking behavior.

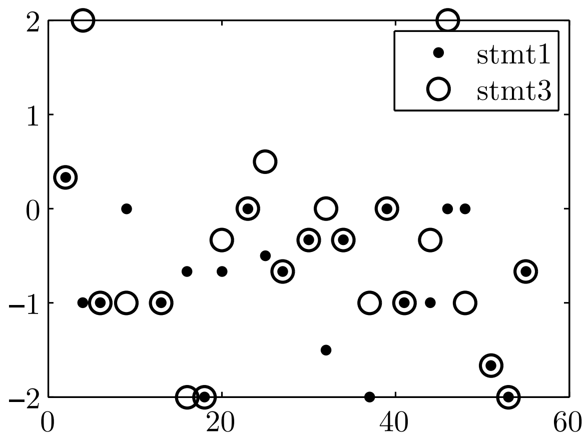

FIG. 18. Drinks and model variables of patient 6009's drinking behavior.

8. Concluding remarks. We began our modeling efforts by attempting (unsuccessfully) to use standard statistical methods (PCA, factor analysis, etc.) to identify dimensions or factors within each individual data record ($n = 56$ longitudinal data responses to 41 questions) that would serve to reduce our data to inform potential model variables. We subsequently continued our efforts by selecting a cohort of 6 participants based on their trajectories of drink reduction. We then constructed individual categorical models based on hypothesized causal relationships and attempted to build mathematical models. We began with patient 6029, and we were able to develop a dynamic mathematical model that provided a plausible set of dynamic relationships to explain his drinking. This model fits a self-regulation framework, with desire driving up drinking and with several empirical supported control variables such as commitment and self-efficacy driving down drinking. Upon attempting to work toward an agreement between model solutions and this patient's data, we found that a simple model with an input function representing the observed time pattern (weekday/weekend) seen in the patient's desire data provides a good fit to the drinking data. In developing a model for the next patient, 6009, input functions were again used to represent the trends seen in the supporting variables. From these, the drinking data for this patient were also reproduced well. Therefore, we offer this work as strong support of mathematical modeling to describe dynamic drinking behavior, and as a potential for understanding the mechanisms underlying behavior change.

In the process of developing dynamic models for these two patients, it became clear that to use inverse problem techniques such as parameter estimation in the usual sense was inappropriate at this stage. This is not only because of the initial stages of development of mathematical modeling in this area, but it is also due to the integrity of the data. That is to say, while it is rich in longitudinal information, and it clearly shows that dynamic processes are occurring, the method of quantification leaves room for improvement. This is not uncommon when a data set is collected, as this one was, for a different purpose, and only later is mathematical modeling used with the data to gain further insight. Further, the shortcomings of these data are indicative of quantification challenges universal in the psychological sciences, as experimental design and data collection require starkly different approaches than those taken in the physical and more tangible life sciences.

On the other hand, the wealth of information available from automated data collection techniques allows us to observe various individuals as they move through their lives and complex environments, and as a result of dynamic interacting factors, make decisions about their behavior. This is a tremendous advantage to data typically available in other fields, as collection is inexpensive and noninvasive and these data sets have much potential for modeling. Inverse problem techniques can improve upon understanding and data collection techniques/experimental design, as they can be used to inform the amount, type and form of data to be collected.

Several conclusions from our present efforts will be of substantial assistance in future efforts. First, there can be little doubt of the overall usefulness in using a dynamic modeling approach to aid in the understanding of sociological/psychological processes involving mechanisms for behavioral change in alcohol use/abuse. The usual statistical regression methodologies which often involve averaging responses over several individuals or over several similar questions (as was done in the earlier efforts reported on here) do not lead to a more useful or better understanding of mechanisms of change. Indeed, it is well known in the inverse problem community that averaging data leads (even with a dynamic modeling approach) to loss of detail in the understanding of mechanisms. For example, the efforts in [5] illustrate that one should not average data (either across individuals or variates, or across covariates in longitudinal data), as this will diminish the ability to detect refined features involving rates, time dependence of mechanisms, etc., as represented in the data.

Our modeling here focuses on trajectories or dynamic variation in change across time in the important variable or variables of interest and then attempts to relate that to more important "hypothesized" variables that may be influencing drinking in an individual. Developing models in such complex environments is an iterative process [11, Chapter 1] that involves beginning with an initial "hypothesized scenario" (such as our initial model in each case with its many compartments) and investigating the feasibility of this scenario as a representation for the observed processes. This was done by carrying out formal model refinement through consideration of the data and a set of iterative steps involving development of categorical models, and then carefully examining the data and developing models to explain the data in a more simple manner.

The advantages of such a modeling approach in the context of the present investigation are clear: behavioral change is a complex process that occurs over time and involves interactions of different factors. Change appears not to be linear, either change in drinking or the relationship of drinking to other hypothesized key variables like desire, and may involve cumulative and/or delayed effects. The relationship between drinking and key variables and the relationship of key variables to each other may change over time in nonlinear ways. This motivates the need for nonlinear, dynamic and possibly non-autonomous (with time-dependent mechanisms/parameters) systems such as differential equations. Thus, typical methods that begin by aggregating individuals together (creating means, SDs) probably serve to obscure the true nature of what is happening. An obvious example that comes out of our study is that by that averaging across individuals we may be missing key "pivotal" or "transformative" events or nonlinear change and what is the immediate influence on and of these critical events. In each case, we do see evidence of these, but the information we collected in these specific data sets is insufficient to completely understand what is happening.

Appendix A. IVR Questionnaire. Here we have included the transcript of the IVR questionnaire to which the subjects of Project MOTION called in each day.

Thank you for participating in this study. During this interview, please press the pound or number key once you have entered your answer. If you would like to have the **current question repeated** at any time, press $*7$. If you would like to have the **previous question repeated**, please press $*1$ at any time. You may repeat a question as many times as you need to. If you would like to **pause the survey**, press $*9$. If you accidentally terminate the questionnaire, please call back immediately to continue.

Q1-11: ACTIVITIES SINCE YESTERDAY

For the next 11 questions, I'm going to ask you about things that you've done since this time yesterday. For each question, answer using a scale of 0 to 3 where:

$0 =$ **no,**
$1 =$ **yes,**
$2 =$ **yes - but last night only, or**
$3 =$ **yes, both last night and today.**

(Stressful events)

1) Did you have or nearly have an argument or disagreement with anyone?
2) Did anything else happen at home, work or school that you felt was stressful?
3) Did anything else happen to you that most people would consider stressful?

(Pleasant events)

4) Did you meet a goal or complete a task that left you with a sense of accomplishment?
5) Did you have a pleasant interaction with a family member?
6) Did you have a pleasant interaction with a friend or colleague?

(Encountered Drinking Situation)

7) Did anyone pressure you to drink?
8) Were you in a situation where you commonly drink?
9) Were you at a celebration or a party?
10) Were you at a nightclub or bar?
11) Were you on a date?

CURRENT MOOD

Now I'm going to ask you about your mood right now. On a scale from 0 to 4 where:

$0 =$ **not at all**
$1 =$ **slightly**
$2 =$ **moderately**
$3 =$ **very much**
$4 =$ **extremely**

Right now, do you feel:

1) Active?
2) Sad?
3) Nervous?
4) Tense?
5) Lonely?
6) Happy?
7) Angry?
8) Enthusiastic?
9) Bored?

10) Tranquil?
11) Relaxed?

PERCEIVED STRESS

1) That you are unable to control the important things in your life?
2) Confident about your ability to handle your personal problems?
3) That things are going your way?
4) That difficulties are piling up so high that you can not overcome them?

YOUR DESIRE TO DRINK RIGHT NOW

Next I'm going to ask you about your desire to drink right now. Please answer the following questions rating your current desire to drink from 0 to 4, where:
$0 =$ **definitely false**
$1 =$ **slightly false**
$2 =$ **neither true nor false**
$3 =$ **slightly true**
$4 =$ **definitely true**

Right now...

1) I really don't feel like drinking.
2) I feel like I could really use a drink.
3) The idea of drinking is appealing.

COMMITMENT AND CONFIDENCE

On a scale of 0 to 4, where:
$0 =$ **not at all**
$1 =$ **somewhat**
$2 =$ **moderately**
$3 =$ **very**
$4 =$ **totally**

1) How confident are you that you can resist drinking heavily (that is, resist drinking more than 5 drinks) over the next 24 hours?
2) How committed are you not to drink heavily (that is, not to drink more than 5 drinks) over the next 24 hours?
3) How committed are you not to drink AT ALL over the next 24 hours?

DRINKS

Now I would like to ask you some questions about your drinking over the past day. Let's start with last night's drinking. Please indicate how many standard drinks you had in each beverage category **last night** - from the time you took the survey yesterday until you went to sleep.

How many beers did you drink last night? Remember, a standard drink is 12 ounces of beer: _____
$0 =$ **no beer**

What kind of beer was it mainly?
$1 =$ **light beer**
$2 =$ **regular beer**
$3 =$ **ale or malt liquor**

How many standard drinks of wine did you have last night? Remember, a standard drink is 5 ounces of wine: ______
$0 =$ **no wine**

How many standard drinks of liquor did you have last night? Remember, a standard drink is 1 $^{1}/_{2}$ ounces of liquor: ______
$0 =$ **no liquor**

Now let me ask you about your drinking **today**. Please indicate how many standard drinks you had in each beverage category today.

How many beers did you drink today? A standard drink is 12 ounces of beer: _____

0 = **no beer**

What kind of beer was it mainly?

1 = **light beer**

2 = **regular beer**

3 = **ale or malt liquor**

How many standard drinks of wine did you have today? A standard drink is 5 ounces of wine: ______

0 = **no wine**

How many standard drinks of liquor did you have today? A standard drink is 1 $^1/_2$ ounces of liquor: ______

0 = **no liquor**

ASSESSMENT OF DRINKING AS EXCESSIVE

Do you consider the total amount you have had to drink since this time yesterday to be excessive? That is, was it more than you think you should have had?

0 = **definitely not**

1 = **possibly**

2 = **probably**

3 = **definitely**

Please press **one** to exit this survey.

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