Dynamic Modelling of a Wind Turbine with Doubly Fed Induction Generator

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Abstract-As a result of increasing environmental concern, more and more electricity is generated from renewable sources. One way of generating electricity from renewable sources is to use wind turbines. A tendency to erect more and more wind turbines can be observed. As a result of this, in the near future wind turbines may start to influence the behaviour of electrical power systems. Therefore, adequate models to study the impact of wind turbines on electrical power system behaviour are needed.
In this paper, a dynamic model of an important contemporary wind turbine concept is presented, namely a doubly fed (wound rotor) induction generator with a voltage source converter feeding the rotor. This wind turbine concept is equipped with rotor speed, pitch angle and terminal voltage controllers. After derivation of the model, the wind turbine response to two measured wind sequences is simulated.

Keywords: variable speed operation, wind turbine, modelling, simulation, doubly fed induction generator, voltage source converter, grid interaction, voltage control

I. INTRODUCTION

As a result of increasing environmental concern, the impact of conventional electricity generation on the environment is being minimized and efforts are made to generate electricity from renewable sources. The main advantages of electricity generation from renewable sources are the absence of harmful emissions and the in principle infinite availability of the prime mover that is converted into electricity. One way of generating electricity from renewable sources is to use wind turbines that convert the energy contained in flowing air into electricity. Up to this moment, the amount of wind power integrated into large scale electrical power systems only covers a small part of the total power system load. The rest of the power system load is for the largest part covered by conventional thermal, nuclear and hydro power plants.

Wind turbines often do not take part in voltage and frequency control and if a disturbance occurs, the wind turbines are disconnected and reconnected when normal operation has been resumed. Thus, notwithstanding the presence of wind turbines, frequency and voltage are maintained by controlling the large power plants as would have been the case without any wind turbines present. This is possible, as long as wind power penetration is still low. However, a tendency to increase the amount of electricity generated from wind can be observed. Therefore, the penetration of wind turbines in electrical power systems will increase, they may begin to influence overall power system behaviour and it will no longer be possible to run a power system by only controlling large scale power plants. It is therefore important to study the behaviour of wind turbines in an electrical power system and their interaction with other generation equipment and with loads.

In this paper a dynamic model of a variable speed (VS) wind turbine (WT) with doubly fed (wound rotor) induction generator (DFIG) and back to back voltage source converter (BVSC) and its controls is presented. Speed control, pitch control and voltage control are included in the model. The model is suitable for integration in a large scale power systems simulation software package, thus facilitating the investigation of the impact of large amounts of wind turbines on the behaviour of a large scale electrical power system.

The paper is organized as follows. First, the system to be modelled is described. Then, the equations describing the behaviour of the various subsystems are derived and the controllers are described. To conclude, the system response to two measured wind speed sequences is investigated.

II. SYSTEM DESCRIPTION

The core of a WT consists of a rotor that extracts energy from the wind and converts it into mechanical power and a generator that converts mechanical power into electrical power. In most systems, the rotor shaft and the generator shaft are coupled through a gearbox, because there is a difference between the optimal rotor and generator speed ranges. However, also direct drive VS WT exist, in which the rotor is coupled directly to the generator. In most systems, the generator is coupled to the grid through a transformer and/or a power electronic converter, because the characteristics of the generator output do not match the characteristics of the grid with respect to frequency and voltage. Furthermore, controllers and protection systems are part of modern WT. More information can be found in the documentation provided by WT manufacturers and in textbooks containing a more detailed description of modern WT and their various subsystems [1-3].

In this paper, one kind of WT is studied, namely a VS WT with DFIG. In this kind of WT, the mechanical power generated by the rotor is converted into electrical power using a DFIG. The stator winding of the generator is coupled directly to the grid. The rotor winding is connected to a BVSC. The system is depicted in figure 1.

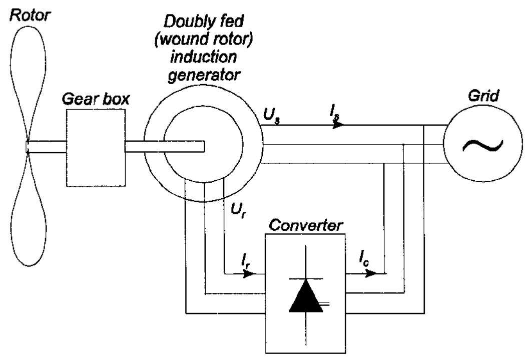

Figure 1. Doubly fed (wound rotor) induction generator with back to back voltage source converter feeding the rotor winding

The DFIG has been used in wind turbines for a long time. In the past, the converter connected to the rotor consisted of a rectifier and inverter based on thyristor bridges [4]. However, this technology is becoming outdated for the power range in which modern WT fall. Nowadays, normally a BVSC is used [5]. This has advantages with respect to speed control and enables voltage control [1].

The system is equipped with a number of controllers, namely:

- speed controller
- pitch angle controller
- terminal voltage controller

The speed controller influences the speed of the rotor by controlling the generator electrical torque according to a speed versus power control characteristic. The controller samples the generator speed and the generator torque set point is adjusted in accordance with the speed control characteristic.

The pitch angle controller controls the rotor speed as well. However, the pitch angle controller becomes operational only if the speed controller can not control the rotor speed anymore, which is the case in high wind speeds. Controlling the rotor speed in high wind speeds by increasing the generator torque would lead to overloading the rotor converter and the generator. In these circumstances, not all energy the WT could extract from the wind can be used. Instead, the rotor blades are pitched in order to decrease the power extracted from the wind.

Terminal voltage control is a feature that is not available on most commercial turbines yet. Older constant speed (CS) WT do not offer possibilities for voltage control. These WT use a squirrel cage induction generator that is directly coupled to the grid. In larger WT, the reactive power consumed by the squirrel cage induction generator is compensated by capacitors, whose size is determined assuming that the WT generates nominal power. However, if the WT generates less than nominal power, the size of the capacitors can often not be changed and no terminal voltage control is possible. On the other hand, a WT equipped with DFIG enables terminal voltage control. Nowadays, however, most VS WT with DFIG are operated at a constant power factor and do not control the grid voltage actively. Because voltage control will become more important when more WT are integrated in the electrical power system, it is considered appropriate to incorporate a voltage controller in the model presented here.

III. SYSTEM EQUATIONS

A. Assumptions

In this paragraph, the equations describing the subsystems of a VS WT with DFIG and BVSC as depicted in figure 1 will be developed. The equations for the rotor, the generator and the converter will be given here. The equations have been developed using the following assumptions:

- All rotating mass is represented by one element, the so-called 'lumped-mass' representation. Elastic shafts and resulting torsional forces are neglected.
- A quasi static approach is used for the description of aerodynamic part of the WT.
- Magnetic saturation in the DFIG is neglected.
- Flux distribution is sinusoidal.
- Dynamic phenomena in the BVSC are neglected.

These assumptions reduce the complexity of the modelling task and the amount of system data that is needed. As reliable data are often hard to obtain, this is considered an important advantage. Furthermore, under these assumptions the computation speed can be increased, which is also considered an advantage, particularly when large systems are to be simulated.

B. Rotor equations

The rotor converts the energy contained by the wind into mechanical energy. The following well known equation between wind speed and power extracted from the wind holds [1-3]

$$P_w = \frac{\rho}{2} c_p(\lambda,\theta) A_R v_w^3 \tag{1}$$

with P_w the power extracted from the airflow [W], ρ the air density [kg/m^3], c_p the performance coefficient or power coefficient, λ the tip speed ratio v_t/v_w, the ratio between blade tip speed v_t and wind speed upstream the rotor v_w [m/s], θ the pitch angle of rotor blades [deg], and A_r the area covered by the rotor [m^2].

Now, the performance coefficient c_p that is a function of the tip speed ratio λ and the pitch angle θ will be investigated further. The calculation of the performance coefficient requires the use of blade element theory [1, 2]. As this requires knowledge of aerodynamics and the computations are rather complicated, numerical approximations have been developed [1]. Here the following function will be used

$$c_p(\lambda,\theta) = 0.22\left(\frac{116}{\lambda_i} - 0.4\theta - 5\right)e^{\frac{-12.5}{\lambda_i}} \tag{2}$$

with

$$\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08\theta} - \frac{0.035}{\theta^3 + 1} \tag{3}$$

This leads to the $c_p(\lambda,\theta)$ versus λ characteristics for various values of θ as depicted in figure 2. Using the actual values of the wind and rotor speed, which determine λ, and the pitch angle, the mechanical power extracted from the wind can be calculated from equations (1) to (3).

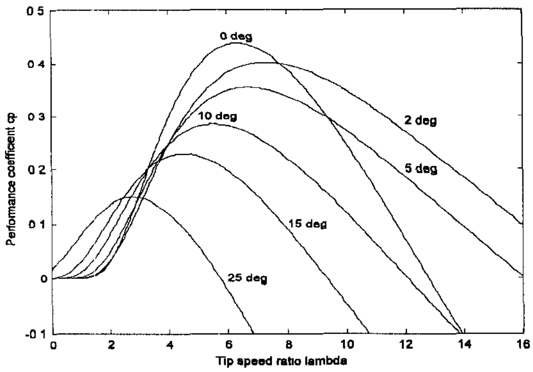

Figure 2 Performance coefficient c_p as a function of tip speed ratio λ with pitch angle θ as a parameter

C. Generator equations

The equations describing a doubly fed induction machine can be found in literature [6, 7]. However, note that in figure 2 both stator and rotor current are outputs. When modelling the DFIG, the generator convention will be used, which means that the currents are outputs instead of inputs and real power and reactive power have a positive sign when they are fed into the grid. Using the generator convention, the following set of equations results,

$$\begin{aligned} v_{ds} &= -R_s i_{ds} - \omega_s \psi_{qs} + \frac{d\psi_{ds}}{dt} \\ v_{qs} &= -R_s i_{qs} + \omega_s \psi_{ds} + \frac{d\psi_{qs}}{dt} \\ v_{dr} &= -R_r i_{dr} - s\omega_s \psi_{qr} + \frac{d\psi_{dr}}{dt} \\ v_{qs} &= -R_r i_{qr} + s\omega_s \psi_{dr} + \frac{d\psi_{qr}}{dt} \end{aligned} \tag{4}$$

with v the voltage [V], R the resistance [Ω], i the current [A], ω_s the stator electrical frequency [rad/s], ψ the flux linkage [Vs] and s the rotor slip.

In (4) the indices d and q indicate the direct and quadrature axis components and s and r indicate stator and rotor quantities. All quantities in (4) are functions of time. The d-q reference frame is rotating at synchronous speed with the q-axis 90° ahead of the d-axis. The position of the d-axis coincides with the maximum of the stator flux, which means that v_{qs} equals the terminal voltage e_t and v_{ds} equals zero. The flux linkages in (4) can be calculated using the following set of equations in per unit [6]

$$\begin{aligned} \psi_{ds} &= -(L_s + L_m) i_{ds} - L_m i_{dr} \\ \psi_{qs} &= -(L_s + L_m) i_{qs} - L_m i_{qr} \\ \psi_{dr} &= -(L_r + L_m) i_{dr} - L_m i_{ds} \\ \psi_{qr} &= -(L_r + L_m) i_{qr} - L_m i_{qs} \end{aligned} \tag{5}$$

with L_m the mutual inductance and L_s and L_r the stator and rotor leakage inductance respectively. In (5) the generator convention is used again. The rotor slip s is defined as [6]

$$s = \frac{\omega_s - \frac{p}{2}\omega_m}{\omega_s} \tag{6}$$

in which p is the number of poles and ω_m is the mechanical frequency of the generator [rad/s].

From (4) and (5) the voltage current relationships of the DFIG can be derived. In doing this, the rotor and stator transients, represented by the last terms in equation (4) are neglected. The reasons for this are:

- In power systems simulation software, the network is modelled by an admittance matrix. Transients are neglected to increase the computation speed. To get a consistent set of equations, the stator transients must be neglected as well [4].
- Taking into account the rotor transients would require detailed modelling of the converter, which is considered beyond the scope of this paper. Instead, the converter is modelled as a controllable current source.
- Taking into account rotor and/or stator transients would require a much smaller time step than is required when neglecting these transients.

A more complex model of the system studied, taking into acount the $d\psi/dt$ terms of (4), can be found in [8].

Keeping the above in mind, the following voltage current relationship of the DFIG can be derived from (4) and (5)

$$\begin{aligned} v_{ds} &= -R_s i_{ds} + \omega_s((L_s + L_m) i_{qs} + L_m i_{qr}) \\ v_{qs} &= -R_s i_{qs} - \omega_s((L_s + L_m) i_{ds} + L_m i_{dr}) \\ v_{dr} &= -R_r i_{dr} + s\omega_s((L_r + L_m) i_{qr} + L_m i_{qs}) \\ v_{qr} &= -R_r i_{qr} - s\omega_s((L_r + L_m) i_{dr} + L_m i_{ds}) \end{aligned} \tag{7}$$

Equations (1) to (3) and equation (7) are linked by the equations giving the active power P and reactive power Q generated by the DFIG [6]

$$\begin{aligned} P &= v_{ds} i_{ds} + v_{qs} i_{qs} + v_{dr} i_{dr} + v_{qr} i_{qr} \\ Q &= v_{qs} i_{ds} - v_{ds} i_{qs} + v_{qr} i_{dr} - v_{dr} i_{qr} \end{aligned} \tag{8}$$

Equations (7) and (8) describe the electrical part of a DFIG. However, also the mechanical part should be taken into account in developing a dynamic model. The following equation gives the electro mechanical torque generated by the DFIG [4,5]

$$T_e = \psi_{dr} i_{qr} - \psi_{qr} i_{dr} \tag{9}$$

The mechanical torque can be calculated by dividing the power extracted from the wind that results from equations (1) to (3) by the mechanical generator frequency ω_m. The changes in generator speed that result from a difference in electrical and mechanical torque can be calculated using the generator equation of motion

$$\frac{d\omega_m}{dt} = \frac{1}{2H}(T_m - T_e) \tag{10}$$

in which H is the inertia constant [s] and T_m is the mechanical torque.

IV. CONTROLLERS

A. Speed controller

VS operation of wind turbines has a number of advantages, namely [2, 9, 10]

- Substantial reduction of torque ripple in the wind turbine drive train and therefore a better quality of output power.
- Attenuation of torsional mode resonances and mechanical stresses.
- Reduced noise emission, mainly at low speed.
- Increased energy capture in a large range of wind speeds because of the ability to operate at a rotational speed that maximizes WT efficiency.

In the speed controller used in the model presented here, the last consideration will be used to develop the speed control characteristic. In figure 3, the power versus rotor speed characteristic that results in maximal energy capture is depicted [9]. First, the rotor speed is kept at its minimum. Then, the rotor speed is proportional to the wind speed and thus with the cubic root of the power, according to equation (1). When the rotor speed reaches its maximum value, it is kept at its maximum.

Controlling the power according to this speed characteristic, however, causes some problems, because the desired power is not uniquely defined at maximum and minimum rotor speed and because if the rotor speed decreases from slightly above nominal speed to slightly below nominal speed or from slightly above minimal speed to slightly below minimal speed, the change in generated power is very large. This leads to large power fluctuations when the rotor speed is around its nominal or minimal value. To solve these problems, a control characteristic that is similar to the characteristic that leads to optimal energy capture but solves the problems associated with it will be used here. This control characteristic is also depicted in figure 3.

The speed controller is controlling the electro mechanical torque. The reason for not controlling the power, but the torque, is that the torque is directly dependent on the quadrature component of the rotor current, when stator resistance is neglected [1]. From equations (5), (7) and (9) it can be derived, that the following relation between torque and i_{qr} holds, in which e_t is the terminal voltage

$$T_e = -\frac{L_m e_t}{L_s + L_m} i_{qr} \tag{11}$$

The rotor speed controller is implemented as follows:

- Every 0.05 s, the actual rotor speed measured.
- From this value, the set point for generated power is derived using the control characteristic.
- The set point for the electro mechanical torque is calculated by dividing the power set point through the rotor speed.
- The value of i_{qr} needed to realize the desired electro mechanical torque is calculated using equation (11). The resulting i_{qr} is fed into the DFIG.

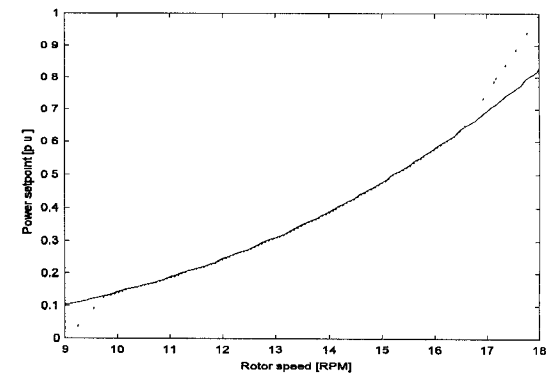

Figure 3. Optimal (straight line) and implemented (dotted line) rotor speed control characteristic

B. Pitch angle controller

As said previously, together with the rotor speed controller the pitch angle controller controls the rotor speed. However, the latter is only active during high wind speeds. In those circumstances, the rotor speed can not be controlled by increasing the electromechanical torque anymore, as this would lead to overloading the generator and the converter. To prevent the rotor speed from becoming too high, which would result in mechanical damage, the blade pitch angle is changed in order to reduce c_p.

From equations (2) and (3) it can be concluded that the pitch angle needs to be increased to reduce c_p. Furthermore, it should be taken into account that the pitch angle control can not change immediately, but at a finite rate, which may be quite low due to the size of modern WT rotor blades and the desire to save money on the drives turning the blades. In figure 4, the pitch angle controller used here is depicted.

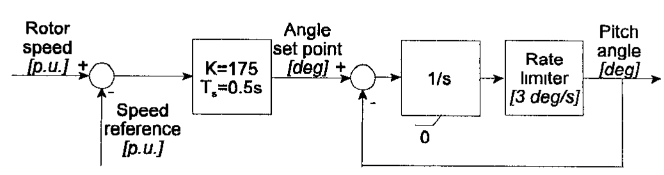

Figure 4. Pitch angle controller

C. Terminal voltage controller

The WT presented here is equipped with a terminal voltage controller. Although nowadays most WT do not take part actively in voltage control, this might change in the future, when more WT are integrated in the electrical power system. It is therefore considered important to incorporate a terminal voltage controller in the WT model being developed here, which is meant for studying the impact of large amounts of WT on an electrical power system.

When stator resistance is neglected, the reactive power generated by the wind turbine is directly dependent on i_{dr} [1]. As can be derived from equations (7) and (8) the following equation gives the relation between reactive power generated and i_{dr}

$$Q_{grid} = -\frac{L_m(i_{dr,magn} + i_{dr,gen})}{L_s + L_m} - \frac{e_t^2}{\omega_s(L_s + L_m)} \tag{12}$$

In (12), the direct component of the rotor current has been split into a part that magnetizes the generator and a part that determines the nett reactive power exchange with the grid. The value of the direct component of the rotor current necessary to magnetize the generator itself, $i_{dr,magn}$, has the following value

$$i_{dr,magn} = -\frac{e_t^2}{\omega_s L_m} \tag{13}$$

The value of $i_{dr,gen}$, the reactive power generating part of i_{dr}, determines whether nett reactive power is generated or consumed.

The terminal voltage will increase when more reactive power is delivered to the grid. From (13) it can be concluded that to increase the generated reactive power Q_{gen}, $i_{dr,gen}$ should be decreased. Therefore, the voltage controller should fulfill the following requirements:

- The reactive power consumed by the DFIG should be compensated by $i_{dr,\ magn}$.
- If the terminal voltage is too low or too high when compared to the reference value, $i_{dr,gen}$ should be adjusted appropriately.

A voltage controller that fulfills these requirements is depicted in figure 5. When the value of K is changed to zero, a controller keeping the power factor equal to one results.

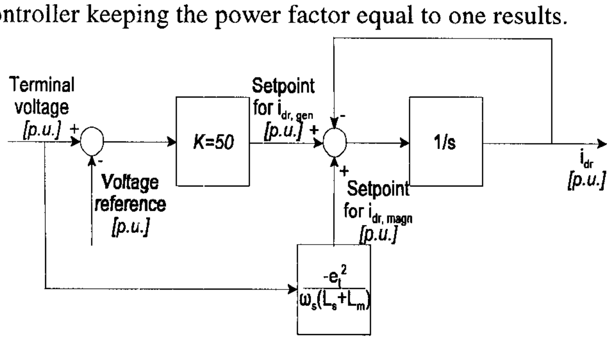

Figure 5. Terminal voltage controller

V. SIMULATION RESULTS

A. System characteristics

In table 1, the characteristics of a fictive 2 MW WT are given. The characteristics of the DFIG and the connection to the grid are given in table 2. All rotating mass is concentrated on the low speed side of the gearbox.

Table 1. Characteristics of wind turbine used in example calculations

WT Characteristic	Value
Rotor diameter	75 m
Area covered by rotor	4418 m2
Rotor speed	9-21 rpm
Nominal power	2 MW
Nominal wind speed	12 m/s
Cut-in wind speed	3.5 m/s
Gear box ratio	1:100
Total moment of inertia	$5.9 \cdot 10^6$ kgm2

Table 2. Characteristics of DFIG and connection used in example

DFIG Characteristic	Value
Number of poles	4
Generator speed	900-2100 rpm
Mutual inductance Lm	3.0 p.u.
Stator leakage reactance Ls	0.10 p.u.
Rotor leakage reactance Lr	0.08 p.u.
Stator resistance Rs	0.01 p.u.
Rotor resistance Rr	0.01 p.u.
Line inductance	0.1 p.u.
Line resistance	0.01 p.u.

B. Response to measured wind sequences

Now the response to two measured wind sequences is be simulated. The wind sequences were measured with a frequency of 2 Hz. In figure 6 the wind speed, the rotor speed, the pitch angle, the output power and the terminal voltage are depicted. In all graphs, the straight line correspond to the low speed wind sequence, the dotted line to the high speed wind sequence.

VI. CONCLUSIONS

In this contribution, a model of a VS WT with DFIG and BVSC is presented. It was shown that it is possible to develop a set of equations describing the behaviour of the WT. Furthermore, controllers for the rotor speed, the pitch angle and the terminal voltage were developed. The behaviour of the system was investigated using two measured wind sequences.

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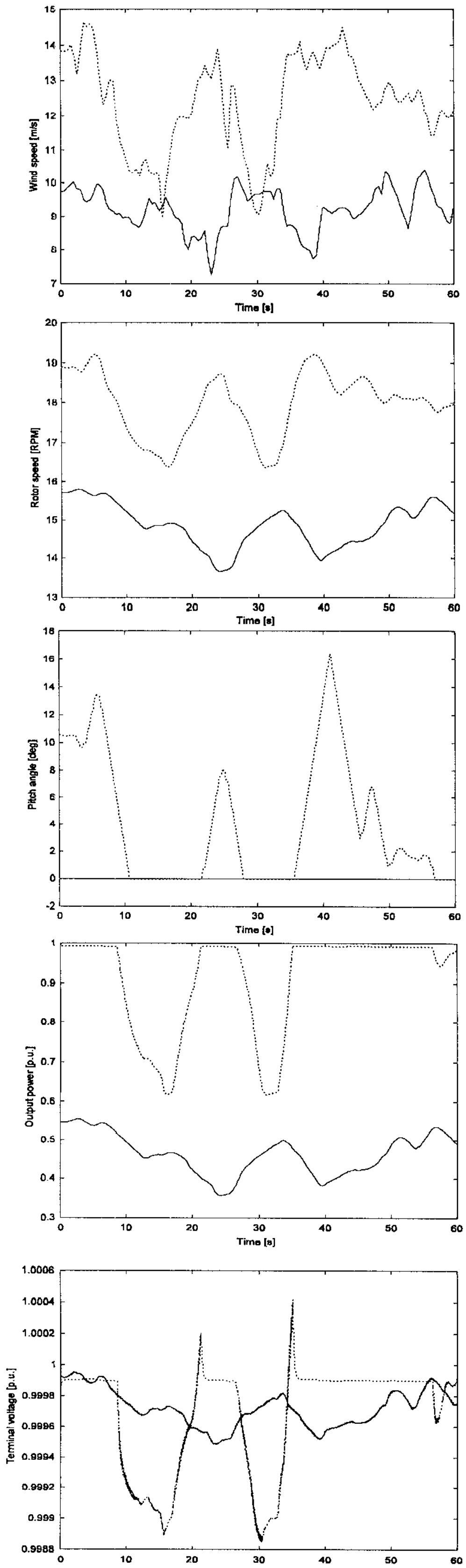

Figure 6. Starting from above: wind speed, rotor speed, pitch angle, output power and terminal voltage. The straight lines correspond to the low speed wind sequence, the dotted lines to the high speed wind sequence.

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BIOGRAPHIES

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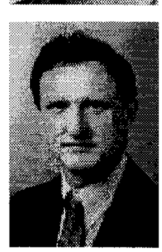
H. Polinder received his MSc degree in electrical engineering in 1992 and his PhD degree in 1998 both from Delft University of Technology. Currently, he is an assistant professor at the Electrical Power Processing Laboratory at the same university, where he gives courses on electrical machines and drives. His main research interest is the field of generator systems in renewable energy, such as wind energy and wave energy



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