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Dynamic Response of Embedded Strip and Rectangular Foundations Using a Poroelastic BEM

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Abstract

The dynamic response of embedded strip and rectangular shallow foundations subjected to time-harmonic vertical excitations is studied. A dynamic poroelastic plane strain BEM formulation is utilized to solve the strip foundation problems while a full three dimensional BEM is used for square and rectangular foundations. The paper aims to extend the previous work on the dynamic response of strip footings on the surface of poroelastic soil media to account for foundation embedment. The effect of different embedment ratios and permeability values of the soil on the vertical and horizontal compliance of such foundations is investigated.

1 Introduction

The dynamic behavior of poroelastic media, such as soils and rocks, is an important class of problems which are often solved on the basis of a single-phase linear elastic model in the study of soil-structure interaction. The boundary element method (BEM) has been applied with much success to the solution of elastodynamic soil-structure interaction. The present paper aims at

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modeling the soil in a more realistic manner by accounting for the inertia effects within the pore water phase and the interaction of the soil skeleton and the pore water under dynamic loads. It is found that the response within an infinite medium may be described adequately by a single phase model with suitable damping, but the effect of layering of the soil adds more complexity. BEM is ideally suited for the study of the dynamic behavior of semi-infinite media and the present multi-region algorithm is also capable of handling layered media.

Dynamic poroelastic analysis is based on the effective stress theory of Terzaghi. Biot [1] extended the work of Terzaghi to a general theory governing the behavior of two-phase fluid-filled materials such as soils. The correctness of this theory in the linear range has been confirmed by other approaches such as a two-scaled analysis of Navier-Stokes equations and the theory of mixtures. The coupled phenomenon, however, precludes the development of analytical solutions for all but the simplest of geometry and boundary conditions. The finite element method has also been used extensively with Biot's theory for poroelastic applications.

BEM was first applied to dynamic poroelastic analysis through the use of six unknowns (solid skeleton displacements and the average relative solid-fluid displacement). Subsequently, Cheng et al. [2] and Dominguez [3] developed two-dimensional frequency domain BEM solutions for the dynamic case using only four independent variables (the solid displacements and pore pressure). Chen and Dargush [4] presented a complete transient BEM for both 2-D and 3-D dynamic poroelastic analysis. Recently, the authors have presented a BEM for axisymmetric dynamic problems and have studied the response of surface (Dargush and Chopra [5]) and embedded (Chopra and Dargush [6]) circular footings on poroelastic media. The 2-D and 3-D dynamic poroelastic BEM (Chen and Dargush [4]) is used in this paper to study the dynamic compliance of strip and rectangular foundations under vertical and horizontal excitation, with particular emphasis on the effects of foundation embedment. The effect of various embedment ratios and permeability values corresponding to a viscous pore fluid-filled soil mass is investigated. The dynamic behavior of embedded foundations in a semi-infinite half-space and a layer of soil overlying a hard stratum is studied.

2 Boundary Element Formulation

The governing equations for dynamic poroelastic analysis based upon Biot's theory may be expressed in frequency domain as balances of momentum and mass of a fluid-filled medium, as follows:

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$$\mu \tilde{u}_{i,jj} + (\lambda + \mu)\tilde{u}_{j,ij} + \omega^2 \rho_1 \tilde{u}_i - \alpha_1 \tilde{p}_{,i} + \tilde{f}_i = 0 \tag{1a}$$

$$\zeta \tilde{p}_{,ii} - (i\omega / Q)\tilde{p} - i\omega\alpha_1 \tilde{u}_{i,i} + \tilde{\psi} = 0 \tag{1b}$$

where u_i is the displacement of the solid skeleton, p denotes the pore water pressure. The parameters λ and μ are the drained Lame constants, κ is the permeability coefficient of the soil (i.e. $\kappa = k / \eta$) where η is the fluid viscosity and k the specific permeability of the soil. Other Biot parameters include:

$$\alpha_1 = \alpha - i\omega\rho_f\zeta \; ; \; \rho_1 = \rho - i\omega\rho_f{}^2\zeta \; ;$$
$$\zeta = (1/\kappa + i\omega m)^{-1} \tag{2}$$

where ω is the rotational frequency. The effective permeability becomes a complex valued function of ω. The quantities ρ and ρ_f denote the total and fluid densities respectively. The parameters α and Q are parameters accounting for the material compressibility while ψ and f_i are the volumetric body source rate and body force respectively. Lastly, m is a parameter arising from the generalized Darcy's Law and is related to the inertial effects of the fluid behavior. The superposed tilde denotes variables transformed to the frequency domain.

An exact boundary integral equation, in the absence of body forces and sources, may be expressed as follows for a poroelastic volume V bounded by a surface S (Chen and Dargush [4]) :

$$C_{\beta\alpha}(x)\, \tilde{u}_\beta(x;w) = \int_S [G_{\beta\alpha}(x,x;w)\, \tilde{t}_\beta(x;w) - F_{\beta\alpha}(x,x;w)\, \tilde{u}_\beta(x;w)]\, dS(x) \tag{3}$$

where $\tilde{t}_\beta = \{\tilde{t}_1, \tilde{t}_2, \tilde{t}_3, \tilde{q}\}^T$ and $\tilde{u}_\beta = \{\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{p}\}^T$ are the generalized tractions and the generalized displacements in three-dimensions. $G_{\beta\alpha}$ is the displacement kernel and the traction kernel $F_{\beta\alpha}$ is derived from the displacement kernel by using the stress-strain and the strain-displacement relations. The matrix $C_{\beta\alpha}$ depends upon the local geometry of the boundary. Detailed expressions for the kernels are provided in Reference [4].

Equation (3) is an exact boundary integral equation but is difficult to solve analytically for anything but very simple problems. Temporal and spatial discretization of this equation is carried out in the standard manner. Details of the numerical implementation may be found in other works [5]. It should be noted that no special treatment is required for the incompressible response under undrained conditions nor for the satisfaction of radiation boundary conditions.

3 Numerical Applications

3.1 Dynamic Response of a Strip Footing Embedded in Poroelastic Soil

As the first illustration, the method is now applied to investigate the dynamic compliance of a smooth, rigid, impermeable strip footing of width 2B embedded within a poroelastic soil. Two cases are studied, namely, semi-infinite soil media and a finite layer of soil resting on a hard stratum. The soil is modeled as a two-phase poroelastic medium and the properties are selected from Dargush and Chopra [5] as: Poisson's Ratio $\nu = 1/3$; damping coefficient D=0.05; Porosity n=0.30; the shear wave velocity c_s=152 m/s; the pore water wave velocity c_w =1439 m/s; solid grain weight density 25.9 kN/m^3; pore water unit weight 9.81 kN/m^3.

Since actual soils have a finite permeability, a second time scale enters the picture [5] and the response, in general, depends upon a characteristic length (e.g., B). A dimensionless permeability χ is introduced where $\chi = c_v/(c_s R)$ and c_v is the coefficient of consolidation from the quasistatic formulation which is related to the permeability as described in Dargush and Chopra [5]. Different degrees of permeability, ranging from drained to undrained behavior, are considered.

The response of the strip footing embedded in a homogeneous poroelastic half space under vertical excitation is first studied. Figure 1 presents the vertical compliance with the dimensionless frequency a_o where $a_o = \omega B / c_s$. Different levels of the non-dimensional parameter χ are shown for a surface strip footing (i.e. d/B = 0 where d is the depth of embedment). The solutions presented by Gazetas and Roesset [7] for a strip footing on an elastic half-space are also shown for comparison. Large values of χ are associated with very permeable soils or very small footings resting on thin soil layers. The frequency range covered in Figure 1 is from zero to 60 Hz. Other solutions shown include a completely drained and undrained elastic BEM analyses. The result for very small permeability, i.e. χ = 0.0067, coincided with the undrained response and is not shown for clarity. For a given footing size, the compliance for intermediate permeability values lies between the drained and undrained responses throughout the frequency range considered and the lower permeability cases tend toward the undrained behavior. This example also provides confidence in the BEM formulation for the elastodynamic solution.

Next, the response of the strip footing on a poroelastic layer is presented as a function of different embedment ratios. The thickness of the single homogeneous elastic layer of soil is expressed as H and for the present study, $H/B = 2$. The role of layer resonance in the overall footing response is readily seen in plots involving the dynamic compliance of the foundations. The compliance values are normalized using the corresponding elastic static stiffness for the embedded foundations under vertical excitation. Four levels of embedment, corresponding to d/B = 0, 0.5, 1.0 and 1.5, are considered. The sides of the embedded footing are considered to be fully bonded with the soil. This approximation was found to be suitable in several previous works (e.g. Gazetas and Tassoulas [8]) for elastic analyses of embedded foundations. For performing the normalization, the following stiffness values were determined from fully drained BEM analyses: $K_o = 4.95\mu B$ for d/B = 0; $K_o = 6.81\mu B$ for d/B = 0.5; $K_o = 9.81\mu B$ for d/B= 1.0 and $K_o = 18.3\mu B$ for d/B = 1.5. Figure 2 shows the variation of the normalized compliance with the dimensionless frequency for the various embedment ratios for a viscous pore fluid with χ = 0.067. The frequency range covered in Figure 2 is from zero to 152 Hz. The frequency dependence and the dynamic amplification of the foundation response is clearly evident from the figure. It is notable that the amplitude of the dynamic amplification is progressively reduced due to the presence of adjoining soil mass for an embedded footing. With increasing depths of embedment, the dynamic amplification begins to shift to the right to higher frequencies. This phenomenon is associated with the propagation of generalized Rayleigh waves within the elastic soil layer. The dynamic amplification of the response due to surface wave propagation near the frequency $a_o \approx \pi/2$ is evident.

The poroelastic response of a surface strip footing at different permeability levels under horizontal excitation was also studied. Due to limitations of space, this response is not plotted here. For a given footing size, the response for soils with higher permeability at lower frequencies follows the drained response but in all cases converges to the undrained response at higher frequencies since the material does not have a chance to respond to the motion.

3.2 Dynamic Response of Square and Rectangular Footing Embedded in Poroelastic Media

The second application of the dynamic poroelastic BEM is the analysis of the response of square and rectangular footings to harmonic excitations. The soil properties are identical to those used in the previous section for strip footing analysis. First, a square footing of side 2B is studied. Figure 3 presents the response of the square footing on the surface of a half-space for the frequency range 0 to 96 Hz. Different permeability ratios are shown including the extreme cases of fully drained and undrained behavior. The drained solution agrees well with the solution presented in published literature (e.g. Wong and Luco, [9]). For this plot, the vertical compliance values are not normalized with the static stiffness values in order to compare with other available results. Once again, the response of a highly permeable material transitions from the drained response at low frequencies to the undrained response at higher frequencies. The soil with low permeability always behaves like an incompressible material irrespective of the frequency levels.

The next figure, Figure 4, shows the normalized compliance of a square foundation as a function of the embedment ratios while embedded in a poroelastic layer of $H/B = 4$. For performing the normalization, the following stiffness values were determined from fully drained BEM analyses: $K_O = 5.1\mu B$ for d/B = 0; $K_O = 7.45\mu B$ for d/B = 1.0 and $K_O = 9.53\mu B$ for d/B= 2.0. Three embedment ratios, namely, d/B = 0, 1, 2, are considered. The effect of layer resonance on the dynamic compliance is once again clearly evident. It is notable that, for this layer depth, the case d/B = 1 does not differ sharply from the surface response while d/B = 2 shows a substantial variation with a reduction in magnitude due to the embedment effect and a shift in the resonance frequency to a higher value.

The next example involves the investigation of a rectangular foundation with length to width ratio of 2 (i.e. L/B = 2). A detailed parametric study was conducted for both the square and rectangular cases. However, only a selected few results are presented due to space limitations. The complete treatment of these cases will be presented in a forthcoming work. Figure 5 shows the effect of embedment on the dynamic compliance of the rectangular footing under vertical excitation. Once again, three embedment ratios of 0, 1 and 2 are considered and the thickness of the poroelastic layer is $H/B = 4$. The following static stiffness values are used for normalizing the dynamic response of the rectangular footing of L/B = 2: for the vertical case, $K_o = 8.42\mu B$ for d/B = 0; $K_o = 12.3\mu B$ for d/B = 1.0 and $K_o = 16.5\mu B$ for d/B= 2.0 while for the horizontal case, $K_o = 5.06\mu B$ for d/B = 0; $K_o = 9.32\mu B$ for d/B = 1.0 and $K_o = 14.04\mu B$ for d/B= 2.0. In a manner similar to the square footing behavior, the smaller embedment ratio responds like a surface footing while the higher embedment ratio shows the damped response due to surrounding soil. Finally, the dynamic response of the same rectangular footing to horizontal harmonic forcing motion is investigated in Figure 6. This response is interesting since this is the first case where embedment causes an increase in the dynamic behavior. Several peaks are evident in the response corresponding to the natural frequencies of the soil layer. Further details the response of all three foundations will be presented in a forthcoming parametric study.

4 Conclusions

A dynamic poroelastic BEM is used for the analysis of the compliance of strip, square and rectangular foundations subjected harmonic excitations. The layer resonance at the natural frequencies of the layers is evident and the variation of the dynamic response for a fluid-filled soil medium between the fully drained and incompressible limits is observed.

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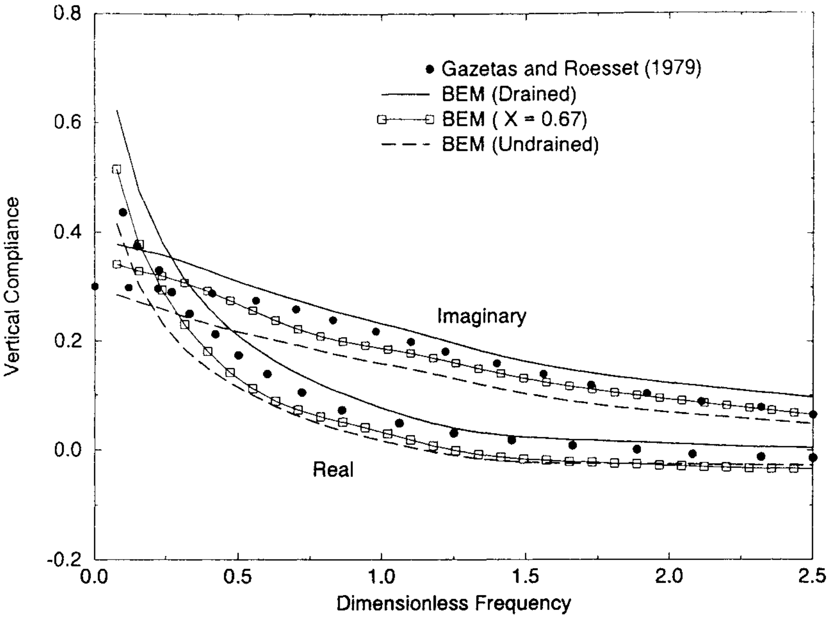

Figure 1: Dynamic Response of a Strip Footing on the Surface of a Poroelastic Half-Space.

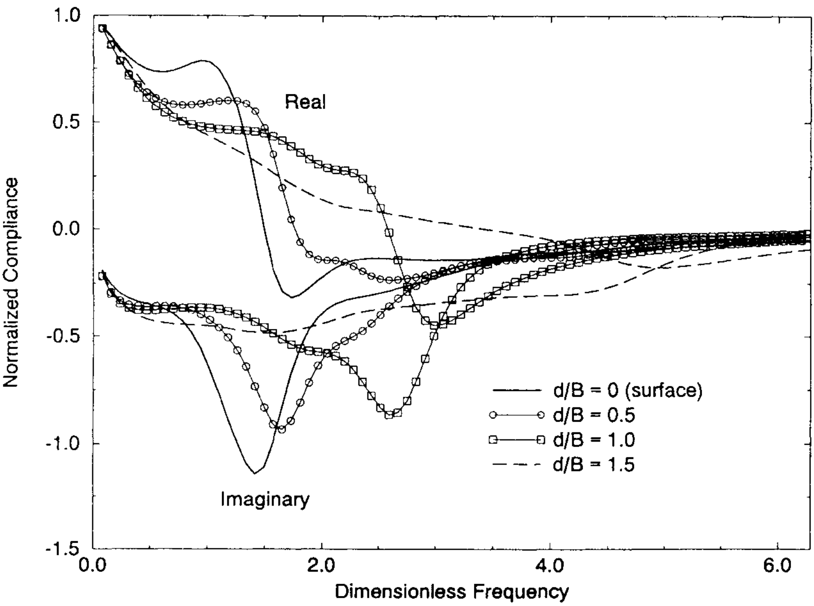

Figure 2: Effect of Embedment on the Normalized Vertical Compliance of a Strip Footing (at $\chi = 0.067$).

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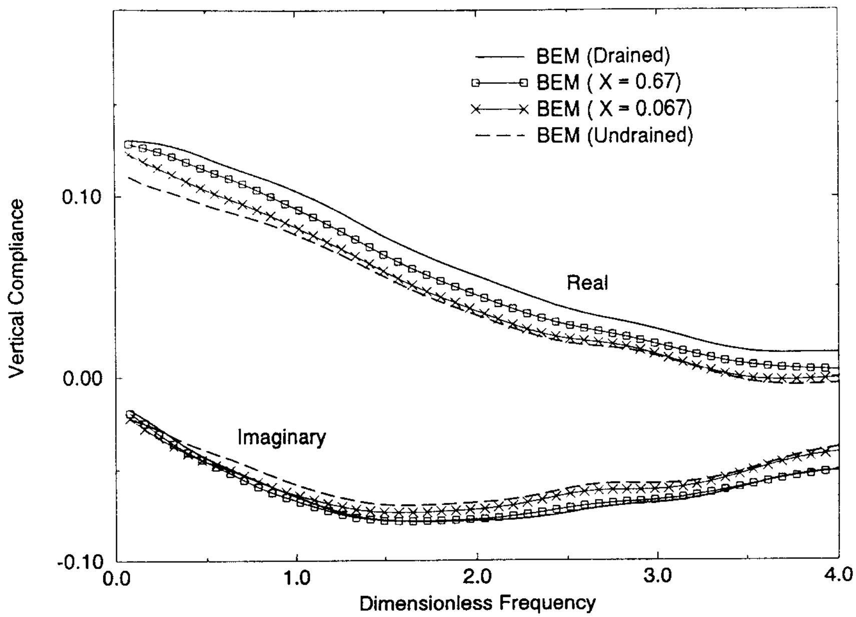

Figure 3: Dynamic Response of a Square Footing on the Surface of a Poroelastic Half-Space.

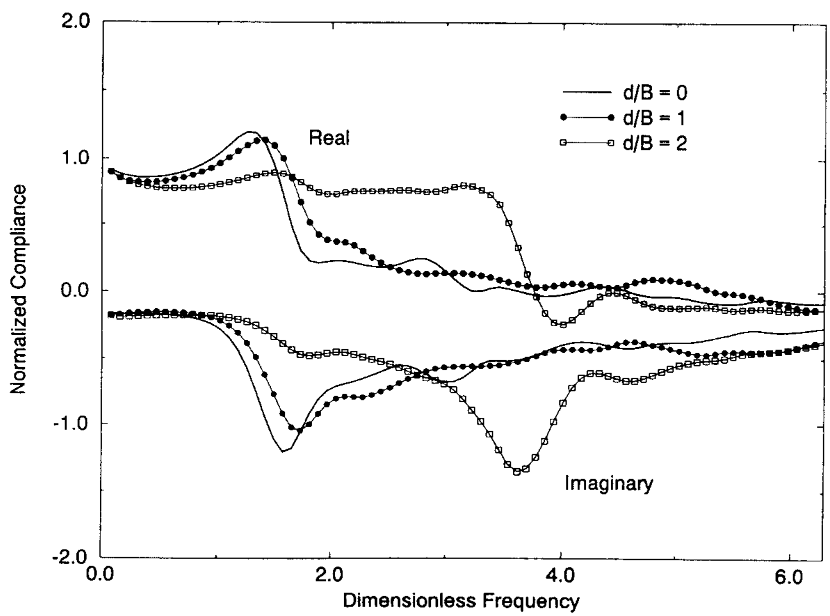

Figure 4: Effect of Embedment on the Normalized Vertical Compliance of a Square Footing (at $\chi = 0.067$).

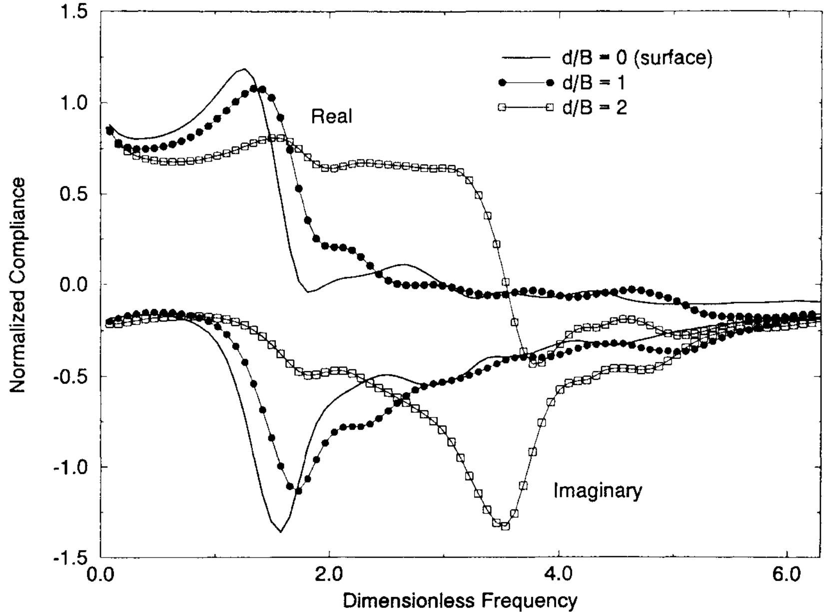

Figure 6: Effect of Embedment on the Normalized Vertical Compliance of a Rectangular Footing with L/B = 2 (at $\chi = 0.067$).

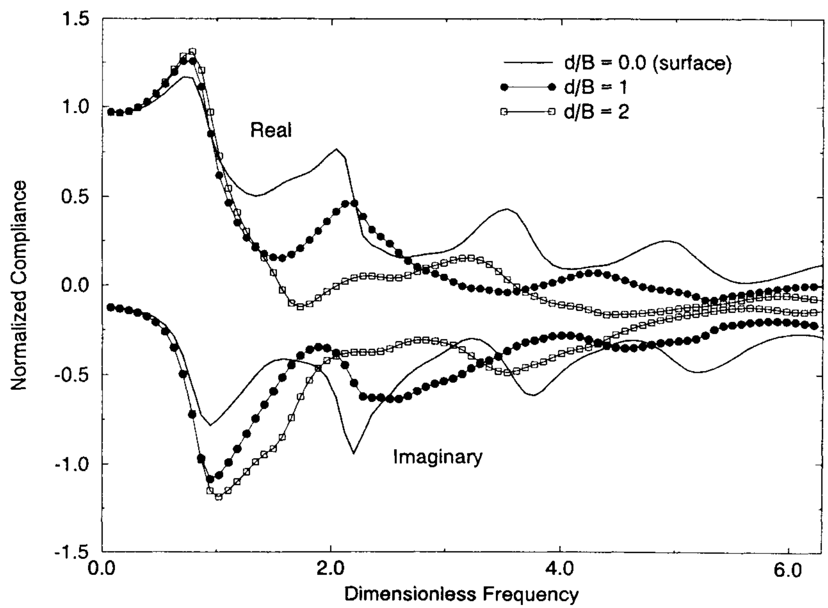

Figure 7: Effect of Embedment on the Normalized Horizontal Compliance of a Rectangular Footing with L/B = 2 (at $\chi = 0.067$).