Chapter 3

Dynamic Resonant Tunneling

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Additional information is available at the end of the chapter

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Abstract

The physics of dynamic resonant tunneling is investigated.

First, the resonant tunneling effect through an opaque barrier via a delta-function well is illustrated. Then, it is shown that, even in the adiabatic regime, where the dynamics can be governed by an analytic solution, the particle can be activated to higher energies. If the well varies quickly enough that the particle cannot escape from the well during the energetic elevation, the activation can be enhanced, as was anticipated by Azbel. However, and this is the main result of this work, the quasi-bound state of the well can even "reduce" the activation. In fact, because the resonant energy of the well matches twice the incoming particle's energy, and if the contribution to the wave function from both parts destructively interferes, then the particle cannot dwell in the well and activation is suppressed.

This effect can be utilized in frequency-controlled transistors, and it is even speculated that it may explain the reason that humans can distinguish between tens of thousands of different odors with merely few hundreds of odor receptors.

Lastly, the short time dynamics of a very fast perturbative well is also discussed.

Keywords: Resonant Tunneling, Dynamic Tunneling, Vibrational Tunneling, Odor detection, Olfactory, Forbidden activation, Selected Activation

1. Introduction

Resonant tunneling is a fascinating quantum phenomenon. It manifests the ability of quantum particles to pass with high probability through an opaque barrier by traveling via a semibound state [1–3].

The high sensitivity of the current on the bound-state parameters suggested harnessing this effect to heterostructure devices in general and transistors in particular [4–10].

Resonant tunneling is usually described as a one-dimensional (1D) phenomenon; however, resonant tunneling in higher number of dimensions was also investigated (see, for example, [11, 12]). Nevertheless, because the main features of resonant tunneling appear in 1D, most of the research was concentrated on the simplest 1D systems.

Tunneling and resonant tunneling are rarely stationary processes. They are affected by thermal noises, and clearly, the accumulation of particles in the bound state varies the potential. It is well known that tunneling in the presence of an oscillating barrier can cause activation (higher energy) and therefore can increase substantially the tunneling current.

This phenomenon was investigated in electronics [13], nanotechnology [14–16], the foundations of quantum mechanics [17–33], and even biology and biochemistry [34–39].

The resonant tunneling effect occurs when the incoming particle's energy coincides with the eigenenergy of the quasi-bound state. In case the barrier is very opaque, the particle remains inside the well at the quasi-bound state for exponentially long time. Therefore, when the particle is quasi-trapped inside the well, its state has to vary with the changes in the well, and its energy varies with the eigenenergy of the quasi-bound state because it does not have the time to escape from the well. Therefore, it was conjectured (see refs. [17, 20]) that a decrease in the perturbation time-scale will enhance the activation. However, not in any energy the particle can remain within the well. Destructive interference can prevent particle trapping and therefore suppress particle activation [23, 33]. In the next several sections, we will elaborate on the delicate structure of these effects.

2. Stationary tunneling

Let us begin with the propagation of a quantum particle through an opaque but stationary barrier. The Schrödinger equation is then

$$-\frac{\partial^2}{\partial x^2}\psi(x,t)+U(x)\psi(x,t)=i\frac{\partial\psi(x,t)}{\partial t} \tag{1}$$

Hereinafter for simplicity, we adopt the units, where the electron mass is half and the reduced Planck constant is unity (i.e., m=1/2 and $\hbar$=1).

In the stationary case (i.e., when the potential is time independent), there is no change in the incoming particle's energy. For any incoming energy ω, the generic stationary solution looks like

$$\psi(x,t)=\varphi_{\omega}(x)\exp(-i\omega t) \tag{2}$$

where $\varphi_\omega(x)$ are the solutions of the stationary Schrödinger equation:

$$-\frac{\partial^2}{\partial x^2}\varphi_\omega(x)+\left[U(x)-\omega\right]\varphi_\omega(x)=0\,. \tag{3}$$

Therefore, every solution can be written as a superposition of incoming $[\varphi_\omega^+(x)]$ and outgoing $[\varphi_\omega^-(x)]$ solutions, where

$$\begin{aligned}\varphi_\omega^+(x)&=\begin{cases}\exp\left(i\sqrt{\omega}x\right)+r_\omega\exp\left(-i\sqrt{\omega}x\right) & x\to-\infty\\ t_\omega\exp\left(i\sqrt{\omega}x\right) & x\to\infty\end{cases}\\ \varphi_\omega^-(x)&=\begin{cases}t_\omega\exp\left(-i\sqrt{\omega}x\right) & x\to-\infty\\ \exp\left(-i\sqrt{\omega}x\right)+r_\omega\exp\left(i\sqrt{\omega}x\right) & x\to\infty\end{cases}\end{aligned} \tag{4}$$

Physically, $\varphi_\omega^+(x)$ and $\varphi_\omega^-(x)$ stand for beams of particles coming from the left and right respectively. $\boldsymbol{t}_\omega$ and $\boldsymbol{r}_\omega$ are the transmission and reflection coefficients of the barrier for energy ω, respectively. In the WKB approximation (see, for example, ref. [19]), the transmission coefficient can be evaluated as

$$t_\omega\sim\exp\left(-\int_{x_L}^{x_R}\left[U(x)-\omega\right]dx\right) \tag{5}$$

where x_L and x_R are the left and right boundaries of the barrier. For a rectangular barrier, an exact expression can be derived [19]:

$$t_\omega=\frac{\exp(-2ikL)}{\cosh(2KL)+\frac{1}{2}i\left(K/k-k/K\right)\sinh(2KL)} \tag{6}$$

where $k\equiv\sqrt{\omega}$ and $K\equiv\sqrt{U-\omega}$. Therefore, when the barrier is opaque [i.e., $\sqrt{U-\omega}L$ >>1 (high and/or wide)], the transmission is exponentially small.

3. Resonant tunneling via a delta-function well

Let us introduce a delta-function well in the barrier (at $x = x_0$; in **Figure 1**, x_0=0). Then, the Schrödinger equation is

$$-\frac{\partial^2}{\partial x^2}\varphi_\omega(x)+\left[U(x)-\omega-f_0\delta(x-x_0)\right]\varphi_\omega(x)=0 \tag{7}$$

It is convenient to use the outgoing Green function $G_\omega^+(x,x_0)$, which is a solution of the equation:

$$-\frac{\partial^2}{\partial x^2}G_\omega^+(x,x_0)+\left[U(x)-\omega\right]G_\omega^+(x,x_0)=\delta(x-x_0) \tag{8}$$

with the boundary conditions:

$$G_\omega^+(x,x_0)\sim\exp\left(i\sqrt{\omega}|x-x_0|\right)\text{ for }|x|\to\infty\,. \tag{9}$$

Therefore, the outgoing Green function reads

$$G_\omega^+(x,x_0)=\begin{cases}\dfrac{\varphi_\omega^-(x_0)\varphi_\omega^+(x)}{\varphi_\omega^+(x_0)\varphi_\omega^{-\prime}(x_0)-\varphi_\omega^-(x_0)\varphi_\omega^{+\prime}(x_0)} & x>x_0\\ \dfrac{\varphi_\omega^-(x)\varphi_\omega^+(x_0)}{\varphi_\omega^+(x_0)\varphi_\omega^{-\prime}(x_0)-\varphi_\omega^-(x_0)\varphi_\omega^{+\prime}(x_0)} & x<x_0\end{cases}. \tag{10}$$

Using the Green function, we can easily construct a solution for the wave equation with the combined potential (the barrier with the delta-function well). In which case, the solution reads

$$\varphi_\omega(x)=\varphi_\omega^+(x)+\frac{f_0\varphi_\omega^+(x_0)}{1-f_0G_\omega^+(x_0,x_0)}G_\omega^+(x,x_0)\,, \tag{11}$$

which for $x>x_0$ is

$$\varphi_\omega(x)=\frac{\varphi_\omega^+(x>x_0)}{1-f_0G_\omega^+(x_0,x_0)}=\frac{\varphi_\omega^+(x>x_0)}{1-f_0/\left(\varphi_\omega^{-\prime}(x_0)/\varphi_\omega^-(x_0)-\varphi_\omega^{+\prime}(x_0)/\varphi_\omega^+(x_0)\right)}.$$

In case of a rectangular barrier, i.e.,

$$U(x)=\begin{cases}U & |x|\le L\\ 0 & |x|>L\end{cases} \tag{12}$$

then

$$\varphi_\omega^+(x>L)=t_\omega\exp(ikx) \tag{13}$$

where $2L$ is the width of the barrier, and t_ω is taken from (6), i.e.,

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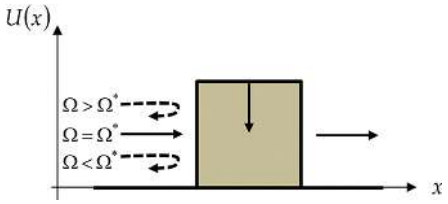

Figure 1. In a stationary resonant tunneling process, only when the particle's energy is equal to the quasi-eigenstate energy can the particle penetrate the barrier with high probability.

$$\varphi_{\omega}(x) = i\frac{\exp\left(ik(x-2L) - i\arctan\left[\left(K^2-k^2\right)/2kK\right]\right)}{1+i\left(1-\frac{f_0}{2K}\right)/4\left(\frac{kK}{U}\right)\exp(-2KL)}. \tag{14}$$

At the resonance $\omega=\Omega^*=U-f_0^2/4$, $T=|t_{\omega}|^2=1$, (see **Figure 2**).

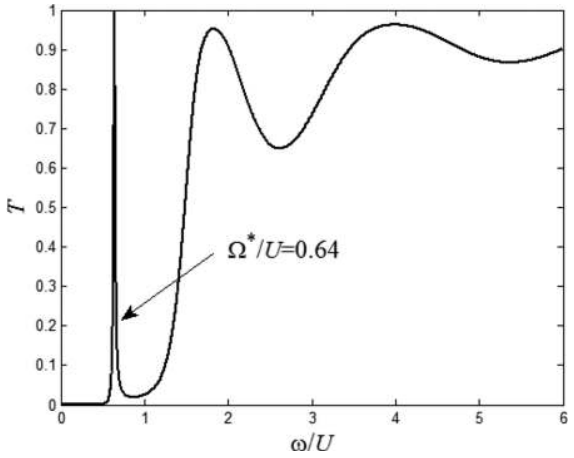

Figure 2. Transmission $T=|t_{\omega}|^2$ as a function of the particles energy ω for the parameters $U=1$, $L=4$ and $f_0=1.2$.

4. Adiabatic transition

Now, let us take a varying potential well,

$$f(t/\tau) = \frac{\lambda_0}{\tau}\exp\left[-(t/\tau)^2\right]. \tag{15}$$

In the adiabatic approximation, τ is longer than any other time-scale of the problem. The relevant time-scale (i.e., the longest one) is the resonance time or the dwelling time of the resonant state. Therefore, in the adiabatic approximation,

$$\tau U >> \exp\left(2\sqrt{U-\omega}\left(L-|x_0|\right)\right) \tag{16}$$

the potential can be regarded as stationary, and the wavefunction simply reads

$$\varphi_\omega(x)=\varphi_\omega^+(x)-\frac{\varphi_\omega^+(x_0)}{1-\tau\exp\left[(t/\tau)^2\right]/\left[\lambda_0 G_\omega^+(x_0,x_0)\right]}\frac{G_\omega^+(x,x_0)}{G_\omega^+(x_0,x_0)}\,. \tag{17}$$

The resonance energy of this system varies in time $\Omega^*(t)=U-f^2(t/\tau)/4$; therefore, large transmission occurs for $\omega=\Omega^*(t)$ and, in principle, can be as high as 1 when the well is located at the center of the barrier, i.e.,

When the lowest eigenenergy of the well

$$\Omega_{\min}^* \equiv \min\left\{\Omega^*(t)\right\}=U-f^2(0)/4=U-\lambda_0^2/4\tau^2 \tag{18}$$

is higher than the incoming energy ω, then the effect of the varying well is negligible; however, if the lowest eigenenergy of the well is lower than the incoming energy ω, then the eigenstate crosses the incoming energy twice. At the vicinity of the crossing time $t\cong t_0=\pm\tau\sqrt{\ln\left(\frac{\lambda_0}{2K\tau}\right)}$, the solution has the Lorentzian shape:

$$\psi(x,t)=i\frac{\exp\left(ik(x-2L)-i\arctan\left[\left(K^2-k^2\right)/2kK\right]-i\omega t\right)}{1\mp it_0(t-t_0)/2\tau^2(kK/U)\exp(-2KL)}\,. \tag{19}$$

5. The general scenario

In principle, in the adiabatic approximation, the outgoing energy is equal to the incoming energy (i.e., $\omega_{out}=\omega_{in}\pm\tau^{-1}$), where τ^{-1} should be exponentially small $\tau^{-1}<<U\exp\left(-2\sqrt{U-\omega}(L-|x_0|)\right)$. However, in practice, the situation can be quite different due to the exponential decay of low energies in the tunneling process.

In general, the generic Schrödinger equation (see refs. [17, 33])

$$-\frac{\partial^2}{\partial x^2}\psi(x,t)+\left[U(x)-f(t/\tau)\delta(x-x_0)\right]\psi(x,t)=i\frac{\partial\psi(x,t)}{\partial t} \tag{20}$$

can be solved by a superposition of solutions of the type (11), namely,

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$$\psi(x,t)=\varphi_{\Omega}(x)\exp(-i\Omega t)+\int_{-\infty}^{\infty}d\omega a(\omega)G_{\omega}^{+}(x,x_0)\exp(-i\omega t) \tag{21}$$

where it is taken that the incoming energy is Ω.

By substituting solution (21) into Eq. (20) (and after spatial integration), the solution can be reduced to the integral equation [17]:

$$-\frac{1}{2\pi}\varphi_{\Omega}(x_0)f(\Omega-\omega)+a(\omega)-\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega''\,f(\omega''-\omega)a(\omega'')G_{\omega''}^{+}(x_0,x_0)=0 \tag{22}$$

where $f(\omega)=\int_{-\infty}^{\infty}dtf(t/\tau)\exp(-i\omega t)$ is the Fourier transform of the perturbation's amplitude.

6. Adiabatic and slow variations

In general, due to the complex structure of the Green function, this is a complex integral equation; nevertheless, as long as the spectrum of the function is mainly concentrated near the incoming energy Ω, i.e., $|a(|\omega-\Omega|\tau>>1)|<<|a(\omega=\Omega)|$, the contribution to the integral of the components $|\omega-\Omega|\tau>>1$ is negligible; therefore, we can replace $G_{\omega}^{+}(0)$ with $G_{\Omega}^{+}(0)$. In this case, the integral equation reduces to

$$-\frac{1}{2\pi}\varphi_{\Omega}(x_0)f(\Omega-\omega)+a(\omega)-\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega''\,f(\omega''-\omega)a(\omega'')G_{\Omega}^{+}(x_0,x_0)=0 \tag{23}$$

which is merely a convolution equation; therefore, the inverse Fourier transform of the solution

$$a(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega a(\omega)\exp(i\omega t) \tag{24}$$

obeys [24, 33]

$$a(t)=-\frac{\varphi_{\Omega}(x_0)f(t/\tau)\exp(i\Omega t)}{1-f(t/\tau)G_{\Omega}^{+}(x_0,x_0)}\,. \tag{25}$$

Clearly, resonance occurs when $f(t/\tau)\Re G_{\Omega}^{+}(x_0,x_0)=1$. We will see in the next sections that nontrivial effects occur when $f(0)\Re G_{\Omega}^{+}(x_0,x_0)>1$. In this regime, it is invalid to substitute G_{Ω}^{+}

(x_0,x_0) for $G_\omega^+(x_0,x_0)$. However, in the $f(0)\Re G_\Omega^+(x_0, x_0)<1$ (where it is taken that $f(0) = \max\{f(t/\tau)\}$) regime, this approximation is still valid.

It is clear, for example, that, when $f(0)\Re G_\Omega^+(x_0, x_0)<<1$, which is equivalent to the adiabatic regime,that

$$a(t)\cong -\varphi_\Omega(x_0)f(t/\tau)\exp(i\Omega t), \tag{26}$$

and thus

$$a(\omega)\cong -\frac{1}{2\pi}\varphi_\Omega(x_0)f(\omega-\Omega),$$

which means that the spectrum broadening is exactly similar to the spectrum of the perturbation; in which case the solution is simply

$$\psi(x,t)\cong \varphi_\Omega(x)\exp(-i\Omega t)-\frac{\varphi_\Omega(x_0)}{2\pi}\int_{-\infty}^{\infty}d\omega f(\omega-\Omega)G_\omega^+(x,x_0)\exp(-i\omega t). \tag{27}$$

7. Activation

From Eq. (27), it is evident that elevation to higher energies is still a possibility even in the adiabatic and slowly varying cases, and because the Green function $G_\omega^+(x,x_0)$ increases with the energy, there is still a possibility that the mean exit energy will be considerably higher than the incoming one. The outcome depends only on the specific functional shape of the perturbation spectrum.

More importantly, when the perturbation becomes more energetic and τ decreases so that $f(0)\Re G_\Omega^+(x_0, x_0)$ approaches 1 from below (i.e., there is still no intersection), the adiabatic Eq. (25) is still approximately valid. In this case, however, one can take

$$f(t/\tau)\cong(\lambda_0/\tau)\left[1-(t/\tau)^2\right]. \tag{28}$$

Then, after substituting Eq. (28) in Eq. (25) and both of them in Eq. (24),

$$\begin{aligned} a(\omega) &= -\varphi_\Omega(x_0)\frac{\lambda_0}{\tau}\int\frac{\exp(-i(\omega-\Omega)t)}{1-(\lambda_0/\tau)G_\Omega^+(x_0,x_0)+(t/\tau)^2}dt = \\ &-\varphi_\Omega(x_0)\lambda_0\sqrt{\frac{\pi}{2}}\exp\left(-\sqrt{1-\beta}\tau|\omega-\Omega|\right) \end{aligned} \tag{29}$$

where $\beta \equiv (\lambda_0/\tau) G_{\Omega}^{+}(x_0, x_0)$. Therefore,

$$\psi(x,t) \cong \varphi_{\Omega}(x)\exp(-i\Omega t) - \varphi_{\Omega}(x_0)\lambda_0\sqrt{\frac{\pi}{2}}\int_{-\infty}^{\infty} d\omega \exp\left(-\sqrt{1-\beta}\tau|\omega-\Omega|\right) G_{\omega}^{+}(x,x_0)\exp(-i\omega t). \tag{30}$$

Now, because the Green function can be written (beyond the barrier) approximately as

$$G_{\omega}^{+}(x > L, x_0) \sim \exp\left(-\sqrt{U-\omega}(L-x_0)\right) \tag{31}$$

then the exponent in the integrand, which can be regarded as an approximate evaluation of the spectrum of the outgoing wavefunction, consists of two main terms (two peaks):

$$\exp\left(-\sqrt{1-\beta}\tau|\omega-\Omega| - \sqrt{U-\omega}(L-x_0)\right) \tag{32}$$

The first peak occurs around the incoming energy Ω (suppressed activation) and the second one occurs around the barrier's height U (activation).

Each one of these peaks has a different height. The higher one will determine whether activation will occur. Therefore, the probability to tunnel through the barrier and to exit with energy ω_{out} is approximately [33]

$$P(\omega_{out}) \propto \left|a(\omega_{out}) G_{\omega_{out}}^{+}(x > L)\right|^2 \propto \exp\left(-2\sqrt{1-\beta}\tau|\omega_{out}-\Omega| - 2\sqrt{U-\omega_{out}}(L-x_0)\right) \tag{33}$$

which means that the activation probability peak is proportional to

$$P(\omega_{out} \cong U) \propto \exp\left(-2\sqrt{1-\beta}\tau|U-\Omega|\right) \tag{34}$$

whereas the inactivation probability peak (i.e., the probability for suppressed activation) goes like

$$P(\omega_{out} \cong \Omega) \propto \exp\left(-2\sqrt{U-\Omega}(L-x_0)\right). \tag{35}$$

In **Figure 3**, we demonstrate the fact that the spectrum is governed by two maxima (peaks) and that Eq. (33) is a good approximation to the numerical solution at the vicinity of these maxima. Therefore, activation occurs when

$$P(\omega_{out} \cong U) \propto \exp\left(-2\sqrt{1-\beta}\tau|U-\Omega|\right) > P(\omega_{out} \cong \Omega) \propto \exp\left(-2\sqrt{U-\Omega}(L-x_0)\right). \tag{36}$$

This occurs approximately at the time scale

$$\tau_T \cong \frac{\lambda_0 + \sqrt{\lambda_0^2 + 16L^2}}{4\sqrt{U-\Omega}}, \tag{37}$$

or, for a given τ, the transition occurs for the following incoming energy:

$$\Omega_T \cong U - \frac{2\lambda_0\left(\lambda_0 + \sqrt{\lambda_0^2 + 16L^2}\right) + 16L^2}{16\tau^2}. \tag{38}$$

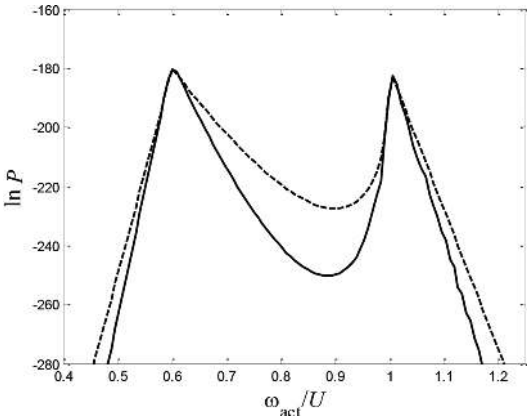

Figure 3. Comparison between the logarithm of the exact numerical solution (solid line) and the analytical approximation (dashed line), i.e., Eq. (33), for $\lambda_0=100, L\sqrt{U}=150, \ \Omega/U=0.6=\Omega_T, \tau=\tau_T=280U$.

Clearly, this energy is lower than the minimum resonance energy

$$\Omega_T < \Omega_{\min}^{*} = U - \frac{\lambda_0^2}{4\tau^2}. \tag{39}$$

In **Figure 4**, the dependence of the spectrum on the transition time-scale τ is presented for three different values: below τ_T where activation prevails, above τ_T when simple tunneling wins, and when they are equal, and the outgoing particle's spectrum has two equally probable outgoing energies.

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It should be stressed that, because these peaks are exponentially narrow, the transition is extremely abrupt (i.e., the process resembles a phase transition). The identification of the process as a phase transition was first suggested by Azbel [17].

However, it was wrongly assumed that, if the particle's incoming energy matches the quasi-eigenstate energy, then an eigenstate-assisted activation (EAA) effect occurs (i.e., if $\Omega>\Omega_{min}$*, then activation will definitely increase). In fact, it will be shown that this process is more complicated, and at some energies (above Ω_{min}*), activation is "totally" suppressed.

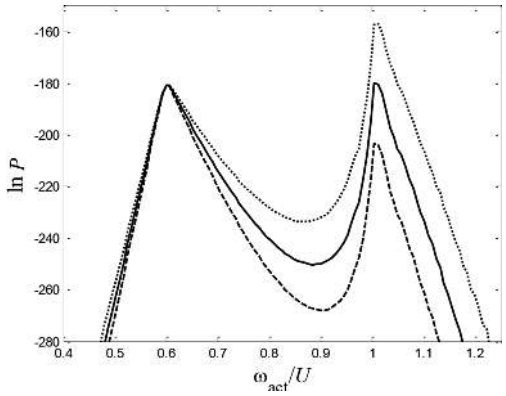

Figure 4. The (logarithm of) the exit probability as a function of the activation energy ω_{act} for the parameters λ_0=100, $L\sqrt{U}$=150, Ω/U=0.6, for three different perturbation time scales: $\tau = \tau_T$ (solid line), $\tau = \tau_T + 20$ (dashed line) and $\tau_T - 20$ (dotted line).

8. Selected elevations and forbidden activations

For $\Omega>\Omega_{min}$*, the spectrum's shape becomes more complicated. Instead of only two peaks, it has a more complex structure. There is a clear difference between the $\omega_{act}<\Omega$ (i.e., the under-activated regime) and the $\omega_{act}>\Omega$ (i.e., the activated one). The former oscillates as a function of ω_{act} but almost independent of the incoming Ω, whereas the latter oscillates as a function of the incoming Ω but has a mild dependence on the outgoing ω_{act}. In **Figure 5**, there is an illustration of this behavior, where a small change in the incoming particle's energy has an enormous effect on the activated regime. In **Figure 6**, a numerical example illustrates this phenomenon where a ~6.7% change in the incoming energy made a dramatic change from full activation to suppressed one.

As a consequence, it is clear that, for specific incoming particle's energies, the entire activated part of the spectrum is suppressed. To illustrate this point, we define the mean activated energy

$$\left\langle \omega_{act} \right\rangle \equiv \frac{\int \omega_{act} P(\omega_{act}, \Omega) d\omega_{act}}{\int P(\omega_{act}, \Omega) d\omega_{act}} \tag{40}$$

where $P(\omega_{act}\Omega)$ is the probability of an incoming particle with energy Ω to exit the barrier with the energy ω_{act}.

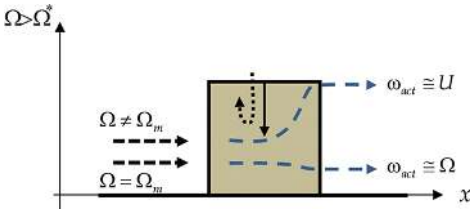

Figure 5. Schematic illustration of the suppressed activation. For most energies, activation occurs (i.e., $\omega_{act} \cong U$); however, for the specific energies (i.e., $\Omega=\Omega_m$), activation is suppressed and $\omega_{act} \cong \Omega = \Omega_m$.

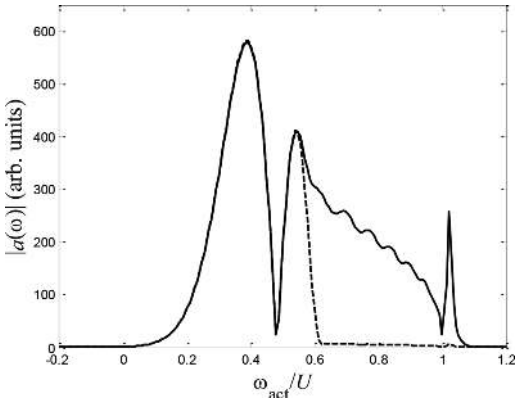

Figure 6. Absolute value of the transmission coefficient $a(\omega)$ as a function of the activation energy ω_{act}. The dashed curve corresponds to the case Ω/U = 0.6 and the solid line corresponds to Ω/U = 0.56. The other parameters are $\lambda_0=100,\ L\sqrt{U}=10,\ \tau U=60.6$.

In **Figure 7** the mean activation energy $<\omega_{act}>$ is plotted as a function of the perturbation time scale τ, and in **Figure 8**, $<\omega_{act}>$ is plotted as a function of the incoming particle's energy Ω. It is clearly seen that activation ($<\omega_{act}> \cong U$) occurs mainly below $\tau<\tau_T$. However, even below this time-scale, there are specific values of τ, for which activation is suppressed (i.e., $<\omega_{act}> \cong \Omega$). Similarly, activation occurs $<\omega_{act}> \cong U$ mainly above $\Omega>\Omega_T$; however, even in the activation regime, there are specific energies for which $<\omega_{act}> \cong \Omega$ (i.e., suppressed activation).

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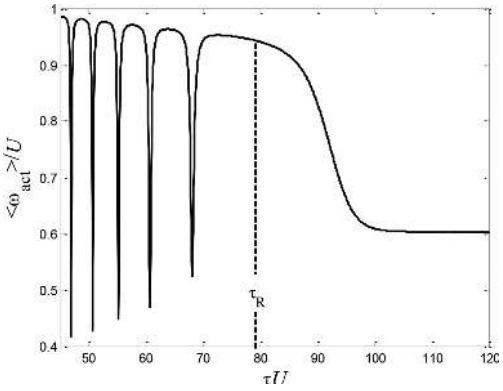

Figure 7. Mean activation energy as a function of the time-scale τ for the parameters: $\Omega/U = 0.6$, $L\sqrt{U} = 6$, $\lambda_0 = 100$.

When $\tau<\tau_T$, two important things occur: (1) At two specific times, the particle's incoming energy is equal to the eigenenergy of the quasi-bound state of the varying well. (2) The well varies quickly enough so the particle has no time to escape from the well.

As a consequence of these two, the particle's state changes with the well's eigenstate; therefore, it is easier to excite the particle energetically. That was the logic that led Azbel to predict the EAA effect. Indeed, this effect does occur, and it is clearly seen (see **Figure 4**) that, when $\tau<\tau_T$, then, for most values of τ, the spectrum's energy is concentrated around the barrier's height $\boldsymbol{U}$. However, this process cannot last if the particle cannot dwell inside the quasi-bound state. This event occurs when there is destruction interference inside the well.

Had it been a stationary eigenstate with an eigenenergy Ω_0 the eigenstate would accumulate a linear phase [i.e., $\exp(-i\Omega_0 t)$].

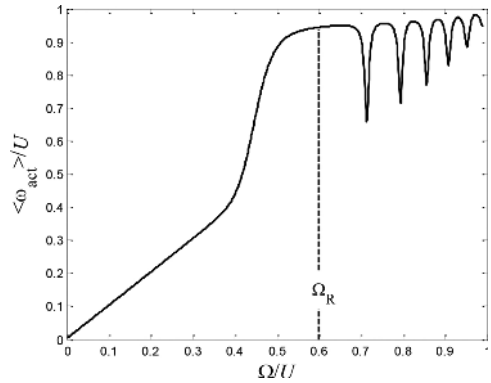

Figure 8. Mean activation energy as a function of the incoming energy Ω/U for $\tau U = 79$, $L\sqrt{U} = 6$, $\lambda_0 = 100$, Ω_R is the resonance energy

However, because the quasi-bound state evolves in time, it gains the integral

$$\exp\left(-i\int dt'\Omega^*(t')\right) \tag{41}$$

When the incoming energy is above the minimum eigenenergy (i.e., $\Omega>\Omega_{min}^*=U-\lambda_0^2/4\tau^2$), there are two times, in which $\Omega=\Omega^*(\boldsymbol{t}_1)=\Omega^*(\boldsymbol{t}_2)$ (see **Figure 9**), and due to the temporal symmetry of the perturbation $\boldsymbol{t}_1=-\boldsymbol{t}_2$. Therefore, the particle has two options to be temporally bounded to the quasi-eigenstate: it can either begin at $\boldsymbol{t}_1$ and gain the phase $\exp\left(-i\int_{t_1}^{t} dt'\Omega^*(t')\right)$ or at $\boldsymbol{t}_2$ and gain the phase $\exp\left(-i\Omega(t_2-t_1)-i\int_{t_2}^{t} dt'\Omega^*(t')\right)$. If the two components are out of phase and a destructive interference occurs [33], i.e.,

$$\int_{t_1}^{t_2} dt'\left[\Omega^*(t')-\Omega\right]=\pi(2m+1) \text{ for } m=0,1,2,... \tag{42}$$

the particle cannot survive within the well, and activation is frustrated.

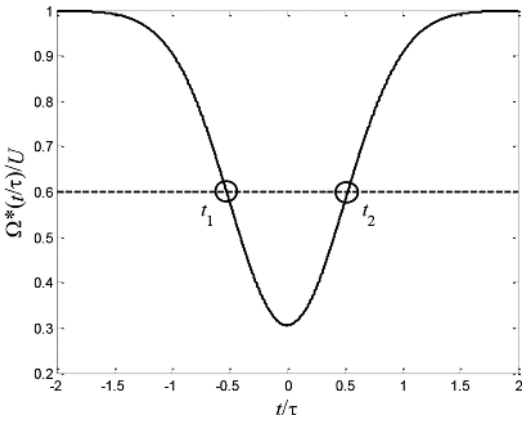

Figure 9. When the minimum of the resonance energy of the perturbation is lower than the incoming energy Ω, then the instantaneous resonance energy Ω^*t crosses the incoming energy twice (at $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$). For successful activation, the cumulative phase between these two events must be constructive.

In our case, at the vicinity of the parabola peak,

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$$\Omega^*(t) = U - \left(\frac{\lambda_0}{2\tau}\right)^2 \exp\left(-\frac{2t^2}{\tau^2}\right) \cong U - \left(\frac{\lambda_0}{2\tau}\right)^2 \left(1 - \frac{2t^2}{\tau^2}\right) \tag{43}$$

After substituting (43) into (42), the values of the forbidden energies Ω_m, for which destructive interference occurs and the activation is suppressed, are directly given

$$\Omega_m \cong \Omega^*_{\min} + \frac{1}{2}\left[\frac{3\lambda_0}{\tau^2}\left(m + \frac{1}{2}\right)\pi\right]^{2/3} \quad \text{for } m = 0,1,2,... \tag{44}$$

In each one of these energies, the activation is suppressed.

To determine these energies more accurately, we take advantage of the fact that, at the vicinity of the minimum Ω_{min}*, the instantaneous resonance energy has a parabola shape; therefore, any varying potential with the same parabola should have approximately the same suppressed energies. Therefore, we replace the Gaussian with a parabolic function, that is, we choose Eq. (28) for the perturbation, namely, $f(t/\tau) \cong (\lambda_0/\tau)(1 - t^2/\tau^2)$, then

$$f(\omega) \cong \int \frac{\lambda_0}{\tau}\left(1 - \frac{t^2}{\tau^2}\right)\exp(-i\omega t)dt = 2\pi\frac{\lambda_0}{\tau}\left[\delta(\omega) + \frac{1}{\tau^2}\delta''(\omega)\right]. \tag{45}$$

Therefore, the integral equation

$$-\frac{1}{2\pi}\varphi_\Omega(x_0) f(\Omega - \omega) + a(\omega) - \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega'' f(\omega'' - \omega) a(\omega'') G^+_{\omega''}(x_0, x_0) = 0 \tag{46}$$

reduces to the differential equation

$$\frac{d^2 s(n)}{dn^2} + s(n)\left[1 - \frac{1}{G^+_{\omega''}(x_0, x_0)}\frac{\tau}{\lambda_0}\right] = \varphi_\Omega(x_0)\lambda_0\delta(n) \tag{47}$$

where we used the dimensionless parameters $n=(\omega-\Omega)\tau$ and .

After linearization of the Green function, Eq. (47) can be approximated to

$$\frac{d^2 s(n)}{dn^2} + s(n)\left[1 - 2K\frac{\tau}{\lambda_0} + \frac{n}{\lambda_0 K}\right] = \varphi_\Omega(x_0)\lambda_0\delta(n) \tag{48}$$

where again $K \equiv \sqrt{U - \Omega}$.

The solution that maintain the boundary conditions that $s(n \to \infty) \to 0$ is

$$s(\xi) = -i\pi\varphi_{\Omega}(x_0)\lambda_0(2\lambda_0 K)^{1/3}\begin{cases} \mathrm{Ai}(-\xi)\left[\mathrm{Ai}(-\xi_0) + i\mathrm{Bi}(-\xi_0)\right] & \text{for} \quad \xi < \xi_0 \\ \mathrm{Ai}(-\xi_0)\left[\mathrm{Ai}(-\xi) + i\mathrm{Bi}(-\xi)\right] & \text{for} \quad \xi > \xi_0 \end{cases} \tag{49}$$

where $\xi \equiv (n + K(\lambda_0/\tau - 2K)\tau)/(2\lambda_0 K)^{1/3}$, $\xi_0 \equiv (K(\lambda_0/\tau - 2K)\tau)/(2\lambda_0 K)^{1/3}$ and Ai and Bi are the Airy functions [40].

Therefore, it is clear that activation is suppressed when

$$\mathrm{Ai}(-\xi_0) = 0\,, \tag{50}$$

which, in the slowly varying approximation (i.e., large τ), correspond to (see [40])

$$\cos^2\left[\frac{2}{3}\left(1 - \frac{2K}{\lambda_0}\tau\right)^{3/2} K\lambda_0 - \frac{\pi}{4}\right] = 0\,. \tag{51}$$

Therefore, the incoming energies for which $\Omega_{act} \cong \Omega$, and thus no activation occurs, are approximately (see **Figures 10** and **11**)

$$\Omega_m = \Omega^*_{\min} + \frac{1}{2}\left[\frac{3\lambda_0}{\tau^2}\left(m + \frac{3}{4}\right)\pi\right]^{2/3}. \tag{52}$$

Therefore, Eq. (42) should be rewritten more accurately as

$$\int_{t_1}^{t_2} dt'\left[\Omega^*(t') - \Omega\right] = \pi\left(2m + \frac{3}{2}\right), \tag{53}$$

like destructive interference condition in the WKB approximation (see, for example, ref. [19]). Eq. (53) can be applied to any varying potential whose temporal shape has minima.

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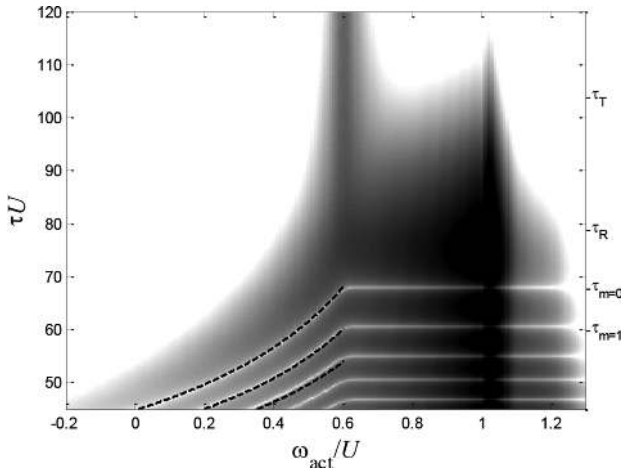

Figure 10. Approximate analytical expression of the forbidden time-scales (dashed curves) with the exact numerical solution of the probability density (the darker the spot the higher probability it represents). τ_m are the time scales for suppressed activations (52). τ_R is the minimum time scale $\tau_R \equiv -\lambda_0 G_\Omega^+(0)$, and τ_T is the transition time when activation wins. With the parameters: $\Omega/U = 0.6,\ \lambda_0 = 100,$ and $L\sqrt{U} = 10$.

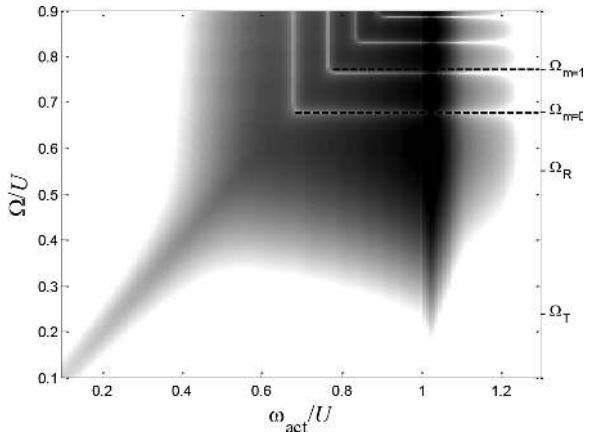

Figure 11. Presentation of the analytical suppressed activation energies Ω_m (dashed lines) on top of the numerical solution of the probability to be activated to energy ω_{act} (the darker the spot, the higher the probability it represents). Ω_T is the transition energy when activation wins, and Ω_R is the minimum resonant energy. The other parameters are $\tau U = 75,\ \lambda_0 = 100$ and $L\sqrt{U} = 10$.

Because (52) was derived for any potential, which can be approximated by a parabola [Eq. (28)], then the same conclusions and the same suppression of activation are valid to periodic potentials of the type

$$f(t/\tau)=\frac{\lambda_0}{\tau}\cos(2t/\tau)\delta(x-x_0)\,. \tag{54}$$

See ref. [23] for a more extensive study on these potentials; nevertheless, the suppressed activation energies (52) and (53) are still valid for these potentials.

9. Instantaneous changes

Lastly, we are going to investigate the scenario in which the perturbation appears instantaneously. In this case, the Schrödinger equation reads

$$-\frac{\partial^2\psi}{\partial x^2}-f_0u(t)\delta(x-x_0)\psi+U(x)\psi=i\frac{\partial\psi}{\partial t} \tag{55}$$

where $u(t)=\begin{cases}1 & t\geq 0\\ 0 & t<0\end{cases}$ is the Heaviside step function.

It was shown in ref. [41] that the effect of such a perturbation has a universal pattern in the short time domain and in fact depends only on the product of the strength of the delta-function (f_0 in this case) and the local value of the initial wavefunction. The idea is [41] that an instantaneous delta-function perturbation is equivalent to a discontinuity in the wavefunction, and it was proven elsewhere [42, 43] that such a discontinuity has, in the short time, a universal pattern even in the presence of potentials [44]. Therefore, if the initial state was

$$\psi(x,t<0)=\varphi_\Omega^+(x)\exp(-i\Omega t) \tag{56}$$

then, after the instantaneous perturbation, the short time dynamics is simply [41]

$$\psi(x,t)\cong\varphi_\Omega^+(x)\exp(-i\Omega t)-2\sqrt{\frac{-i}{\pi}}\frac{t^{3/2}}{(x-x_0)^2}\exp\left(i\frac{(x-x_0)^2}{4t}\right)f_0\varphi_\Omega^+(0)\,. \tag{57}$$

Because the first term is exponentially smaller than the second one, then very quickly the second term becomes dominant. In fact, it becomes dominant as early as

$$\frac{t^3}{(x-x_0)^4}\left(\frac{Uf_0^2}{U-\Omega}\right)\exp(K(L-x_0))>1\,. \tag{58}$$

If the delta-function well is turned on instantaneously but only for a period of 2τ, then the Schrödinger equation can be written as

$$-\frac{\partial^2\psi}{\partial x^2}-\frac{\lambda_0}{2\tau}\left[u(t+\tau)-u(t-\tau)\right]\delta(x-x_0)\psi+U(x)\psi=i\frac{\partial\psi}{\partial t}\,, \tag{59}$$

and the solution is

$$\psi(x,t)\cong\varphi_\Omega^+(x)\exp(-i\Omega t)- \sqrt{\frac{-i}{\pi}}\frac{\lambda_0\varphi_\Omega^+(0)}{\tau(x-x_0)^2}\left\{(t+\tau)^{3/2}\exp\left(i\frac{(x-x_0)^2}{4(t+\tau)}\right)-(t-\tau)^{3/2}\exp\left(i\frac{(x-x_0)^2}{4(t-\tau)}\right)\right\} \tag{60}$$

which can be further simplified (provided $\tau<<t$) to

$$\psi(x,t)\cong\varphi_\Omega^+(x)\exp(-i\Omega t)+i\sqrt{\frac{-i}{\pi}}\frac{t^{-1/2}\lambda_0\varphi_\Omega^+(0)}{2}\exp\left(i\frac{(x-x_0)^2}{4t}\right)\mathrm{sinc}\left(\frac{\tau(x-x_0)^2}{4t^2}\right) \tag{61}$$

where $\mathrm{sinc}(x)\equiv\sin(x)/x$.

This result again suggests that, if the variation occurs quickly enough, there is no dependence on the incoming particle's energy Ω. In fact, the variation term vanishes for

$$\frac{\tau(x-x_0)^2}{4t^2}=m\pi\quad\text{for } m=1,2,3,... \tag{62}$$

Moreover, when $\frac{\tau(x-x_0)^2}{4t^2}<<1$, the second term has a totally universal pattern, which is even independent of τ:

$$\psi(x,t)\cong\varphi_\Omega^+(x)\exp(-i\Omega t)+i\sqrt{\frac{-i}{\pi}}\frac{t^{-1/2}\lambda_0\varphi_\Omega^+(0)}{2}\exp\left(i\frac{(x-x_0)^2}{4t}\right). \tag{63}$$

10. Applications

(A) The effect of controlled activation can be used in dynamic heterostructures, which can be used as frequency effect transistors. A schematic presentation of the system is presented in **Figure 12**. The barrier can be constructed by a semiconductor, and its potential shape can be controlled by the gate's voltage. In such a device, the current from source to drain will be governed by the frequency of the gate and therefore can be much more sensitive than any other transistor (because frequency, unlike voltage or currents, can be determined with great accuracy).

In **Figure 13**, the current $j \propto |i\psi'(x,t)\psi^*(x,t) - c.c.|$ [where $\psi(x,t)$ is the solution of Eq. (21)] of such a device is plotted as a function of the incoming energy Ω and τ. The forbidden activations values are clearly seen by the white stripes. They are exponentially narrow; therefore, the device's current can be controlled by small variations in τ.

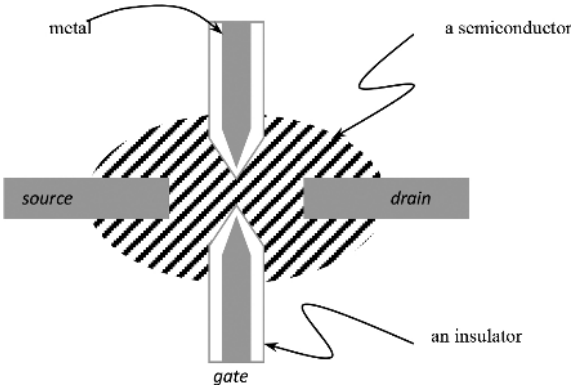

Figure 12. Schematic illustration of a frequency effect transistor. The semiconductor functions as an effective barrier, and the gate voltage oscillates with frequency ω to create the oscillating region. The insulating layer prevents current leakage from the gate to the drain.

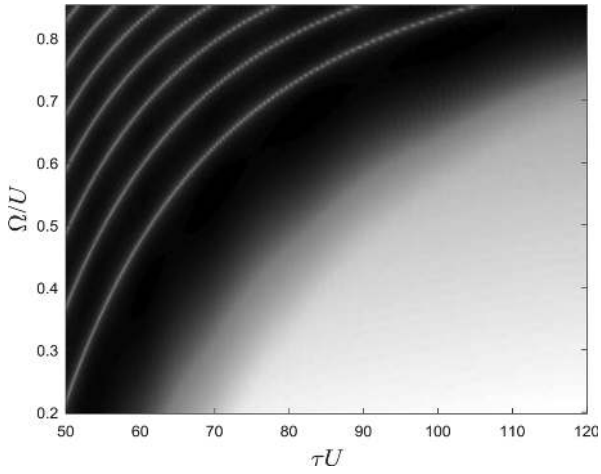

Figure 13. Plot of the current $j \propto |i\psi'(x,t)\psi^*(x,t) - c.c.|$ as a function of the Ω and τ for $\lambda_0 = 100$ and $L\sqrt{U} = 10$. The darker the color, the higher is the current.

(B) As mentioned in the Section 1, there are evidences that odor detection is governed by dynamic resonant tunneling. It is known that odor receptors are, in some sense, like resonant

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tunneling devices—only specific molecules, with the right chemical properties, "activate" the receptor, which sends a signal to the brain. However, it was recently recognized that the olfactory system could distinguish between two molecules, which have the same chemistry but have different mechanical properties. That is, in cases where one of the atoms in a molecule is replaced with one of its isotopes (e.g., deuterium instead of hydrogen), a different odor is detected. It was therefore suggested that the receptor actually operates as a "dynamic" resonant tunneling device [38], which is sensitive on the molecule vibrations. If this is indeed the case, an enigma still remains: how humans can distinguish between approximately 10,000 different smells while they have only few hundred receptors. If every receptor is calibrated to a specific molecule with a specific vibration, then only several hundred odors should have been detected. The dynamics, which is presented by us in this chapter by the forbidden activation energy, suggests that each molecule has a different fingerprint, and every molecule can activate several receptors in a specific combination, which characterizes only the specific molecule. From this specific combination (instead of a single specific receptor), the brain can identify the specific detected molecule (see **Figure 14**).

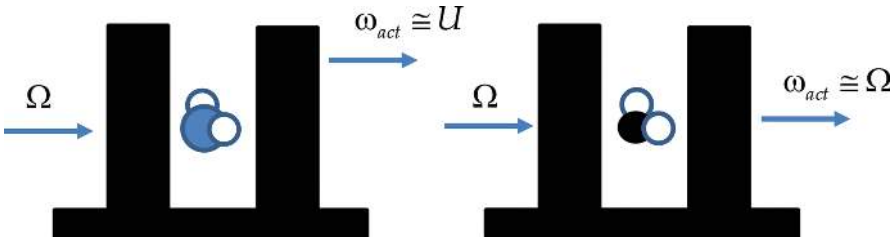

Figure 14. A receptor as a resonant tunneling system distinguishes between molecules, which are identical chemically but different mechanically. The heavier the molecule, the lower the vibrational frequencies and therefore have a different impact on each one of the receptors. Thus, every molecule would have a unique combination of activated receptors.

11. Summary

Resonant tunneling is a fascinating quantum phenomenon. In this chapter, we have focused on the transmission of a quantum particle through an opaque (mostly rectangular) barrier via a delta-function potential. We have discussed all dynamics regimes—stationary resonant tunneling, adiabatic changes, activation, forbidden activation, and instantaneous changes. The generic dynamic problem is highly complex. Each one of these regimes has its specific characteristics.

The main results are related to the domain in which the incoming particle's energy is higher than the minimum eigenenergy of the changing well and the changes are fast enough to prevent particle escape. Unlike previous predictions, the fact that the particle can be trapped in the well does not mean by itself that it will necessarily be activated. In fact, in some cases,

the particle's wavefunction experiences "destructive interference" inside the well, and as a consequence, activation is suppressed. We show that all the cases in which the potential's temporal minima has a parabola shape have the same forbidden energies, and for the first time, we present a generic solution for this case.

Furthermore, the instantaneous case is also discussed for the first time. In this case, the dependence on the specific incoming energy vanishes, and a generic universal pattern appears.

Finally, we suggest testing this effect as a frequency effect transistor, which have the potential to be used as a highly accurate transistor.

Moreover, it is suggested that the effect of forbidden activation energies may explain the reason that humans can distinguish between 10,000 different odors while they have only several hundred odor receptors. According to this suggestion, every molecule can trigger different receptors, and only the combination of the activated ones creates the perception of the right smell. In any case, this research shows that dynamic resonant tunneling, in general (and forbidden activation, in particular), has a major, and totally nontrivial, role in the olfactory mechanism.

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References

[1] Bohm D., Quantum Theory (Dover Publication, New York, 1989).

[2] Ricco B. and Azbel M.Ya., Phys. Rev. B 29, 1970 (1984).

[3] Granot E., Eur. J. Phys. 27, 985–993 (2006).

[4] Kane E.O., Tunneling Phenomenon in Solids (Plenum, New York, 1969).

[5] Price P.J., IEEE Trans. Electron. Dev. 36, 2340 (1989).

[6] Stollen T.C., Goodhue W.D., Parker C.D., Tannenwald T.E., and Peck D.D., Appl. Phys. Lett. 43, 588 (1983).

[7] Sugiyama Y., Inata T., Muto S., Nakata Y., and Hiyamizu S., Appl. Phys. Lett. 52, 314 (1988).

[8] Tsu R. and Esaki L., Appl. Phys. Lett. 22, 562 (1973).

[9] Azbel M.Ya., Phys. Rev. B. 28, 4106 (1983).

[10] Singh J., Physics of Semiconductors and Their Heterostructures (McGraw-Hill, Singapore 1993).

[11] Granot E. and Azbel M.Ya., Phys. Rev. B 50, 8868 (1994).

[12] Granot E. and Azbel M.Ya., J. Phys. Condensed Matter, 11, 4031 (1999).

[13] Kwok K.Ng., Complete Guide to Semiconductor Devices, 2nd ed., pp. 75–83 (Wiley, 2010).

[14] Settnes M., Power S.R., Petersen D.H., and Jauho A.P., Phys. Rev. Lett. 112, 096801 (2014).

[15] Qi Z., Bahamon D.A., Pereira V.M., Park H.S., Campbell D.K., and Neto A.H.C., Nano. Lett. 13, 2692–2697 (2013).

[16] Kashcheyevs V. and Timoshenko J., Phys. Rev. Lett. 109, 216801 (2012).

[17] Azbel M.Ya., Phys. Rev. Lett 68, 98 (1992).

[18] Giles R., J. Math. Phys. 11, 2139–2160 (1970)

[19] Merzbacher E., Quantum Mechanics, (Wiley, 1970).

[20] Azbel M.Ya., Europhys. Lett. 18, 537 (1992).

[21] Azbel M.Ya., Phys. Rev. B. 46, 7596 (1992).

[22] Azbel M.Ya., Phys. Rev. B. 43, 6847 (1991).

[23] Granot E., Europhys. Lett. 61, 817 (2003).

[24] Granot E., Physica E 14, 397–401(2002).

[25] Kälbermann G., Phys. Rev. C. 77, 041601 (2008).

[26] Zangwill A. and Soven P., Phys. Rev. Lett. 45, 204 (1980).

[27] Ivlev B.I. and Mel'nikov V.I., Phys. Rev. Lett. 55, 1614 (1985).

[28] Fisher M.P.A., Phys. Rev. B 37, 75 (1988).

[29] Bagwell P.F. and Lake R.K., Phys. Rev B 46, 15329 (1992).

[30] Aleiner I.L. and Andreev A.V., Phys. Rev. Lett. 81, 1286 (1998).

[31] Buttiker M. and Landauer R., Phys. Rev. Lett. 49, 1739 (1982).

[32] Stefanucci G. and Almbladh C.O., Phys. Rev. B 69, 195318 (2004).

[33] Zangwil G. and Granot E., Physica B 461, 140–146 (2015).

[34] Gray H.B. and Winkler J.R., Q. Rev. Biophys. 36, 341–372 (2003).

[35] Lloyd S., J. Phys. 302, 012037 (2011).

[36] Turin L., Chem. Senses. 21, 773 (1996).

[37] Dyson G.M., Chem. Ind. 57, 647 (1938).

[38] Brookes J.C., Hartoutsiou F., Horsfield A.P., and Stoneham A.M., Phys. Rev. Lett. 98, 038101 (2007).

[39] Pavlicek N., Swart I., Niedenfuhr J., Meyer G., and Repp J., Phys. Rev. Lett. 110, 136101 (2013).

[40] Granot E. and Marchewka A., Physica B 459, 62–68 (2015).

[41] Granot E. and Marchewka A., Europhys. Lett. 72, 341–347 (2005).

[42] Marchewka A., Granot E., and Schuss Z., Opt. Spectrosc. 103, 330–335 (2007).

[43] Granot E. and Marchewka A., Phys. Rev. A 76, 012708 (2007).