Paper from: *Computers in Railways VIII*, J Allan, RJ Hill, CA Brebbia, G Sciutto and S Sone (Editors).
ISBN 1-85312-913-5

Dynamic properties of GNSS/ INS based train position locator for signalling applications

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Abstract

The paper deals with dynamic properties of GNSS (Global Navigation Satellite System) and INS (Inertial Navigation System) based train position locator (TPL) intended for railway signalling applications. Main attention is paid to the cost-effective on-board train positioning and routing detection on a "dark" track where GNSS SIS (Signal-in-Space) is not temporally available. The route map-matching technique is considered as a key method to improve the accuracy and the reliability of the entire locator system. The *double* heading differences fully compensating the drift of a gyro are introduced in order to estimate the reliability of the routing decision process by means of Bayes' theorem. The sensor data validation process based on the route map is also discussed. On-board detection of the characteristic elements of the switch is proposed to achieve higher safety standards and reduce cost of signalling. The presented experimental results have been achieved by means of 3 kV DC electric locomotive and computer controlled track rover.

1 GNSS for signalling on "dark" tracks

Within several last years a number of R&D train position determination projects based on GPS or GNSS have been carried out world-wide. These initiatives were mainly focused on non-safety applications, where accuracy of GPS receiver and temporally absence of SIS were not critical.

Quite different demands on an on-board GNSS based TPL are required by railway safety related applications, mainly those concerning signalling and train control. An on-board TPL must be able to recognise on which of two nearby parallel tracks the train is located. Therefore it has been already specified and experimentally verified [1], that an on-board GNSS receiver with a horizontal accuracy of about 1 meter in code mode is needed for the most demanding

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signalling applications. This accuracy can be currently achieved by a GNSS receiver operating only in the local differential mode with the RTCM-104 corrections generated by a local track-side differential GNSS reference station. The existing ESTB (EGNOS System Test Bed) WAD (Wide Area Differential) corrections enable to achieve a horizontal positional accuracy of about 2 meters and have already been employed within a railway oriented R&D process – e.g. within the EC DG-XIII's APOLO project [1]. It is expected that future railway safety related applications would employ the navigation system GALILEO with guaranteed a sub-metre horizontal accuracy and information on integrity [2].

On the other hand, there are many kilometres of lines world-wide, where GPS SIS is not available due to masking by track-side objects and landscape profile. Although the GALILEO system will add about 30 additional navigational MEO satellites on orbits and thus increase SIS availability significantly, some "dark" territories uncovered by GNSS SIS will still remain. It is not so critical if SIS is not temporally available on the track where the train doesn't change its direction of movement – i.e. on the track without switches. In this case the instant position of the train can be estimated by means of an odometric system and a map-matching technique from the last absolute position provided by a GNSS receiver. After SIS is available again, the relative positioning is replaced by the absolute one and thus the accumulated error of the odometer is corrected.

However, the more difficult task is, if the train changes its direction of movement on a "dark" switch (e.g. under a bridge, in a tunnel, etc.), or even worse, if train routing must be detected just after the train passed a "dark" area with length of several kilometres. Train routing is then detected by using a gyroscope together with an odometer and route map-matching.

The following paragraphs of this paper describe techniques and selected experimental results related to train position determination and routing detection on the track where GNSS SIS is unavailable.

2 Train routing detection on "dark" switch

Two routing detection models are discussed in this paragraph: 1) one-dimensional (1D) probability and 2) two-dimensional (2D) semi-deterministic one. Both these models are based on travelled distance and heading measurement and employ a precise route map.

2.1 One-dimensional probability model

The probability approach is based on a fact that differences between two subsequent measured heading values of train movement are mutually independent. Then the measured heading differences are compared with the corresponding heading differences computed from the track axis map. Bayes' theorem employing conditional probabilities is used as an arbiter in the decision making process.

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2.1.1 Double heading differences

The differences between two successive heading measurements and the travelled distance data provided by an odometer can be used for train trajectory calculation (dead reckoning) from the known initial point. However, these heading differences are still influenced by the drift of the gyro, which introduces an error in the computed trajectory. In order to eliminate the drift of the gyro completely, the *double* heading differences have been introduced with an aid of the precise reference track axis map. The *double* heading difference $\delta^n(t)$ according to eqn. (1) means the difference between the measured heading difference $\Delta\varphi^{meas}(t)$ of the vehicle and the corresponding heading difference $\Delta\varphi^{ref\_n}(t)$ computed from

$$\begin{aligned}\delta^n(t) &= \Delta\varphi^{meas}(t) - \Delta\varphi^{ref\_n}(t), \text{ where}\\ \Delta\varphi^{meas}(t) &= \varphi^{meas}(t-\Delta t) - \varphi^{meas}(t) \text{ and} \\ \Delta\varphi^{ref\_n}(t) &= \varphi^{ref\_n}(t-\Delta t) - \varphi^{ref\_n}(t).\end{aligned} \tag{1}$$

the precise *n-th* reference trajectory, where Δt is a time interval between two heading measurements. Obviously, the consistency of the measured and the calculated heading differences on the switch mainly depends on the accuracy of travelled distance measurement.

2.1.2 Statistical independence of the measured heading data

The measured heading data of the train can be considered as a stochastic process. In order to apply the measured heading data for railway safety related applications is important to verify that only one realisation of the stochastic process completely describes the entire process. In other words to demonstrate that the stochastic process is the ergodic one.

It is evident that the existing drift of a gyro influences the measured heading data. The typical drift rate for the fibre optic gyro (FOG) is 0.002 deg/s and 0.017-0.083 deg/s for cheaper piezo gyros. Since the *double* heading differences eliminate the drift of the gyro, then the stochastic process can be considered as the stationary one. If the autocorrelation function (2) of the stationary process

$$R(\tau) \approx \frac{1}{T-\tau}\int_0^{T-\tau} \delta^n(t)\,\delta^n(t+\tau)\,dt \tag{2}$$

tends to zero for $\tau \rightarrow \infty$, it confirms the ergodic feature of the stochastic process. The autocorrelation function (2) can also be expressed in a discrete form

$$R(\tau) = R\left(\frac{mT}{N}\right) = \frac{1}{N-m}\sum_{i=1}^{N-m} \delta^n(i\Delta t)\,\delta^n[(i+m)\Delta t] \tag{3}$$

(3), where N is the total number of samples, $m=0,\ 1,\ \ldots,\ N\text{-}1$, $\delta^n(i\Delta t)$ and $\delta^n[(i+m)\Delta t]$ are the *double* heading differences, T is the record time. The

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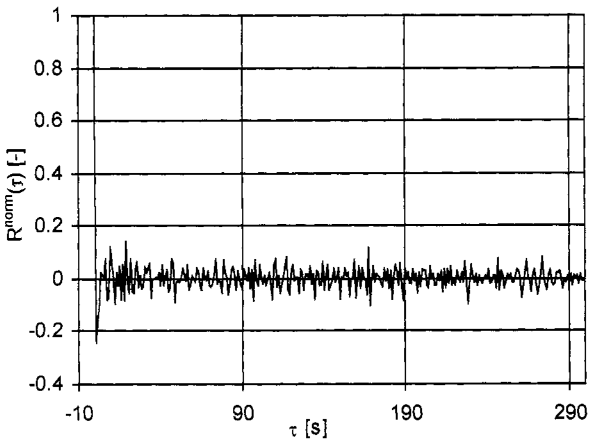

Figure 1: The autocorrelation function of the *double* heading differences.

normalised autocorrelation function $R^{norm}(mT/N)$ is obtained when equation (3) is divided by $R(0)$. The statistical experience considers the ordinates of the stochastic process as statistically independent for the specified correlation time of τ_0, if $R^{norm}(\tau_0) \leq 0.05$. An example of the normalised autocorrelation function calculated from the real measured heading data on the track is shown in a diagram in Fig. 1. It's evident that this experimental function steeply tends to zero. Therefore, the heading data can be considered as the ergodic process and employed for the train routing detection probability model as described below.

2.1.3 Probability of train routing detection

The decision process evaluating train routing detection on a switch is based on the conditional probabilities and Bayes' theorem (4). H_1 and H_2 are two

$$P(H_1 \mid A) = \frac{P(H_1)P(A \mid H_1)}{P(H_1)P(A \mid H_1) + P(H_2)P(A \mid H_2)} \tag{4}$$

inconsistent hypotheses. The hypotheses H_1 and H_2 mean that the train is located on the tracks No. 1 and No. 2, respectively. The term $P(H_1)$ is the *prior* probability (known before measurement) that train is located on the track No. 1. The term $P(H_2)$ means the same but for the track No. 2. Before the decision process of routing detection begins (the first axle of a vehicle is located on the blade of the switch), the *prior* probabilities $P(H_1)$ and $P(H_2)$ equal 0.5. The term $P(A| H_1)$ means the conditional probability that the train is located on the track No. 1 after the experiment A , i.e. heading and travelled distance measurements were performed. The term $P(A| H_2)$ is the same conditional probability for the track No. 2. The conditional probabilities $P(A| H_1)$ and $P(A| H_2)$ can be expressed by means of the successive *double* heading differences (1) as follows

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$$P(A \mid H_1) = \frac{|\Delta\varphi^{meas} - \Delta\varphi^{ref\_2}|}{|\Delta\varphi^{meas} - \Delta\varphi^{ref\_1}| + |\Delta\varphi^{meas} - \Delta\varphi^{ref\_2}|} \quad (5)$$

$$P(A \mid H_2) = \frac{|\Delta\varphi^{meas} - \Delta\varphi^{ref\_1}|}{|\Delta\varphi^{meas} - \Delta\varphi^{ref\_1}| + |\Delta\varphi^{meas} - \Delta\varphi^{ref\_2}|} \quad (6)$$

Finally, the *posterior* probability $P(H_1|A)$ that the train is located on the track

$$P(H_1 \mid A) = \frac{P(H_1)\,|\Delta\varphi^{meas} - \Delta\varphi^{ref\_2}|}{P(H_1)\,|\Delta\varphi^{meas} - \Delta\varphi^{ref\_2}| + P(H_2)\,|\Delta\varphi^{meas} - \Delta\varphi^{ref\_1}|} \quad (7)$$

No. 1 is derived in eqn. (7). According to (7), the resulting probability evaluating routing detection on the switch can be computed from the heading data and the instant position of the train in the route map determined by means of the odometric data. In each following step, the computed conditional probabilities (5) and (6) replace $P(H_1)$ and $P(H_2)$, respectively. The error of the odometer taken as a parameter enables to investigate the relation between the odometric error and the final *posterior* probability computed by means of eqn. (7).

2.2 Two-dimensional probability model

When the train rides on the track section without a switch, its position can be determined only by means of the data provided by an odometer and route map matching technique. The heading data provided by a gyro is not needed for calculation of train's position in this case. Thus 2D problem is reduced to 1D task and the error in position of the train mainly depends on the error of odometer. After the train arrives to the switch blade and continues in its movement towards the frog, the gyro data together with the odometer data are employed for train's position calculation and the decision process evaluating detection of train routing. The front of the switch blade is taken as the last verified initial point employed for estimation of train‘s position. Thus on the switch, 1D position determination problem is transferred to 2D routing detection one. However, the drift of the gyro introduces an error into the calculated train's position.

The gyro drift is mainly represented by time-dependent displacement of mean value of the heading probability density function. In final effect, the displacement results in degradation of 2D positional accuracy characterised by the corresponding standard deviations σ_x and σ_y . Then the routing detection process can be e.g. evaluated by means of the actual difference between the reference track axes of the switch and the probability distribution of the estimated position. It has been already experimentally demonstrated [3] that both more expensive FOG and cheap piezo gyro can detect the routing process on the "dark" switch if a route map-matching technique is employed. It is not so

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critical, if the train rides the switch at a low speed (e.g. 8 -20 km/ hour) or passes it repeatedly several times, as it is common during shunting operations. However, the influence of very low speed (less than 8 km/ hour) on the routing decision process is still under investigation. Complete elimination of the gyro drift can be done by means of the *double* heading differences and accurate track axis map, as it has been already proposed in 2.1.1. An error in 2D position of the train due to the place of TPL installation on the locomotive body, the locomotive dimensions and design can be reduced by re-computing of the original position generated by the TPL.

3 On-board sensor data validation on "dark" track

Future applications of GNSS/INS based TPL in signalling require highly reliable data provided by on-board sensors. Therefore, the entire sensor data validation becomes crucial. This requirement is mainly essential in a such TPL operational mode where the absolute positioning fails due to the absence of GNSS SIS and other track-side train position determination systems can not be applied due to their high cost.

3.1 Gyro data validation on "dark" track

A basic tool for gyro data validation seems a very precise reference track axis map surveyed with a centimetre level accuracy. As it has been already mentioned above, no gyro data is needed for train position calculation on the track out of a switch area. However, the gyro data is needed for its entire validation. Continuous evaluation of the *double* heading angle differences, introduced in 2.1.1, provides the relevant information on the gyro health status. More complicated case happens after the train arrives to the switch and the routing decision process must be performed. No doubt it is also desirable to provide the gyro data validation during the train routing process on the switch after the single-track section is splinted into two tracks. A precise map of the switch is employed in such a way that the heading differences are simultaneously propagated along the both reference trajectories in dependence on the travelled distance. If the measured data doesn't fit neither first nor second reference trajectory of the switch, a fatal error in the gyro data is indicated.

Another gyro data validation technique based on on-board detection of the characteristic features of the switch has been experimentally investigated [3]. The position of switch blades, guide rail or switch frog provides additional information on the real heading of a vehicle. These switch elements can be detected e.g. by eddy current, laser or other smart sensors. However, an influence of the environmental effects such as snow, rain, dust or dirt must be investigated.

3.2 On-board odometric data validation

It is evident that the error of the axle odometer significantly influences the error in train's position and thus also the reliability of routing detection on the switch.

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It is known that the odometric error can achieve as much as 5% of the travelled distance due to a wheel slip or slide under the most unfavourable conditions (rain, snow, glaze on the rail, etc.). This odometric error can be e.g. greatly reduced by a system, which integrates two axle odometric sensors installed on different locomotive axles and an accelerometer. Additional non-contact sensors such as microwave Doppler, eddy current or laser speedometers can also improve the reliability and the performance of the travelled distance measurement system significantly.

3.2.1 Semi-deterministic method

Another possibility how to effectively validate the odometric data is by means of 2D semi-deterministic method. This method is based on knowledge of the exact positions on the track where the train changes its direction of movement – in the track curves and on switches. These significant points are included in the precise route map. The dead-reckoned trajectory of the train computed by means of the odometric and the gyro data is continuously compared with the real position of train estimated by means of the odometric data and the exact reference trajectory stored in the on-board computer. In case that the shapes of these two trajectories do not match each other in a given time period, the odometric data are corrected (position in the route map is shifted) in such a way, until these both trajectories match each other again. This technique seems very efficient mainly for low-density lines with a number of track curves and limited GNSS SIS availability.

3.2.2 Probability approach

The probability approach employs differences of subsequent heading measurement provided by a gyro similarly as in case of routing detection described in 2.1.3. When the train rides on the straight track, the differences oscillate around zero. When a curve or a change in direction of movement on a switch occur, the mean value of the measured heading differences diverts from zero and corresponds to the differences computed from the route map. The place where the heading differences start to divert means the beginning of the curve.

3.2.3 Low-cost track marks

In case that there is no curve on the track and thus the above SW methods for validation of the odometer fail, then the odometer can be calibrated by means of low-cost track marks. For example, the same guide rail employed for gyro data validation on the switch can be used for odometer calibration on a “dark” track.

4 Experiments & results

The train position determination and routing detection experiments on a “dark” track have been performed by means of 3 kV DC electric locomotive and the remotely controlled track rover. While the locomotive was used for tests at an

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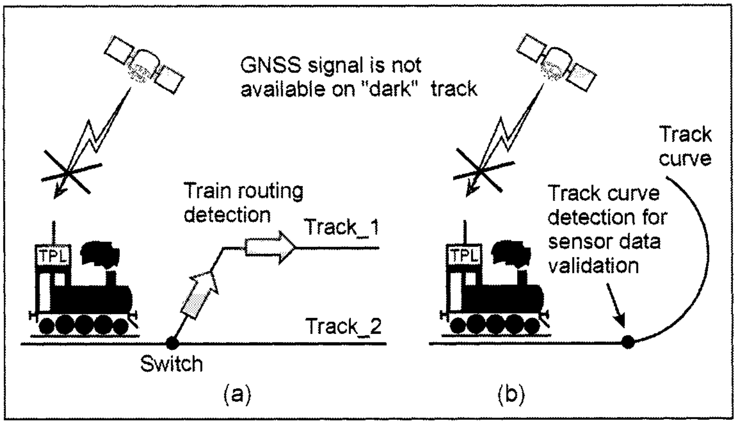

Figure 2: Field trials: (a) Detection of routing, (b) Sensor data validation.

ordinary operational speed ranging from 10 to 100 km/ hour, the track rover with very precise control of its movement by steppers was employed for tests at low speed (below 10 km/hod) when influence of a gyro drift was investigated. The reference trajectory was generated by the DGPS RTK method with a cm level accuracy. Two basic experiments on a "dark" track are presented in this paper: 1) Routing detection on a switch and 2) Track curve detection for sensor data validation, as shown in Fig. 2.

A diagram in Fig. 3 shows the relation among the heading differences and the travelled distance within the routing detection experiment. The data was recorded on a switch with a crossing angle of 7° 46'03'' at speed of 40 km/hour. The locomotive was passing the switch in the deflection direction. Zero value on the travelled distance axis in this and a next diagram means the front of the switch blade. From this diagram is evident that the heading differences measured by the KVH FOG and the reference heading differences computed by means of

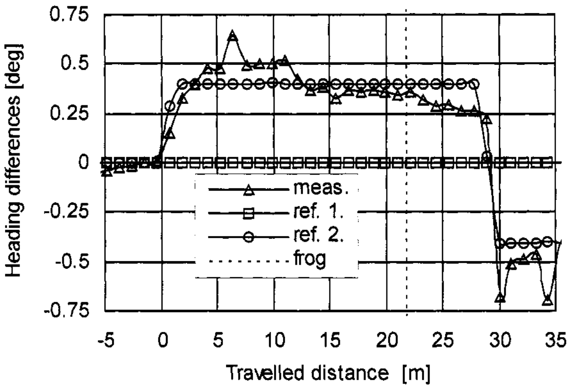

Figure 3: The measured and the reference heading differences on the switch.

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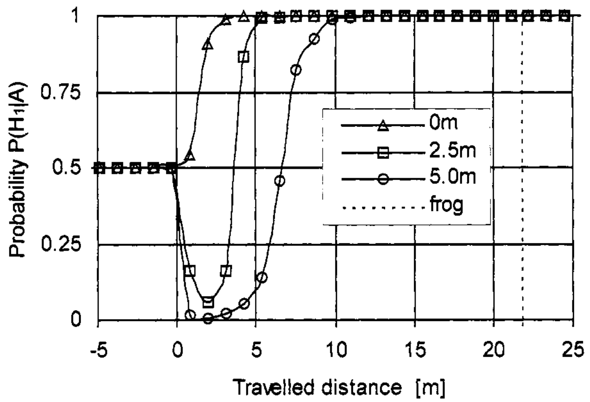

Figure 4: Probability of routing detection on the switch.

the precise map match well. Further, the reference heading data in the straight direction of movement can be employed for the sensor data validation process in order to detect a fatal error in heading measurement. The odometric data was compared with the reference RTK data and no error in it was observed in this case.

The same heading difference data shown in Fig. 3 was employed for evaluation of the routing detection process by means of the conditional probabilities and Bayes' theorem according to the eqn. (5)-(7). A diagram in Fig. 4 shows three computed probabilities $P(H_1|A)$ of routing v.s. travelled distance on the switch for the following error modes of the odometer: a) the real recorded odometric data – i.e. without an error, b) the intentionally introduced error of 2.5 meters and c) the introduced error of 5 meters. According to the error mode a) $P(H_1|A)$ achieved value of 0.99999 after the locomotive travelled a distance of 11.7 meters from the front of switch blade. The same levels in modes b) and c) were achieved for the travelled distances of 13.9 and 17.4 meters, respectively. These results confirm the fact the routing decision process depends on the performance of odometric system. Since the distance between the front of the switch blade and the frog is 21.8 m, the above specified probability level is achieved before the train arrives to the frog. Therefore the odometric error of +/- 5 meters seems also acceptable for the routing decision process.

In a diagram in Fig. 5 there are compared the measured and the computed heading differences versus the travelled distance at the beginning of the track curve with a radius of 250 meters. The beginning of the track curve can be determined with an accuracy of about +/-1 metre in this case. Experimental detection of track curves with larger radii including cubic transition curves is currently under investigation. The preliminary estimations and the initial experiments indicate that 2D semi-deterministic approach seems more efficient for detection of track curves with larger radii. However, 2D approach is effected by a drift of a gyro which must be compensated by a drift model, e.g. by a shaping filter.

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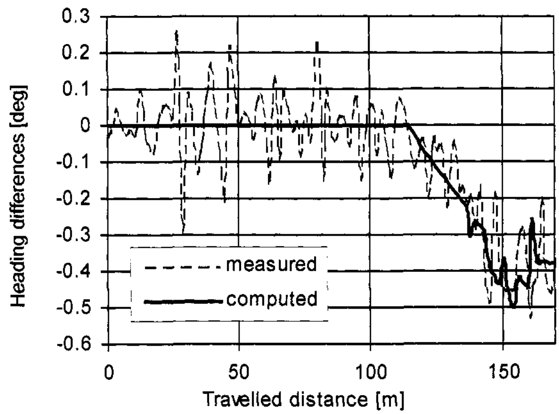

Figure 5: The heading differences at the beginning of the track curve.

5 Conclusion

In this paper, detection of train routing on a "dark" switch based on Bayes' theorem has been demonstrated. The conditional probabilities included in this theorem have been computed by means of the *double* heading differences, which completely eliminate a drift of a gyro. The influence of the odometric error in the routing decision process has been evaluated. On-board detection of track curves and turnouts has been proposed for sensor data validation without need of extra hardware. A robust multi-sensorial odometric system and a precise route map are crucial for future GNSS/INS based safety related applications on "dark" tracks.

Acknowledgement

Supported by Grant Agency of Czech Republic under contract No.102/02/0641.

References

[1] Filip, A., Mocek, H., Bažant, L., GPS/GNSS based train positioning for safety critical applications. *Signal und Draht*, pp. 16-21, pp.51-55. May 2001 (in English and German).
[2] Filip, A.., Global Navigation Satellite System in Low-Cost Signalling. *Fourth International Conference on Communications-Based Train Control*, Washington Marriott, Washington, D.C., May 8-9, 2001.
[3] Filip, A., Bažant, L., Mocek, H., Taufer, J. & Maixner, V., GNSS-1/INS based train positioning trials in „dark" area, *NavSat conference*, Nice Acropolis, France, Nov. 13-15, 2001.