Dynamic Pricing for Hotel Revenue Management Using Price Multipliers

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1 Abstract

In this paper we propose a new dynamic pricing approach for the hotel revenue management problem. The proposed approach is based on having "price multipliers" that vary around "1" and provide a varying discount/premium over some seasonal reference price. The price multipliers are a function of certain influencing variables (for example hotel occupancy, time till arrival, etc). We apply an optimization algorithm for determining the parameters of these multipliers, the goal being to maximize the revenue, taking into account current demand, and the demand-price sensitivity of the hotel's guest. The optimization algorithm makes use of a Monte Carlo simulator that simulates all the hotel's processes, such as reservations arrivals, cancellations, duration of stay, no shows, group reservations, seasonality, trend, etc, as faithfully as possible. We have tested the proposed approach by successfully applying it to the revenue management problem of Plaza Hotel, Alexandria, Egypt, as a case study.

Keywords: *Revenue management system, dynamic pricing, price elasticity, Monte Carlo simulation, hotel room forecasting.*

2 Introduction

Revenue management is the science of managing a limited amount of supply to maximize revenue, by dynamically controlling the price/quantity offered. Revenue management systems have recently gained significant worldwide adoption in the hotel industry, at least for higher-rated hotels. The typical hotel guest has probably noticed in the past few years a progressively evolving level of the dynamism in the quoted room prices. This is indicative of the sophisticated nature of the algorithms behind these reservation systems. However, the hotel revenue management models are still in their infancy, and there is a need for further development and improvement of these systems. By the sheer number of worldwide hotels every percent of revenue improvement will add up considerably for the top line and much more so for the bottom line because of the thin margins in this industry.

Hotel revenue systems can be partitioned into two major groups (Abdel aziz et al., 2011; Ingold et al., 2000; Talluri and Van Ryzin, 2005). In the first group, the quantity control approach, the rooms are segmented by categories, such as by rate, guest type, room type, and/or length of stay. Each category has a fixed price, but the number of rooms allocated to the category is dynamically controlled in a way that maximizes revenue. The other group, the dynamic pricing approach, groups all similar rooms in one category, and applies a price that is continually adjusted with time based on the supply and demand variations. The dynamic price is set so as to maximize revenue, taking into account the hotel occupancy, and the current and expected demand. The dynamic pricing approach is particularly prevalent in some online hotel reservations. The online nature makes updating the price periodically quite manageable. So, it is expected that the dynamic pricing approach will in the future overcome the quantity based approach in adoption.

In this paper we propose a novel dynamic pricing approach. It is based on having a seasonal reference price, and control variables in the form of multipliers. Each multiplier will adjust the price up or down around the reference price based on a certain influencing variables (for example hotel occupancy, time till arrival, etc). The parameters of these multipliers are optimized. The goal is to maximize the revenue, taking into account current demand, and the demand-price sensitivity of the hotel's guest. Embedded in the optimization model is a Monte Carlo simulation that simulates all the hotel's processes, such as reservations arrivals, cancellations, duration of stay, no shows, group reservations, seasonality, trend, etc, as faithfully as possible.

The proposed dynamic pricing model has the advantage of framing the price in terms of carefully optimized premiums and discounts over a time varying or seasonal reference price set by the hotel. This could conquer some of the hurdles that make some mid-range hotels reluctant in adopting revenue management systems. The black-box behavior of some of the other models can at some times be unsettling. Our proposed model allows the hotel manager to give his input that he gained through his long experience. Moreover, this will reduce the uncertainty of where to expect the price, and hence improve the willingness of managers to adopt it.

The rest of the paper is organized as follows. Section 3 presents a review of the literature. Section 4 discusses our proposed revenue management framework. A case study is presented in Section 5. In Section 5 we present a conclusion.

3 Literature Review

A large volume of revenue management research has focused on the capacity control approach, especially in the earlier times (see McGill and van Ryzin (1999), McGill (1989), and Talluri and Van Ryzin (2005) for very valuable reviews of this approach, and Pullman and Rodgers (2010) for a review of capacity management approaches in the hospitality industry). The dynamic pricing approach of revenue management, on which we focus in this review, has recently had its share of research. For example, Feng and Xiao (2000) present a dynamic pricing model, where prices are selected from a set of predetermined prices over time. Zhao and Zheng (2000) present a dynamic pricing model for selling a stock of a perishable product, where the demand is stochastic and it follows a non-homogeneous Poisson process. Gallego and Van Ryzin (1994) investigate the problem of pricing a perishable stock of items. They find an upper bound for general demand. Finally, they extend their results by modeling the demand as a compound Poisson process. Perakis and Sood (2004) present a model for the multi-period pricing problem, in order to maximize the revenue of perishable products in a competitive market, where the multiple periods could correspond to duration of stay in the case of hotels. They assume that the demand is a deterministic function of the prices set by all sellers in that period. Chatwin (2000) introduces a dynamic pricing policy for perishable products. The price is also selected from a predetermined set of discrete prices. Moreover, the author assumes that the demand of the product follows a Poisson distribution with a rate that has some inverse relationship with the price. It is common in the literature to assume Poisson arrivals for the product demand. This is a sensible assumption because of the successive arrival of independent events. The rate of arrival, however, can be time varying, to reflect varying demand periods. Akçay et al. (2010) present a general stochastic dynamic programming pricing model for multi-product problems. They introduce a linear utility framework for modeling the customer's choice.

The majority of revenue management research targeted the airline problem or any general perishable product. However, some research considered the application of revenue management to the hotel industry. Ivanov and Zhechev (2011) present a literature review of the main concepts and methods of hotel revenue management. Also, the review of Vinod (2004) describes the details of the problem of hotel revenue management, gives some definitions and guidelines, and reviews some literature. In this paragraph we will present a review of some of the hotel revenue management work that is based on the capacity control approach. For example, Bitran and Mondschein (1995) develop a model, whereby customers' arrivals are modeled as a non-homogenous Poisson process. In order to have their model as general as possible, they allow for multiple types of rooms and also allow for downgrading between these types. They also consider reservations for multiple nights. They utilize some heuristics when searching for the optimal solution, in order to reduce the computational effort. Bitran and Gilbert (1996) present a hotel room capacity control approach, whereby the reservations' arrivals are modeled as a Poisson process, while no-shows and cancellations are drawn from binomial distributions. They also develop simple heuristics that perform well in practice. Baker and Collier (1999) develop two algorithms that integrate overbooking with the allocation decisions. They aim to produce a realistic hotel operating environment through simulation, where the demand is modeled as a Poisson process. They test and compare the performance of five heuristics-based solutions. They conclude that the simpler heuristics work as well as the more complex ones in many operating environments. Baker et al. (2002) develop a forecasting-allocation approach that explicitly takes into account the dependability between the demand for a service package and the group of service packages available for sale. The demand is modeled using a gamma probability distribution. Choi and Cho (2000) develop a probabilistic rule-based framework for controlling the capacity of hotels. The goal is to provide guidance to hotel managers through a set of decision rules. They take competitors' effects into account, and assume that customers' arrivals follow a truncated Poisson distribution. Goldman et al. (2002) study decision rules for controlling the hotel's capacity, whereby reservations of multiple day stays are taken into account. Also, they deal with overlapping decision periods by optimizing with respect to the complete range of open target booking dates, instead of only a fixed set of booking days. In their approach they model the booking requests' arrivals by a non-homogeneous Poisson process. Koide and Ishii (2005) propose controlling the hotel's capacity using discounts on early reservations. They take into account cancellations and overbooking. Lai and Ng (2005) propose a network optimization model to control hotel's room capacity. This model captures the random nature of the demand using a stochastic programming framework. It takes into account network structure of length of stay, cancellations, different pricing policies, early check-outs, extended stays, and over-booking. El Gayar et al. (2011) present an integrated framework for hotel room revenue maximization. They address group reservations and use forecasted demand that models the hotel's reservation process. Badinelli (2000) presents a model, which mainly targets small hotels. It is based on a dynamic programming formulation of the problem, which allows for general demand patterns, and it gives a simple closed-form solution. Guadix et al. (2010) introduce a decision support system for maximizing hotels' revenue. It includes methods for demand forecasting that consider both individual and group reservations. Also, it integrates deterministic and stochastic mathematical programming models for the decision process. Furthermore, there is a simulator that compares between different heuristics of room inventory control in order to select the best one.

There is also some work concerning the dynamic pricing aspect of hotel room revenue management, and we will review that work below in this paragraph. Gu and Caneen (1998) review the various definitions of hotel revenue management, and then develop two optimization models. One of the two models optimizes the rooms' revenue, while the other takes the costs into account and optimizes the rooms' profit. Other factors such as competition, seasonality and service quality can be taken into account when estimating the parameters of their profit-based model. Abdel Aziz et al. (2011) propose a dynamic pricing model for hotels. They use a Monte Carlo simulator for estimating arrivals, instead of using pre-defined probability distribution. Moreover, their model captures the price-demand elasticity effect, and takes into account the special nature of group arrivals. They use a non-linear programming formulation that can solve realistically sized problems. References (Gallego and Van Ryzin, 1994; Perakis and Sood, 2004; Chatwin, 2000; Akçay et al., 2010), described above, argue that their model, developed for general perishable products applies also to the hotel industry as well. However they did not mention any details about the specifics of the application to hotels.

As can be noted from the above review, there is very little work covering the dynamic pricing approach for the hotel industry. There is essentially a fundamental difference between the airline problem or any other perishable product problem and the hotel room problem. A hotel room reservation typically covers a number of days, and this duration of stay effect has to be taken into account. Moreover, the suitability to the hotel problem of any airline-based approach cannot be taken for granted, and has to be tested on real hotel data. Because of the amount of revenue at stake in the worldwide hospitality industry, we argue that more research is needed to explore hotel revenue management, especially the dynamic pricing aspect, which will gain further importance with the progressively increasing trend towards internet reservation systems. The contribution of the presented work can be summarized in the following points:

- Our model has very little assumptions for the distributions of the hotel room processes. It estimates the distributions, rates, and parameters, all from the data.
- It does not enforce any heuristics or assumptions in order to have an analytically feasible solution. A Monte Carlo simulation is applied, and one can include any details that render the model as realistic as possible.
- It models the pricing function in terms of multipliers that pose the problem in terms of discounts and premiums over a given or computed reference price. This makes the model more appealing for revenue managers, as it allows them to relate the proposed price with their own set reference price. Moreover, this reference price, around which price changes are anchored, could be hard to obtain analytically in the case of absence of competitors' pricing data.
- The model does not assume a predetermined set of discrete price levels. The price can take any continuous value in a range.
- It takes into account the price elasticity, and the effect of price on demand. Very few models consider this effect.

4 The Proposed Revenue Management Framework

4.1 An Overview

The proposed idea is based on having a seasonal reference price that is possibly a piecewise constant function that varies according to the major seasons' classification. This reference price is typically set by the hotel managers. Moreover, we have four multipliers that represent the "control variables". They will be multiplied by the reference price, to obtain the final price. They vary around the value of 1, where a value that is lower than one corresponds to a discount with respect to the reference price (for example 0.9 means the price is 10 % lower). Conversely, a value that is higher than 1 represents a premium over the reference price. The advantage of this formulation is that it will give the hotel manager or the revenue manager a suggested price that has some relation with the price that he has determined during his experience. So, he can relate to the new price by observing its discount/premium in relation to his reference price.

Each of the four multipliers corresponds to a variable that is known to have an influencing effect on pricing decisions. Specifically, the four variables that we selected are:

1. Time from reservation till arrival date.

2. The hotel's remaining capacity at the time of the reservation.
3. The length of stay, abbreviated as LoS.
4. The number of rooms to be reserved (group size).

We construct a multiplier for each of these four influencing variables. To simplify the problem formulation, these multipliers are usually taken as linear or piecewise linear functions of the influencing variables, whose levels and slope are determined by the optimization algorithm. The piecewise linear functions are selected based on logically expected relations identified by experts. For instance, if the remaining capacity is high, then the multiplier should be low in order to attract more customers using the lower price (or else more rooms will stay non-booked). Conversely, if the remaining capacity is small, the hotel prices the rooms higher to better save the remaining few rooms for higher paying customers. So, we use a linear function that is monotonically decreasing with remaining capacity. Similar arguments apply for the other multipliers. We emphasize that these multipliers alter the price *relative* to the reference price set by the hotel manager, which typically varies with time according the season. For example, a weekend reservation in a seaside resort (where weekend is the high demand part of the week) would still be quoted a high price even if the multiplier is below one due to lower than expected capacity. It is just that the quoted price is lower than a normal weekend rate. We will discuss each one of these multipliers in detail later. The final price is given by the product of the reference price and the multipliers, as follows.

$$\boldsymbol{Price = Seasonal\ Reference\ Price\ *\ Time\ multiplier\ *\ Capacity\ multiplier\ *}$$
$$\boldsymbol{LoS\ multiplier\ *\ Group\ Size\ multiplier} \qquad (1)$$

The resulting price will reflect the discounts/premiums resulting from the different values of the influencing variables.

Figure 1 shows the overall proposed dynamic pricing framework and its different components. The components of the block diagram will be explained in detail next. Initially, we design the so-called Monte Carlo room demand simulator that we have proposed in an earlier work (Zakhary et al., 2009). Its function is to simulate all the hotel's processes, such as

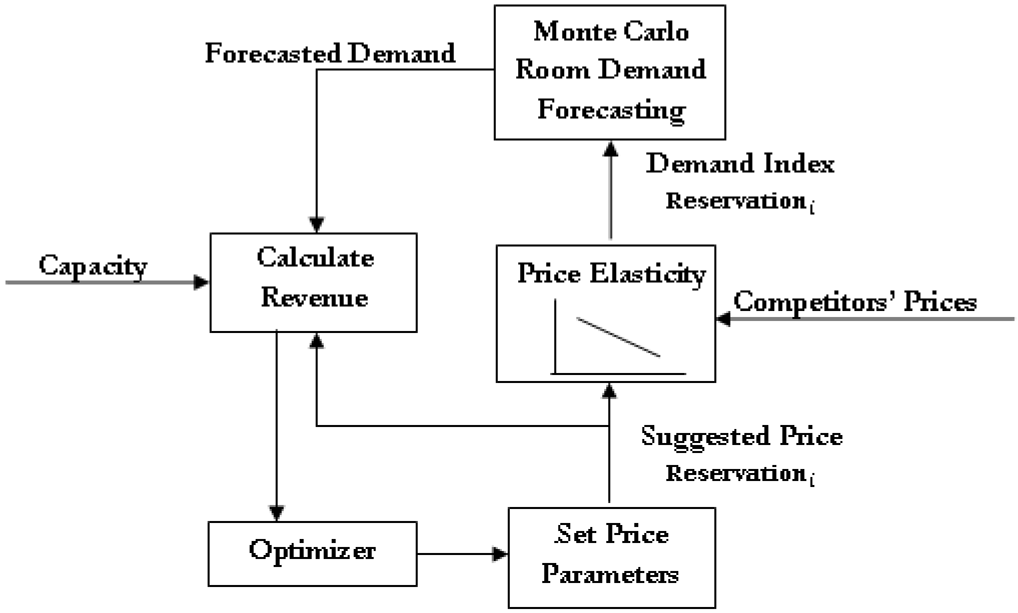


Figure 1: Framework Overview

reservations arrivals, cancellations, duration of stay, no shows, group reservations, seasonality, trend, etc, as faithfully as possible, projecting current and future room demand. This simulation is based on a probabilistic or stochastic modeling of each of the processes, and all parameters of these processes are estimated from the hotel's historical data. This simulator will yield a forecast of future arrivals and occupancy. More details about this component are given in subsection 4.4. Based on the simulator's forecasts, the expected total future revenue is estimated, and is passed forward to the optimization module that attempts to maximize this total revenue. The parameters of the price multipliers (e.g. the line slopes or positions) are the optimization variables. Once they are obtained, they in turn will determine the suggested price through the price multiplier formula (1). To determine how the new suggested price will influence demand, a price elasticity function is estimated from the hotel's historical data. Competitor's prices can be factored in as well. From this relation a new demand factor will be obtained for the price suggested by the optimization algorithm. This demand factor could be greater than 1, representing an excess demand if the suggested price is lower than the reference price, or smaller than 1, representing a reduction demand if the suggested price is higher than the reference price. The new demand will therefore be amplified or attenuated according to this demand factor. Subsequently, the new demand, represented in proportionally higher or lower reservation rate will be taken into account in a new run of the Monte Carlo simulator. This, in turn will produce a new forecast of the future demand, and hence also the expected total revenue. The optimization algorithm will test another set of price parameters, and we continue another iteration of the whole loop (shown in Figure 1). We keep looping around in this optimization/simulator framework for a few iterations, until we end up with hopefully the best parameter selections that lead to maximum total revenue. In a way, it is akin to "control" problems in the field of engineering, where certain "control variables" (the price parameters in our case) are optimally determined to achieve certain objectives on the measured variables (Nagrath, 2005). Once we find the optimal parameters, the final output is the *price of every incoming reservation* (which of course is a function of the obtained parameters).

4.2 Price Multipliers

The following are the four multipliers that determine the final price:

4.2.1 Time Multiplier

The time from the reservation date till the arrival date can be a very important controlling variable for the room's price. Figure 2 shows the proposed price multiplier (y-axis) against the time remaining till the arrival (the x-axis, in units of days). At the beginning of the booking horizon room prices should be low in order to quickly fill up the rooms, so the time multiplier will start from a low level y_2^T. This can be considered as an "early bird" discount. As time goes by and arrival day becomes closer, the discount is gradually lifted, so the multiplier's value increases. This is until a few days before arrival (corresponding to the multiplier's peak value y_3^T). At this point we are confronted with several vacant rooms that have little prospect to be filled up in the remaining short time. For such situation, it is prudent to price them low, in order to avoid them going non-booked as the target date comes. So the multiplier's value decreases until it reaches its minimum value y_1^T on the arrival day.

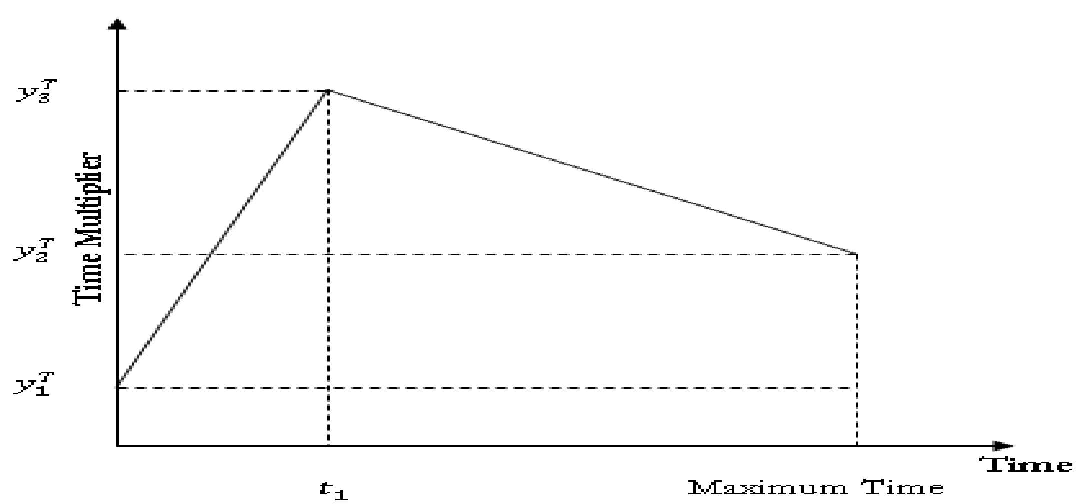


Figure 2: Time Multiplier Curve

4.2.2 Capacity Multiplier

Concerning the capacity multiplier, we assume that it has the downward sloping linear shape, as shown in Figure 3:

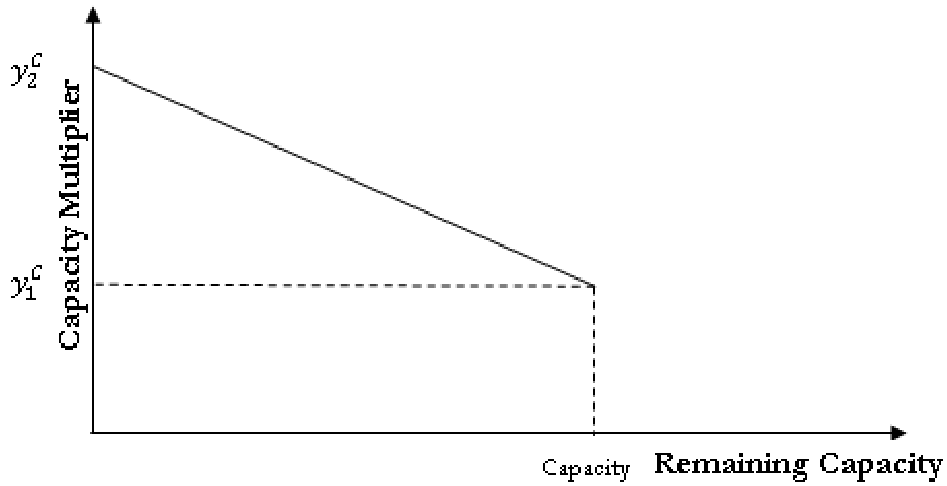


Figure 3: Capacity Multiplier Curve

The x-axis represents the total number of vacant rooms for the target date, and the y-axis represents the value of the multiplier. If there are many vacant rooms, then one has to offer some incentives, so the capacity multiplier is at its lowest value y_1^C. The multiplier's value increases as the remaining capacity of the hotel is decreased, till it reaches the maximum value y_2^C when there are no remaining rooms to be sold.

4.2.3 Length of Stay (LoS) Multiplier:

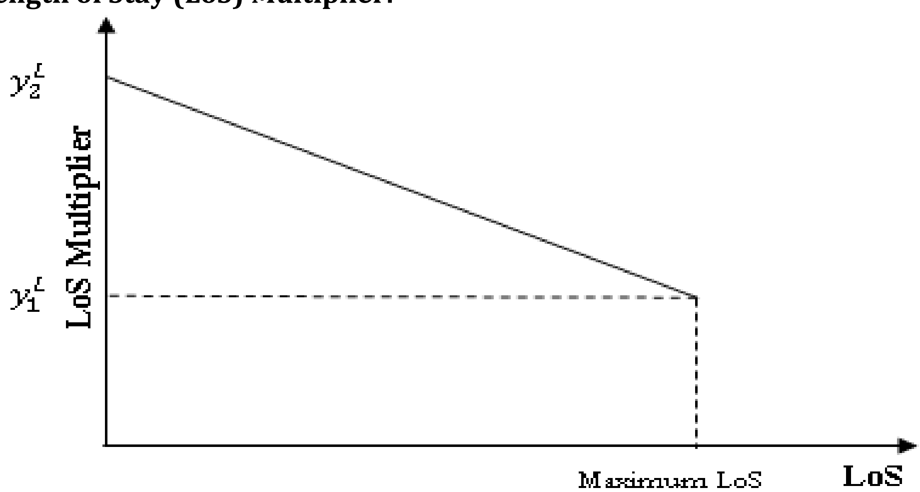


Figure 4: Length of Stay Multiplier Curve

The pricing should be monotonically decreasing with the length of stay, in order to attract longer stays (which therefore yield larger and more guaranteed revenue). Figure 4 shows the suggested relation for the multiplier as a function of the length of stay. From the figure one can see that the multiplier's value varies linearly from a maximum value of y_2^L for a LoS of one day to a minimum value of y_1^L for the typical highest value of LoS encountered by the hotel.

4.2.4 Group Size Multiplier:

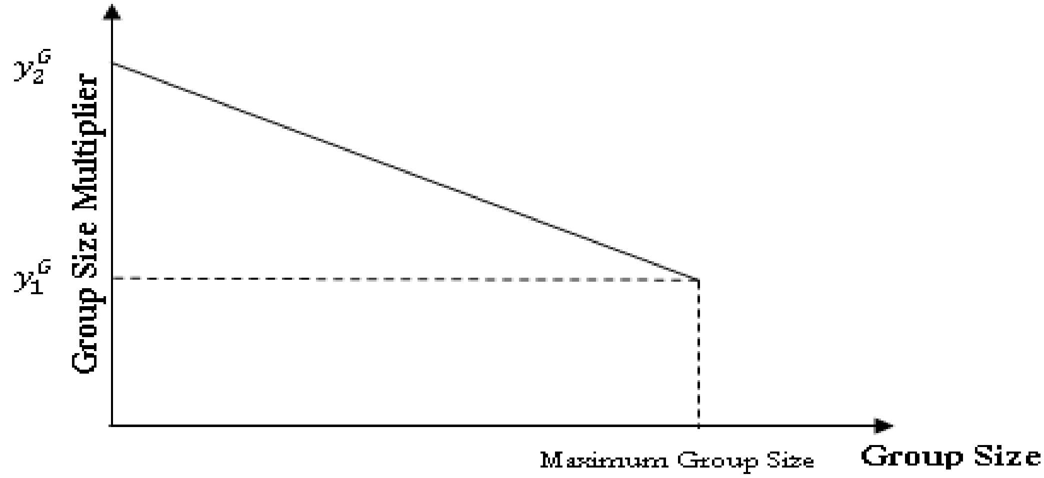


Figure 5: Group Size Multiplier Curve

A large amount of tourism travel nowadays is through pre-arranged tour packages. This way the tour operator can achieve block reservations. In our model we also consider the effect of group size on pricing. We considered the multiplier function as in Figure 5, which varies linearly from a peak value of y_2^G in case of no group (i.e. a single reservation) to a smaller value of y_1^G for the typical maximum size of a group.

4.3 Optimization Variables and Constraints

We assume that the average value of each multiplier function equals one. The reason is that these multipliers are considered correcting factors that will be multiplied by the reference price. So they should vary around one, with a value greater than one signifying a price premium, while a value less than one representing a price discount.

The optimization variables are essentially the following: *time multiplier-based:* y_1^T, y_2^T & t_1; *Capacity Multiplier:* y_2^C; *LoS Multiplier:* y_2^L; *Group Size Multiplier:* y_2^G. These are the variables that the optimization algorithm will determine such that the revenue is maximized. The other multiplier parameters are not independent, as they will be set so that the average value of each multiplier function equals one. These relations are the following:

$$\boldsymbol{y_3^T = 2 - \frac{t_1 * y_1^T + y_2^T * (Maximum\,Time - t_1)}{Maximum\,Time}} \qquad (2)$$

$$\boldsymbol{y_1^K = 2 - y_2^K\,;\ \forall K \in \{C, L, G\}}$$

We also have inequality constraints that will guarantee that the multiplier functions are well-behaved and produce logically accepted relations:

Time Multiplier:	$0 \le y_1^T \le y_2^T \le y_3^T$
	$0 \le t_1 \le T_0$
Capacity Multiplier:	$1 \le y_2^C \le C_0$
LoS Multiplier:	$1 \le y_2^L \le L_0$
Group Size Multiplier:	$1 \le y_2^G \le G_0$

The first constraint is meant to preserve the intended shape of the multiplier function, where the largest price is at the interior time instant t_1. The next constraints are bounds on the parameters that should be determined with the help of the hotel manager or the expert. For example, we took T_0 to equal 20, whereas we took $C_0 = L_0 = G_0 = 1.5$.

In addition to these multiplier-specific constraints, we have a global constraint for the overall price correction (the product of all multipliers). This product should not exceed a certain percentage. In our case we took that percentage to be 40 % (this means that the final price has to be within plus or minus 40 % of the reference price). The rationale for this constraint is to ensure that the suggested price does not deviate too much from the reference price. This will guarantee that the pricing of the room is under control, and does not behave in a peculiar way, should things go wrong in the optimization procedure for one reason or another.

We have also considered another formulation of this latter overall price correction constraint. Namely we use a "soft" rather than a hard constraint, using a probit function. So instead of the hard truncation at +40 % and -40 %, the applied probit function will smoothly map the product of the multipliers to an increasing function in the range of 0.6 to 1.4. This smooth truncation will also be helpful in the optimization procedure, as it leads to a smoother function surface.

4.4 Hotel Simulator Forecasting

In order to obtain the future revenue that is going to be optimized, we need to forecast the future reservations. For this purpose we apply the hotel simulator that we have developed in an earlier work (Zakhary et al., 2009). In this model we simulate all the processes of the hotel as faithful as possible. There are essentially a number of components in this system, as follows:

- Reservations: We model the reservations arrivals as Bernoulli trials, whose mean rate is a function of the time till arrival (i.e. the time between reservation and arrival date). This leads to the so-called *reservation curve* (this curve is a function of time till arrival), whose shape is estimated from the data. We assume that there is a distinct shape for the reservation curve for each of the major seasons' classifications (e.g. high season, low season, etc). Moreover, the level of the reservation curve can go up or down (while shape being the same) as a function of demand (for example due to seasonal effects, or price changes). We call this factor that affects the level the *demand index*. It is a relative variable, in the sense that it measures the overall demand normalized with respect to the mean demand.
- Seasonality: We consider seasons at all time scales, such as weekly and yearly seasonality, and perform a deseasonalization step, to factor out the seasons' effects in the subsequent forecasting step. After the deseasonalization, we end up with a time series of demand indexes.
- Cancellations: We also model cancellations as Bernoulli trials, with a cancellation rate that is a function of time till arrival, leading to a *cancellation curve*, which is also estimated from the data.
- Length of Stay (LoS): We estimate the probability distribution of the length of stay. Using this estimated distribution we can simulate different lengths of stay when we apply the Monte Carlo procedure.
- Group reservations: We also model the occurrence of batch reservations by estimating the group's size distribution.
- Forecasting: We project forward the deseasonalized time series of demand indexes, in order to obtain forecasts that will be used to generate simulated reservations. Any simple forecasting model such as exponential smoothing (Andrawis and Atiya, 2009) or linear regression can be used.
- Parameter estimation: As mentioned, all parameters and distributions, for example the reservation curve, the cancelation curve, the probability distributions of the LoS and group size, the seasonality curve are estimated from the hotel's historical data.

- Monte Carlo simulation: All the processes, including reservations arrivals, cancellations, seasonality, LoS, etc, are simulated forward in time, to obtain reservations scenarios for the future time horizon. The future reservations arrivals are a function of the forecasted demand index, obtained in the forecasting step above (the demand index, as mentioned, will move the reservation curve proportionally up or down). Several Monte Carlo paths are generated (reservations scenarios), and mean occupancy, arrivals, or revenue can be obtained by simple averaging over these reservations scenarios.

4.5 Price Elasticity

The optimization procedure obtains the price multipliers, which in turn determine the overall price. This price affects the demand through the price elasticity relation. We estimate this price elasticity from the data. Rather than using a linear function, we use a probit function, to account for the saturation effects for extreme price levels. However, for most of the price range, we are operating in the linear portion of the probit function. The price elasticity relation is given by:

$$\boldsymbol{Demand\ Index} = \boldsymbol{probit}\left(\frac{\boldsymbol{Normalized\ Price} - \mathbf{1}}{\boldsymbol{a}}\right) + \mathbf{0.5} \qquad (3)$$

where normalized price means normalized with respect to the reference price, and where the demand index is the relative demand (with demand index = 1 corresponding to the mean current demand that we observe when the price equals the reference price). This is why the probit function is taken to be centered around 1. Note that the reference price and reference demand correspond to their seasonal averages that are computed in the hotel simulator phase. (To avoid confusion, note that the probit function used here has a different purpose from that described in subsection 4.3 and used for the smooth truncation.)

The main price sensitivity variable is the scaling parameter a. It is the parameter affecting the slope of the demand-price relation, and as such it should be negative. Figure 6 shows an example of the shape of the probit model for a given price sensitivity slope of value equal to -0.3.

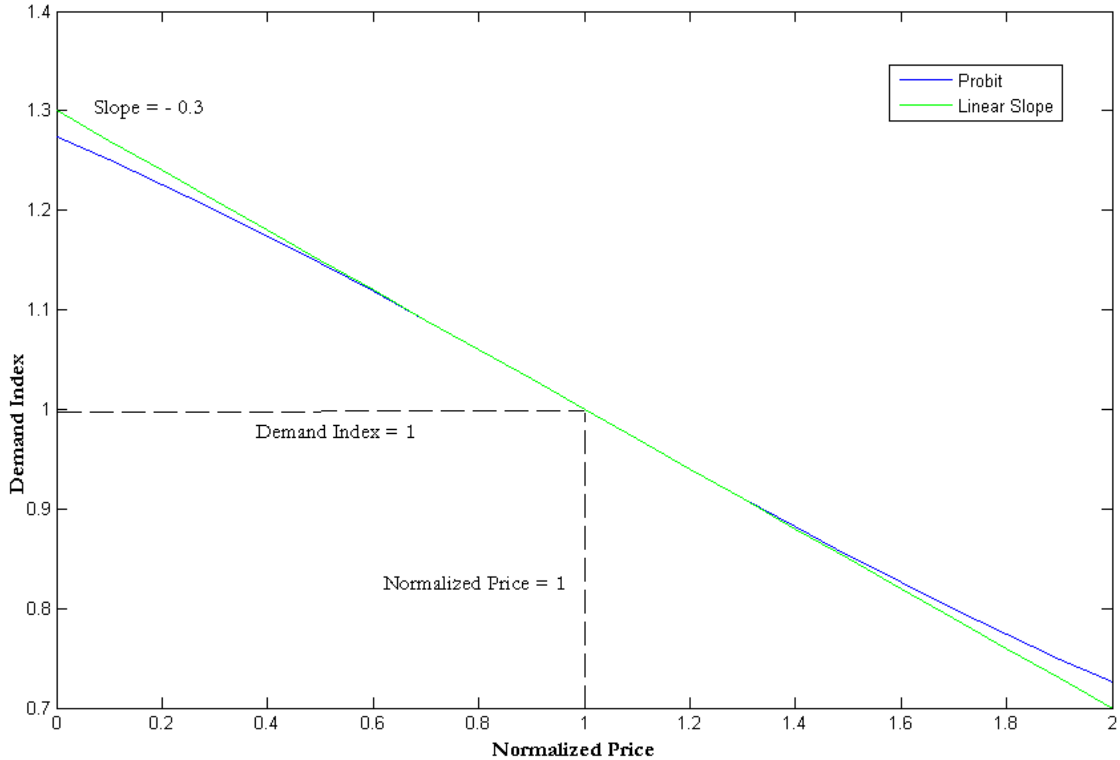


Figure 6: Example of the probit model

Once the optimization procedure suggests a new value for the normalized price, it reads the demand index off the price elasticity curve. Then the reservations scenarios that are generated by the Monte Carlo simulator are adjusted as follows. If the demand index is less than one, for example some value x, then each existing reservation is kept with probability x, otherwise it is discarded. By this way we uniformly reduce the number of reservations according to the new demand index. On the other hand, if the demand index turns out to be greater than one, for example a value $1+x$, then we keep the current reservation and in addition generate a new duplicate one with probability x. By this way we uniformly increase the demand (or number of reservations) by an average of $(1+x)$ times.

The price elasticity function can also be extended to take into account the effect of the competitors' prices. For example, we can use the model proposed by Fibich et al. (2005) that models changes in price elasticity in the presence of reference prices. In our simulations we did not apply this extension because the effect of the competition is already factored into the available price data.

4.6 Optimization

Figure 7 shows a flow chart for the whole optimization framework. As mentioned, the optimization has embedded in it the Monte Carlo simulator. The starting point is to consider the hotel's past and present reservations, and apply the hotel simulator forecasting model to generate future reservations. The room prices for these reservations are initially assumed to follow the seasonal average prices (or reference prices) obtained from the historical reservations. Subsequently, the optimization algorithm tests different multiplier values, which will give new room prices. Note that generally for each reservation we attach a different room price, because each room has its specific conditions, such as time till arrival, length of stay, prevailing capacity, etc. So the demand index for each reservation is generally different. However, when adding up the individual revenue of each reservation, we obtain the collective effect for the specific choice of multipliers on total revenue. The optimization algorithm keeps searching for different sets of multipliers until it reaches the set that optimizes total revenue.

We use a meta-heuristic evolutionary method called Covariance Matrix Adaption Evolution Strategy (CMA-ES). The CMA-ES is one of the most competitive evolutionary algorithms; it has some aspects of self-adaptation in its search strategy. As such, only one parameter (the average range of the parameters) is needed as input from the user. For more details, refer to (Auger and Hansen, 2005; Hansen and Kern, 2004; Hansen, 2009).

5 Case Study

We applied the dynamic pricing model to Plaza Hotel, Alexandria, Egypt, as a detailed case study. Plaza Hotel is a mid-sized (134 rooms) four star sea-side hotel, located on the Mediterranean Sea. Quantitative revenue management approaches have recently started to attract some interest from hotels in Egypt. While a good fraction of five star hotels in Egypt apply some form of revenue management, the technology has not caught up yet for four star hotels and lower. Plaza Hotel plans to implement a revenue management system. Moreover, they need a system where they can incorporate their knowledge and experience in pricing, but at the same time is quantitative in nature in order to avoid human inconsistencies and biases (that they frequently encounter due to personnel changes). As a first step, in collaboration with the hotel, we apply here our proposed dynamic pricing model for their online reservation system, and therefore achieve their sought trade-off between the use of human expert knowledge and algorithm efficiency and capability.

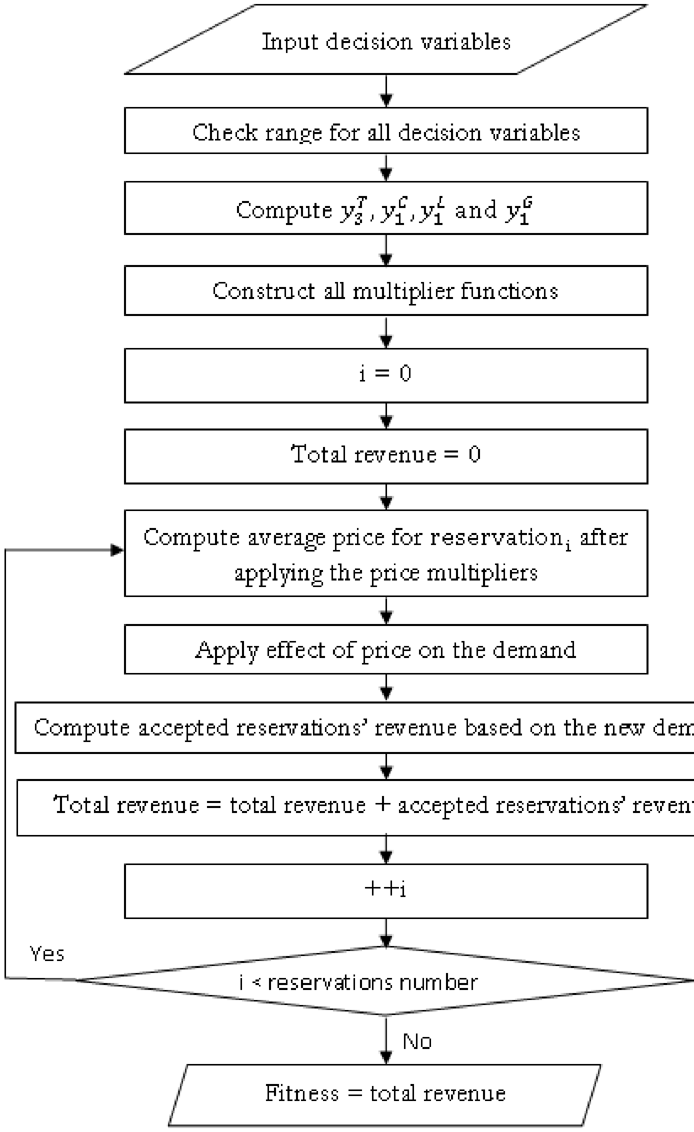


Figure 7: Flow Chart of the Fitness Function

We have obtained a full set of data covering the period from 1-Oct-2006 until 14-Dec-2010. The breadth of the data is extensive, and they include all aspects of the reservations, room prices, with all their details, such as room type, customer category, rate category, etc.

By applying our proposed system to the Plaza Hotel data, we test the effectiveness of this system. The goal is to test if this proposed approach leads to improvement in revenue over the baseline (i.e. over what the hotel would have generated if it kept pricing according to its original strategy).

We apply a walk-forward optimization/test procedure. This means that we divide the data into twelve parts: each consisting of one year of in-sample data, followed by three months for out of sample testing. Because of the probabilistic nature of the proposed algorithm, one thousand runs are applied, and the average revenue is computed over these thousand runs. Assuming that the testing data follows the same price elasticity slope parameter estimated from the training data (slope = -0.4 with respect to normalized price and normalized demand), the revenue improvement percentage for each of the out of sample periods are given as in Table 1 below.

In realistic situations, however, the price elasticity parameter could have an estimation error, it could possibly drift with time, or it could have some other effects mixed up with it. We tried to minimize the latter aspect by using normalized price and normalized demand (i.e. dividing each by its seasonal average). However, there could possibly be some unanticipated factors. So we have to ensure that our proposed system is robust enough, and can withstand such inaccuracies in price elasticity. For this purpose, we examined the revenue in the out of sample periods in situations where the price elasticity parameter is different from the estimated value of -0.4. Specifically, we tested the values of -0.2, -0.3, -0.5, -0.6. Table 2 presents the revenue improvement percentage for such situations. Note that all aspects of the designed pricing use an elasticity factor of -0.4, and only in the testing period we assume that we encounter a different elasticity factor.

Data Set	**Testing Data Improvement Percentage**
Data Set 1	13.39
Data Set 2	13.56
Data Set 3	15.00
Data Set 4	14.65
Data Set 5	17.73
Data Set 6	17.49
Data Set 7	16.91
Data Set 8	17.21
Data Set 9	17.35
Data Set 10	17.56
Data Set 11	17.51
Data Set 12	15.53
Average	**16.16**

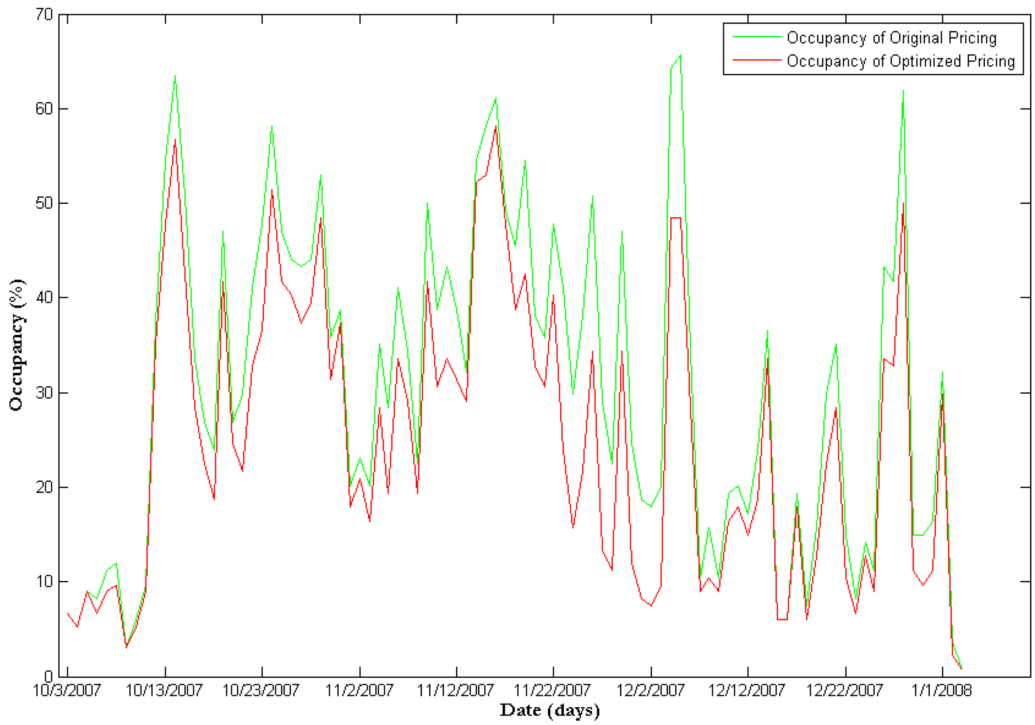
Table 1: Revenue Improvement (in Percent) over the Baseline for each of the out of Sample Periods

In our case simulations, the booking horizon over which we optimize the revenue is set as 90 days. Regarding the probit model of the price elasticity, we obtain the price sensitivity slope value by fitting the model to all the available data using the Matlab curve fitting tool cftool (Open Curve Fitting Tool). We used all available data (for the price elasticity part only), because one year's of data was too little for obtaining an accurate estimate. Figure 9 compares the occupancy using our optimized pricing to that using the original pricing for one of the test sets.

One can observe from the tables that the proposed approach succeeds in improving the revenue about 16 % over the baseline. Moreover, this improvement is consistent across all periods of the data. Also, it is robust, and can withstand any variations in the price elasticity parameter that are unaccounted for. This parameter is critical to the computation of the pricing parameters, and at the same time vulnerable to estimation error. The improvement in revenue is considered to be a large from the point of view of typical hotel revenues. Even though it generally leads to lower occupancy (see Figure 6), this is compensated by generally higher pricing, and an overall higher revenue. The net profit would generally increase even more than the revenue. This is because we have simultaneously higher revenue and higher margins (due to generally higher pricing). These results reveal not only the superiority of the proposed model, but the superiority of the quantitative dynamic pricing general methodology versus human-based pricing. There is a tendency in many hotels to focus on maximizing occupancy, even though unknowingly this does not always translate to better revenues.

Data Set	**Testing Data Revenue Improvement Percentage**			
	Slope = -0.2	**Slope = -0.3**	**Slope = -0.5**	**Slope = -0.6**
Data Set 1	23.32	18.27	8.57	4.20
Data Set 2	23.58	18.55	8.80	4.11
Data Set 3	24.38	19.75	10.60	6.48
Data Set 4	23.02	18.84	10.59	6.90
Data Set 5	28.06	22.70	12.77	8.21
Data Set 6	27.61	22.68	12.69	8.26
Data Set 7	26.79	21.71	12.19	7.83
Data Set 8	27.36	22.33	12.47	7.85
Data Set 9	27.49	22.32	12.6	7.84
Data Set 10	27.92	22.74	12.82	8.03
Data Set 11	27.93	22.52	12.67	8.01
Data Set 12	24.15	19.74	11.09	7.28
Average	**25.97**	**21.01**	**11.49**	**7.08**

Table 2: Revenue Improvement (in Percent) over the Baseline for each of the out of Sample Periods in Case of Encountering a Varying Price Elasticity



y Slope

Figure 6: Occupancy Comparison in the out of Sample Period

6 Conclusion

In this work, we propose a new hotel room dynamic pricing system. The proposed model uses the concept of price multipliers that provide a varying discount/premium within some bands over some seasonal reference price. The transparent way of designing such system, including the knowledge of the variables that affect the pricing, will allay the hotel's concerns regarding the uncertainty of system's outcome. Hotel managers can start out with fairly tight bands for the allowable premium/discount until they gain enough confidence about the system's performance. Moreover, some of the relations regarding the four influencing variables (hotel capacity, time till arrival, length of stay, and group size) can be adjusted or removed, according to the hotel's request. Of course having custom made pricing systems (rather than off-the-shelf) should be the better strategy. However, this is not practical and too costly, so a middle ground could be a good compromise. This middle ground could be achieved using the model proposed.

The proposed model utilizes a Monte Carlo simulator, which provides a faithful emulation of the hotel's processes. So, it is based on a realistic simulation of the hotel's processes, and therefore does not necessitate any simplifying approximations, as is frequently done in other work in order to obtain tractable formulas. The application of the proposed model on the case study indicated a successful and a large improvement of the revenue. Moreover, the improvement is robust to statistical estimation errors, such as errors in computing the price elasticity.

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