# Dynamic Selection: An Idea Flows Theory of Entry, Trade and Growth

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# Motivation

- Productivity dispersion across firms opens two channels for aggregate productivity gains:
  1. Reallocation from low to high productivity firms (Melitz 2003; Hsieh & Klenow 2009)
  2. Technology diffusion between firms (Luttmer 2007; Lucas & Moll 2014)

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  1. Reallocation from low to high productivity firms (Melitz 2003; Hsieh & Klenow 2009)
  2. Technology diffusion between firms (Luttmer 2007; Lucas & Moll 2014)
- What are the effects of trade when there is both reallocation and technology diffusion?

# Technology diffusion

- Technologies are non-rival and partially non-excludable
- Firms learn about the process technologies used by competitors, e.g. managerial methods; organizational structure; production techniques
- But most firms do not adopt frontier technologies
  - Information asymmetries; adaptation costs; learning capacity constraints

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- But most firms do not adopt frontier technologies
  - Information asymmetries; adaptation costs; learning capacity constraints
- Model technology diffusion by introducing knowledge spillovers where:
  1. Spillovers affect entrants' productivity, not entry costs
  2. Spillovers depend upon entire productivity distribution

# Selection & diffusion

- Incorporate knowledge spillovers into dynamic version of Melitz 2003
- Entrants' productivity draws are endogenous to incumbent productivity distribution
- Selection on productivity causes spillovers that increase productivity of future entrants
- Entry increases competition and leads to tougher selection
- Complementarity between selection and diffusion generates endogenous growth as the productivity distribution shifts upwards over time

# Trade liberalization

Free entry condition:

$$\text{Entry cost} = \mathbb{P}(\text{Successful entry}) * \mathbb{E}\left[\text{Profits} \mid \text{Successful entry}\right]$$

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- Static economy: increase in exit cut-off $\rightarrow$ $\mathbb{P}$(Successful entry) falls $\rightarrow$ static selection effect
- With knowledge spillovers tougher selection does not affect $\mathbb{P}$(Successful entry)
- Instead free entry condition implies faster growth of exit cut-off $\rightarrow$ fall in entrants' expected lifespan
- Trade leads to higher growth through dynamic selection generating a new channel for gains from trade

# Related literature

- Growth through technology diffusion
  - Luttmer 2007; Alvarez, Buera & Lucas 2008, 2011; Perla & Tonetti 2014; Lucas & Moll 2014
- Trade, growth & scale effects
  - Grossman & Helpman 1991; Rivera-Batiz & Romer 1991; Jones 1995; Young 1998
- Trade & growth with heterogeneous firms
  - Baldwin & Robert-Nicoud 2008; Perla, Tonetti & Waugh 2014
- Static gains from trade
  - Atkeson & Burstein 2010; Arkolakis, Costinot & Rodríguez-Clare 2012; Melitz & Redding 2013

# Overview

1. Model set-up
2. Evolution of productivity distribution
3. Balanced growth path
4. Gains from trade
5. Extensions

# Environment

- $J + 1$ symmetric economies
- Single sector producing differentiated varieties
- Single consumption good – numeraire
- Continuous time
- Constant population growth $L_t = L_0 e^{nt}$

# Preferences

- Representative household has dynastic preferences:

$$U = \int_0^\infty e^{-\rho t} e^{nt} \frac{c_t^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} dt$$

- Consumption per capita $c_t$
- Intertemporal elasticity of substitution $\gamma > 0$
- Discount rate $\rho > 0$
- Household budget constraint:

$$\dot{a}_t = w_t + r_t a_t - c_t - n a_t$$

- Assets per capita $a_t$; interest rate $r_t$

# Production

- Consumption good produced under perfect competition as a CES aggregate:

$$c_t L_t = \left[ \int_{\omega \in \Omega_t} q_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1$$

$\Omega_t$ set of available varieties

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- Variety production follows Melitz 2003
- Each firm produces a differentiated variety
- Labor is only factor of production
- Monopolistic competition between firms
- Heterogeneity across firms in labor productivity $\theta$
- Fixed production cost $f$ units of labor per period

# Trade

- Fixed export cost $f_x$ units of labor per period per country
- Iceberg variable trade costs $\tau$

# Static profit maximization

- Firm's static optimization problem equivalent to Melitz 2003
- Exit cut-off:

$$\theta_t^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left(\frac{fw_t^{\sigma}}{c_t L_t}\right)^{\frac{1}{\sigma-1}}$$

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- Normalize productivity relative to the exit cut-off:

$$\phi_t \equiv \frac{\theta}{\theta_t^*}$$

- Exit when $\phi_t < 1$
- Export when $\phi_t \geq \tau \left( \frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} \equiv \tilde{\phi}$

Details

# Entry 1

- Entry cost $f_e$ denominated in labor units
- Entrant draws productivity:

$$\theta = x_t \psi$$

$x_t$ function of incumbent productivity distribution $G_t(\theta)$
$\psi$ stochastic component with distribution $F(\psi)$

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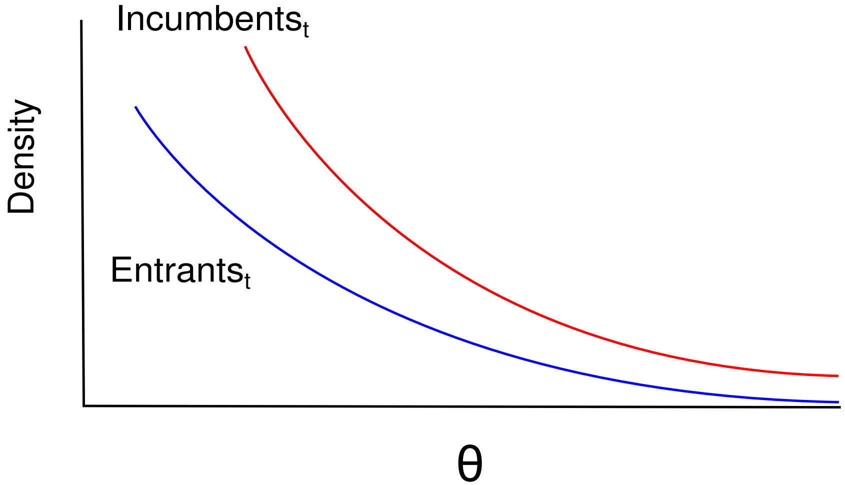
- $x_t$ captures knowledge spillovers from incumbents to entrants
- Assume $x_t$ equals average productivity of incumbents
  1. $x_t$ is a location statistic such that if $G_{t_1}(\theta) = G_{t_0}(\frac{\theta}{\kappa})$ then $x_{t_1} = \kappa x_{t_0}$
  2. $x_t$ is independent of the mass of incumbent firms
  3. $x_t$ is independent of the frontier productivity
- Upwards shift of incumbent firm productivity distribution leads to spillovers that benefit future entrants

# Entry & diffusion



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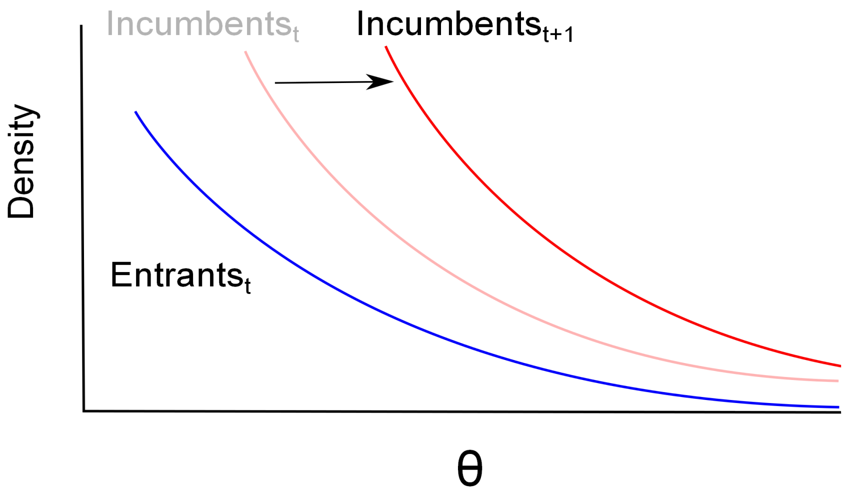

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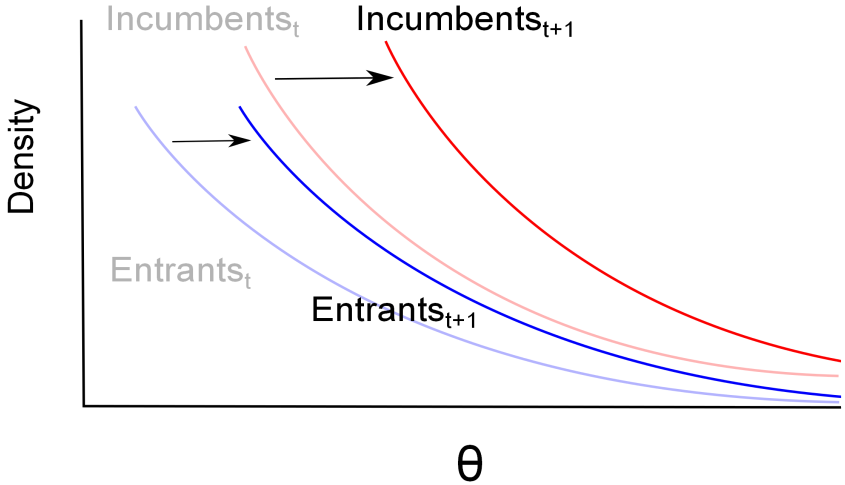

# Entry 2

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- Costless financial intermediation sector pools entry risk across households
- Assume productivity remains constant after entry
- Alternative assumption that leads to same balanced growth path properties in baseline model is:

$$x_t \text{ constant}, \quad F = G_t$$

- Captures technology diffusion when each entrant is randomly matched with an incumbent producer and learns incumbent's technology

# Productivity distribution dynamics 1

- Dynamics of $\phi$:

$$\phi_{t+\Delta} = \frac{\theta_t^*}{\theta_{t+\Delta}^*}\phi_t$$

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$$\phi_{t+\Delta} = \frac{\theta_t^*}{\theta_{t+\Delta}^*}\phi_t$$

- Evolution of relative productivity distribution $H_t(\phi)$:

$$\begin{aligned} M_{t+\Delta}H_{t+\Delta}(\phi) &= M_t\left[H_t\left(\frac{\theta_{t+\Delta}^*}{\theta_t^*}\phi\right) - H_t\left(\frac{\theta_{t+\Delta}^*}{\theta_t^*}\right)\right] \\ &\quad + \Delta R_t\left[F\left(\frac{\phi\theta_{t+\Delta}^*}{x_t}\right) - F\left(\frac{\theta_{t+\Delta}^*}{x_t}\right)\right] \end{aligned}$$

- $M_t$ mass of incumbent firms
- $R_t$ flow of entrants

# Productivity distribution dynamics 2

- Taking the limit as $\Delta \to 0$ gives:

$$\frac{\dot{M}_t}{M_t} = -H_t'(1)\frac{\dot{\theta}_t^*}{\theta_t^*} + \left[1 - F\left(\frac{\theta_t^*}{x_t}\right)\right]\frac{R_t}{M_t}$$

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$$\begin{aligned}\dot{H}_t(\phi) \quad = \quad & \left\{\phi H_t'(\phi) - H_t'(1)\left[1 - H_t(\phi)\right]\right\}\frac{\dot{\theta}_t^*}{\theta_t^*} \\ & + \left\{F\left(\frac{\phi\theta_t^*}{x_t}\right) - F\left(\frac{\theta_t^*}{x_t}\right) - H_t(\phi)\left[1 - F\left(\frac{\theta_t^*}{x_t}\right)\right]\right\}\frac{R_t}{M_t}\end{aligned}$$

# Distribution assumptions

- $F$ Pareto:

$$F(\psi) = 1 - \left(\frac{\psi}{\psi_{\text{min}}}\right)^{-k}$$

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$$\lim_{\theta \to \infty} \frac{1 - G_0(\theta)}{\theta^{-k}} = \kappa$$

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- Define:

$$\lambda = \frac{x_t \psi_{\min}}{\theta_t^*}$$

  Assume $\lambda \leq 1$

- $\lambda$ measures the strength of knowledge spillovers

# BGP equilibrium

Solve for a balanced growth path (BGP) equilibrium on which:

1. Households maximize utility subject to their budget constraints
2. Firms maximize static profits conditional on their productivity levels
3. Free entry
4. Asset, labor and output markets clear
5. Evolution of $M_t$ and $H_t(\phi)$ as above
6. $c_t$, $a_t$, $w_t$, $r_t$, $\theta_t^*$, $W_t(\phi)$, $M_t$ and $R_t$ grow at constant rates
7. Relative productivity distribution is stationary

# Stationary productivity distribution

- Unique stationary relative productivity distribution is Pareto:

$$H(\phi) = 1 - \phi^{-k}$$

- Knowledge spillovers on BGP:

$$x_t = \frac{k}{k-1}\theta_t^* \Rightarrow \lambda = \frac{k}{k-1}\psi_{\min}$$

- And entrants obtain relative productivity draws:

$$\tilde{H}(\phi) = F\left(\frac{\phi\theta_t^*}{x_t}\right) = H\left(\frac{\phi}{\lambda}\right)$$

- Assumption on $G_0(\theta)$ implies $H_t(\phi)$ converges to Pareto in any economy with positive productivity growth

# Sources of growth

- Let $g = \frac{\dot{\theta}_t^*}{\theta_t^*}$ be the dynamic selection rate
- Firm relative productivity $\phi_t$ declines at rate $g$
- Let $\frac{\dot{c}_t}{c_t} = q$. Differentiating definition of exit cut-off gives:

$$q = g + \frac{n}{\sigma - 1}$$

- Two sources of growth
  1. Dynamic selection: growth of exit cut-off causes productivity distribution to shift outwards as a traveling wave and raises average productivity
  2. Population growth drives expansion in mass of varieties produced:

$$\frac{\dot{R}_t}{R_t} = \frac{\dot{M}_t}{M_t} = n$$

# Dynamic selection

- What determines the dynamic selection rate?
- Free entry condition:

$$
\begin{aligned}
f_e w_t &= \int_\phi W_t(\phi) dH\left(\frac{\phi}{\lambda}\right) \\
&= \int_\phi \left[\int_t^\infty \pi_v(\phi_v) e^{-(v-t)r} dv\right] dH\left(\frac{\phi}{\lambda}\right)
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- Because of technology diffusion entrants draw relative productivity from stationary distribution
- Increased profit flow $\pi_t(\phi) \to$ higher returns to entry $\to$ rise in $\frac{R_t}{M_t} \to$ increase in $g$
- Increase in dynamic selection rate shortens entrants' expected lifespan ensuring the free entry condition is satisfied

# Trade & growth

$$\begin{aligned}\pi_t(\phi) &= \pi_t^d(\phi) + \pi_t^x(\phi) I\left[\phi \geq \tilde{\phi}\right] \\ &= f w_t \left(\phi_t^{\sigma-1} - 1\right) + f J \tau^{1-\sigma} w_t \left(\phi^{\sigma-1} - \tilde{\phi}^{\sigma-1}\right) I\left[\phi \geq \tilde{\phi}\right]\end{aligned}$$

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1. Trade integration (higher $J$, lower $\tau$, lower $f_x$) creates new profit opportunities and raises $g$
2. Higher $f$ increases profit flow and raises $g$
   - Profit flow increases due to lower static competition caused by reduction in level of $M_t$

# BGP equilibrium

## Proposition

*The world economy has a unique balanced growth path on which consumption per capita grows at rate:*

$$q = \frac{\gamma}{1+\gamma(k-1)}\left[\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{f_e}\left(1+J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right)+\frac{kn}{\sigma-1}-\rho\right].$$

- Existence of equilibrium assumes parameter restrictions such that $g > 0$ and transversality condition holds Details
- Growth rate increasing in: $n$, $\gamma$, $\lambda$, $f$, $J$
- Growth rate decreasing in: $\rho$, $f_e$, $f_x$, $\tau$

Solution details

# No scale effects

- Growth rate is independent of population $L_t$
- Both $R_t$ and $M_t$ are proportional to $L_t$, but:

$$g = \frac{1}{k}\left(\lambda^k \frac{R_t}{M_t} - n\right)$$

- Larger population increases the number of varieties produced without generating knowledge spillovers (cf. Young 1998)
- In first generation endogenous growth theory trade affects growth because of scale effects and international knowledge spillovers (Grossman & Helpman 1991). Both are absent from this model

# Trade & welfare

- BGP welfare depends on initial consumption level $c_0$ and per capita consumption growth rate $q$ Welfare
- No transition dynamics since $H(\phi)$ independent of trade integration
- Gains from trade $z$ defined by:

$$U\left(zc_0^A, q^A\right) = U\left(c_0, q\right)$$

"$A$" superscript denotes autarky

- Decompose gains from trade into two components $z = z^s z^d$ where:
  1. Static gains $z^s$ – welfare gains holding $q$ constant
  2. Dynamic gains $z^d$ – welfare gains from higher $q$

# Static gains from trade

$$z^s = \left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]^{\frac{1}{k}}$$

- $z^s$ equals total gains from trade in Melitz 2003 if entrants draw productivity from a Pareto distribution
- $z^s$ equals total gains from trade in this paper if there are no knowledge spillovers
- Calibrated value of static gains same as in Arkolakis, Costinot & Rodríguez-Clare 2012:

$$z^s = \left(\frac{1}{1 - IPR}\right)^{\frac{1}{TE}}$$

# Dynamic gains from trade

- Higher growth raises welfare conditional on $c_0$, but has ambiguous effect on $c_0$
  - Reallocation of labor from production to entry has negative effect on $c_0$
  - Higher marginal propensity to consume wealth raises $c_0$ if and only if $\gamma < 1$
- But net effect of $q$ on $z^d$ is always strictly positive
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- But net effect of $q$ on $z^d$ is always strictly positive
- Dynamic selection effect raises the gains from trade
- Why does positive effect of trade on growth increase welfare?
  - Selection has a positive externality on the productivity of future entrants
  - Trade exploits the technology diffusion externality by increasing the dynamic selection rate

# Quantifying the gains from trade

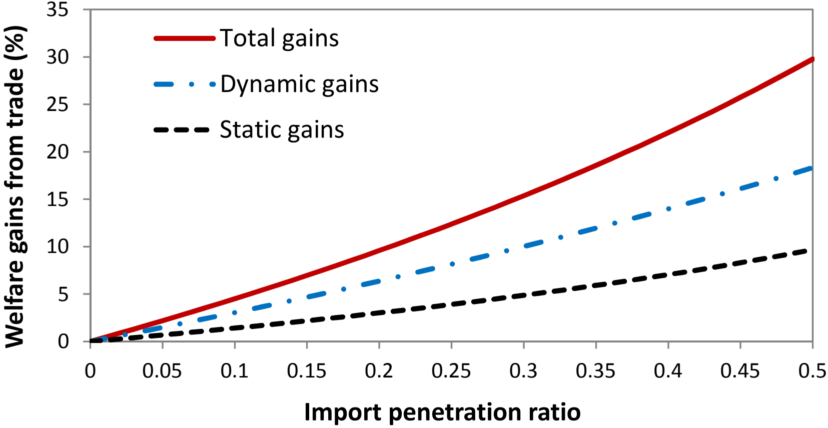
- Calibrate model using U.S. data
- Calibration uses 3 observables and 4 parameters

**Table 1: Calibration observables and parameters**

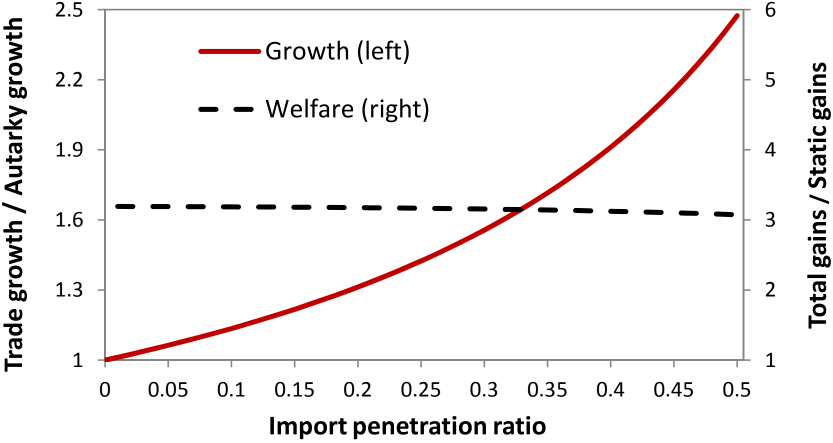
| Observable/parameter | | Value | Source |
|---|---|---|---|
| Import penetration ratio | IPR | 0.081 | U.S. import penetration ratio in 2000 |
| Firm creation rate | NF | 0.116 | U.S. Small Business Administration 2002 |
| Population growth rate | $n$ | 0.011 | U.S. average 1980-2000 |
| Trade elasticity | $k$ | 7.5 | Anderson and Van Wincoop (2004) |
| Elasticity of substitution across goods | $\sigma$ | 8.1 | $\sigma = k/1.06 + 1$ to match right tail index of employment distribution |
| Intertemporal elasticity of substitution | $\gamma$ | 0.33 | García-Peñalosa and Turnovsky (2005) |
| Discount rate | $\rho$ | 0.04 | García-Peñalosa and Turnovsky (2005) |

Calibration

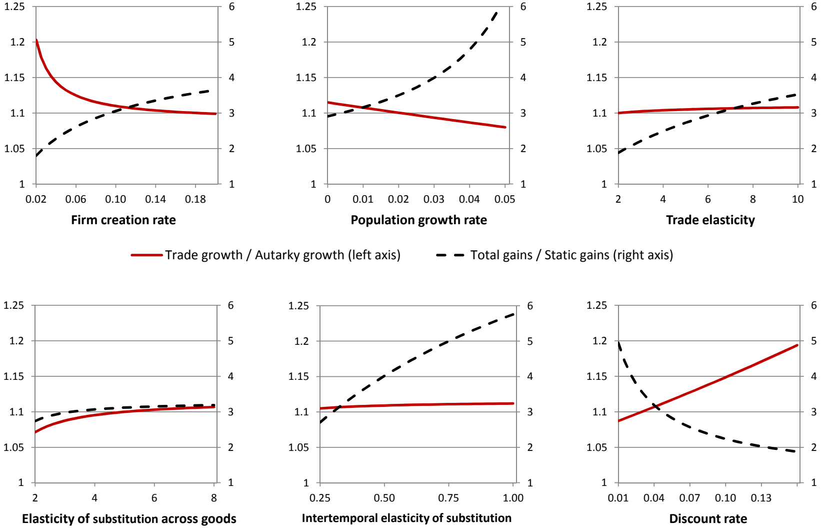
# IPR & gains from trade



# IPR & dynamic gains



# Robustness


1.25
1.2
1.15
1.1
1.05
1
6
5
4
3
2
1
0.02
0.06
0.10
0.14
0.18
Firm creation rate
0
0.01
0.02
0.03
0.04
0.05
Population growth rate
2
4
6
8
10
Trade elasticity
Trade growth / Autarky growth (left axis)
Total gains / Static gains (right axis)
2
4
6
8
Elasticity of substitution across goods
0.25
0.50
0.75
1.00
Intertemporal elasticity of substitution
0.01
0.04
0.07
0.10
0.13
Discount rate

# Extensions

1. International knowledge spillovers Go
2. Non-Pareto productivity distribution & frontier growth Go
3. Firm level productivity dynamics Go
4. Technology diffusion to incumbents Go

# Conclusions

- Introduce technology diffusion into an open economy model with heterogeneous firms
- Selection on productivity leads to endogenous growth through spillovers from incumbent firms to entrants
- Because of free entry trade raises the dynamic selection rate and increases growth
- Gains from trade larger than in static steady state open economy models
- In baseline calibration trade raises growth by 11% and the gains from trade are 3.2 times higher than in static models

# Profits & firm value

- Profit flow from domestic sales:

$$\pi_t^d(\phi_t) = fw_t \left( \phi_t^{\sigma-1} - 1 \right) \mathbb{I}\left[\phi_t \geq 1\right]$$

- Profit flow from exports:

$$\pi_t^x(\phi_t) = J\tau^{1-\sigma} fw_t \left( \phi_t^{\sigma-1} - \tilde{\phi}^{\sigma-1} \right) \mathbb{I}\left[\phi_t \geq \tilde{\phi}\right]$$

- Firm value:

$$W_t\left(\phi_t\right) = \mathbb{E}\left[\int_t^{\infty} \pi_\tau\left(\phi_\tau\right) \exp\left(-\int_t^{\tau} r_s ds\right) d\tau\right]$$

Back

# Parameter restrictions

- To ensure $g > 0$ assume:

$$\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{f_e} > \rho + \frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}$$

- To ensure transversality condition holds assume:

$$\frac{(1-\gamma)(\sigma-1)}{k+1-\sigma}\frac{\lambda^k f}{f_e}\left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right] > \gamma k(n-\rho) - (1-\gamma)\frac{k+1-\sigma}{\sigma-1}n$$

Back

# BGP solution

- On BGP:

$$\frac{\dot{a}_t}{a_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} = q$$

- Household utility maximization implies Euler equation:

$$q = \gamma(r - \rho)$$

- Free entry requires:

$$q = kg + r - \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k}{f_e} \left( f + Jf_x \tilde{\phi}^{-k} \right)$$

- Labor market clearing:

$$L_t = \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} M_t f \left[ 1 + J\tau^{-k} \left( \frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right] + R_t f_e$$

Back

# Welfare

Household welfare on BGP:

$$U = \frac{\gamma}{\gamma - 1}\left[\frac{\gamma c_0^{\frac{\gamma-1}{\gamma}}}{(1-\gamma)q + \gamma(\rho - n)} - \frac{1}{\rho - n}\right]$$

Initial consumption:

$$\begin{aligned} c_0 &= A_1 f^{-\frac{k+1-\sigma}{k(\sigma-1)}}\left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]^{\frac{1}{k}} \\ &\quad * \left[1 + \frac{\sigma-1}{k\sigma+1-\sigma}\frac{n+gk}{n+gk+\frac{1-\gamma}{\gamma}q+\rho-n}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \end{aligned}$$

$$A_1 \equiv (\sigma-1)\left(\frac{k}{k+1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{k+1-\sigma}{k\sigma+1-\sigma}\right)^{\frac{k\sigma+1-\sigma}{k(\sigma-1)}}\hat{\theta}_0^*\hat{M}_0^{\frac{1}{k}}L_0^{\frac{k+1-\sigma}{k(\sigma-1)}}$$

Back

# Calibration

- Firm creation rate:

$$\begin{aligned} NF &= \lambda^k \frac{R_t}{M_t} \\ &= n + gk \end{aligned}$$

- Fixed costs:

$$\begin{aligned} \frac{\lambda^k f}{f_e} &= \frac{k+1-\sigma}{\gamma k(\sigma-1)}(1-IPR)\left\{\left[1+\gamma(k-1)\right](NF-n)\right. \\ &\quad \left. + \frac{k(1-\gamma)}{\sigma-1}n + \gamma k \rho\right\} \end{aligned}$$

Back

# Baseline calibration results

**Table 2: Calibration results**

| Outcome | | Value |
|---|---|---|
| Growth rate - trade | $q$ | 0.0156 |
| Growth rate - autarky | $q^A$ | 0.0141 |
| **Growth (trade vs. autarky)** | $\mathbf{q/q^A}$ | **1.107** |
| Consumption level (trade vs. autarky) | $c_0/c_0^A$ | 1.010 |
| Static gains from trade | $z^s$ | 1.011 |
| Dynamic gains from trade | $z^d$ | 1.025 |
| Total gains from trade | $z$ | 1.036 |
| **Gains from trade (total vs. static)** | $\mathbf{(z-1)/(z^s-1)}$ | **3.2** |

# Parameter restrictions

- To ensure $g > 0$ assume:

$$\begin{aligned} 1 &> \gamma \\ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} &> \rho + \frac{1 - \gamma}{\gamma} \frac{n}{\sigma - 1} \end{aligned}$$

- To ensure transversality condition holds assume:

$$\frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} > n$$

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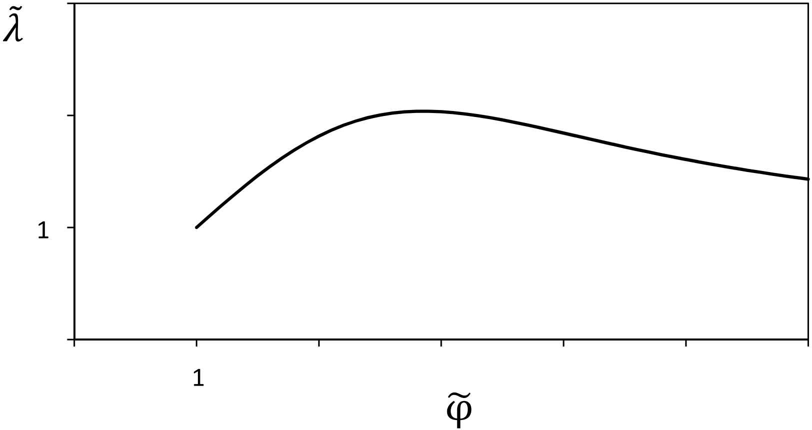
# International knowledge spillovers

- Suppose entrants learn from both domestic and foreign firms
- Let $x_t$ be average productivity of all firms that sell in the domestic market
- Only difference from baseline model is:

$$\lambda = \frac{k}{k-1}\psi_{\min}\tilde{\lambda} \qquad \text{where} \qquad \tilde{\lambda} \equiv \frac{1 + J\tilde{\phi}^{1-k}}{1 + J\tilde{\phi}^{-k}}$$

- Trade increases $\lambda$ because exporters are on average more productive than domestic firms
- Increase in strength of knowledge spillovers is a second channel through which trade raises dynamic selection rate and generates dynamic gains

# Trade & knowledge spillovers

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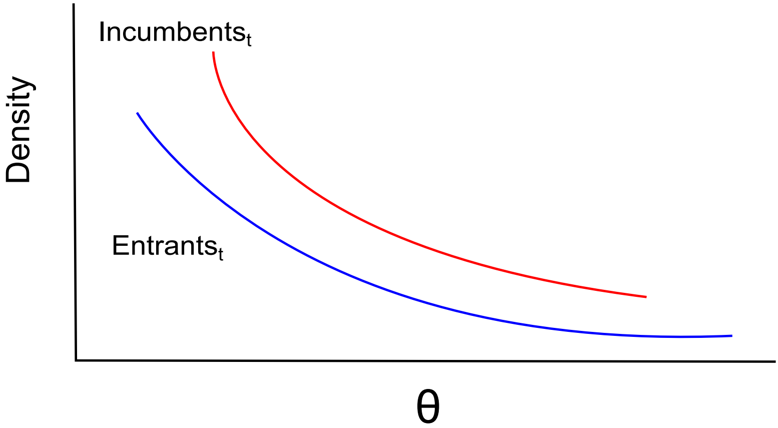
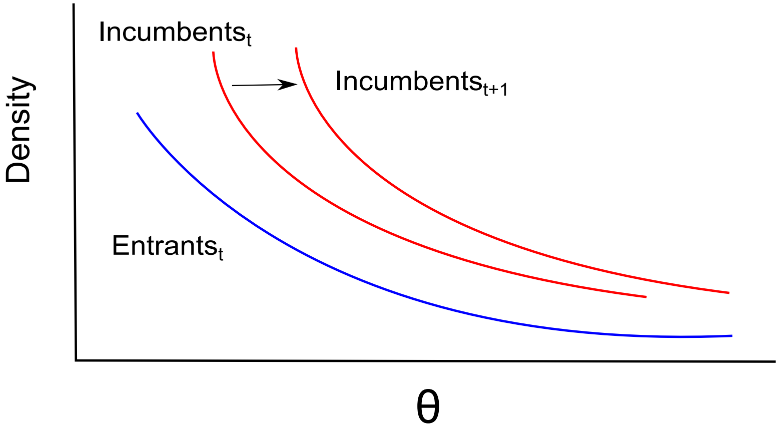
# Non-Pareto productivity distribution & frontier growth

- Let $F$ be a differentiable cumulative distribution function with bounded support $[\psi_{\min}, \psi_{\max}]$
- Assume $x_t = x\theta_t^*$ where $x\psi_{\min} \leq 1$ and $x\psi_{\max} > 1$
- Assume initial productivity distribution $G_0(\theta)$ is bounded above
- Productivity growth results from increases in both the lower and upper bounds of the productivity distribution

# Entry & frontier growth



# Entry & frontier growth



# BGP with frontier growth

- Provided the transversality condition is satisfied and the dynamic selection rate is positive then:
  1. There exists a unique BGP on which the stationary relative productivity distribution satisfies:

$$\phi H'(\phi) = \frac{n}{g}\left[H(\phi) - \frac{F\left(\frac{\phi}{x}\right) - F\left(\frac{1}{x}\right)}{1 - F\left(\frac{1}{x}\right)}\right] + H'(1)\frac{1 - F\left(\frac{\phi}{x}\right)}{1 - F\left(\frac{1}{x}\right)}$$

  2. Sufficient conditions that ensure trade liberalization raises growth are:

$$\rho + \frac{1-\gamma}{\gamma}\frac{n}{\sigma - 1} > 0, \quad (\sigma - 1) + \frac{1-\gamma}{\gamma} > 0$$

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# Firm productivity dynamics

- Dynamic selection effect of trade robust to allowing for general firm level productivity dynamics
- Assume entrants draw both $\phi$ and a set of productivity growth rates $\zeta_t$ from a stationary joint distribution:

$$\frac{\dot{\theta}_t}{\theta_t} = \zeta_t \Rightarrow \frac{\dot{\phi}_t}{\phi_t} = \zeta_t - g$$

- Allows for firm level productivity dynamics that are conditional on firm size
- Assume there exists a BGP with a positive dynamic selection rate
- $\gamma \leq 1$ is a sufficient, but not necessary, condition for trade integration to increase growth

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# Technology diffusion to incumbents

- Assume productivity of all incumbents grows at rate $g$
- Firm's relative productivity is constant as technology diffusion raises the productivity of both entrants and incumbents
- Assume $F$ Pareto, transversality condition satisfied and positive dynamic selection rate
- There exists a unique BGP on which the relative productivity distribution is Pareto and the growth rate is:

$$q = \frac{\gamma}{1-\gamma}\left[\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{f_e}\left(1+J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right)-\rho\right]$$

- $\gamma < 1$ is a necessary and sufficient condition to ensure trade integration raises growth and generates dynamic gains
- If $\gamma \geq 1$ no BGP exists

# Parameter restrictions

- To ensure $g > 0$ assume:

$$\begin{aligned} 1 &> \gamma \\ \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} &> \rho + \frac{1 - \gamma}{\gamma} \frac{n}{\sigma - 1} \end{aligned}$$

- To ensure transversality condition holds assume:

$$\frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} > n$$

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# Alternative extensions

1. Small open economy
   - Perfect competition
   - Homogeneous output sold at higher price in foreign markets
2. Decreasing returns to scale in R&D
   - Flow of entrants $\Psi(R_t, M_t)$ where $\Psi$ homogeneous of degree one
   - Could interpret as congestion in technology adoption process

In both cases:

- Trade increases growth
- Gains from trade can be decomposed into static and dynamic components
- Dynamic gains from trade increase welfare relative to a static steady state version of the model