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DYNAMIC MODELLING and ANALYSIS of a MAGNETICALLY SUSPENDED FLEXIBLE ROTOR

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Abstract

A 12-state lumped-element model is presented for a flexible rotor supported by two attractive force electromagnetic journal bearings. The rotor is modelled as a rigid disk with radial mass unbalance mounted on a flexible, massless shaft with internal damping (Jeffcott rotor). The disk is offset axially from the midspan of the shaft. Bearing dynamics in each radial direction are modelled as a parallel combination of a negative (unstable) spring and a linear current-to-force actuator. The model includes translation and rotation of the rigid mass and the first and second bending models of the flexible shaft, and is unique in that it simultaneously includes internal shaft damping, gyroscopic effects, and the unstable nature of the attractive force magnetic bearings.

The model is used to analyze the dependence of the system transmission zeros and open-loop poles on system parameters. The dominant open-loop poles occur in stable/unstable pairs with bandwidth dependent on the ratios of bearing (unstable) stiffnesses to rotor mass and damping dependent on the shaft spin rate. The zeros occur in complex conjugate pairs with bandwidth dependent on the ratios of shaft stiffnesses to rotor mass and damping dependent on the shaft spin rate. Some of the transmission zeros are non-minimum phase when the spin rate exceeds the shaft critical speed.

The transmission zeros and open-loop poles impact the design of magnetic bearing control systems. The minimum loop cross-over frequency of the closed-loop system is the speed of the unstable open-loop poles. And for super-critical shaft spin rates, the presence of non-minimum phase zeros limits the disturbance rejection achievable at frequencies near or above the shaft critical speed. Since non-minimum phase transmission zeros can only be changed by changing the system inputs and/or outputs, closed-loop performance will be limited for super-critical spin rates unless additional force or torque actuators are added.

This paper reports work performed as part of the author's master's thesis [McCallum 1988], which was completed in the winter of '87 and spring of '88. This work was funded under Draper Lab I.R.&D.

Motivation

- The use of modern control system design methodologies for magnetic bearing applications is the focus of current research.
- In many applications, rotors spin above the first and second critical speeds

  e.g. - jet engines, momentum wheels
- The application of modern design techniques to the active control of high-speed rotors requires a linear state-space model that is simple to use yet includes all important dynamics -
  - shaft flexibility, and *the effects of internal shaft damping*
  - angular dynamics and gyroscopic effects
  - coupling between translational and angular dynamics
  - variation of rotor dynamics with spin rate
  - unstable nature of attractive-force magnetic bearings

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The application of modern, multi-input multi-output (MIMO) control system design methodologies to the control of flexible rotors is the focus of current research. These methodologies include: linear quadratic gaussian (LQG) control; LQG/LTR (Linear Quadratic Gaussian/Loop Transfer Recovery); and H-infinity optimal control.

In many magnetic bearing applications, such as aircraft engines, the suspended rotor spins at speeds exceeding the first and second critical speeds. The application of modern design techniques to the control of these high speed rotors requires a linear system model that is simple yet includes all dynamic effects that are important at high speeds.

When rotors spin above their critical speed(s), rotor flexible body modes can be excited. When the rotor flexes, internal damping can serve as a mechanism of instability [Crandall 1980, Johnson 1986]. Thus, the model used for control design should include rotor flexibility (first and second bending modes) and the effects of internal shaft damping. Angular dynamics are also important at high spin rates since gyroscopic coupling between input axes can be large for high speed rotors. Gyroscopic effects are particularly important for jet engines, where the rotor-bearing system can be subject to relatively large rotation rates. Coupling between angular dynamics and translational dynamics can be significant, and should be included in the model. Finally, the unstable nature of attractive-force magnetic bearings should also be included.

This paper presents a model that simultaneously includes first and second mode bending, gyroscopic effects, the effects of internal shaft damping, and the unstable nature of attractive-force magnetic bearings. The model is used to determine system open-loop poles and transmission zeros and their dependence on spin rate. The paper concludes with a brief discussion of the implications for control system design.

The model presented here is intended to bridge the gap between models available in the literature when the author's thesis research was performed and models used for detailed rotor dynamic analysis. In practice, the model presented here should be augmented with finite element or experimental analyses in a magnetic bearing control system development effort.

Outline

- *Motivation*
- *Outline*
- Model Description
- Analysis
  - Open-loop eigenvalues (poles)
  - Transmission zeros
- Conclusions - implications for control
- Appendices - linearized equations of motion, bibliography

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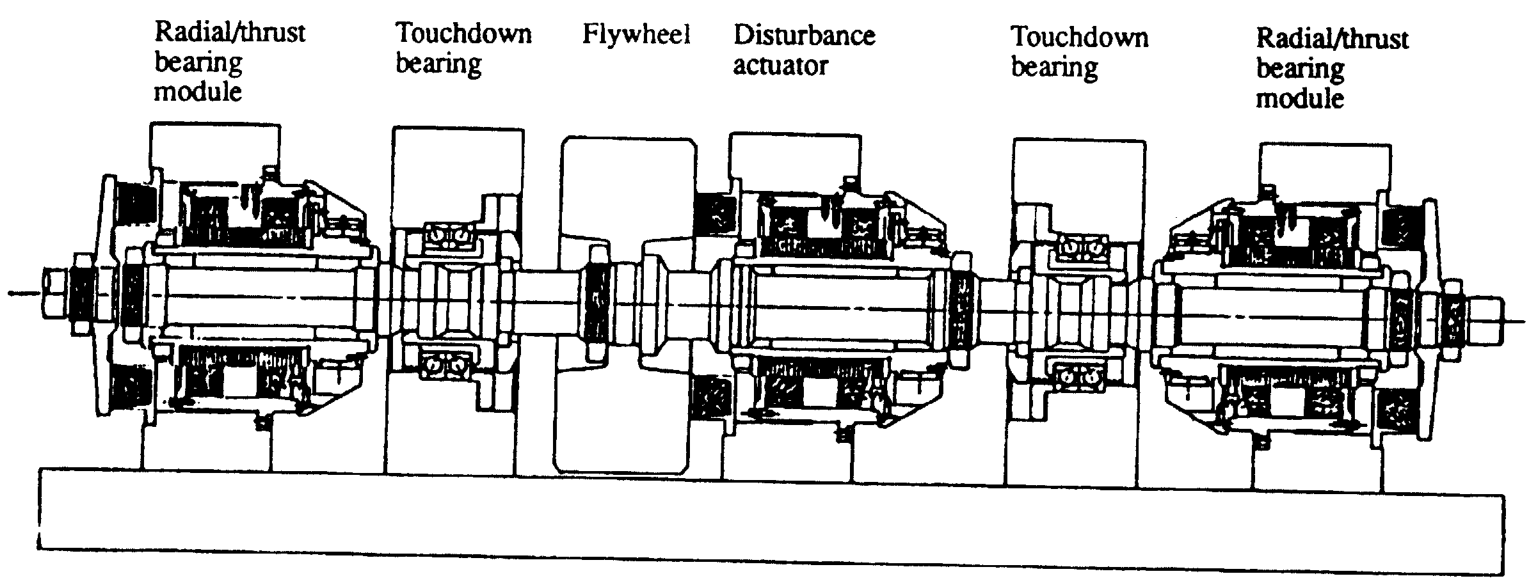
The remainder of this paper is composed of three sections and two appendices. A description of the lumped-element model is presented first. A discussion of the open-loop eigenvalues and transmission zeros, and their variation with shaft spin rate, is then presented. The paper concludes with a brief discussion of the implications of the system dynamics for control.

Linearized equations of motion for the model are presented in the appendix. A complete derivation of these equations, and a more thorough analysis of the dynamics, can be found in [McCallum 1988]. A bibliography also appears in an appendix.

Model Description

- The parameters used in the model are for a testbed designed at Draper Lab. The testbed employs two attractive-force, permanent magnet-biased electromagnetic journal bearings. A similar testbed is now under construction.
- The model includes only radial dynamics. Axial dynamics decouple (to first order).



Parameters used in the modelling effort are from a testbed designed at Draper Lab in 1987-1988. A testbed similar to the one shown above is now under construction.

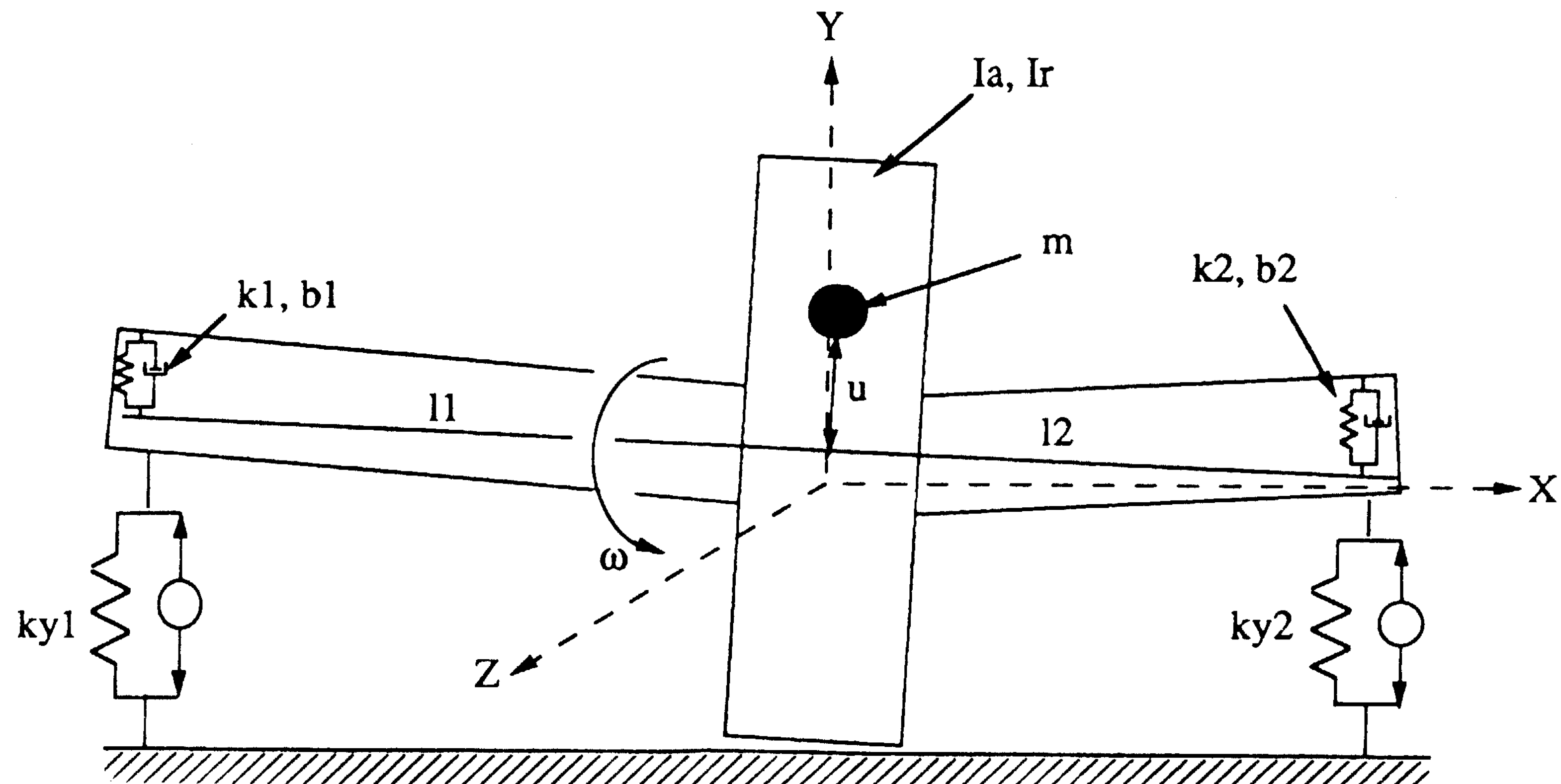
The testbed consists of a flexible rotor supported at its ends by two attractive-force, permanent magnet biased, electro-magnetic journal bearings. Inputs to the testbed are the bearing currents in each axis. Measurements of the shaft-end positions are used for control.

Although model parameters are from the Draper testbed, the model and results presented here can be generalized to other actively-controlled rotors.

The model presented here considers only radial dynamics (radial translation and rotation about shaft radii). Axial dynamics decouple from the radial dynamics to first order.

Model Description (cont.)

- The 12 state, lumped-element model includes radial translation of the flywheel and shaft ends and rotation of the flywheel about its radii.



The rotor is modelled as a rigid flywheel with radial mass unbalance mounted on a flexible, massless shaft with internal damping. In the model, the flywheel is allowed to translate in radial directions and rotate about its radii. Each end of the shaft is modelled by a rigid rod of appropriate length, with a parallel combination of a spring and a damper acting between the rod end and the shaft wall in each of two perpendicular radial directions. The shaft ends are allowed to move (independent of flywheel and of each other) in shaft radial directions.

As desired, this rotor model includes gyroscopic effects, first and second mode bending, the effects of static mass unbalance, and internal shaft damping. Note that, since the rigid rods are not necessarily of equal length, the effects of coupling between angular and translational dynamics are also included.

Assumptions used in modelling the rotor are -

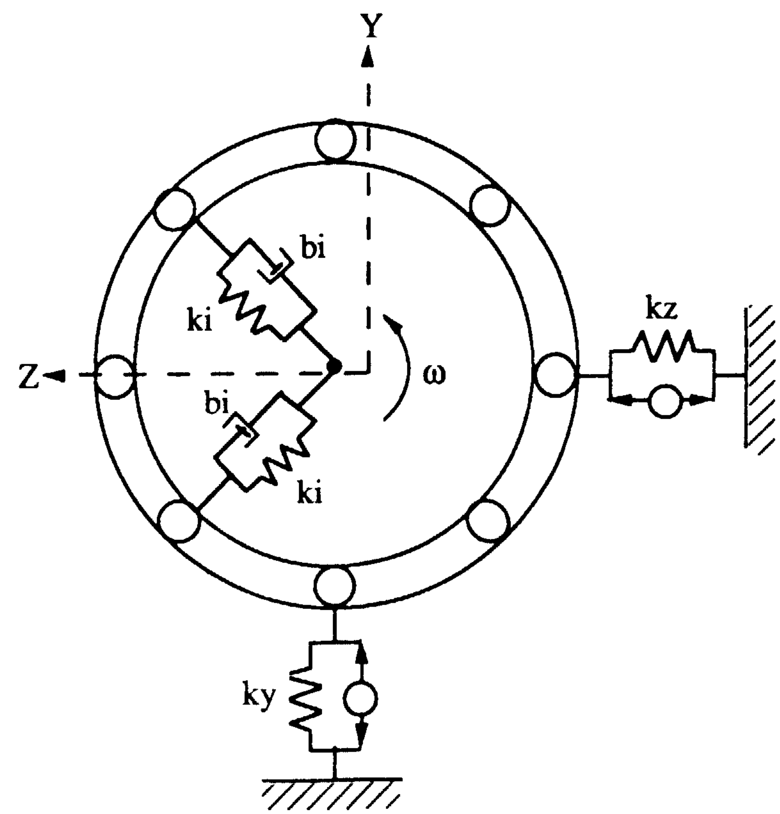
- the rigid flywheel is dynamically balanced
- the shaft ends are axisymmetric, so that stiffness and damping are the same in all radial directions
- shaft stress is linearly dependent on strain-rate (linear internal shaft damping)
- axial dynamics decouple from the radial dynamics

Dynamics of each bearing in each radial direction are modelled by a parallel combination of an unstable spring and a linear current to force actuator. Losses and high frequency roll-off are ignored in the bearing model. Past magnetic bearing designs have demonstrated input current to output force frequency responses that are flat to high frequencies [Traxler 1984, Ulbrich 1984, Maslen, 1988]. This bearing model has appeared often in the literature [ex. Downer 1986].

System inputs are the currents in coils for each axis of the two bearings. It is assumed that (only) shaft end positions at the bearing are available. The measurements and bearing forces are assumed to be in the same radial plane of the shaft - the effects of noncollocation are not considered here.

Model Description (cont.)

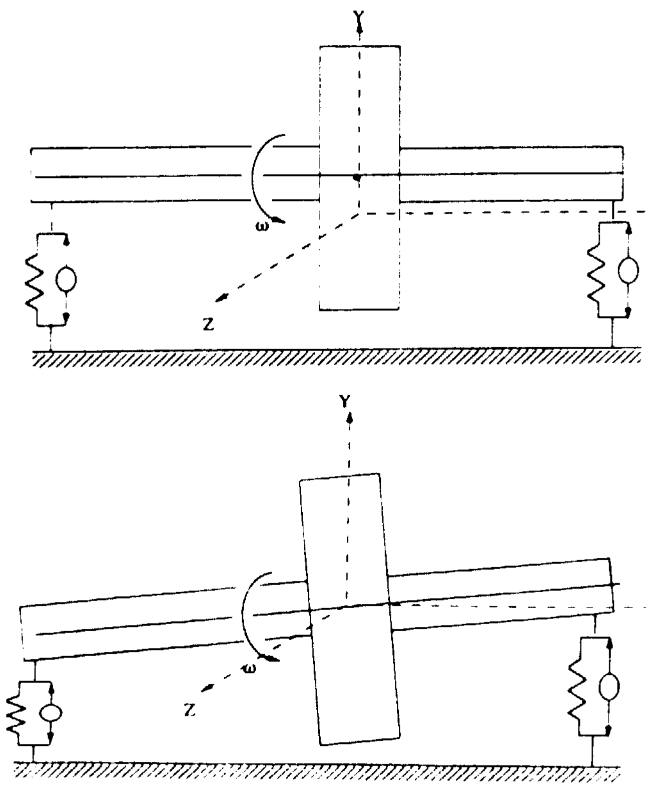
- The equivalent shaft springs and dampers rotate with the shaft (Jeffcott rotor). This is important for modelling flexible body modes, where modal damping varies with the spin rate and is dependent on the direction of whirl.

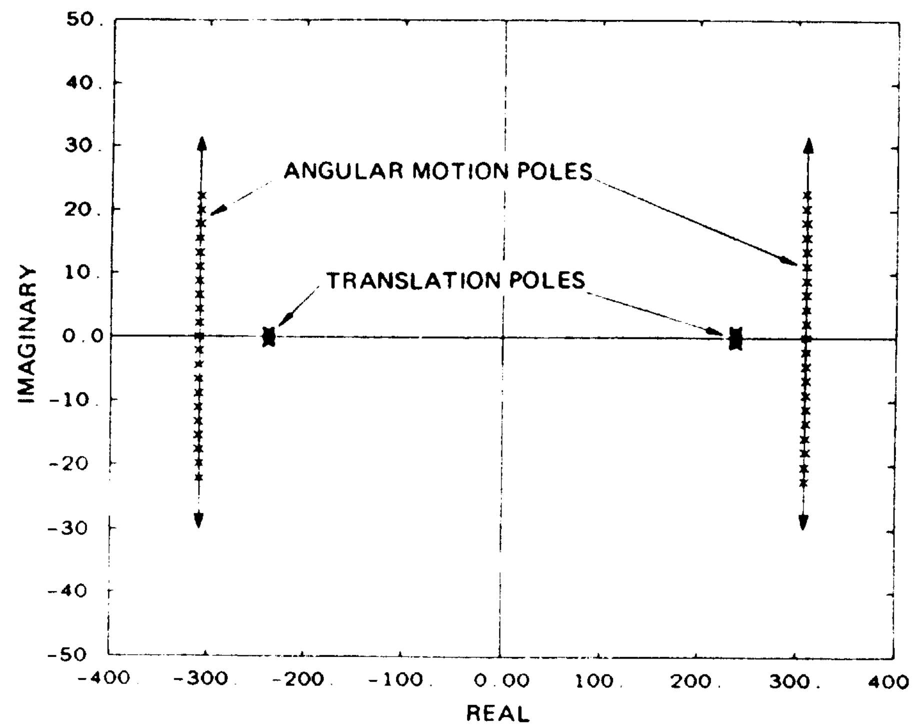


The figure above shows an end view of the model. Note that the shaft equivalent springs and dampers rotate with the shaft. This model is sometimes called the Jeffcott rotor in the literature [Johnson 1986]. Rotation of the dampers with the shaft is important for modelling shaft whirl modes, since the equivalent damping of forward and backward whirl modes is different for non-zero spin rates. In fact, the combination of shaft spin and internal damping can add energy to the system for supercritical spin rates.

Analysis - open-loop eigenvalues

- The model predicts 12 eigenvalues.
- 4 eigenvalue pairs correspond to rigid body rotation.





Linearized equations of motion are derived in detail in [McCallum 1988], and are presented in the appendix of this paper. The model that results for this system has 12 states, and predicts dynamics that are a function of shaft spin rate.

The linearized equations of motion were used to analyze the dependence of system eigenvalues (poles) and transmission zeros on system parameters (stiffness, damping, spin rate). Only the dependence on spin rate is discussed in this paper.

The model predicts 12 eigenvalues. 4 pair of eigenvalues correspond to rigid body motion of the suspended rotor.

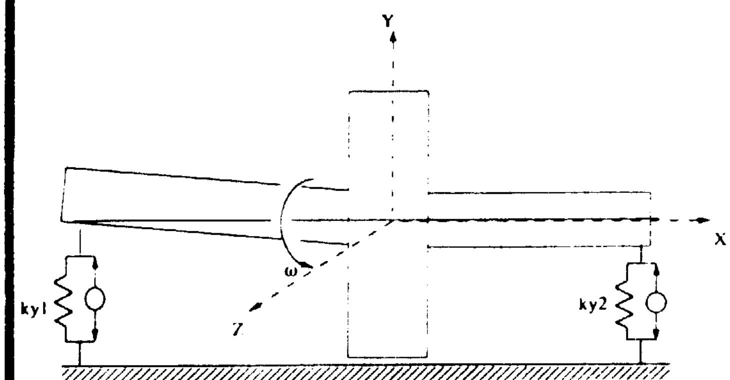
The first 4 rigid body poles correspond to angular motion of the flywheel. These poles occur in 2 stable/unstable pairs. Since the shaft is much stiffer then the bearings, the speed of these poles is dependent on the ratio of the equivalent rotational stiffness of the bearings to the radial moment of inertia of the flywheel. For zero spin rate, these poles lie on the real axis. As spin rate increases, the imaginary component of these poles increases due to gyroscopic coupling.

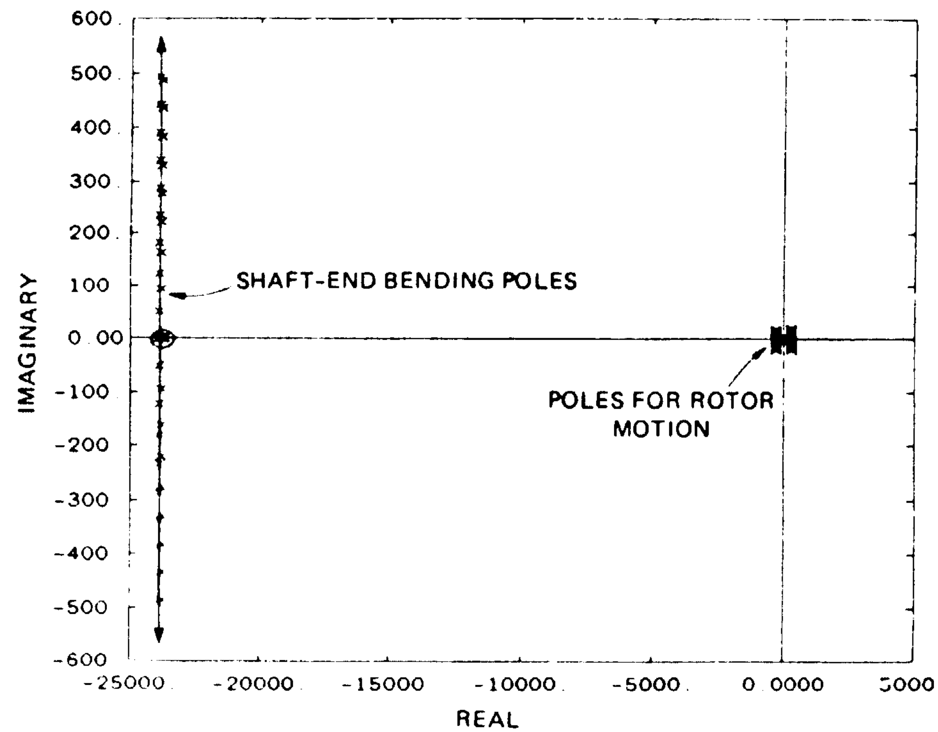
4 poles correspond to translational motion of the flywheel. These poles occur in 2 stable/unstable pairs, each pair corresponding to displacement of the flywheel in the horizontal or vertical direction. The speed of these poles is dependent on the ratio of the bearing (unstable) stiffness to the rotor mass. For zero spin rate, these poles lie on the real axis. As the spin rate increases, the translational poles have a small imaginary component due to coupling with the angular rotational mode. If the flywheel were in the shaft center, the translational motion poles would be independent of spin rate.

As the flywheel moves away from the center, coupling between the angular and translational open-loop modes increases. In these cases, all rigid body modes will be a combination of angular and translational motion.

Analysis - open-loop eigenvalues (cont.)

- 2 eigenvalue pairs correspond to shaft-end bending.

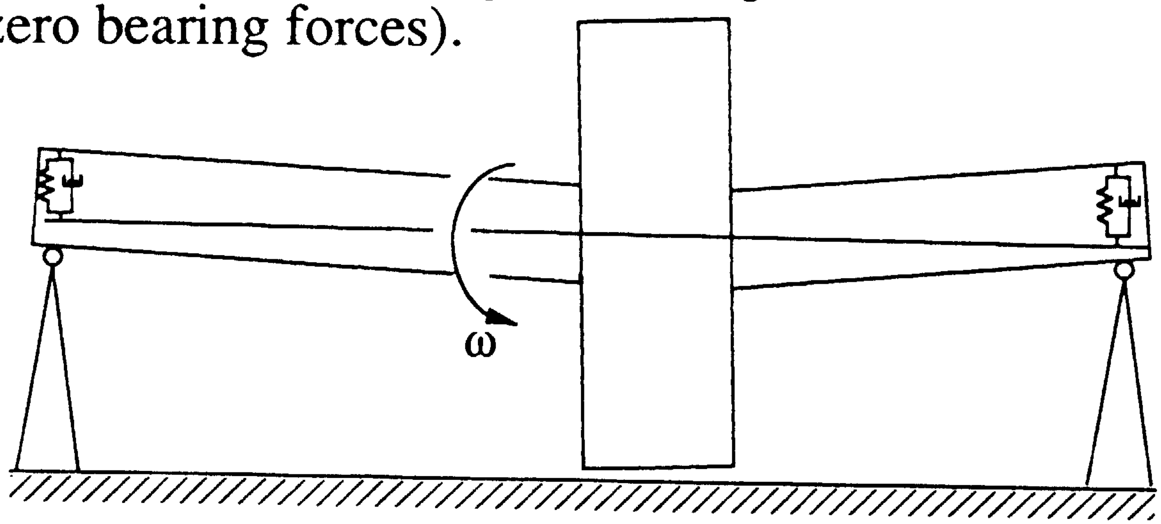




The 4 remaining poles correspond to shaft-end bending. These poles occur in pairs, each pair corresponding to bending of one shaft end. The speed of these poles is the ratio of the equivalent shaft end stiffness to the shaft end damping. For zero spin rate, these poles lie on the real axis. As spin rate increases, the shaft-end bending poles have an imaginary component equal to the shaft spin rate. This is because the projection of the shaft-end deflection in the lab frame changes as the shaft rotates.

Analysis - transmission zeros

- A transmission zero corresponds to a natural system motion where the output vector is zero for a non-zero input.
  - Outputs = shaft end positions
  - Inputs = coil currents, proportional to bearing force when shaft ends do not move
- Frequencies and (part of) directions of transmission zeros are the eigenvalues and eigenvectors the rotor would have if mounted in perfectly rigid bearings (no shaft end motion, non-zero bearing forces).



Transmission zeros occur when, for some initial condition, a natural motion of the system exists for which the output vector is zero for a non-zero input vector.

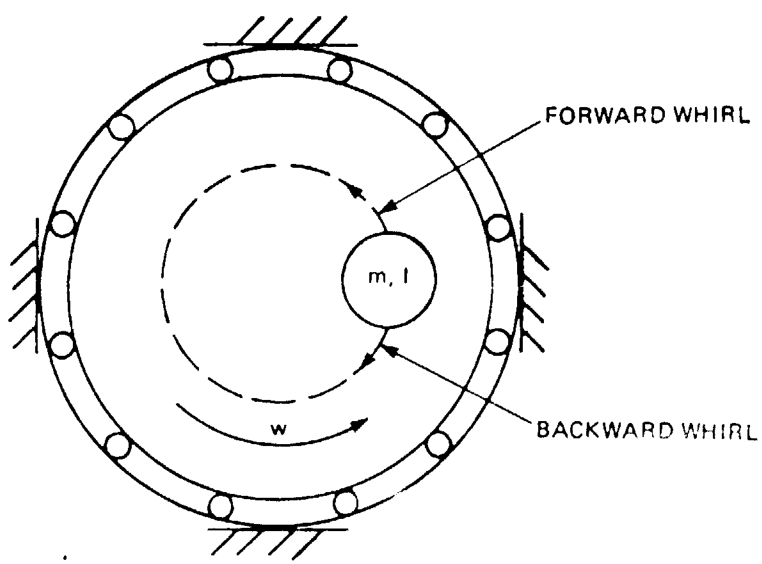
For our model, the outputs are the shaft-end positions and the inputs are the bearing currents. For a transmission zero to exist, the shaft-ends must be stationary. And if the shaft ends are stationary, bearing current is directly proportional to bearing force. Thus, for this model a transmission zero occurs if and only if there is a natural motion of the system where the shaft-ends remain fixed for non-zero bearing forces.

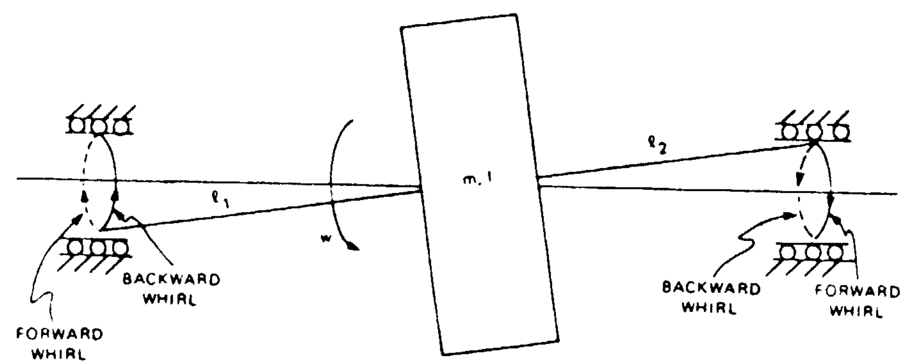
These natural motions correspond to motions the rotor would have if mounted in perfectly rigid bearings. Thus, the frequencies and (part of) the directions of transmission zeros are the same as the eigenvalues and eigenvectors of the rotor with both ends fixed.

The extensive literature on the dynamics of rotors in stiff bearings can be used to predict transmission zeros. Any rotor that is unstable in stiff bearings will have non-minimum phase transmission zeros.

Analysis - transmission zeros (cont.)

- The model predicts 4 pair of transmission zeros.
  - 2 pair correspond to first-mode bending (translational whirl)
  - 2 pair correspond to second-mode bending (rotational whirl)

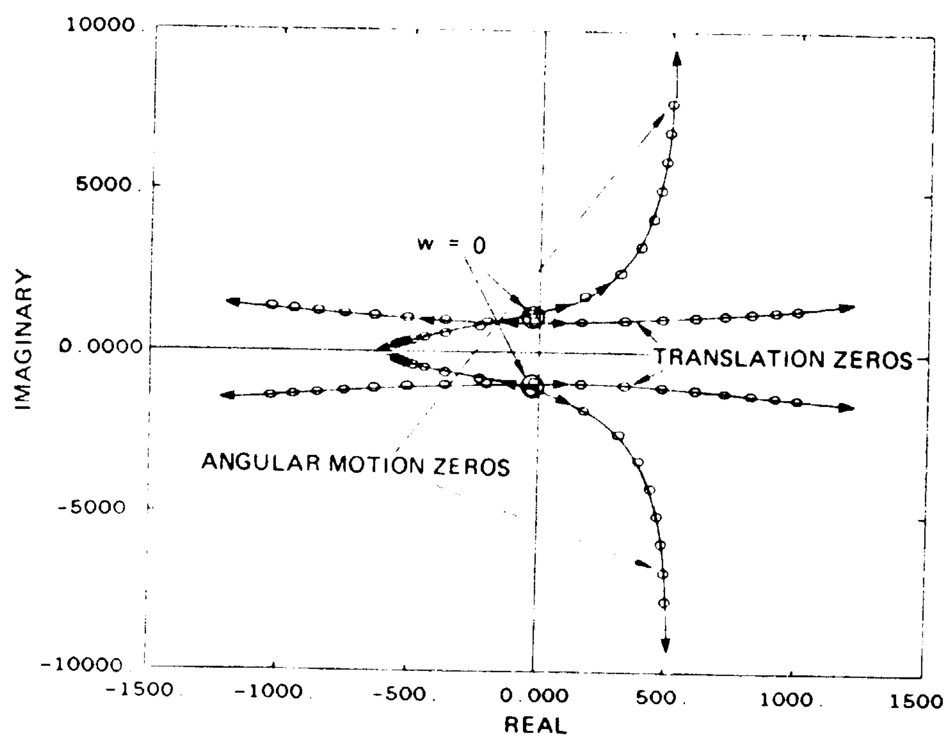




The model predicts 4 pair of transmission zeros. The first 2 pair correspond to translational whirl (first-mode bending). 1 pair corresponds to forward translational whirl. 1 pair corresponds to backward translational whirl. Forward whirl is defined as a circular rotor motion in the same direction as the rotor spin. Backward whirl is defined as circular rotor motion in the direction opposite the rotor spin. Forward and backward translational whirl are illustrated in above.

The second 2 pair of transmission zeros correspond to rotational whirl (second-mode bending). Again, 1 pair corresponds to forward whirl, the other pair correspond to backward whirl. For the rotational whirl case, forward whirl corresponds to motions where the ends of the equivalent rigid rods move in circles in the same direction as the shaft spin. Backward whirl is the opposite motion.

Analysis - transmission zeros (cont.)



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The above figure shows the transmission zeros as a function of spin rate.

The translational motion transmission zeros have a frequency equal to the first critical speed. The frequency of these zeros is only a weak function of rotational speed. However, the damping of these zeros is a strong function of spin rate. Damping of the backward whirl zeros increases with spin rate, while the damping of the forward whirl zeros decreases with spin rate. The forward translational whirl zeros become non-minimum phase for all spin rates exceeding the first critical speed (the translational resonance frequency).

At zero speed, the rotational motion transmission zeros has a frequency equal to the second critical speed. The frequency of these zeros is a strong function of spin rate, with forward whirl zeros asymptotically approaching the gyroscopic whirl frequency. The damping of the forward whirl zeros is a decreasing function of rotational speed - for spin rates exceeding the second critical speed, the forward whirl zeros are non-minimum phase. The damping of the backward rotational whirl zeros increases with spin rate, with the frequency of these zeros asymptotically approaching zero.

Conclusions - implications for control

- The minimum loop cross-over frequency of the closed-loop system is the speed of the unstable open-loop poles.
- When the spin rate reaches a critical speed, a complex-conjugate zero pair will become non-minmum phase, contributing 180 deg. of phase lag (rather than phase lead). This transition plane explains stability problems encountered when spinning through shaft critical speeds.
- Non-minimum phase zeros limit the achievable closed-loop disturbance rejection at frequencies near or above the shaft critical speeds.
- Zeros can only be changed by changing the system inputs or outputs, i.e. adding additional actuators or sensors. *Limitations imposed by n.m.p. zeros are physical limitations that cannot be overcome through feedback.*

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The open-loop system dynamics impact the design and achievable performance of magnetic bearing control systems.

The minimum loop cross-over frequency of the closed-loop system is the speed of the unstable open-loop poles associated with rigid body motion. This result has appeared in the literature [Groom 1979, Downer 1986].

For spin rates at or above the rotors critical speed, the transmission zeros become non-minimum phase. As a result, the achievable disturbance rejection of the closed-loop system is limited at frequencies near or above the shaft critical speeds (the frequencies of the zeros). In addition, when the transmission zeros become non-minimum phase, each complex conjugate pair contributes 180 deg. of phase lag rather than phase lead. This transition explains stability problems encountered when spinning a magnetically suspended rotor through its critical speeds (as for systems with notch filters).

Non-minimum phase zeros cannot be changed through active control. Canceling right-half plane zeros with compensator poles results in a system that it is internally unstable. Zeros can only be changed by changing the system inputs or ouputs, i.e. by adding additional actuators or sensors. The limitations imposed by non-minimum phase zeros are physical limitations that cannot be overcome through feedback.

Appendix A - Equations of Motion

This appendix presents linearized equations of motion for a magnetic bearing - suspended rotor system. Included are a summary of the assumptions used in deriving the equations, descriptions of reference frames, and a list of symbols. A detailed derivation of the equations of motion can be found in [McCallum 1988].

A.1 Summary of assumptions

The magnetically-suspended rotor is modelled as a rigid flywheel with radial mass unbalance mounted on a flexible, massless shaft with internal damping. In the model, the flywheel is allowed to translate in radial directions and rotate about its radii. The shaft ends are allowed to move (independent of the flywheel and of each other) in radial directions. The mass is not assumed to be at the midspan of the shaft, so that the forces at the shaft ends are not equal. The rotor model includes: gyroscopic effects; the first and second bending modes; the effects of internal shaft damping; and differences in bearing loads. The rotor model excludes the effects of axial torque and axial translation. Further, the shaft is assumed to be axially symmetric so that the shaft stiffness and damping are the same in all radial directions. Also, the rotor is assumed to be configured such that the spin axis is aligned with the axial principal axis of the rotor (i.e., the rotor is dynamically balanced) and the radial moment of inertia is the same in all directions.

The magnetic bearings are modelled as a parallel combination of an unstable spring and a linear current-force actuator in each direction[1], i.e.,

$$Fyi = Kyi\ Yi + Ki\ \delta i_{Y1} \qquad (A.1.1)$$

Bearing losses and high frequency dynamics are ignored.

It is assumed that (only) measurements of the shaft end positions are available. Sensor dynamics are excluded. In addition, it is assumed that the bearing forces and the sensor measurements are at the same points on the shaft - non-colocation effects are ignored. Justifications for the assumptions listed here can be found in [McCallum 1988].

[1]This model appears often in the literature.

A.2 Reference frames

The system state is described by the position and velocity of the flywheel's geometric center, the positions of the shaft ends, and rotations of the flywheel about its radii. Positions and velocities are described in the cartesian reference frame XYZ shown in Figure A.1. This frame is fixed to "ground" with its origin at the force-free equilibrium position of the flywheel's geometric center. The X-axis is horizontal and coincident with the centerline of the shaft when the system is in force-free equilibrium. The Y-axis of the frame is vertical. The Z-axis is horizontal and coincident with a radius of the flywheel at equilbrium. Figure A.1 also shows the frames xyz and 123, which are used in [McCallum 1988] to derive the equations of motion. The rotational motions of the flywheel are described in terms of the gyroscopic coordinates illustrated in Figures A.2 and A.3.

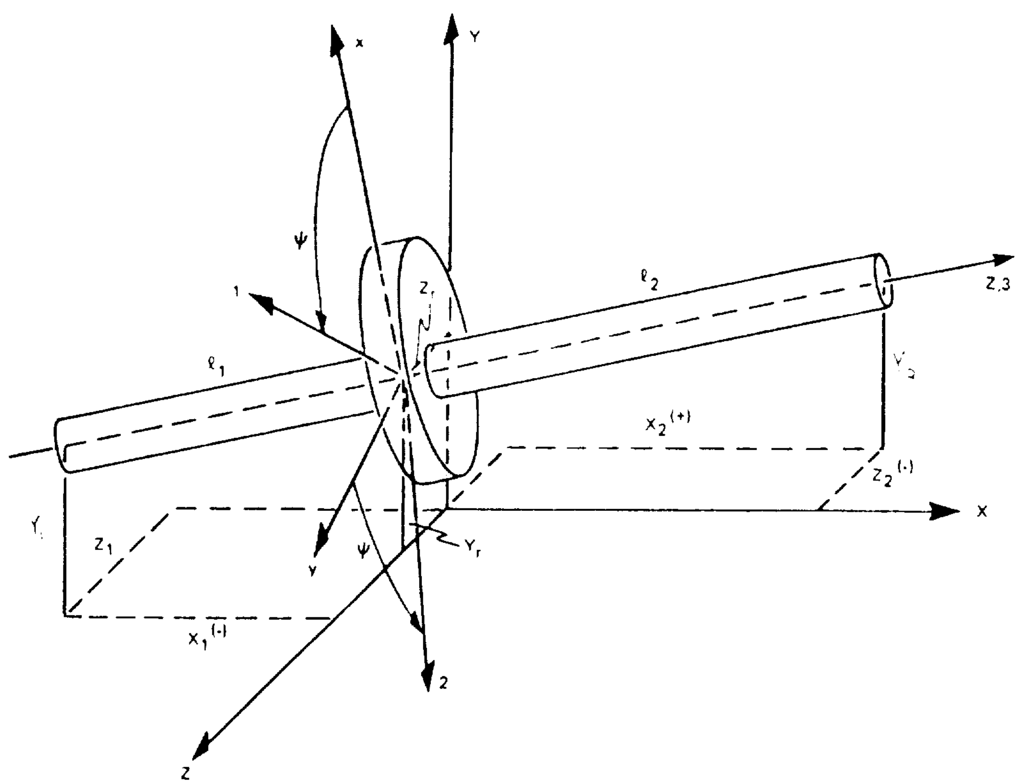

Figure A.1 - Reference frames

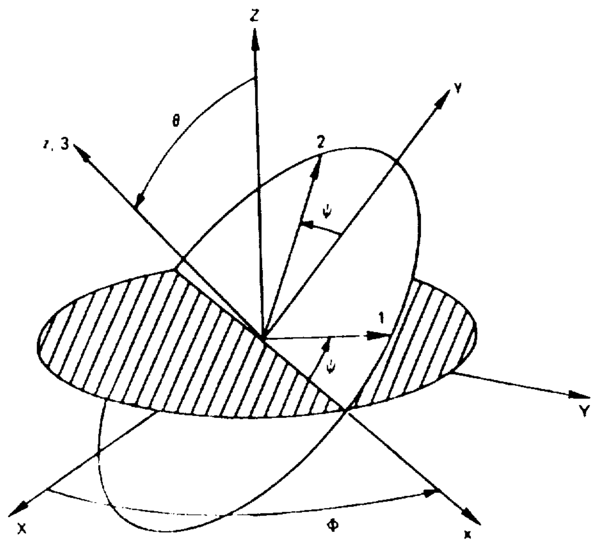

Figure A.2 - Euler angles

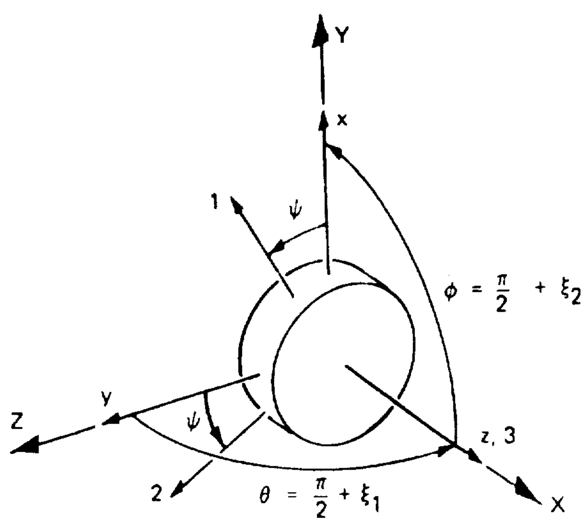

Figure A.3 - Gyroscopic coordinates

A.3 Equations of Motion

The system states (**x**) are chosen as the flywheel translational and rotational positions and velocities and the positions of the shaft ends

$$\mathbf{x} = \left[Y_r \ \dot{Y}_r \ Z_r \ \dot{Z}_r \ \xi_1 \ \dot{\xi}_1 \ \xi_2 \ \dot{\xi}_2 \ Y_1 \ Z_1 \ Y_2 \ Z_2 \right]^T \tag{A.3.1}$$

System inputs are the changes in bearing coil currents from their equilibrium values

$$\mathbf{u} = [\, i_{Y1} \;\; i_{Z1} \;\; i_{Y2} \;\; i_{Z2} \,]^T \tag{A.3.2}$$

System outputs are the positions of the shaft ends

$$\mathbf{y} = [\, Y_1 \; Z_1 \; Y_2 \; Z_2 \,]^T \tag{A.3.3}$$

Linearized equations of motion for the rotor-bearing system can be written in state-space form as:

$$\dot{\mathbf{x}} = \mathbf{A}(\omega)\,\mathbf{x} + \mathbf{B}\;\mathbf{u} + \mathbf{d} \tag{A.3.4}$$

$$\mathbf{y} = \mathbf{C}\;\mathbf{x} + \mathbf{D}\;\mathbf{u} + \mathbf{n} \tag{A.3.5}$$

where

$$\mathbf{d} = \begin{bmatrix} 0 \\ u\,\dot{\omega}\,\mathrm{Sin}\,(\omega t + \theta_0) + u\,\omega^2\,\mathrm{Cos}\,(\omega t + \theta_0) - g + \dfrac{dY_r}{M} \\ 0 \\ -\,u\,\dot{\omega}\,\mathrm{Cos}\,(\omega t + \theta_0) + u\,\omega^2\,\mathrm{Sin}\,(\omega t + \theta_0) + \dfrac{dZ_r}{M} \\ 0 \\ \dfrac{d\xi_1}{Ir} \\ 0 \\ \dfrac{d\xi_2}{Ir} \\ \dfrac{dY_1}{b1} \\ \dfrac{dZ_1}{b1} \\ \dfrac{dY_2}{b2} \\ \dfrac{dZ_2}{b2} \end{bmatrix} \tag{A.3.6}$$

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{Ky1}{M} & 0 & \frac{Ky2}{M} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{Kz1}{M} & 0 & \frac{Kz2}{M} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\omega Ia}{Ir} & 0 & \frac{Kz1\ l1}{M} & 0 & -\frac{Kz2\ l2}{M} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\omega Ia}{Ir} & 0 & 0 & -\frac{Ky1\ l1}{M} & 0 & \frac{Ky2\ l2}{M} & 0 \\
\frac{k1}{b1} & 1 & \omega & 0 & l1\ \omega & 0 & -\frac{k1\ l1}{b1} & -l1 & \frac{Ky1 - k1}{b1} & -\omega & 0 & 0 \\
-\omega & 0 & \frac{k1}{b1} & 1 & \frac{k1\ l1}{b1} & l1 & l1\ \omega & 0 & \omega & \frac{Kz1 - k1}{b1} & 0 & 0 \\
\frac{k2}{b2} & 1 & \omega & 0 & -l2\ \omega & 0 & \frac{k2\ l2}{b2} & l2 & 0 & 0 & \frac{Ky2 - k2}{b2} & -\omega \\
-\omega & 0 & \frac{k2}{b2} & 1 & -\frac{k2\ l2}{b2} & -l2 & -l2\ \omega & 0 & 0 & 0 & \omega & \frac{Kz2 - k2}{b2}
\end{bmatrix}
\tag{A.3.7}
$$

$$
B = \begin{bmatrix}
0 & 0 & 0 & 0 \\
\frac{Ki}{M} & 0 & \frac{Ki}{M} & 0 \\
0 & 0 & 0 & 0 \\
0 & \frac{Ki}{M} & 0 & \frac{Ki}{M} \\
0 & 0 & 0 & 0 \\
0 & \frac{Ki\ l1}{Ir} & 0 & -\frac{Ki\ l2}{Ir} \\
0 & 0 & 0 & 0 \\
-\frac{Ki\ l1}{Ir} & 0 & \frac{Ki\ l2}{Ir} & 0 \\
\frac{Ki}{b1} & 0 & 0 & 0 \\
0 & \frac{Ki}{b1} & 0 & 0 \\
0 & 0 & \frac{Ki}{b2} & 0 \\
0 & 0 & 0 & \frac{Ki}{b2}
\end{bmatrix}
\tag{A.3.8}
$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.3.9}$$

$$\mathbf{D} = \left[0_{(4\times4)}\right] \tag{A.3.10}$$

Note that, in this representation, radial mass unbalance is modelled as an input force disturbance. The effects of dynamic balance could be included similarly. The effects of measurement center offset could be added to this model as additive measurement noise.

A.4 List of Symbols

b_i = equivalent damping of shaft end i

$\mathbf{d}$[2] = vector of input disturbances

$d\xi_1$ = torque disturbance acting on the rotor in the ξ_1-direction

$d\xi_2$ = torque disturbance acting on the rotor in the ξ_2-direction

dY_i = force disturbance acting on shaft end i in the Y-direction

dYr = force disturbance acting on the rotor in the Y-direction

dZ_i = force disturbance acting on shaft end i in the Z-direction

dZr = force disturbance acting on the rotor in the Z-direction

Ia = axial moment of inertia

Ir = radial moment of inertia

i_{Yi} = current in the Y-coil(s) at shaft end i

i_{Zi} = current in the Z-coil(s) at shaft end i

k_i = equivalent stiffness of shaft end i

Kyi = equivalent (unstable) stiffness of bearing at shaft end i in the Y-direction

Kzi = equivalent (unstable) stiffness of bearing at shaft end i in the Z-direction

li = equivalent length of shaft end i

M = rotor mass

n = sensor noise

u = control inputs

u = radial mass unbalance distance

ω = shaft spin rate

[2]**Bold** denotes a vector.

$\mathbf{x}$ = state vector

ξ_1 = gyroscopic coordinate, approximately equal to rotation about the +Y-axis

ξ_2 = gyroscopic coordinate, approximately equal to rotation about the +Z-axis

$\mathbf{y}$ = system outputs

Y_i = displacement of shaft end i in the Y-direction

Y_r = displacement of the rigid rotor in the Y-direction

Z_i = displacement of shaft end i in the Z-direction

Z_r = rotor displacement in the Z-direction

t = time

θ_0 = orientation of the radial unbalance vector with respect to the +1 axis

θ, ϕ, ψ = Euler angles describing rotor orientation

Appendix B - Bibliography

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