# Appendix I

## Multi-axle vehicular generated loads

The vehicular generalized load can be written as follows:

$$P(t) = \int_0^L \phi_n(x)\,\delta(x - vt_i)\,p(x,t)\,dx = \phi_n(vt_i)\sum_{i=1}^{m} p_{si}\cos(\omega_{pi}t) \tag{I.1}$$

Substituting the mode shape (i.e., Equation (7a)) into Equation (A1), one has:

$$P(t) = \sum_{i=1}^{m} p_{si}\cos(\omega_{pi}t)\sin\left(\frac{n\pi(vt - d_{i-1})}{L}\right) \tag{I.2}$$

Using the trigonometric relationship, Equation (A2) can be expressed as follows:

$$P(t) == \sum_{i=1}^{m}\left[\begin{array}{l} p_{si}\cos\left(\frac{n\pi d_{i-1}}{L}\right)\sin\left(\frac{n\pi vt}{L}\right)\cos(\omega_{pi}t)\text{-} \\ p_{si}\sin\left(\frac{n\pi d_{i-1}}{L}\right)\cos\left(\frac{n\pi vt}{L}\right)\cos(\omega_{pi}t) \end{array}\right] \tag{I.3}$$

Similarly, using trigonometric functions, Equation (A3) can be simplified as follows:

$$P(t) = \sum_{i=1}^{m}\left[\frac{p_{sci}}{2}\left(\sin(\gamma_{1in}t) - \sin(\gamma_{2in}t)\right) - \frac{p_{ssi}}{2}\left(\cos(\gamma_{1in}t) + \cos(\gamma_{2in}t)\right)\right] \tag{I.4}$$

$$\gamma_{1in} = \omega_{pi} + \frac{n\pi v}{L} \;\; and \;\; \gamma_{2in} = \omega_{pi} - \frac{n\pi v_i}{L} \qquad (i = 1 \ldots n) \tag{I.5}$$

$$p_{sci} = p_{si}\cos\left(\frac{n\pi d_{i-1}}{L}\right) \; and \; p_{ssi} = p_{si}\sin\left(\frac{n\pi d_{i-1}}{L}\right) \tag{I.6}$$

where $\gamma_{1in}$ and $\gamma_{2in}$ are two disturbance frequencies from moving vehicles.

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# Appendix II

## Parameter standard values of the special vehicle

A1. Special vehicle parameters

| Parameters | Unit | Values |
|---|---|---|
| $m_2$ (Mass of the car body) | *kg* | $4.8\times10^5$ |
| $J_{2\theta}$ (Roll mass moment of the car body) | $kg.m^{-2}$ | $9\times10^6$ |
| $J_{2\varphi}$ (Pitch mass moment of the car body) | $kg.m^{-2}$ | $7\times10^4$ |
| $m_{1i}$ (Mass of the bogie) | *kg* | $8\times10^4$ |
| $J_{1\theta}$ (Roll mass moment of the bogie) | $kg.m^{-2}$ | $1.014\times10^7$ |
| $J_{1\varphi}$ (Pitch mass moment of the bogie) | $kg.m^{-2}$ | $1.8\times10^5$ |
| Vertical stiffness $k_1$ of the primary suspension system | $N.m^{-1}$ | $1.2\times10^6$ |
| Vertical damping $c_1$ of the primary suspension system | $N.s.m^{-1}$ | $2.4\times10^4$ |
| Vertical stiffness $k_2$ of the secondary suspension system | $N.m^{-1}$ | $8.044\times10^6$ |
| Vertical damping $c_2$ of the secondary suspension system | $N.s.m^{-1}$ | $2.4\times10^4$ |
| Longitudinal distance $l_1$ of wheelsets | *m* | 1.5 |
| Effective length $l_2$ of the car body | *m* | 39 |
| Longitudinal space $l_3$ of the car body | *m* | 1.8 |
| Transverse distance $b_2$ of the 2nd springs | *m* | 1.2 |
| Transverse distance $b_1$ of wheelsets | *m* | 1.8 |

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## Appendix III

### Special vehicle system matrices and load vectors

The investigated vehicle's DOFs are written as follows:

$$\{U_v\} = \left(Z_2, \theta_2, \varphi_2, Z_{11}, \theta_{11}, \varphi_{11}, Z_{12}, \theta_{12}, \varphi_{12}\right) \quad \text{(III.1)}$$

Based on the invariable principle of the system's potential energy (Zeng Q. Y., 2000), the calculus of variations for the system is only conducted on the elastic strain and displacement in Equation (23), and the matrices of the special vehicle are obtained as follows:

$$[M_v] = diag\left[m_2, J_{\theta 2}, J_{\varphi 2}, m_{11}, J_{\theta 11}, J_{\varphi 11}, m_{12}, J_{\theta 12}, J_{\varphi 12}\right] \quad \text{(III.2)}$$

Vehicle stiffness matrix is:

$$[K_v] = \begin{bmatrix} 4k_2 & 0 & 0 & -2k_2 & 0 & 0 & -2k_2 & 0 & 0 \\ 0 & l_2^2 k_2 & 0 & -l_2 k_2 & 0 & 0 & l_2 k_2 & 0 & 0 \\ 0 & 0 & b_2^2 k_2 & 0 & 0 & D & 0 & 0 & D \\ -2k_2 & -l_2 k_2 & 0 & B & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A & 0 & 0 & 0 & 0 \\ 0 & 0 & D & 0 & 0 & H & 0 & 0 & 0 \\ -2k_2 & l_2 k_2 & 0 & 0 & 0 & 0 & B & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A & 0 \\ 0 & 0 & D & 0 & 0 & 0 & 0 & 0 & H \end{bmatrix} \quad \text{(III.3)}$$

$$\begin{aligned} A &= \frac{n(n+1)(n+2)}{12} l_1^2 k_1, \ \ B = 2k_2 + 2\left(n+1\right)k_1 \\ H &= \frac{b_1^2}{2}\left(n+1\right)k_1 + \frac{b_2^2}{2} k_2, \ \ D = -\frac{b_2^2}{2} k_2 \end{aligned} \quad \text{(III.4)}$$

Vehicle damping matrix is:



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$$[C_v]=\begin{bmatrix} 4c_2 & 0 & 0 & -2c_2 & 0 & 0 & -2c_2 & 0 & 0 \\ 0 & l_2^2c_2 & 0 & -l_2c_2 & 0 & 0 & l_2c_2 & 0 & 0 \\ 0 & 0 & b_2^2c_2 & 0 & 0 & L & 0 & 0 & L \\ -2c_2 & -l_2c_2 & 0 & M & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E & 0 & 0 & 0 & 0 \\ 0 & 0 & L & 0 & 0 & N & 0 & 0 & 0 \\ -2c_2 & l_2c_2 & 0 & 0 & 0 & 0 & M & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A & 0 \\ 0 & 0 & L & 0 & 0 & 0 & 0 & 0 & N \end{bmatrix} \tag{III.5}$$

$$\begin{aligned} E &= \frac{n(n+1)(n+2)}{12} l_1^2 c_1, \quad M = 2k_2 + 2(n+1)c_1 \\ N &= \frac{b_1^2}{2}(n+1)c_1 + \frac{b_2^2}{2}c_2, \quad L = -\frac{b_2^2}{2}c_2 \end{aligned} \tag{III.6}$$

Vehicle load vector is:

$$\begin{aligned} &\{F_v\} = \{0 \quad 0 \quad 0 \quad F_{fv1} \quad F_{fv2} \quad F_{fv3} \quad F_{fv4} \quad F_{fv5} \quad F_{fv6}\}^T \\ &F_{fv1} = -\sum_{i=1}^{n+1} k_1 (Z_{R1i} + Z_{L1i}), \quad F_{fv2} = -\sum_{i=1}^{n+1} k_1 \left(\frac{n}{2} - (i-1)\right) l_1 (Z_{R1i} + Z_{L1i}), \\ &F_{fv3} = -\sum_{i=1}^{n+1} \frac{b_1}{2} k_1 (Z_{R1i} + Z_{L1i}), \quad F_{fv4} = -\sum_{i=1}^{n+1} k_1 (Z_{R2i} - Z_{L2i}), \\ &F_{fv5} = -\sum_{i=1}^{n+1} k_1 \left(\frac{n}{2} - (i-1)\right) l_1 (Z_{R2i} + Z_{L2i}), \quad F_{fv6} = -\sum_{i=1}^{n+1} \frac{b_1}{2} k_1 (Z_{R2i} - Z_{L2i}) \end{aligned} \tag{III.7}$$





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## Appendix IV

### Frequencies of the special vehicle and bridge

B1. Frequencies of the special vehicle and bridge

| No. of modes | Vehicle frequency (Hz) | Bridge frequency (Hz) | Ratio $\beta$ |
|---|---|---|---|
| 1 | 0.8196 | 0.6618 | 1.238 |
| 2 | 0.9511 | 1.5398 | 0.618 |
| 3 | 1.191 | 1.7323 | 0.688 |
| 4 | 1.191 | 2.1091 | 0.565 |
| 5 | 2.0084 | 2.4856 | 0.808 |

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