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Abstract

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"*Dynamic Response of a Deepwater Riser Subjected to Combined Axial and Transverse Excitation by the Nonlinear Coupled Model*",

authored by

Song Lei, Xiang Yuan Zheng, D. Kennedy

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I look forward to hearing from you.

Yours sincerely

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Dynamic Response of a Deepwater Riser Subjected to Combined Axial and Transverse Excitation by the Nonlinear Coupled Model

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Abstract

In offshore engineering long slender risers are simultaneously subjected to both axial and transverse excitations. The axial load is the fluctuating top tension which is induced by the floater's heave motion, while the transverse excitation comes from environmental loads such as waves. As the time-varying axial load may trigger classical parametric resonance, dynamic analysis of a deepwater riser with combined axial and transverse excitations becomes more complex. In this study, to fully capture the coupling effect between the planar axial and transverse vibrations, the nonlinear coupled equations of a riser's dynamic motion are formulated and then solved by the central difference method in the time domain. For comparison, numerical simulations are carried out for both linear and nonlinear models. The results show that the transverse displacements predicted by both models are similar to each other when only the random transverse excitation is applied. However, when the combined axial dynamic tension and transverse wave forces are both considered, the linear model underestimates the response because it ignores the coupling effect. Thus the coupled model is more appropriate for deep water. It is also found that the axial excitation can significantly increase the riser's transverse response and hence the bending stress, especially for cases when the time-varying tension is located at the classical parametric resonance region. Such time-varying effects should be taken into account in fatigue safety assessment.

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1. Introduction

The offshore oil and gas industry is moving into deeper waters and facing the challenge of harsher environments. The last three decades have seen a number of floating offshore structures such as tension leg platform (TLP), spar, and semi-submersible platforms deployed in the Gulf of Mexico, South China Sea, and offshore Brazil pre-salt fields. As an important system supporting drilling and production activities, long slender marine risers are widely used in these regions. A riser system can be essentially a conductor pipe or a cluster of pipes connecting the floater on the sea surface and the wellheads at the seabed. For the safety of risers in their service lives, it is important to look into their dynamic characteristics of vibration and the related responses under environmental loads.

In order to enhance the geometrical stiffness of a marine riser in practical marine operation, a considerably large axial tension load is usually imposed by tensioners at the riser's top end. Such top-tensioned risers (TTR) have been qualified for use in water depths up to 1500m. When the water depth reaches 2000m and beyond, there are specific concerns that prevent the offshore industry from using TTRs. One of these concerns is related to the fluctuation of the top tension which is caused by the heave motion of the floater in waves. Although heave compensators are used to significantly reduce the fluctuation, the time-varying tension is a potential threat for operation. For a particular parametric excitation such as the varying periodically top tension, it may destabilize the vertical equilibrium of the riser with merely a small disturbance in the lateral direction. This effect is the so-called Mathieu instability [1]. Thus, it is important to investigate the effect of time-varying tension on the riser’s dynamic motion.

Many researchers have paid attention to parametric excitation induced instability problems in offshore engineering. Hsu [1] was the first to report the parametric resonance phenomenon of offshore cables. Patel and Park [2] investigated the dynamics of the tethers with reduced pre-tension of a TLP according to the chart of Mathieu stability. Patel and Park [3] then investigated the tether response combined with the forcing excitation and parametric excitation at the top end. Chatjigeorgiou and Mavrakos [4] applied a numerical approach to study the transverse motion of offshore cables with parametric excitation at the top. Chatjigeorgiou [5] further discussed the damping effect on riser stability of parametric excitation, while Chatjigeorgiou and Mavrakos [6] subsequently presented a closed-form solution for a parametrically excited riser based on the first two modes. Kuiper et al. [7] discussed two qualitatively different mechanisms of stability loss of TTRs suffering fluctuating top tension. Xu et al. [8] gave the Hill instability analysis of TLP tethers subjected to combined platform surge and heave motions. Franzini and Mazzilli [9] employed a three-mode reduced-order model to analyze the non-linear behavior which affects the region of Mathieu instability. In addition to the above theoretical and numerical investigations regarding the Mathieu instability of risers, Franzini et al. [10] presented an experimental model which was designed with a high level of dynamic similitude to a real riser. A curious finding is that the Mathieu instability may simultaneously occur in more than one mode, leading to interesting but complicated dynamic behavior. Yang et al. [11] predicted the parametric instability of TTRs in irregular waves using a multi-frequency excitation. The instability diagram differed significantly from that for single-frequency parametric excitation.

From the above review, it is seen that a number of investigations have focused on the Mathieu instability problem while ignoring transverse loading on the TTRs. It should be recognized that when the risers are used in deep waters, ocean waves not only induce motion of the floater, but also generate hydrodynamic loading directly on the riser. The wave-induced transverse vibration causes bending and stresses in risers, associated with fatigue damage. As shown in the industry standards (API [12], DNV [13]), dynamic analysis and fatigue assessment of risers by waves is an important routine procedure for design. To assess the accurate dynamic response of risers by waves, Spanos et al. [14] developed a model for a TTR system in deep water conditions. Based on the concepts of equivalent linearization and a time averaging method they obtained the maximum stress. Han and Benaroya [15, 16] developed a nonlinear model for the coupling axial and transverse displacements of tendons of a TLP. A comparison of linear and nonlinear responses to random wave forces was obtained by a numerical method in the time domain. Further work has been done by Gadagi and Benaroya [17] who analyzed the effect of end tensions on a TLP tendon, both for a reduced model and for an actual tether. Wang et al. [18] gave a static analysis of a marine riser during installation and suggested that increasing the motion of the floating platform can greatly increase the total riser stress. Mao et al. [19] established a dynamic model considering the actual riser string configuration to analyze the mechanical behavior of a drilling riser under ocean environment loadings. However, in all these models, the top tension was constant or time invariant, which is inconsistent with the actual situation.

As a matter of fact, a riser suffers time-varying top tension and wave load simultaneously, but only a few works have reflected this. Park and Jung [20] presented a numerical analysis of lateral responses of a riser under combined parametric and forcing excitations employing finite element method. Lei et al. [21] calculated the frequency domain responses of the parametrically excited riser subjected to the random wave. Wang et al. [22] investigated the dynamic response of a marine riser under combined forcing and parametric excitation. Nevertheless, in these works the riser is modeled as a linear beam, in which the axial extension and the geometric nonlinearity are ignored.

It is therefore the main interest of this research to simulate the riser dynamic behavior more consistently with the actual situation. It should be noted that the axial and transverse motions of a riser are coupled if the geometric nonlinearity is considered [15, 16, 23, 24]. When a riser is long, the coupling between the axial and transverse motions can be significant. Hence, this paper aims to investigate the behavior of a riser under the combination of fluctuating top tension and transverse wave excitation using the more comprehensive coupled analysis, which is derived from the geometrically nonlinear strain displacement relation. This paper hereafter confirms that the nonlinear coupled axial and transverse vibration model is more suitable than the classical linear Euler-Bernoulli beam or Rayleigh beam models, especially for cases where the deepwater riser is subjected to combined axial and transverse excitation.

2. Modeling of a marine riser

2.1 *Formulation of the nonlinear coupled model*

As shown in Fig. 1, a TTR is connected to a floating platform by means of a heave compensator at the sea surface, and the bottom end is connected to a wellhead at the seabed. For a proper mathematical modeling of the dynamics of TTRs, the following assumptions are introduced to describe the motions of the floating platform and the environmental loads.

- The riser moves only in the plane of the figure, where the x axis is along the vertical body of riser and the y axis is parallel to the wave travelling direction. The unique source of external load in the transverse direction is waves and the current speed is assumed to be small though in is some waters the current speed is comparable to the fluid velocity. With this assumption and the use in engineering practice of vortex suppression instrumentation like thin strakes, the notorious vortex-induced vibration and related non-planar motions are not treated in this study.
- The riser is assumed to be ideally uniform from the bottom end to the top end, so the physical properties are identical along the x axis. Also, the riser length L is much greater than its diameter (i.e. L/r>>1, where r is the gyration radius of section) so that the effects of shear on the riser dynamics can be ignored.
- In consideration of the effect of mooring constraints and dynamic positioning systems, the horizontal surge and sway motions of the platform are assumed to be very small, so that the primary motion affecting the riser's axial load is heave. Coupling of risers and floater is not the main concern of this study.
- Generally speaking, the wave-induced heaving motion of the platform is a stationary random process. However, this paper is focused on the special cases where heave motion is modeled by a harmonic fluctuation. It is a simplified model for narrow-banded random sea wave conditions [7, 21, 25]. Thus the associated time-varying top tension can be expressed as

$$T_0(t) = T_s + T_d = fwL + k_c a \cos(\omega_d t) \tag{1}$$

where T_s is the static pretension; T_d is the dynamic component; T_s can be determined by the pretension factor f, the submerged weight of riser per unit length w and the total length L; k_c is the equivalent stiffness of the heave compensator; a and ω_d are the amplitude and frequency of platform heave motion. The natural frequency of heave motion is determined by the physical characteristics of the platform, and should be shifted out of the wave frequency region during design [26].

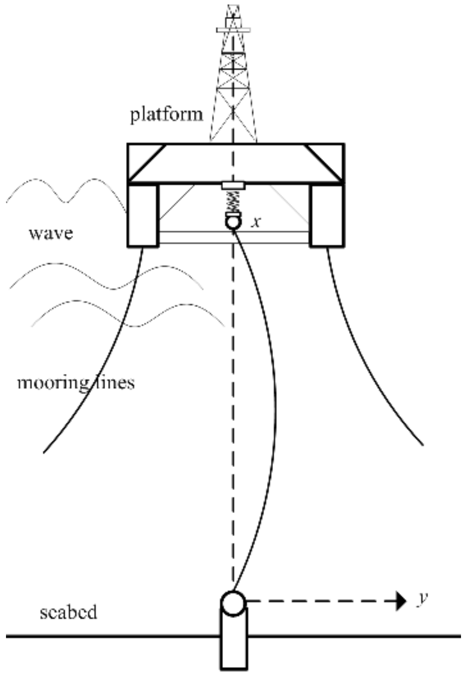

Fig. 1 Sketch of a top-tensioned riser hinged to a floating platform and wellhead in deep water showing (x, y) coordinates

The governing equations of motion and boundary conditions are derived based on the beam theory using Kirchhoff's hypothesis [27], which includes the nonlinear relation between displacements and strains. The displacement field is expressed as

$$u_1(X,Y,t)=u(X,t)-Y\frac{\partial v(X,t)}{\partial X} \tag{2}$$

$$u_2(X,Y,t)=v(X,t) \tag{3}$$

where u_1 and u_2 are displacements in the x and y directions, and $u(X, t)$ and $v(X, t)$ are the deflections of the mid-plane in the x and y directions. X and Y are Lagrangian coordinates. Then, the Green's strains are

$$\varepsilon_{XX} = \frac{\partial u_1}{\partial X} + \frac{1}{2}\left(\frac{\partial u_2}{\partial X}\right)^2 \tag{4}$$

$$\varepsilon_{YY} = \frac{\partial u_2}{\partial Y}, \qquad \varepsilon_{XY} = \frac{1}{2}\left(\frac{\partial u_1}{\partial Y} + \frac{\partial u_2}{\partial X}\right) \tag{5}$$

Substituting the displacement field into the above equations, the Green's strains are obtained as follows.

$$\varepsilon_{XX} = \frac{\partial u}{\partial X} - Y\frac{\partial^2 v}{\partial X^2} + \frac{1}{2}\left(\frac{\partial v}{\partial X}\right)^2 \tag{6}$$

$$\varepsilon_{YY} = 0\,, \quad \varepsilon_{XY} = 0 \tag{7}$$

It follows that the potential energy is given by

$$\begin{aligned} \mathrm{PE} &= \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij}\,\mathrm{d}V = \frac{1}{2}\int_V E\varepsilon_{XX}\varepsilon_{XX}\,\mathrm{d}V \\ &= \frac{E}{2}\int_X\int_A \left(\frac{\partial u}{\partial X} - Y\frac{\partial^2 v}{\partial X^2} + \frac{1}{2}\left(\frac{\partial v}{\partial X}\right)^2\right)^2 \mathrm{d}A\,\mathrm{d}X \end{aligned} \tag{8}$$

where E is Young's modulus and A is the cross-sectional area of the riser. As the cross-section is symmetric, $\int_A Y\,\mathrm{d}A = 0$. If the area moment of inertia $I = \int_A Y^2\,\mathrm{d}A$ is introduced, then Eq. (8) is further expanded as

$$\mathrm{PE} = \frac{1}{2}\int_0^L \left[EA\left(u' + \frac{1}{2}v'^2\right)^2 + EIv''^2\right]\mathrm{d}X \tag{9}$$

where the prime notation is used for partial derivatives with respect to X. The kinetic energy of the riser is given by

$$\begin{aligned} \mathrm{KE} &= \frac{1}{2}\int_0^L\int_A \rho\left(\dot{u}_1^2 + \dot{u}_2^2\right)\mathrm{d}A\,\mathrm{d}X \\ &= \frac{1}{2}\int_0^L \left[\bar{m}\left(\dot{u}^2 + \dot{v}^2\right) + J\dot{v}'^2\right]\mathrm{d}X \end{aligned} \tag{10}$$

where $\bar{m} = \rho_r A + \rho_f A_i$ is the wet mass of riser per unit length, ρ_r is the density of the riser, ρ_f is the density of internal fluid, A_i is the inner section area; $J = \rho_r I + \rho_f I_i$ is the rotatory inertia, I_i is the inner section inertia moment; and the dot notation is used for derivatives with respect to time t.

The total virtual work is given by

$$\delta W = \int_0^L \left(p(X,t)\delta u + f(X,t)\delta v \right) \mathrm{d}\, X + T_0 \delta u(L,t) \tag{11}$$

where $p(X,t)$ = $-w$ is the axial force per unit length. $f(X,\ t)$ is the transverse hydrodynamic wave force that will be detailed later. The Hamilton principle for the TTR is given by

$$\delta \int_{t_1}^{t_2} \mathrm{L}_\mathrm{a}\, \mathrm{d}\, t + \int_{t_1}^{t_2} \delta W\, \mathrm{d}\, t = 0 \tag{12}$$

where L_a=KE-PE is the known Lagrangian function. Assuming that the variations with time at both endpoints are zero, partial integration of the first term of variation in Eq. (12) leads to

$$\begin{aligned} \delta \int_{t_1}^{t_2} \mathrm{L}_\mathrm{a}\, \mathrm{d}\, X\, \mathrm{d}\, t = {} & \int_{t_1}^{t_2} \int_0^L \left(EA \left(u' + \frac{1}{2} v'^2 \right)' - \bar{m}\ddot{u} \right) \delta u \\ & + \left(EA \left[\left(u' + \frac{1}{2} v'^2 \right) v' \right]' - EIv'''' - \bar{m}\ddot{v} + J\ddot{v}'' \right) \delta v\, \mathrm{d}\, X\, \mathrm{d}\, t \\ & - \int_{t_1}^{t_2} EA \left(u' + \frac{1}{2} v'^2 \right) \delta u \Big|_0^L \mathrm{d}\, t - \int_{t_1}^{t_2} EIv'' \Big|_0^L \delta v'\, \mathrm{d}\, t \\ & - \int_{t_1}^{t_2} \left(EA \left(u' + \frac{1}{2} v'^2 \right) v' - EIv''' + J\ddot{v}' \right) \delta v \Big|_0^L \mathrm{d}\, t \end{aligned} \tag{13}$$

Substituting equations (11) and (13) into Eq. (12), the nonlinear coupled axial and transverse motion of the riser are obtained as follows.

$$\bar{m}\ddot{u} - EA\left(u'' + v'v''\right) = p\left(X,t\right) \tag{14}$$

$$\bar{m}\ddot{v} - J\ddot{v}'' - EA\left(u''v' + u'v'' + \frac{3}{2} v'^2 v'' \right) + EIv'''' = f\left(X,t\right) \tag{15}$$

The boundary conditions are

$$u(0,t) = 0\,, \quad EA(u' + \frac{1}{2} v'^2)\Big|_{L,t} - T_0 = 0 \tag{16}$$

$$v(0,t) = 0\,, \quad EIv''\left(0,t\right) = 0 \tag{17}$$

$$v(L,t) = 0\,, \quad EIv''\left(L,t\right) = 0 \tag{18}$$

For the nonlinear coupled model developed above, similar equations have been also obtained by Yigit and Christoforou [23], Adrezin and Benayora [24], Han and Benayora [15,16], but the boundary conditions in Eq. (16) to account for the time-varying top tension (axial excitation) were ignored in these works.

2.2 *Linear model*

If the axial motion and nonlinear coupling effects are ignored, the riser is assumed to be extensible as a quasi-static beam. Equation (14) is reduced to

$$EAu'' = p(X,t) \tag{19}$$

Integrating from X to L and using the boundary condition Eq. (16) gives

$$EAu' = T_0(t) + w(X-L) \tag{20}$$

Substituting Eq. (20) into Eq. (15), one can obtain

$$EIv'''' - \left[\left(T_0(t) + w(X-L)\right)v'\right]' + \bar{m}\ddot{v} - J\ddot{v}'' = f(X,t) \tag{21}$$

which is a Rayleigh model under axial tension. If the rotatory inertia term involving J is ignored, Eq. (21) is further reduced to the familiar Euler-Bernoulli beam vibration model

$$EIv'''' - \left[\left(T_0(t) + w(X-L)\right)v'\right]' + \bar{m}\ddot{v} = f(X,t) \tag{22}$$

Thus, the Rayleigh model and the Euler-Bernoulli model can be regarded as two special cases of the nonlinear coupled model.

2.3 *Transverse force due to random waves*

The transverse force $f(X,t)$ in Eq. (11) can be expressed by the Morison's equation as follows.

$$f(X,t) = \frac{1}{2}\rho D_{\mathrm{o}} C_{\mathrm{D}} \left(v_{\mathrm{w}}^{n} - v_{\mathrm{R}}^{n}\right)\left|v_{\mathrm{w}}^{n} - v_{\mathrm{R}}^{n}\right| + C_{\mathrm{M}} \frac{\pi}{4}\rho D_{\mathrm{o}}^{2}\dot{v}_{\mathrm{w}}^{n} - \left(C_{\mathrm{M}} - 1\right)\frac{\pi}{4}\rho D_{\mathrm{o}}^{2}\dot{v}_{\mathrm{R}}^{n} \tag{23}$$

where ρ is water density; D_{o} is the outer diameter of the riser; C_{D} is drag coefficient; C_{M} is inertia coefficient; v_{w}^{n}, v_{R}^{n} and $\dot{v}_{\mathrm{R}}^{n}$ are the normal components of the instantaneous water particle velocity, riser velocity and riser acceleration, respectively. For deep water, the hydrodynamic damping incurred by the relative velocity term in Eq. (23) is usually much larger than the structural damping. This is the reason why both linear and nonlinear models ignore the contribution of structural damping terms in Eqs. (15), (21) and (22).

The normal components can be written as

$$v_{\mathrm{w}}^{n} = v_{wy} - v_{wx} v' \tag{24}$$

$$v_{\mathrm{R}}^{n} = \dot{v} - \dot{u} v' \tag{25}$$

$$\dot{v}_{\mathrm{R}}^{n} = \ddot{v} - \ddot{u} v' \tag{26}$$

where v_{wx} and v_{wy} are the water particle velocities in the x and y directions, respectively. Using the linear Airy wave theory, the free surface of the wave η is a harmonic function of time t

$$\eta(t) = \frac{H}{2}\cos(\omega t) \tag{27}$$

and the horizontal and vertical water particle velocities are expressed as

$$v_{wx} = \omega \frac{\sinh kx}{\sinh kd} \frac{H}{2} \sin(\omega t) \tag{28}$$

$$v_{wy} = \omega \frac{\cosh kx}{\sinh kd} \frac{H}{2} \cos(\omega t) \tag{29}$$

where d is water depth, H is wave height, k is wave number, ω is the angular frequency of the wave, $x \leq d$. For a linear wave, k satisfies the wave dispersion relation

$$\omega^2 = gk \cdot \tanh kd \tag{30}$$

In reality, the free surface η is random and contains more than one frequency component. Thus, the wave spectrum $S_{\eta\eta}(\omega)$ is used to express the wave energy distribution at various frequencies. There are a number of wave spectrum models for ocean engineering. In this paper the JONSWAP spectrum is used [28], which is defined as

$$S_{\eta\eta}(\omega) = 487(1 - 0.287 \ln \gamma) H_{\mathrm{s}}^2 T_{\mathrm{p}}^{-4} \omega^{-5} \exp(-1948 T_{\mathrm{p}}^{-4} \omega^{-4}) \gamma^{\exp\left[-\frac{(0.159\omega T_{\mathrm{p}} - 1)^2}{2\sigma^2}\right]} \tag{31}$$

where σ=0.07 for $\omega \leq 2\pi/T_{\mathrm{p}}$ and σ=0.09 for $\omega > 2\pi/T_{\mathrm{p}}$, H_s is the significant wave height, T_{p} is the peak period and γ is a peakedness factor. When given these three parameters in the spectrum, the random wave surface and velocities can be simulated by applying the superposition principle of sinusoidal waves [29].

3. Numerical solution

The nonlinear coupled model can be solved in two steps. The first step transforms the partial differential equations into ordinary differential equations. As shown in Fig. 2, the length of the riser is divided into N segments with nodes placed at equal distances along its length. The length of each segment is $h=L/N$. The first node (0) is at $x=0$, and the last node (N) is at $x=L$. By extending the riser length at both ends, virtual node indices -1 and $N+1$ are then introduced. The spatial derivatives of v in Eqs. (14-18) are discretized using the standard second-order central difference scheme as follows.

$$v' = \frac{v_{j+1} - v_{j-1}}{2h}, v'' = \frac{v_{j+1} - 2v_j + v_{j-1}}{h^2}, v'''' = \frac{v_{j+2} - 4v_{j+1} + 6v_j - 4v_{j-1} + v_{j-2}}{h^4} \tag{32}$$

Using the boundary conditions in Eqs. (17-18), it is easy to obtain the relationship between virtual nodes.

$$v_0 = v_N = 0, \quad v_{-1} = -v_1, \quad v_{N+1} = -v_{N-1} \tag{33}$$

The spatial derivatives of u can be discretized using a similar procedure. Applying the finite difference scheme, $2N$ second order differential equations are obtained, which are expressed in terms of the matrix form as follows.

$$\mathbf{M}\ddot{\mathbf{S}} + [\mathbf{K}_0 + \mathbf{K}_G]\mathbf{S} = \mathbf{F} \tag{34}$$

where $\mathbf{S}=[u_1, ..., u_N, v_1, ..., v_N]^T$ is the total displacement vector; $\mathbf{M}$ is the mass matrix, including the added mass of water; $\mathbf{K}_0$ is the linear stiffness matrix; $\mathbf{K}_G$ is the nonlinear stiffness matrix dependent on the unknown vector $\mathbf{S}$; $\mathbf{F}$ is the generalized force vector, also a function of vector $\mathbf{S}$ because of the nonlinear drag term in Morison's equation.

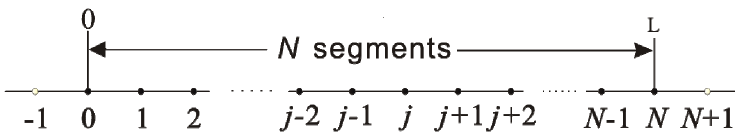

Fig. 2 Sketch of discretized nodes along the riser

In the present study, the ordinary differential equations defined by Eq. (34) are integrated numerically in the time domain. Here the Newmark integration method is used. This is an unconditionally stable method with high integration precision [27]. Due to the nonlinearities of stiffness and drag force, Newton-Raphson iteration is needed at each time step. The linear models are solved with the same scheme, but with less computational effort than the nonlinear model.

4. Results and discussion

Numerical studies are carried out to analyze the response characteristics of a TTR subjected to the periodic time-varying top tension and random wave force. The properties of the riser and waves are given in Table 1.

Table 1: Properties of the TTR riser and the random wave

Outer diameter, D_o	0.5588 m
Inner diameter, D_i	0.5080 m
Density of riser, ρ_r	7850 kg/m^3
Density of internal fluid, ρ_f	1200 kg/m^3
Density of sea water, ρ	1025 kg/m^3
Drag coefficient, C_D	0.8
Inertia coefficient, C_M	2.0
Elastic modulus, E	2.1×10^{11} N/m^2
Pretension factor, f	1.3
Significant wave height, H_s	8.7 m
Peak period, T_p	12.3 s
Peakedness factor in JONSWAP spectrum, γ	3.3

Two typical lengths, 500m and 2000m, are considered in the simulation. The stiffness of the heave compensator is dependent on the efficiency of the tensioner system at the top end of the riser [7, 30]. The stiffness is defined by

$$k_c = wL / a_c \tag{35}$$

where a_c is the critical amplitude, usually set at 10m. This implies that if a relative heave motion of the platform of 10m occurs, the compensator will generate an additional tension force in the riser, equal to its total submerged weight.

4.1 *Natural frequencies with time-varying top tension*

Based on the linear model, natural frequencies of the riser's transverse vibration can be obtained. With the properties given in Table 1, it can be found that the natural frequencies obtained by the Rayleigh model and the Euler-Bernoulli model under constant pretension force are almost identical. This indicates that the rotatory inertia has negligible influence on the dynamic characteristics of a long slender riser. Without the dynamic component of tension on the TTR, natural frequencies ω_n (n=1,2,3, …) for the 500m and 2000m cases based on the Euler-Bernoulli beam model are listed in Table 2, for the pretension factor f=1.3.

Table 2 Natural frequencies of risers of two typical lengths under constant pretension

Length (m)	Natural frequencies (rad/s)							
	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8
500	0.2347	0.4802	0.7442	1.0322	1.3484	1.6963	2.0790	2.4990
2000	0.1164	0.2329	0.3496	0.4666	0.5839	0.7017	0.8200	0.9389

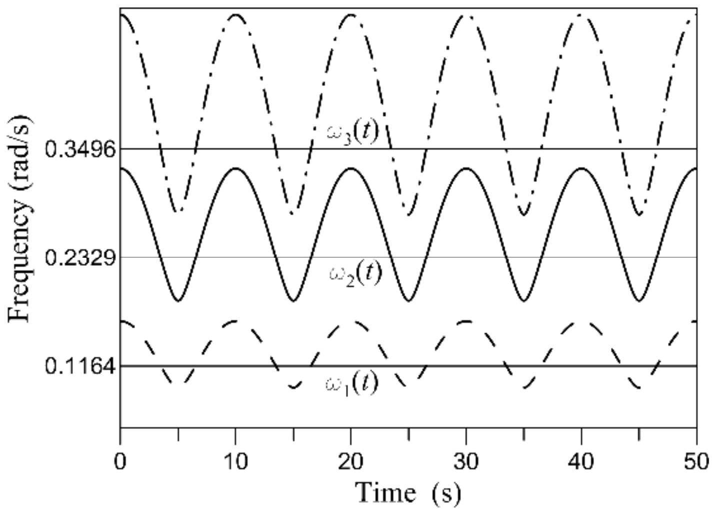

Fig. 3 Natural frequencies of a 2000m TTR subjected to a harmonic time-varying top tension (a=3m, ω_d=2π/10 rad/s)

On the contrary, when the dynamic tension is taken into account, it is clear that the time-varying behavior affects the natural frequencies in that the tension alters the geometrical stiffness. If the tension load is periodic like the harmonic formulation in Eq. (1), the natural frequencies are also periodic, as shown in Fig. 3. However, it should be noted that the frequencies do not vary as trigonometric sine or cosine functions. Moreover, the maximum value and the minimum values of the frequencies within one cycle are not symmetric about the natural frequency obtained under constant pretension. This is because the natural frequencies are not a linear but a square root function of the top tension [31].

4.2 *Dynamic equilibrium with only parametric excitation*

Prior to investigating the TTRs' response when subjected to combined axial and transverse excitations, the dynamic equilibrium of parametrically excited TTRs (with axial load only) in calm water must be addressed. Generally for a linear system, if the combination of amplitude and frequency in a parametric excitation is initially located in an unstable zone, even a small perturbation would trigger an unbounded response to the system and no steady states would be reached (Chatjigeorgiou and Mavrakos, 2002). However, both the TTR and its loadings are nonlinear, see Eqs. (14-15). Even if the TTR structure is modeled as a linear beam, it should be recognized that the transverse hydrodynamic wave loading in Eq. (23) is nonlinear and contributes to the nonlinearity in the system's damping. Consequently, the response of a parametrically excited TTR will reach a dynamic equilibrium when the energy input by the heave motion equals the energy consumption. The dynamic equilibrium can be simulated by applying the following initial conditions.

$$v(X,0) = 0.01\sin(\frac{\pi X}{L}), \quad v'(X,0) = 0 \tag{36}$$

Based on numerical simulation for the linear model of a 2000m TTR, it is found that with the increase of amplitude and frequency of the parametrical excitation, the dynamic equilibrium exhibits very different scenarios. These scenarios, reflecting the complicated mechanisms of nonlinear vibration, were previously discussed by Kuiper et al. (2008) and summarized into the following three distinct categories.

Sub-critical local dynamic buckling;

Classical parametric resonance;

Super-critical local dynamic buckling.

Figures 4-6 show the abovementioned three typical scenarios for vibration of the TTR in 2000m calm water. In these figures, the two horizontal axes correspond to the location of TTR elements and time, whereas the vertical axis gives the transverse displacement. Figure 7 gives the root-of-mean-square (RMS) of transverse displacement along the TTR, which is a useful index for describing the response energy.

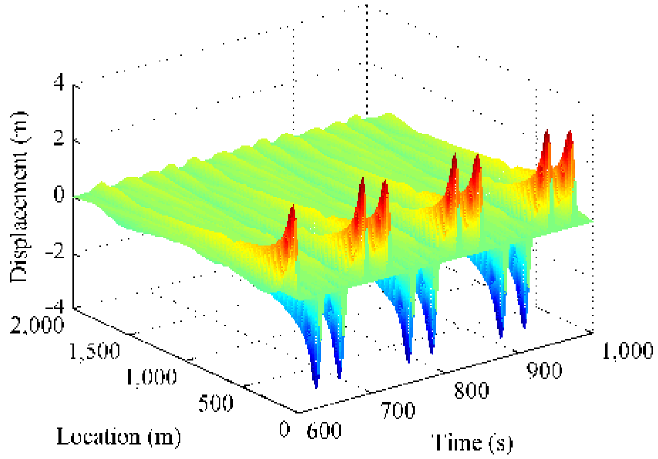

Fig. 4 Transverse dynamic response of a 2000m TTR subjected to parametric excitation (a=5m, ω_d=0.2 rad/s) in calm water

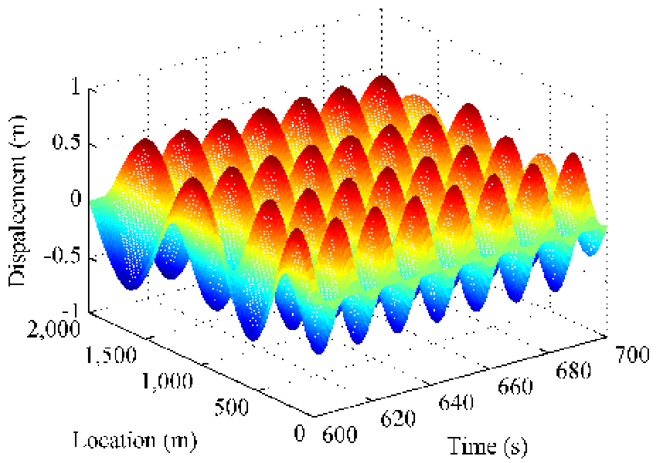

Fig. 5 Transverse dynamic response of a 2000m TTR subjected to parametric excitation (a=2m, ω_d=2ω_4) in calm water

In Fig. 4, the 'sub-critical local dynamic bucking' scenario, occurs at a low frequency and with large heave amplitudes. A sequence of short pulses is located at the segment near the sea bed whose displacement is much larger than elsewhere. These pulses, of similar patterns, occur periodically in clusters and each cluster consists of two pulses. The displacement between two neighboring clusters is very small, indicating that there is little memory effect between them.

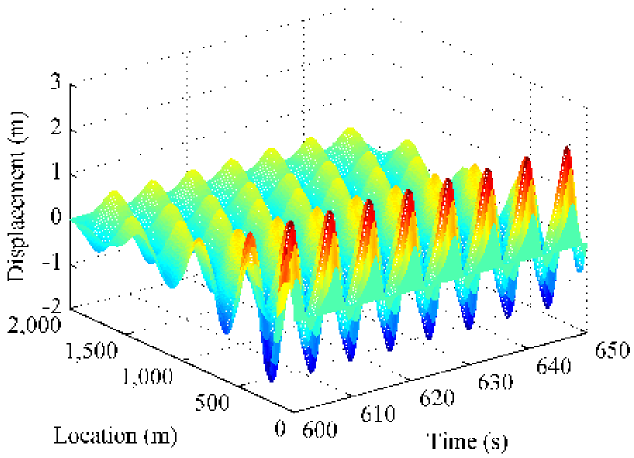

Fig. 6 Transverse dynamic response of a 2000m TTR subjected to parametric excitation (a=5m, $\omega_d=2\omega_4$) in calm water

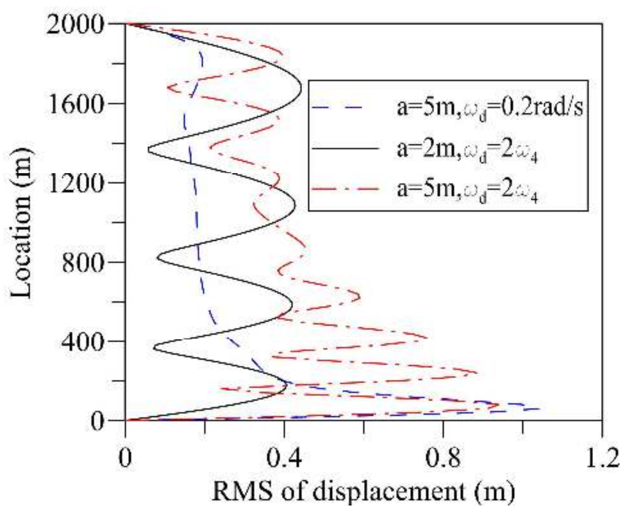

Fig. 7 RMS of transverse displacement of a 2000m TTR for three parametric vibration scenarios in calm water

The 'classical parametric resonance' scenario occurs even with moderate platform heave amplitudes. A steady response with frequency about half the excitation frequency develops.

Figure 5 shows this scenario, in which the specified frequency of heave excitation is twice the fourth natural frequency (ω_4) given in Table 2 and the amplitude is 2m. The parametric vibration of the TTR is dominated by the 4th-mode which is manifested in Fig. 7.

When the heave amplitude is significantly increased to 5m, the third typical scenario, so-called super-critical local dynamic buckling, comes into effect, as shown in Fig. 6. It can be seen that large pulses are generated at the bottom segments of the TTR, and these gradually decay towards the top end. The vibration frequency is different from the classical parametric resonance in Fig. 5. It depends on the periodic generation of induced pulses near the bottom end. Also, the shape of parametric vibration of the riser is in the 8th mode, in that the excitation frequency is close to the eighth natural frequency (see Table 2).

Sub- and super-critical local dynamic buckling occur with large amplitudes of disturbance (a=5m). The mechanism of buckling is attributed to compression of the TTR at the bottom segment, because the submerged weight of the riser makes the riser axial tension force decrease from the top to the bottom. Thus, if the amplitude of platform heave motion is sufficiently large, the harmonic axial force within a periodic cycle will become an additional compressive load on the bottom end. Then, when the local compression deformation is large and cannot be restored, the local buckling occurs. In fact, these two scenarios belong to the class of instability problems at extreme situations [32, 33]. Above a certain threshold of the local displacement, the local stress in the riser will exceed the yield stress, causing the riser to fail due to insufficient strength. For failure analysis, the geometrical nonlinearity and material plasticity should be taken into account in the developed numerical model, but this is not the focus of the present study. Also, noting that the nonlinear geometry and stiffness are associated with singularity of the characteristic matrix during the time simulation process, in the next section when TTR is subjected to combined axial and transverse excitations the analysis of the response will be confined to parametric excitation for the 'classical parametric resonance' scenario.

4.3 ***Dynamic response driven by combined axial and transverse excitation***

In this section, typical cases are studied to investigate how the periodic time-varying tension affects the response of the TTR under random waves that are specified by the spectral parameters in Table 1. The differences between the nonlinear and linear models are also analyzed. As the heave motion period of a floating or semi-submersible platform is commonly between 5s and 30s, the frequency of axial excitation falls into the range from 0.2rad/s to 1.25rad/s. Attention is paid to the scenario of classical parametric resonance which is of interest to the engineering community. Hence only moderate heave amplitudes will be used in the axial excitation and no sub- or super-critical buckling will be treated in this section.

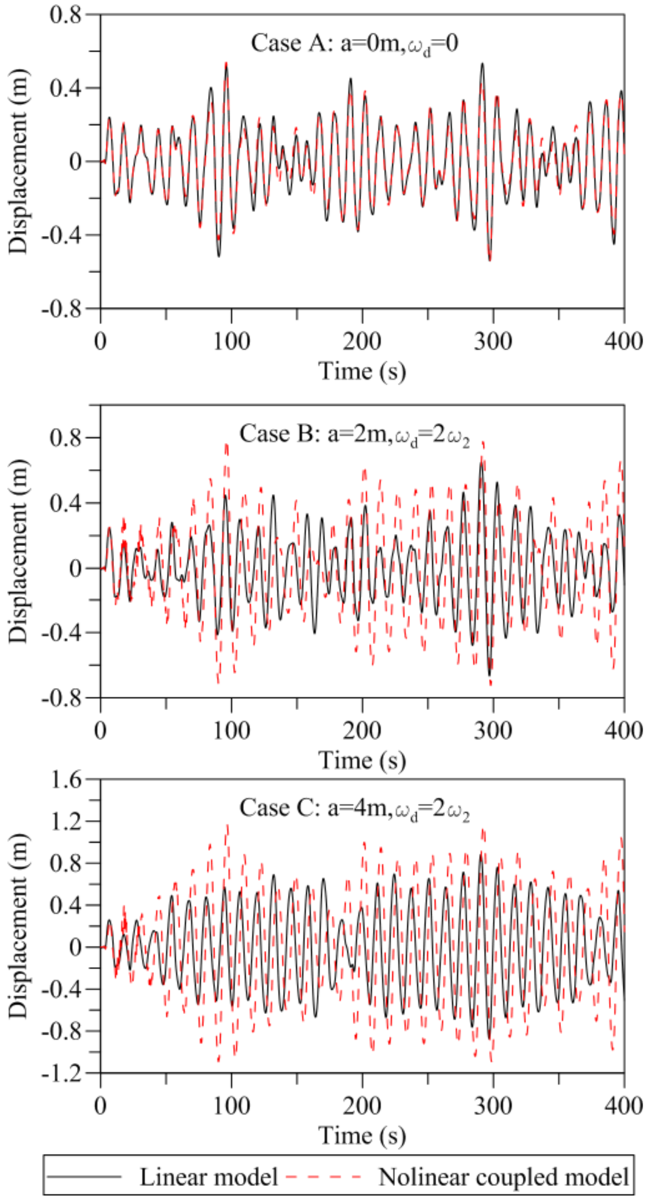

Fig. 8 Midpoint transverse displacement of the 500m TTR

For 500m and 2000m TTRs, three typical cases of parametric excitation are specified as follows. Note that Cases A and D correspond to the scenarios of constant top tension.

500m TTR: Case A: a=0m, ω_d=0; Case B: a=2m, $\omega_d=2\omega_2$; Case C: a=4m, $\omega_d=2\omega_2$.

2000m TTR: Case D: a=0m, ω_d=0; Case E: a=1m, $\omega_d=2\omega_4$; Case F: a=2m, $\omega_d=2\omega_4$.

Time-domain dynamic computations are implemented for both the nonlinear coupled model of Section 2.1 and the reduced linear model of Section 2.2. Comparisons of midpoint (X=L/2) transverse displacements are given in Figs. 8 & 9, respectively. Simulation showed that the responses of linear models, i.e. Rayleigh and Euler-Bernoulli beams, were almost identical to each other although for brevity the details are omitted. Thus the sectional rotation of the riser can be ignored in the governing equation. The linear model results in these two figures therefore refer to the Euler-Bernoulli model.

For the 500m TTR, in Case A which corresponds to the situation of a constant top tension, it is clearly shown in Fig. 8 that the linear and nonlinear models give very close results. This is consistent with the conclusion given by Han and Benaroya [16] for cases in which the random transverse force is applied with zero initial conditions. However, when axial parametric excitation is considered (as in Case B and C), the displacement of the nonlinear coupled model is larger than that of the linear model. This is a useful conclusion which has not been found in previously published papers. In Case C, where the parametric excitation amplitude is doubled while the frequency is still $2\omega_2$, the difference in peaks between the linear and nonlinear models is more pronounced. In case B, the maximum response of the nonlinear model within 400s duration is 0.7946m, 14% higher than the maximum linear response of 0.6999m. When the excitation amplitude increases to 4m in Case C, the two models show greater deviations in response. In the linear model, the maximum displacement is 0.7760m, but in the coupled model, this value increases to 1.173m, or 51% larger. Nonetheless, the response amplification is disproportional to the excitation amplitude. Furthermore, in Cases B and C, it can be observed that as time proceeds, a phase difference develops between the two models. The nonlinear model vibrates slower than the linear model, due to nonlinearities in the system and also to the model's flexibility in the axial direction. Comparing the responses for Cases A, B and C， it is easily observed that the parametric excitation has boosted the transverse response of the riser in waves for both the linear and nonlinear models. In addition, as illustrated by the time histories in Cases B and C, the response of the TTR has gained amplification due to the parametric excitation even for the linear model. The peak response in Case A (constant top tension) is 0.5348m, while in Cases B and C they are 0.6999m and 0.7760m, respectively.

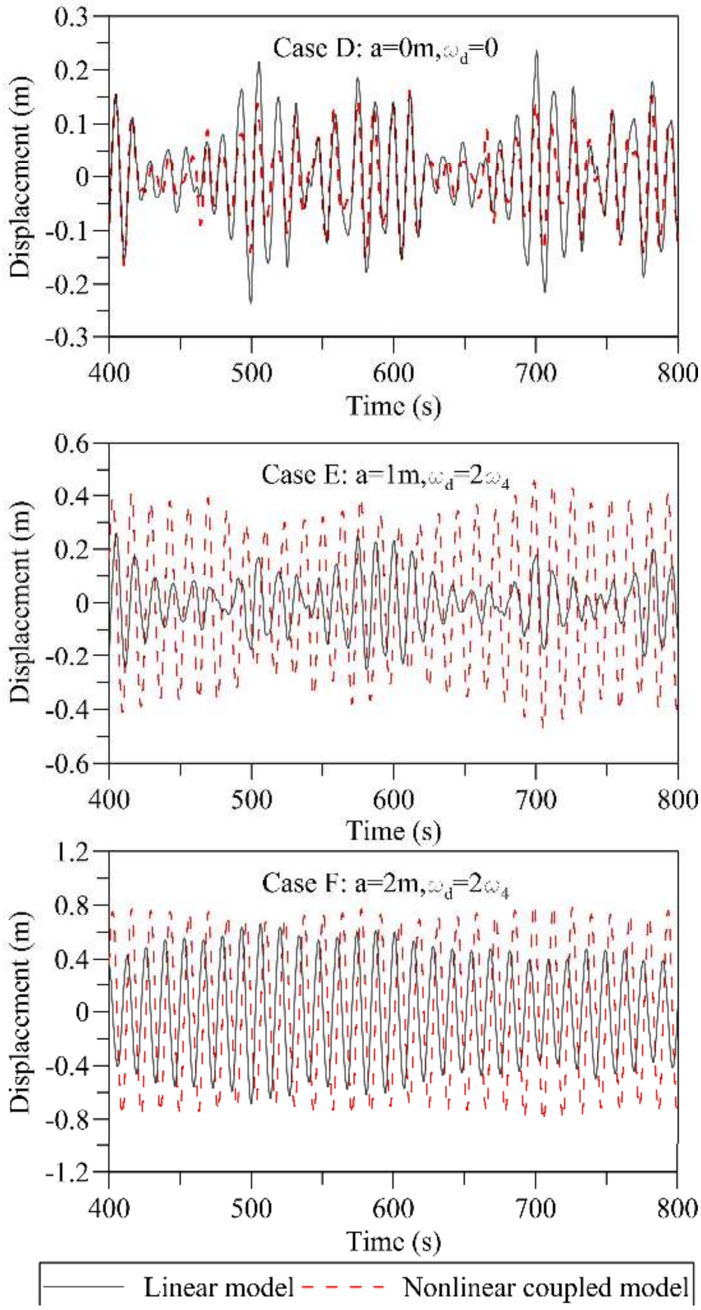

Fig. 9 Midpoint transverse displacement of the 2000m TTR

For the 2000m TTR, when the parametric excitation is neglected (Case D) the difference in response between linear and nonlinear models is more obvious than for the 500m TTR (Case A). In Fig. 9 the peaks of the nonlinear coupled model are much smaller than the linear ones, since the geometrical nonlinearity increases the transverse stiffness. It can be also be explained by the conservation of energy that the external wave energy applied to the TTR is converted to axial and transverse kinetic energy in the coupled model, but in the linear model no coupling is considered and only the transverse kinetic energy is accounted for. When the time-varying top tension is taken into account in Cases E and F, the parametric resonance involves both axial and transverse motions and their coupling gives rise to the more pronounced response in the direction of the wave. It is also found that the vibration behavior in Case E is different from that in Case F. In Case E, the response of the linear model is more random than the response of the nonlinear model. In Case F, both linear and nonlinear model responses exhibit periodic characteristics.

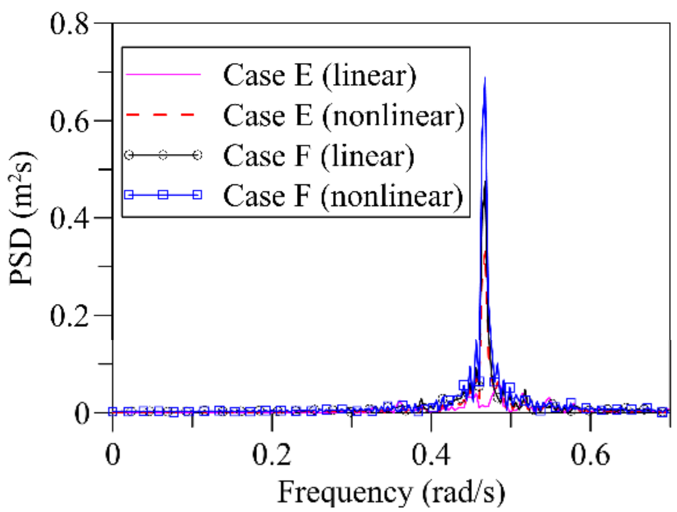

Fig. 10 PSD of midpoint transverse displacements of the 2000m TTR in Cases E and F

The response power spectral densities (PSD) depicted in Fig. 10 also demonstrate that the majority of the response energy is concentrated at the frequency of ω_4. Thus for risers operating in very deep water, their vibrations are sensitive to the axial parametric excitation. For the platform's heave motion, the 'classical parametric resonance' scenario can be easily triggered in a periodic pattern and in that situation the transverse wave load acts just like a continuously applied random disturbance.

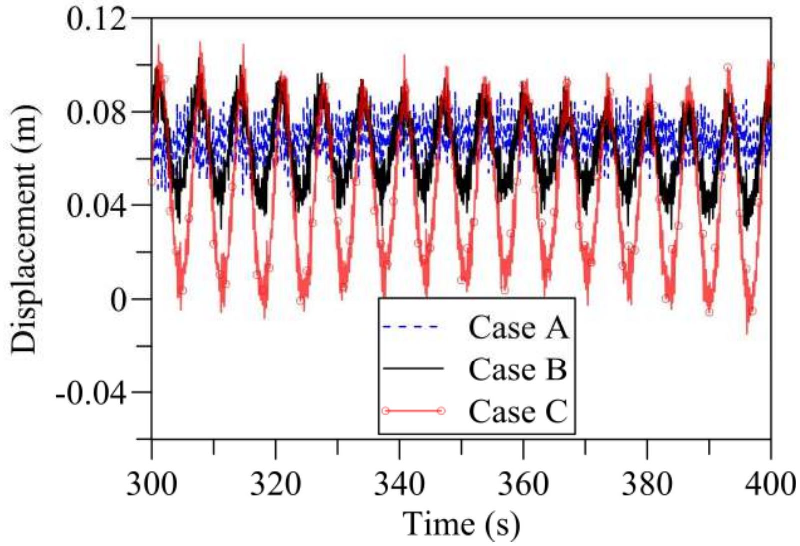

Fig. 11 Top end axial displacement of the 500m TTR simulated using the nonlinear coupled model

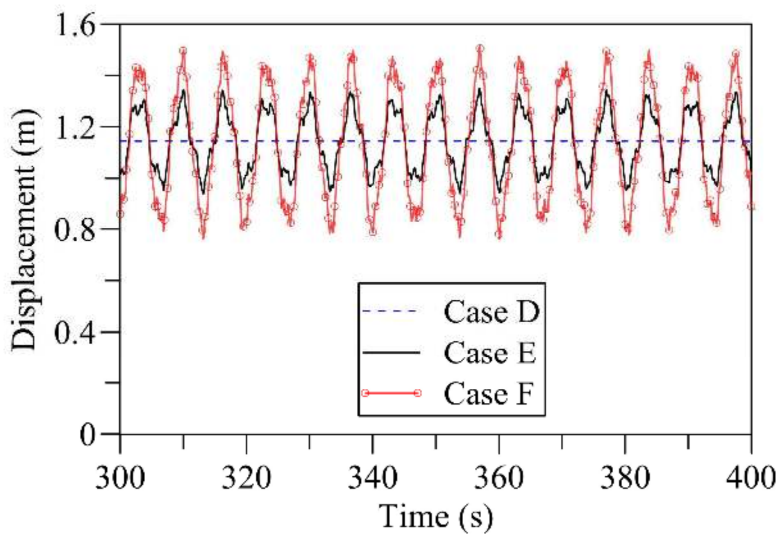

Fig. 12 Top end axial displacement of the 2000m TTR simulated using the nonlinear coupled model

The axial displacement of the 500m TTR top end simulated using the coupled model is shown in Fig. 11. The axial displacement oscillates about the initial extension location, which relates to the static top pretension. Due to the coupling action, the axial displacement turns into a stochastic process although the axial top tension is time-varying in a harmonic form. Comparing the three cases, it is seen that the axial displacement is amplified further with increasing platform heave amplitude. It should be also noted that in Cases A and B the axial displacements are always positive while in Case C the displacement becomes negative at some instants of time, indicating the occurrence of compressive deformation. If the heave amplitude were even larger, there is no doubt that local buckling would occur at the bottom segment of the riser and the vibration would then enter the super-critical buckling domain. For the 2000m riser in Fig. 12, the axial displacement at the top end is much higher than that of the 500m TTR in Fig. 11. This is due to a larger top tension and a decrease of tensile stiffness with the increasing length of the TTR. Further, for the combined excitation in Cases E and F, the axial vibration of the top end is in a form similar to the periodically time-varying tension, with little influence from the transverse load.

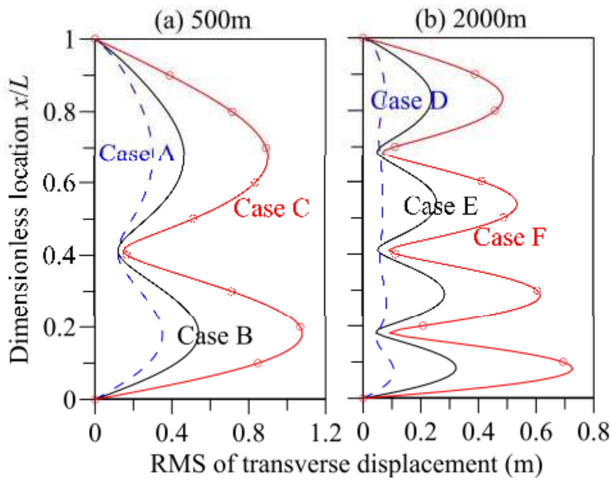

Fig. 13 RMS of transverse displacement of TTR simulated using the nonlinear coupled model

The RMS of the transverse displacement is shown in Fig. 13. It can be seen from Fig. 13(a) that the response of the 500m riser is dominated by the second mode of beam vibration. The reason lies in the peak period of the wave spectrum (12.30s) which is close to the second mode period (13.08s) of the riser. The RMS has two peak values at sections 100m and 350m upwards from the bottom end. When the periodic time-varying top tension is admitted, as in Cases B and C, the levels of RMS along the riser are significantly boosted. The maximum values of RMS for Cases A, B, and C are respectively 0.3506m, 0.5390m and 1.0780m, which suggests that the axial time-varying tension strongly affects the transverse wave response. For the 2000m riser, the response amplification phenomenon is analogous to the 500m riser, except that the response is dominated by the fourth vibration mode.

From the analysis of the above three cases it can be concluded that if the top tension is constant, the transverse displacement of the linear model is slightly larger than that of the nonlinear coupled model. However, if the tension is dynamic, the linear model will underestimate the response because it ignores the coupling effect. Hence the coupled model is recommended for use in deep water. As the time-varying tension can significantly increase the wave induced transverse response, it should be taken into account in safety assessments. Also, as revealed by the illustrated numerical results, for very long risers with moderate platform heave motions, it is the classic parametric resonance rather than wave load which contributes more to the vibration magnitude of risers.

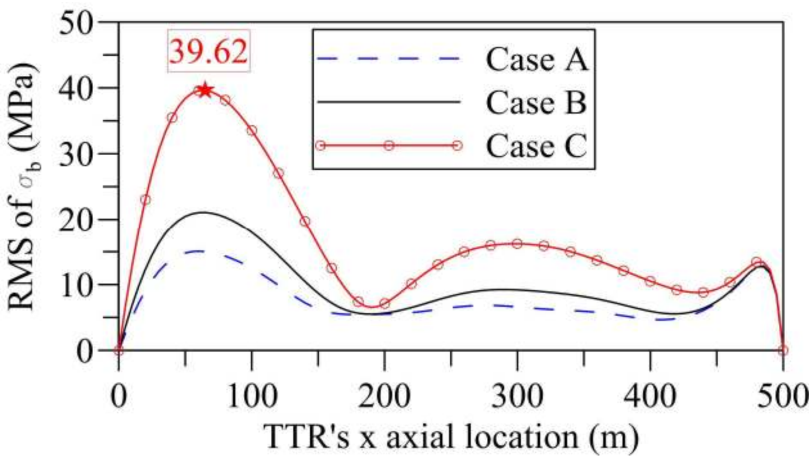

Fig. 14 RMS of bending stress of the 500m TTR simulated using the nonlinear coupled model

4.4 ***Bending stress analysis***

In practice, the designer is more concerned about the stress than the displacement of the riser. In the TTR, the bending moment, axial tension and radial pressure all contribute to the total stress. Here, only the dominant bending stress is considered. The bending stress at the outer diameter is calculated as

$$\sigma_{\mathrm{b}}(X) = \frac{ED_{\mathrm{o}}}{2}\frac{\partial^2 v}{\partial X^2} \tag{37}$$

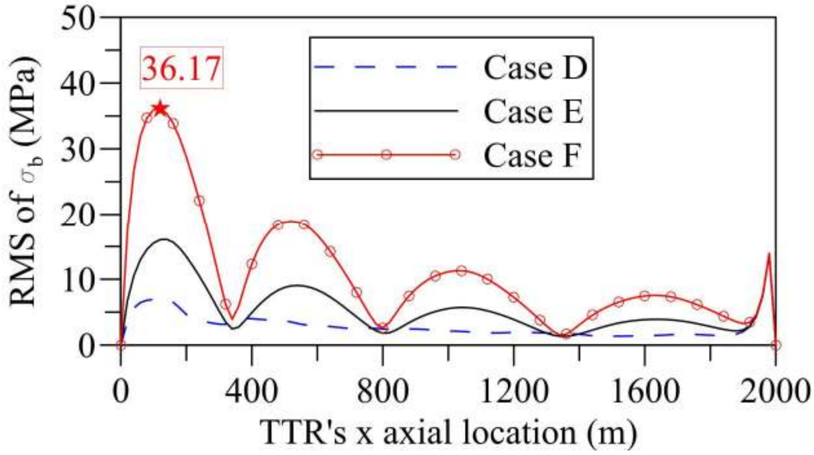

Fig. 15 RMS of bending stress of the 2000m TTR simulated using the nonlinear coupled model

For the 500m TTR, the RMS of bending stresses for three cases are given in Fig. 14. It is clear that a peak value is located near the sea surface (*X*=500m) where the greatest wave load is applied. Near the bottom of the riser, the bending stress is also considerably large. This is because the axial tension is decreased from the sea surface to bottom due to the wet weight of riser, thereby increasing the flexibilities of the bottom segments. When the influence of heave motion is considered, the bending stress increases with the heave amplitude, especially at the bottom section. The peak values of the bottom segment in Case B and C are 21.07MPa and 39.62MPa, i.e. respectively 1.4 times and 2.6 times the value in Case A. Such increase in stress can be also found for the 2000m TTR, as shown in Fig. 15. The extreme stress is about 4RMS, or about 160MPa, assuming that the stress is a weekly non-Gaussian process. This extreme value is still below the yield stress for a steel riser (around 380MPa for instance). However, RMS of 39.62MPa and 36.17MPa for the 500m and 2000m TTRs imply that significant fatigue damage can be induced by the parametric excitation during the service lifetime.

5. Concluding remarks

The dynamic response of a single long slender top-tensioned riser subjected to random waves loading is analyzed, taking into account the periodic time-varying tension caused by the platform heave motion. In contrast to most previous works on riser dynamics which only treated an axial or transverse excitation on the linear model, this paper represents the axial top tension by a periodic process and, the nonlinear model of the riser vibration driven by coupled axial and transverse loadings are derived using Hamilton's principle. The numerical solution scheme for the stochastic response is given. The time-domain simulation results of transverse displacement and bending stress for TTR in 500m and 2000m of water have not only demonstrated significant differences between the coupled model and the reduced linear model but also the importance of the dynamic tension to account for such differences. The nonlinear coupled model, although more comprehensive, provides a rational approach to better assess the dynamic response of a riser subjected to the non-negligible time-varying top tension load. As revealed by a number of illustrations showing significant increases in displacement and bending stress, the parametric resonance (which could contribute even more to the nonlinear dynamic response than the transverse wave loading) does occur within the service lifetime of risers and may unfavorably reduce the structural fatigue life.

Future work will be extended to evaluate the effect of top tension more accurately by a narrow banded random process and the interaction of risers among a cluster. Also, as the present study is confined to planar vibration while in a real sea state waves have varying directionality, it would be of practical interest to study the three-dimensional parametric vibration by both numerical and experimental approaches.

Acknowledgements

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Figure Captions

Fig. 1 Sketch of a top-tensioned riser hinged to a floating platform and wellhead in deep water showing (x, y) coordinates

Fig. 2 Sketch of discretized nodes along the riser

Fig. 3 Natural frequencies of a 2000m TTR subjected to a harmonic time-varying top tension (a=3m, ω_d=2π/10 rad/s)

Fig. 4 Transverse dynamic response of a 2000m TTR subjected to parametric excitation (a=5m, ω_d=0.2 rad/s) in calm water

Fig. 5 Transverse dynamic response of a 2000m TTR subjected to parametric excitation (a=2m, ω_d=2ω_4) in calm water

Fig. 6 Transverse dynamic response of a 2000m TTR subjected to parametric excitation (a=5m, ω_d=2ω_4) in calm water

Fig. 7 RMS of transverse displacement of a 2000m TTR for three parametric vibration scenarios in calm water

Fig. 8 Midpoint transverse displacements of the 500m TTR

Fig. 9 Midpoint transverse displacements of the 2000m TTR

Fig. 10 PSD of midpoint transverse displacements of the 2000m TTR in Cases E and F

Fig. 11 Top end axial displacement of the 500m TTR simulated using the nonlinear coupled model

Fig. 12 Top end axial displacement of the 2000m TTR simulated using the nonlinear coupled model

Fig. 13 RMS of transverse displacement of TTR simulated using the nonlinear coupled model

Fig. 14 RMS of bending stress of the 500m TTR simulated using the nonlinear coupled model

Fig. 15 RMS of bending stress of the 2000m TTR simulated using the nonlinear coupled model

Highlights as follows:

- The nonlinear coupled model of a riser subjected to time varying axial and transverse loading are formulated.
- The nonlinear coupled model is more appropriate than the linear one to account for nonlinear riser dynamics in deepwater.
- The axial excitation significantly boosts the riser's transverse response.