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Dynamic screening and collective excitation of an electron gas under intense terahertz radiation

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By using time-dependent wave functions for electrons under an intense laser, we calculated the charge-density fluctuation of an electronic system under a weak probing potential. The dielectric function of the system as a function of the laser frequency and intensity is derived. The spectrum of the collective excitation is calculated. The spectrum exhibits the contribution of various multiphoton processes.

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Recently there has been a great deal of interest in terahertz phenomena in electronic materials. This is mainly due to the rapid development of high-power, long-wavelength, and tunable laser sources such as free electron lasers (FEL's). Very recently, a new mechanism for population inversion in semiconductors under an applied electric field was predicted[1] and a widely tunable continuous-wave terahertz (THz) generation was achieved experimentally in strained Ge.[2] These radiation sources can provide linearly polarized laser radiation in the terahertz regime.[3–10] THz lasers have been applied to the experimental investigation of nonlinear transport and optical properties in electron gases such as low-dimensional semiconductor systems. Many interesting terahertz phenomena have been investigated, including resonant absorption,[3] photon-enhanced hot-electron effect,[6] THz photon-induced impact ionization,[7] LO-phonon bottleneck effect,[8] THz photon-assisted tunneling,[9] and THz cyclotron resonance.[10]

In view of this rapid development of terahertz phenomena, a theoretical formalism describing strongly coupled electron-photon systems becomes urgently required. The most useful quantity in understanding the transport and optical properties of an electronic system is the dielectric function. In this paper, we present a theoretical investigation of the electronic and dielectric properties of an electron gas strongly coupled to a THz radiation field. We first calculate the charge fluctuation of the system using the time-dependent perturbation technique. The dielectric function can then be derived. It is shown that the plasma frequency of the system is strongly dependent on the intensity and frequency of the THz laser field. Various multiphoton processes can be identified.

Let us consider an electron gas under intense laser radiation. We choose the laser field to be along the x direction, $\mathbf{E}(t)=E\cos(\omega t)\mathbf{e}_x$, where E and ω are the amplitude and frequency of the laser field. For the notational convenience, both $\hbar$ and the speed of light c have been set to unity. In the absence of a laser field, the Schrödinger equation for a single electron is given as

$$i\frac{\partial}{\partial t}\psi(\mathbf{r},t)=\frac{\mathbf{p}^2}{2m^*}\psi(\mathbf{r},t). \tag{1}$$

The wave function of the system is simply a plane wave

$$\psi(\mathbf{r},t)=\exp(-i\epsilon_k t)\exp(i\mathbf{k}\cdot\mathbf{r}), \tag{2}$$

where $\epsilon_k=k^2/2m^*$.

In the presence of an intense laser, the electrons are strongly coupled to the photon field. Let us choose the vector potential for the laser field to be in the form

$$\mathbf{A}=(E/\omega)\sin(\omega t)\mathbf{e}_x. \tag{3}$$

The time-dependent Schrödinger equation is given as

$$i\frac{\partial}{\partial t}\psi(\mathbf{r},t)=H\psi(\mathbf{r},t)=\frac{(\mathbf{p}-e\mathbf{A})^2}{2m^*}\psi(\mathbf{r},t). \tag{4}$$

It can be shown that Eqs. (1) and (4) are related by a simple unitary transformation $U=\exp(i2\gamma_1\omega t)\exp\{i\gamma_0 k_x[1-\cos(\omega t)]\}\exp[i\gamma_1\sin(2\omega t)]$,

$$U\left[i\frac{\partial}{\partial t}-\frac{\mathbf{p}^2}{2m^*}\right]U^\dagger=i\frac{\partial}{\partial t}-\frac{(\mathbf{p}-e\mathbf{A})^2}{2m^*}, \tag{5}$$

and the time-dependent wave function can be written as

$$\psi_{\mathbf{k}}(\mathbf{r},t)=U\exp(-i\epsilon_k t)\exp(i\mathbf{k}\cdot\mathbf{r}), \tag{6}$$

where $\gamma_0=(eE)/m^*\omega^2$ and $\gamma_1=(eE)^2/(8m^*\omega^3)$.

We now employ this time-dependent wave function to calculate the electronic state in a local potential (to be determined self-consistently) and to derive the dielectric properties of the system. The zeroth-order wave function is given by Eq. (6),

$$\psi_{\mathbf{k}}^{(0)}(r,t)=e^{iF(t)}e^{i\gamma_0 k_x(1-\cos\omega t)}e^{i\mathbf{k}\cdot\mathbf{r}}e^{i\epsilon_k t}, \tag{7}$$

where $F(t)=2\gamma_1\omega t+\gamma_1\sin(2\omega t)$. Equation (7) forms an orthonormal set,

$$\langle\psi_{\mathbf{k}}^{(0)}(r,t)|\psi_{\mathbf{k}}^{(0)}(r,t)\rangle=\delta_{\mathbf{k},\mathbf{k}'}.$$

There is no charge fluctuation even in the presence of the laser field, i.e.,

$$\rho_k^{(0)}=-e|\psi_{\mathbf{k}}^{(0)}(r,t)|^2=-e, \tag{8}$$

where e is the charge of an electron. The wave function of an electron under a local potential can be expanded using the above orthonormal set,

$$\psi(r,t)=\sum_{\mathbf{k}}a_{\mathbf{k}}(t)e^{iF(t)}e^{i\gamma_0 k_x(1-\cos\omega t)}e^{i\mathbf{k}\cdot\mathbf{r}}e^{i\epsilon_k t}. \tag{9}$$

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,The coefficient $a_{\mathbf{k}}(t)$ will be determined below using the time-dependent perturbation method.

We now consider a local potential

$$\phi(r,t)=\int d\mathbf{q}'\int d\Omega e^{i\mathbf{q}'\cdot\mathbf{r}}e^{i\Omega t}\phi(\mathbf{q}',\Omega)+\text{c.c.},\qquad(10)$$

and seek the change of the electron density and the induced potential. In the above equation, ''c.c.'' denotes the complex conjugate of the preceding term. We assume that the local potential is weak and use the time-dependent perturbation to calculate the change of electronic state. The time-dependent Schrödinger equation is now given as

$$i\frac{\partial\psi}{\partial t}=(H-e\phi)\psi,\qquad(11)$$

where H is given in Eq. (4). Upon using Eq. (9), we obtain the first-order equation

$$i\frac{\partial a_{\mathbf{k}'}(t)}{\partial t}=-ee^{i\gamma_0(k_x-k_x')[1-\cos(\omega t)]}e^{-i(\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}})t}\times\int d\mathbf{r}e^{-i\mathbf{k}'\cdot\mathbf{r}}\phi(\mathbf{r},t)e^{i\mathbf{k}\cdot\mathbf{r}}\qquad(12)$$

and

$$a_{\mathbf{k}'}(t)=ie\int_{-\infty}^{t}dte^{i\gamma_0(k_x-k_x')[1-\cos(\omega t)]}e^{-i(\epsilon_{\mathbf{k}'}-\epsilon_{\mathbf{k}})t}\times\int d\mathbf{r}e^{-i\mathbf{k}'\cdot\mathbf{r}}\phi(\mathbf{r},t)e^{i\mathbf{k}\cdot\mathbf{r}}.\qquad(13)$$

Substituting the Fourier expansion given in Eq. (10) and making use of the generating function of the Bessel function,

$$e^{i\alpha\cos x}=\sum_m i^mJ_m(\alpha)e^{imx},$$

we obtain

$$a_{\mathbf{k}+\mathbf{q}}(t)=-(ie)e^{-i\gamma_0q_x}\sum_{m,\Omega}i^mJ_m(q_x\gamma_0)\phi(\mathbf{q},\Omega)\times\frac{e^{i(\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}-\Omega-m\omega)t}}{\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}-\Omega-m\omega-i\eta}\quad(\eta\to0^+).\qquad(14)$$

Now the wave function up to first order is given as

$$\begin{aligned}\psi_{\mathbf{k}}(\mathbf{r},t)&=\psi_{\mathbf{k}}^{(0)}(\mathbf{r},t)+\sum_{\mathbf{q}}a_{\mathbf{k}+\mathbf{q}}(t)\psi_{\mathbf{k}+\mathbf{q}}^{(0)}(\mathbf{r},t)\\&=e^{iF(t)}e^{i\gamma_0k_x(1-\cos\omega t)}e^{i\epsilon_kt}\\&\quad\times\Bigg\{e^{i\mathbf{k}\cdot\mathbf{r}}-e\sum_{\mathbf{q},\Omega}e^{-i\gamma_0q_x\cos\omega t}\sum_m i^mJ_m(q_x\gamma_0)\\&\quad\times\frac{\phi(\mathbf{q},\Omega)e^{-i(\Omega+m\omega)t}}{\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}-\Omega-m\omega-i\eta}e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}}\Bigg\}.\end{aligned}\qquad(15)$$

The fluctuation of the charge distribution (induced charge density) can now be calculated,

$$\rho_k(\mathbf{r},t)=-e[\psi_{\mathbf{k}}^*(\mathbf{r},t)\psi_{\mathbf{k}}(\mathbf{r},t)-1].\qquad(16)$$

Neglecting high-order terms in ϕ, we obtain

$$\begin{aligned}\rho_k(\mathbf{r},t)&=-e^2\sum_{\mathbf{q},\Omega}\sum_m i^m\phi(\mathbf{q},\Omega)J_m(q_x\gamma_0)\\&\quad\times\Bigg[\frac{e^{-i\gamma_0q_x\cos\omega t}e^{-i(\Omega+m\omega)t}}{\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}-\Omega-m\omega-i\eta}e^{i\mathbf{q}\cdot\mathbf{r}}\\&\quad+\frac{(-1)^me^{i\gamma_0q_x\cos\omega t}e^{i(\Omega+m\omega)t}}{\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}-\Omega-m\omega-i\eta}e^{-i\mathbf{q}\cdot\mathbf{r}}\Bigg].\end{aligned}\qquad(17)$$

The contribution to the induced charge density due the c.c. part of the local potential can be calculated with the same method. After some rearrangement we obtain

$$\begin{aligned}\rho_k(\mathbf{r},t)&=-e^2\sum_{\mathbf{q},\Omega}e^{i\mathbf{q}\cdot\mathbf{r}}\phi(\mathbf{q},\Omega)e^{-i\gamma_0q_x\cos\omega t}e^{-i\Omega t}\\&\quad\times\sum_m i^mJ_m(q_x\gamma_0)e^{-im\omega t}\\&\quad\times\Bigg[\frac{1}{\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}-\Omega-m\omega-i\eta}\\&\quad+\frac{1}{\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}+\Omega+m\omega+i\eta}\Bigg]+\text{c.c.}\end{aligned}\qquad(18)$$

Now the total density fluctuation of the system is given as

$$\rho(\mathbf{r},t)=\sum_{\mathbf{k}}f_{\mathbf{k}}\rho_{\mathbf{k}}(\mathbf{r},t),\qquad(19)$$

where $f_{\mathbf{k}}=\{\exp[(\epsilon_{\mathbf{k}}-\mu-E_\gamma)/k_BT]+1\}^{-1}$ is the Fermi distribution function. Here $E_\gamma=2\gamma_1\omega$ is the energy of the laser field, μ is the chemical potential, and k_B is the Boltzmann constant. Substituting Eq. (18) into Eq. (19),

$$\begin{aligned}\rho(\mathbf{r},t)&=e^2\sum_{\mathbf{q},\Omega}e^{i\mathbf{q}\cdot\mathbf{r}}\phi(\mathbf{q},\Omega)e^{-i\gamma_0q_x\cos\omega t}e^{-i\Omega t}\\&\quad\times\sum_m i^mJ_m(q_x\gamma_0)e^{-im\omega t}\Pi(q,\Omega+m\omega),\end{aligned}\qquad(20)$$

where $\Pi(q,\Omega)$ is the electron polarizability,

$$\Pi(q,\Omega)=\sum_{\mathbf{k}}\frac{f_{\mathbf{k}+\mathbf{q}}-f_{\mathbf{k}}}{\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}}-\Omega-i\eta}.\qquad(21)$$

After decomposing the time-dependent factor $e^{-i\gamma_0q_x\cos\omega t}$ into successive harmonics, the electron density fluctuation can be written as

$$\rho(\mathbf{r},t) = -e^2 \sum_{\mathbf{q},\Omega} e^{i\mathbf{q}\cdot\mathbf{r}} \phi(\mathbf{q},\Omega) e^{-i\Omega t} \sum_{m,m'} i^{m-m'} \times J_m(q_x \gamma_0) J_{m'}(q_x \gamma_0) e^{-i(m-m')\omega t} \times \Pi(q,\Omega+m\omega). \quad (22)$$

From Poisson's equation, the induced potential can be calculated from the density fluctuation,

$$\nabla^2 \phi_{ind}(\mathbf{r},t) = -4\pi\rho(\mathbf{r},t). \quad (23)$$

Performing the Fourier expansion for the induced potential, we obtain

$$\nabla^2 \phi_{ind}(\mathbf{r},t) = -\sum_{\mathbf{q},\Omega} q^2 e^{i\mathbf{q}\cdot\mathbf{r}} e^{i\Omega t} \phi_{ind}(\mathbf{q},\Omega) + \text{c.c.} \quad (24)$$

Combining Eqs. (20), (23), and (24), we obtain the Fourier component of the induced potential,

$$\phi_{ind}(\mathbf{q},\Omega) = \frac{4\pi e^2}{q^2} \phi(\mathbf{q},\Omega) \sum_m J_m^2(q_x \gamma_0) \Pi(q,\Omega+m\omega). \quad (25)$$

We see immediately that terms with $m \neq m'$ in the density fluctuation do not contribute to the induced potential.

The local potential is the sum of the external potential and the induced potential in terms of Fourier components, we have

$$\phi(\mathbf{q},\Omega) = \phi_{ext}(\mathbf{q},\Omega) + \phi_{ind}(\mathbf{q},\Omega), \quad (26)$$

which leads to

$$\phi(\mathbf{q},\Omega) = \frac{\phi_{ext}(\mathbf{q},\Omega)}{D(\mathbf{q},\Omega)}, \quad (27)$$

where the dielectric function is given as

$$D(\mathbf{q},\Omega) = 1 - \frac{4\pi e^2}{q^2} \sum_m J_m^2(q_x \gamma_0) \Pi(q,\Omega+m\omega). \quad (28)$$

The induced potential and the dielectric function derived above are valid for any strength of the laser field at any electron densities and temperatures, provided the probing potential $\phi_{ext}(\mathbf{r},t)$ (and the resulting local potential) is weak. The theory presented here is basically a linear response theory for the probing potential but includes infinite orders of electron-photon coupling. The above result can be applied to both bulk systems and low-dimensional systems. For two-dimensional systems, one should replace $4\pi e^2/q^2$ by $2\pi e^2/q$ in Eq. (28).

The modes of the collective excitation of the system are determined by the solution of

$$D(\mathbf{q},\Omega) = 0.$$

For the present system, the plasma frequency will be strongly dependent on the frequency and intensity of the laser field. Most recent experimental work in terahertz phenomena are carried out in two-dimensional semiconductor systems. Here we perform some numerical computations of plasma frequencies for a two-dimensional GaAs semiconductor system. The results are plotted in Figs. 1–3. In all these calculations, we used the following parameters: $m^* = 0.067 m_e$, $r_s = m^* e^2/k_F = 0.8$, and $\epsilon_F = 12$ meV. The presented plasmon energies are for the case where $q_x = q_y$. Several interesting properties can be discussed.

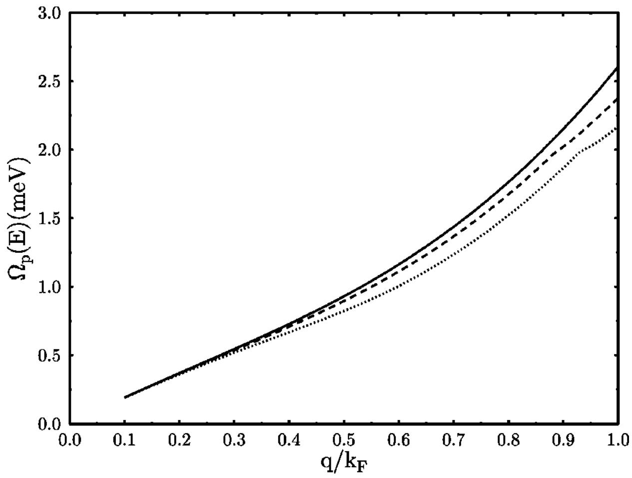

FIG. 1. Plasma dispersion along the direction of $q_x = q_y$ at fixed laser frequency ($\omega = 1$ meV) and for three different values of the laser intensities. The solid line is for $R = 0$, the dashed line is for $R = 1.0$, and the dotted line is for $R = 2.0$. All parameters are given in the text.

Figure 1 is the dispersion of the plasmon frequency for different laser intensities along the direction of $q_x = q_y$. The frequency of the laser field is 1 meV. We use a dimensionless quantity $R = k_F e E/m^*$ (THz)^2 to specify the field strength. At small wave vectors, the coupling between the electron and photon is weak (high-order J_m is negligible). Therefore the plasma energy is mainly determined by the electrons. As q increases, the electron-photon coupling increases and high-order photon processes start to contribute. The effect of electron-photon coupling is to lower the plasmon frequency.

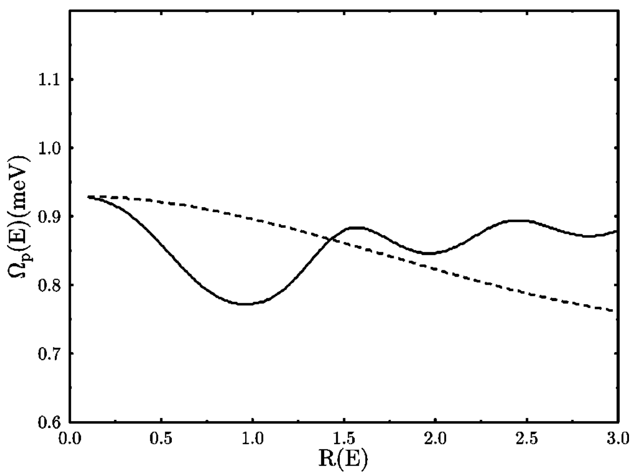

FIG. 2. Dependence of the plasma frequency on the intensity of laser field. The solid line is for $q = 0.5 k_F$ and $\omega = 0.3$ meV; the dashed line is for $q = 0.5 k_F$ and $\omega = 1.0$ meV.

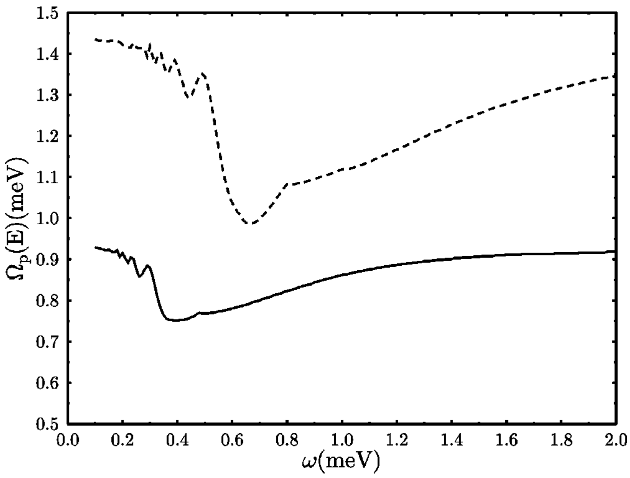

FIG. 3. Plasma frequency as a function of laser frequency (or photon energy). The dashed line is for $q=0.7k_F$ and $R=3.0$; the solid line is for $q=0.5k_F$ and $R=1.5$.

The long-wavelength plasmon frequency in the absence of the radiation field is related to the high-frequency electrical conductivity through $\omega_p^2=2\pi\sigma(\omega)\omega q$. Therefore the reduction of plasmon frequency under THz laser radiation can also be understood as the suppression of electrical conductivity under THz radiation. In other words, when electrons are strongly coupled to a photon field, they become less mobile. This effect is consistent with the experimental observation of suppressed conductivity in a two-dimensional electron gas under an intense THz radiation.[6] If the laser intensity is strong, the multiphoton processes start to contribute at a smaller value of q.

Figure 2 depicts the plasma frequency as a function of laser intensity. For any value of q and ω, the plasmon frequency is always lower than the plasmon frequency in the absence of a radiation field. The coupling of electrons to the mth-order photon process is proportional to the square of $J_m(\gamma_0 q_x)$. For given q_x and ω, the Bessel functions oscillate with the laser field intensity. If the laser frequency is lower (solid line), the electron-photon coupling is strong and the plasmon frequency will reflect the oscillatory behavior of the electron-photon coupling. The electron-photon coupling is weaker at higher frequencies. In this case one can observe a slow variation of the plasmon frequency as the laser intensity increases.

Figure 3 shows the dependence of the plasma frequency on the laser frequency. Here, again, when the electron-photon coupling is strong at low frequencies, the plasmon frequency exhibits rapid oscillations. At high laser frequencies, the plasmon frequency gradually approaches the zero-field value as the frequency increases.

It is well known that only the long-wavelength plasmon is free of Laudau damping. For a uniform system in the absence of THz radiation, the critical wave vector q_c beyond which the plasmon is damped is given as $q_c=\omega_p/v_F$ where v_F is the Fermi velocity. $q_c\approx 0.75k_F$ for the present system in the absence of THz radiation. Under THz radiation, the plasma frequency increases as the radiation intensity increases. As a result, the plasmon is long lived up to q_c' ($>q_c$). In Fig. 1 the q_c' for three dispersion curves at three different radiation intensities are $0.79k_F$, $0.98k_F$, and $1.15k_F$. In Figs. 2 and 3, the wave vectors are smaller than q_c' and therefore the plasmons presented here are free of Landau damping.

In conclusion, we have studied the dielectric properties and spectrum of collective excitations for an electron gas under intense laser radiation. Successive multiphoton processes can be identified from the spectrum of collective excitations.

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