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Dynamic response of axially loaded Euler-Bernoulli beams

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1. Introduction

The Euler–Bernoulli theory of beams provides a reasonable explanation of the bending behavior of long isotropic beams. It is based on the assumption that a relationship between bending moment and the beam curvature exists.

Kopmaz et al. [1] considered different approaches to describing the relationship between the bending moment and curvature of a Euler-Bernoulli beam undergoing a large deformation. Then, in the case of a cantilevered beam subjected to a single moment at its free end, the difference between the linear theory and the nonlinear theory based on both the mathematical curvature and the physical curvature was shown. Biondi and Caddemi [2] studied the problem of the integration of static governing equations of the uniform Euler-Bernoulli beams with discontinuities, considering the flexural stiffness and slope discontinuities.

Many researchers have addressed the nonlinear vibration behavior of beams, theoretically [3-6]. The vibration problems of uniform Euler-Bernoulli beams can be solved by analytical or approximate approaches [7, 8]. Failla and Santini [9] presented the eigenvalue problem of Euler-Bernoulli discontinuous beams. Specifically, for stepped beams with internal translational and rotational springs, they proved that a formulation of well-established lumped-mass methods employing exact influence coefficients is readily feasible, based on appropriate Green's functions yielding the response of the discontinuous beam to a static unit force. Yeih et al. [10] obtained the natural frequencies and natural modes for an Euler-Bernoulli beam using a dual multiple reciprocity method (MRM) and the singular value decomposition method. Yeih's method was able to avoid the spurious eigenvalue problem and modes resulted from applying the conventional MRM.

A recent innovative method in solving these problems is presented by Lai et al. [11]. Through their contribution, the Adomian Decomposition Method was employed to obtain the natural frequencies and mode shapes for the Euler- Bernoulli beam under various supporting conditions. The technique used is based on the decomposition of a solution of nonlinear operator equation in a series of functions. Each term of the series is obtained from a polynomial generated from an expansion of an analytic function into a power series. Liu and Gurram [12] utilized variational iteration method (VIM) to solve free vibration of Euler-Bernoulli beam under various supporting conditions. The technique they used is based on the use of restricted variations and correction functionals which has found a wide application for the solution of nonlinear ordinary and partial differential equations. The proposed method does not require the presence of small parameters in the differential equation, and provides the solution (or an approximation to it) as a sequence of iterates.

Recently, researchers have been concentrated on approximate analytical methods such as Parameter Expansion Method [13,14], Adomian Decomposition Method [15], Differential Transform Method [16], VIM [17,18], Homotopy Perturbation Method [19-24], Max-Min Approach [25-27] and other analytical techniques [28-30].

He [31] gave a comprehensive review of the recently developed nonlinear analytics techniques for solving nonlinear oscillations problems, which comprise the relatively newer family of solutions which lie within the framework of periodic analytical solutions. Other methods have also been developed in recent years which seem to be just as promising in obtaining accurate solutions to generally more difficult nonlinear problems. Energy balance method [32] is one such method, which is actually a heuristic approach valid not only for weakly nonlinear systems, but also for strongly nonlinear ones [33-35].

The main objective of this study is to obtain analytical expressions for geometrically nonlinear vibration of Euler-Bernoulli beams. First, the governing nonlinear partial differential equation is reduced to a single nonlinear ordinary differential equation. It is assumed that only the fundamental mode is excited. The latter equation is solved analytically in time domain using Energy Balance Method (EBM).

2. Mathematical formulation

Consider a straight Euler-Bernoulli beam of length L, a cross-sectional area A, the mass per unit length of the beam m, a moment of inertia I, and modulus of elasticity E that is subjected to an axial force of magnitude P as shown in Fig. 1. The equation of motion including the effects of mid-plane stretching is given by

$$m\frac{\partial^2 w'}{\partial t'^2}+EI\frac{\partial^4 w'}{\partial x'^2}+\bar{P}\frac{\partial^2 w'}{\partial x'^2}-\frac{EA}{2L}\frac{\partial^2 w'}{\partial x'^2}\int_0^L\left(\frac{\partial^2 w'}{\partial x'^2}\right)^2 dx'=0 \quad (1)$$

For convenience, the following nondimensional variables are used

$$x=x'/L,\ \ w=w'/\rho,\ t=t'\left(EI/ml^4\right)^{1/2},\ P=\bar{P}L^2/EI$$

where $\rho=\left(I/A\right)^{1/2}$ is the radius of gyration of the cross-section. As a result Eq. (1) can be written as follows

$$\frac{\partial^2 w}{\partial t^2}+\frac{\partial^4 w}{\partial x^2}+P\frac{\partial^2 w}{\partial x^2}-\frac{1}{2}\frac{\partial^2 w}{\partial x^2}\int_0^L\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx=0 \quad (2)$$

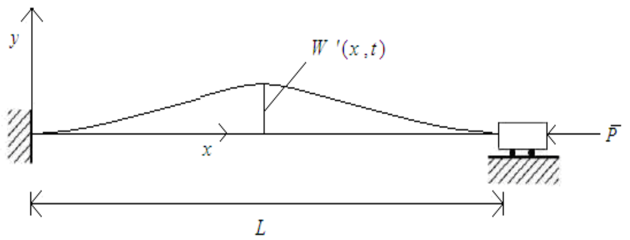


Fig. 1 A schematic of an Euler-Bernoulli beam subjected to an axial load

Assuming $w(x,t)=W(t)\phi(x)$ where $\phi(x)$ is the first eigenmode of the beam [36] and applying the Galerkin method, the equation of motion is obtained as follows

$$\frac{d^2W(t)}{dt^2}+(\alpha_1+P\alpha_2)W(t)+\alpha_3W^3(t)=0 \tag{3}$$

Eq. (3) is the differential equation of motion governing the nonlinear vibration of Euler-Bernoulli beams. The center of the beam is subjected to the following initial conditions

$$W(0)=W_{max},\ \frac{dW(0)}{dt}=0 \tag{4}$$

where W_{max} denotes the nondimensional maximum amplitude of oscillation.

Under the transformation $\tau=\omega t$, the Eq. (3) can be written as

$$\omega^2\ddot{W}+(\alpha_1+P\alpha_2)W+\alpha_3W^3=0 \tag{5}$$

where ω is the nonlinear frequency and double-dot denotes differentiation with respect to τ and α_1,α_2 and α_3 are as follows

$$\alpha_1=\frac{\left(\int_0^1\left(\frac{\partial^4\phi(x)}{\partial x^4}\right)\phi(x)\,dx\right)}{\int_0^1\phi^2(x)dx} \tag{6a}$$

$$\alpha_2=\frac{\left(\int_0^1\left(\frac{\partial^2\phi(x)}{\partial x^2}\right)\phi(x)\,dx\right)}{\int_0^1\phi^2(x)\,dx} \tag{6b}$$

$$\alpha_3=\frac{\left(\left(-\frac{1}{2}\right)\int_0^1\left(\frac{\partial^2\phi(x)}{\partial x^2}\int_0^1\left(\frac{\partial^2\phi(x)}{\partial x^2}\right)^2dx\right)\phi(x)\,dx\right)}{\int_0^1\phi^2(x)\,dx} \tag{6c}$$

Post-buckling load-deflection relation for the problem can be obtained from Eq. (5) by substituting $\omega=0$ as

$$P=\left(-\alpha_1-\alpha_3W^2\right)/\alpha_2 \tag{7}$$

Neglecting the contribution of W in Eq. (7), the buckling load can be determined as

$$P_c=-\alpha_1/\alpha_2\,. \tag{8}$$

3. Basic idea of energy balance method

In the present paper, we consider a general nonlinear oscillator in the form [32]

$$u''+f(u(t))=0 \tag{9}$$

In which u and t are generalized dimensionless displacement and time variables, respectively. Its variational principle can be easily obtained

$$J(u)=\int_0^t\left(-\frac{1}{2}u'^2+F(u)\right)dt \tag{10}$$

where $T=\frac{2\pi}{\omega}$ is period of the nonlinear oscillator, $F(u)=\int f(u)\,du.$

Its Hamiltonian, therefore, can be written in the form

$$H=\frac{1}{2}u'^2+F(u)+F(A) \tag{11}$$

or

$$R(t)=-\frac{1}{2}u'^2+F(u)-F(A)=0 \tag{12}$$

Oscillatory systems contain two important physical parameters, (i.e., the frequency ω and the amplitude of oscillation A). So let us consider such initial conditions

$$u(0)=A,\ u'(0)=0 \tag{13}$$

We use the following trial function to determine the angular frequency ω

$$u(t)=Acos\omega\, t \tag{14}$$

Substituting (14) into u term of (12), yield

$$R(t)=\frac{1}{2}\omega^2A^2sin^2\omega\, t+F(Acos\,\omega\, t)-F(A)=0 \tag{15}$$

If, by chance, the exact solution had been chosen as the trial function, then it would be possible to make R zero for all values of t by appropriate choice of ω. Since Eq. (14) is only an approximation to the exact solution, R cannot be made zero everywhere. Collocation at $\omega\, t=\pi/4$ gives

$$\omega=\sqrt{\frac{2F(A)-F(Acos\omega\, t)}{A^2sin^2\omega\, t}} \tag{16}$$

Its period can be written in the form

$$T=\frac{2\pi}{\sqrt{\frac{2F(A)-F(Acos\,\omega\, t)}{A^2sin^2\omega\, t}}}. \tag{17}$$

4. Application of the energy balance method

Consider the Eqs. (3) and (4) for the vibration of an Euler-Bernoulli beam. Free oscillation of the system without damping is a periodic motion and under the transformation $W(t) = V(\tau)$, Eqs. (3) and (4) become as follows

$$\omega^2 \frac{d^2V(\tau)}{d\tau^2} + (\alpha_1 + P\alpha_2)V(\tau) + \alpha_3 V^3(\tau) = 0 \tag{18}$$

$$V(0) = W_{max}, \quad \frac{dV(0)}{d\tau} = 0 \tag{19}$$

Its variational formulation can be readily obtained as follows

$$J(V) = \int_0^t \left(\begin{array}{l} -\frac{1}{2}\omega_0^2 \frac{dV(\tau)}{d\tau} + \frac{1}{2}(\alpha_1 + P\alpha_2) \times \\ \times V^2(\tau) + \alpha_3 V^4(\tau) \end{array} \right) d\tau \tag{20}$$

Its Hamiltonian, therefore, can be written in the form

$$H = -\frac{1}{2}\omega_0^2 \frac{dV(\tau)}{d\tau} + \frac{1}{2}(\alpha_1 + P\alpha_2)V^2(\tau) + \alpha_3 V^4(\tau) \tag{21}$$

and

$$H_{t=0} = \frac{1}{2}W_{max}^2(\alpha_1 + P\alpha_2) + \frac{1}{4}\alpha_4 W_{max}^4 \tag{22}$$

$$H_t - H_{t=0} = \frac{1}{2}\omega_0^2 \frac{dV(\tau)}{d\tau} + \frac{1}{2}(\alpha_1 + P\alpha_2)V^2(\tau) +$$
$$+ \alpha_3 V^4(\tau) - \frac{1}{2}W_{max}^2(\alpha_1 + P\alpha_2) - \frac{1}{4}\alpha_4 W_{max}^4 \tag{23}$$

We will use the trial function to determine the angular frequency ω, i.e.

$$V(\tau) = A cos\omega\, \tau \tag{24}$$

If we substitute Eq. (24) into Eq. (23), it results the following residual equation

$$\frac{1}{2}\omega_0^2(-W_{max}\omega\, sin(\omega t))^2 + \frac{1}{2}(\alpha_1 + P\alpha_2) \times$$
$$\times (W_{max}\, cos(\omega t))^2 + \frac{1}{2}\alpha_3 (W_{max}\, cos(\omega t))^4 -$$
$$-\frac{1}{2}W_{max}^2(\alpha_1 + P\alpha_2) - \frac{1}{4}\alpha_4 W_{max}^4 = 0 \tag{25}$$

If we collocate at $\omega t = \frac{\pi}{4}$ we obtain

$$\frac{1}{4}\omega_0^2 W_{max}^2\, \omega^2 - \frac{1}{4}W_{max}^2(\alpha_1 + P\alpha_2) - \frac{3}{16}\alpha_3 W_{max}^4 = 0 \tag{26}$$

The nonlinear natural frequency and deflection of the beam centre become as follows

$$\omega_{NL} = \frac{\sqrt{4(\alpha_1 + P\alpha_2) + 3\alpha_3 W_{max}^2}}{2\omega_0} \tag{27}$$

According to Eq. (14) and Eq. (27), we can obtain the following approximate solution

$$\upsilon(t) = W_{max} cos\left(\frac{\sqrt{4(\alpha_1 + P\alpha_2) + 3\alpha_3 W_{max}^2}}{2\omega_0} \right) \tag{28}$$

Its period can be written in the form

$$T_{EBM} = \frac{4\pi\ \omega_0}{\sqrt{4(\alpha_1 + P\alpha_2) + 3\alpha_3 W_{max}^2}} \tag{29}$$

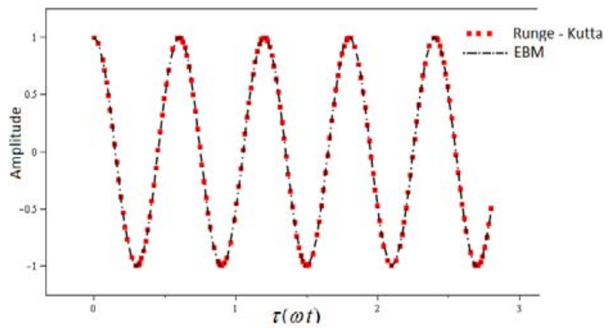
5. Results and discussions

The simply supported and clamped beams are used to demonstrate the accuracy and effectiveness of the Energy Balance Method, as the procedure explained in previous sections. Table shows the comparison of nonlinear to linear frequency ratio (ω_{NL}/ω_L) with those reported in the literature. It has illustrated that there is an excellent agreement between the results obtained from the energy balance method and those reported by Azrar et al. [37] and

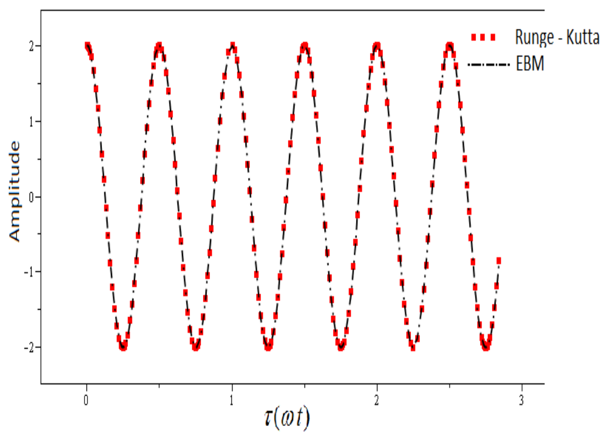
Table

The comparison of nonlinear to linear frequency ratio (ω_{NL}/ω_L)

W_{max}	Simply supported			Clamped		
	Azrar [36]	Qaisi [30]	Present study	Azrar [36]	Qaisi [30]	Present study
1	1.0891	1.0897	1.0897	1.0221	1.0628	1.0572
2	1.3177	1.3229	1.3228	1.0856	1.2140	1.2125
3	1.6256	1.6394	1.6393	1.1831	1.3904	1.4344
4	-	-	1.9999	1.3064	1.5635	1.6171



a



b

Fig. 2 Variation of the nondimensional amplitude ratio versus τ for $W_{max} = 1$ and $W_{max} = 2$

Qaisi [30]. The difference between the nonlinear frequency and linear frequency increases when the amplitude of vibration is increased. In general, large vibration amplitude will yield a higher frequency ratio. It can be easily seen that for high-amplitude ratios the present method overestimates the frequencies of clamped beams but gives close agreement with published results for simply supported beams. The reason is because of using the trigonometric base functions in the application of energy balance method, which means that we assumed the general form of solution is a combination of trigonometric functions. Since the eigenmodes for simply supported beams involve only the sinusoidal component, the energy balance method gives more accurate results in comparison with clamped beams which have hyperbolic component in their eigenmodes. To demonstrate the accuracy of the obtained analytical results we also calculate the variation of nondimensional amplitude ratio versus τ for the beam center using fourth-order Runge-Kutta method. Fig. 2 illustrates the comparison between these results. As can be seen in the figure, the results obtained using the energy balance method have a good agreement with numerical results.

6. Conclusions

In this study, the energy balance method was employed to obtain analytical expressions for the nonlinear fundamental frequency and deflection of Euler-Bernoulli beams. These expressions are valid for a wide range of vibration amplitudes, unlike the solutions obtained by the other analytical techniques such as perturbation methods. The energy balance method solution converges quickly and its components can be simply calculated. Also, compared to other analytical methods, it can be observed that the results of energy balance method require smaller computational effort and only a first-order approximation leads to accurate solutions. Beside all the advantages of the energy balance method, there are no rigorous theories to direct us to choose the initial approximations, auxiliary linear operators, auxiliary functions, and auxiliary parameter. However, further research is needed to better understand the effect of these parameters on the solution quality.

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AŠINE KRYPTIMI APKRAUTŲ EULERIO IR BERNULIO SIJŲ DINAMINIS ATSPARUMAS

R e z i u m ė

Nagrinėjamas naujo analitinio metodo, vadinamo energijos balanso metodu (EBM), taikymas netiesiniams uždaviniams spręsti. Energijos balanso metodas taikomas netiesinių svyravimų poveikio Eulerio ir Bernulio sijoms, apkrautoms ašiniais krūviais, analitiniam sprendimui gauti. Pasiūlytos sijų geometriškai netiesinių virpesių analitinės išraiškos. Aptarta virpesių amplitudės įtaka netiesiniam dažniui. Energijos balanso metodo rezultatų palyginimas su literatūros rezultatais patvirtina šio metodo tikslumą. Šiuo metodu, priešingai nei įprastiniais metodais, tik vienu priartėjimu gaunamas labai tikslus sprendinys, galiojantis plačiame svyravimo amplitudžių intervale.

M. Bayat, A. Barari, M. Shahidi

DYNAMIC RESPONSE OF AXIALLY LOADED EULER-BERNOULLI BEAMS

S u m m a r y

The current research deals with application of a new analytical technique called Energy Balance Method (EBM) for a nonlinear problem. Energy Balance Method is used to obtain the analytical solution for nonlinear vibration behavior of Euler-Bernoulli beams subjected to axial loads. Analytical expressions for geometrically nonlinear vibration of beams are provided. The effect of vibration amplitude on the nonlinear frequency is discussed. Comparison between Energy Balance Method results and those available in literature demonstrates the accuracy of this method. In Energy Balance Method contrary to the conventional methods, only one iteration leads to high accuracy of the solutions which are valid for a wide range of vibration amplitudes.

М. Баиат, А. Барари, М. Схахиди

ДИНАМИЧЕСКОЕ СОПРОТИВЛЕНИЕ БАЛОК ЭУЛЛЕРА-БЕРНУЛИ НАГРУЖЕННЫХ ОСЕВОЙ НАГРУЗКОЙ

Р е з ю м е

Настоящее исследование рассматривает применение нового аналитического метода, называемого методом баланса энергии (МБЭ) для решения нелинейных проблем. Метод баланса энергии применяется для аналитического решения влияния нелинейных колебаний балок Эуллера-Бернули, нагруженных осевой нагрузкой. Составлены аналитические зависимости для геометрически нелинейных колебаний балок. Рассмотрено влияние амплитуды колебаний на нелинейную частоту. Сопоставление результатов метода баланса энергии с результатами литературы подтверждает точность этого метода. Этот метод в противоположность известным методам, уже после одного приближения дает точные результаты для широкого интервала амплитуд колебаний.

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