DYNAMIC SIMILARITY OF TRANSPORT PHENOMENA

by

M.S. Yalin,
Professor, Dept. of Civil Engineering,
Queen's University, Kingston, Canada
(Consultant, Dept. of Ocean Engineering, University of Hawaii)

and

F. Gerritsen
Professor of Ocean Engineering
University of Hawaii

ABSTRACT

The discharges issued by OTEC plants, thermal power plants and other engineering devices, give rise to the transport of "foreign" properties and substances into the natural ocean environment. In order to predict the functioning of such structures and assess their environmental impact, physical modelling has already been utilized. Since the simultaneous fulfillment of both Reynolds and Froude criteria is impossible (in a conventional small scale model operating with the prototype fluid) in the models mentioned the transport phenomenon was reproduced on the basis of the densimetric Froude number (Fr) only, the influence of the Reynolds number (Re) being neglected. On the other hand, the identification of the scale of Fr (viz λ_{Fr}) with unity can lead to substantial differences between the model and prototype values of Re. (Because $\lambda_{Fr} = 1$ yields $\lambda_{Re} = \lambda_\ell^{3/2}$ where λ_ℓ is the linear model scale.) Yet many of the pertinent aspects of a turbulent diffusion (energy dissipation, thickness of mixing zones, separation processes, etc.) are strongly dependent on Re, and therefore an appreciable distortion of Re ($\lambda_\ell^{3/2} << 1$) can lead to some substantial errors with regard to the similarity of these aspects.

The central theme of the approach presented in this paper can be outlined as follows: Why should any of λ_{Fr} or λ_{Re} necessarily be identified with unity? If Π_A is the dimensionless version of a quantitative property A of the transport phenomenon, then $\Pi_A = \Phi_A$ (Re, Fr,...) The modelling of A means the approach of λ_{Π_A} to unity as close as possible. But, if so, then there should exist such (optimal) scales $\lambda_{Fr} = \alpha$ and $\lambda_{Re} = \beta$ (both $\neq 1$) which would yield

$$| \lambda_{\Pi_A} - 1 | \to \text{minimum}$$

The proposed determination of α and β rests on experimental basis; it rests on the calibration of the model by adjusting the model velocity v" so that the last equation becomes valid.

Since $\frac{\partial A}{\partial Re}$ cannot be expected to be the same for all the properties of the transport phenomenon each A will in general require "its own" scales α and β.

1. INTRODUCTION

Transport phenomena form a large class of problems whereby discharges from a source induce certain changes in the natural environment. The transported element may be matter (salinity) or temperature but may also refer to fluid properties such as momentum. The overall objective of this paper is to describe a method by which transport phenomena can be tested in a hydraulic model. In such phenomena both Froude number and Reynolds number play a role in their model simulation.

The method developed in this paper may also be useful in other hydraulic phenomena, in which both the Reynolds number and the Froude number play a role.

The reason to undertake this study was the present development of O.T.E.C. plants in the Hawaiian environment. In addition to the vast number of technological problems that have to be solved before O.T.E.C. plants can be operable and can deliver power to the network, the induced circulation and the effect of OTEC plants on the ocean environment have to be evaluated quantitatively. This is particularly important if one visualizes a large number of OTEC plants operating in the tropical oceans.

To the authors' opinion the environmental impact aspects of OTEC power plants have not been given adequate consideration in the research planning by the DOE and other agencies involved in OTEC developments.

Studies on environmental impact of these plants can be accomplished in three different ways:

- by field studies on pilot plants
- by laboratory investigations with hydraulic models
- by mathematical modelling.

The question is often raised which of those methods would be preferable. In the authors' opinion all three are necessary in order to arrive at satisfactory and reliable answers.

Of these three methods a mathematical model is the most versatile tool. After the formulation of the model has been worked out and the algorithm has been developed, a large number of varying conditions can be investigated at a relatively low cost. There is no doubt that the mathematical model will be the ultimate method of analysis for the future. However such model utilizes physical concepts, parameters and constants, the values of which are only known for a limited degree of accuracy for a given situation. Of interest is the numerical model developed by Yamashita (Yamashita, 1979). This model has been developed for a shore-based OTEC plant at Keahole Point on the island of Hawaii, but can easily be extended into an offshore floating OTEC plant.

The answers that the mathematical model provides depend heavily on the numerical values of essential parameters and therefore verification is required. There is no doubt that the best way to verify the value of physical constants is from the prototype. However, such verification requires a large number of points for the measurement of velocities, temperatures and salinities, both in the near field and in the far field.

Such program will be extremely costly, if at all feasible.

The scope of the field experiment can be strongly reduced if a combination of laboratory and field studies is employed to verify the physical constants. In the field, measurements can then be limited to observations in a number of strategically located points. The laboratory studies will provide the opportunity for taking a great number of detailed measurements in a controlled environment, whereby verification with the field conditions is possible from data obtained in corresponding points of prototype and model.

Having stated that hydraulic model experiments are an indispensible tool in the study of OTEC-related phenomena we will have to determine which scale relationships to apply to convert model data to prototype conditions. Before we discuss this matter in detail we will give a short description of the OTEC concept.

2. DESCRIPTION OF OTEC CONCEPT

Ocean Thermal Energy Conversion (OTEC) is presently one of the major development thrusts for energy extraction from the ocean. Other areas in which efforts may become fruitful are tides, currents, waves and salinity gradients.

In the OTEC concept the temperature difference between the warm surface water and the cold bottom water is utilized as a source of power generation.

The vast expanse of the oceans covers nearly three-quarters of the earth's surface and stores sufficient energy to fulfill the needs of everyone for many years to come (Richards and Vadus, 1980).

The study of OTEC-related problems in Hawaii has come to the forefront because Hawaii has been chosen for three major R&O OTEC research projects: the Mini-OTEC (50 kWe), OTEC-1 (1 MWe) and OTEC 10/40 MWe pilot plants.

Of these pilot projects the Mini-OTEC pilot project was in operation in Hawaiian waters during the summer and fall of 1979. It proved that the OTEC concept is workable.

Temperature, density and salinity profiles for offshore Hawaiian waters are summarized in Table 1. See also Bathen (1975).

Mini-OTEC was the world's first at-sea OTEC plant to produce net power.

Table 1. Density Profile (Offshore Hawaiian Waters)

Depth	Field Data		Computation with Constant Salinity (35‰)
	Temperature	Density	
0m	24.30°C	1.0236 (gm/liter)	1.0237
200	17.40	1.0254	1.0254
400	8.40	1.0267	1.0271
600	6.35	1.0270	1.0274

The system was developed by a priviate consortium of organizations including the University of Hawaii, Lockheed Missiles and Space Co., Oillingham Construction & Dredging Co., and Alfa-Laval.

An interesting feature of Mini-OTEC was that the cold water pipe (length 630m, diameter .6m and built of polyethelene), through which cold water is pumped from the deep to the plant, also serves as a part of a single mooring system.

As another step in the development, OTEC-1, a converted government-owned T-2 tanker, the Chepachet, will arrive at its test site off Keahole Point, westside of the island of Hawaii, in the summer of 1980.

The schematics of an OTEC seawater system is shown in Figure 1.

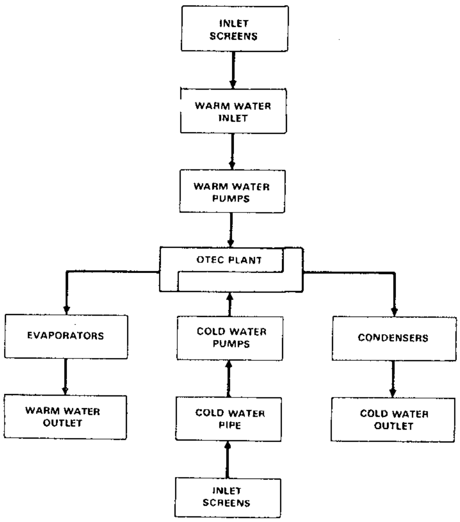

Figure 1. Schematics of OTEC Seawater System (from Richards and Vadus, 1980)

Warm water of the surface layers of the ocean is used to evaporate a working fluid (ammonia) in an evaporator.

The vapor drives a turbogenerator, after which it passes through a condensor and a pressurizer. The condensor is cooled with the cold water pumped from the deep ocean layers.

The above-described system is the so-called closed cycle, which has been emphasized in ongoing research. Other cycles such as the open cycle, the hybrid cycle and the lift cycle offer other possibilities (Richards and Vadus, 1980).

When passing through the installation the warm water loses some of its heat, whereas the cold water gets warmer. Both are being discharged into the ambient ocean, either separately in two outlets or mixed in one discharge opening.

The location (depth) of the discharge jets is to be chosen in such a manner that no shortcuts in the circulation will develop; such shortcuts will necessarily lead to a reduction in the available potential.

The efficiency of an OTEC system is primarily determined by the difference in temperature between the warm and cold water. In Hawaiian waters this difference is about 20°C. The second law of thermodynamics specifies that the best efficiency of an OTEC system is of the order of 6%. If energylosses are taken into account (for pumping and other system components) the expected energy producing efficiency is 2-3%.

Despite the low efficiency the system may still be economically feasible, since no fuel is required for its operation.

Present research on OTEC systems is concentrated on the technological aspects of system components. Major problem areas are the biofouling of heatexchangers, and the design of the cold water pipe, and of the mooring system.

So far only little attention has been given to the aspects of induced ocean circulation and environmental impact, particularly if more than one plant will be built in a specified area. This area of study needs more attention in the near future. Laboratory studies can assist in providing the required information.

3. PHYSICAL MODELLING OF OTEC PLANTS

3.1. Objectives of Study

In the study of the functioning of OTEC plants by means of laboratory investigations (hydraulic models) two types of problems are particularly relevant:

problems concerned with the circulation near the plant (nearfield) with the objective of determining the required distance between intake and discharge opening in order to avoid short circuiting, having adverse effects on the available temperature potential.

problems of the flow field away from the plant (far-field) with the principal objective to determine the ocean area affected, and the required distance between plants if several plants are to be built in the same region.

Changes induced by the plants in the far field furthermore will be indicative of the effects, if any, on climatological conditions.

In order to achieve a reliable prediction from mathematical models it has been noted that the quantitative formulation of the phenomenon must be known. In the case of OTEC plants, it is the discharge flow, which manifests itself in the form of a turbulent jet, that is the main source of the flow phenomenon and its consequences.

At present the turbulent flow in general and the "free turbulence" (to which the jet flows belong) in particular have not yet been understood completely.

Consequently the mathematical formulation of the free turbulence is also far from being complete and thus reliable. This is especially so for the conditions presented by OTEC plants, where the fluid which constitutes the discharge flow has different temperature (and thus density and viscosity) in comparison to the ambient fluid and where the ambient fluid is not homogeneous (temperature decreases with the depth) and very often not even static (ocean currents). Considering this it will not be difficult to realize that mathematical modelling whose output accuracy is completely dependent on the knowledge and thus accuracy of the mathematical formulations forming the impact, cannot be regarded as more reliable than physical modelling which does not depend on the mathematical formulations, i.e. on the knowledge of the quantitative relations among the parameters involved but which is dependent only on the knowledge of the parameters themselves. Admittedly very often the knowledge of the parameters and thus of the criteria of similarity does also not mean that a reliable physical model can be immediately designed. Indeed, some criteria of similarity may turn out to be conflicting (and in the case of OTEC plants where both the densimetric Froude number (Fr) and the Reynolds number (Re) are involved this is just so), nonetheless to find a "way out" in such cases by means of a special research is considerably more feasible than the formulation of the free turbulence and its consequences for heterogeneous fluids; it should be sufficient to recall that the research on turbulence is being carried out since the beginning of the century and yet, with the exception of some very simple cases (parallel flows, rectilinear flow boundaries, homogeneous fluids, etc.), no generally accepted formulation of this elusive phenomenon has been found so far.

In view of the above it is not surprising that efforts dealing with the physical modelling of OTEC plants have been limited.

To the authors' opinion the work done by Jirka et al. (1977) has been the only work of this kind, and in their view the approach used in their study is very innovative.

These authors have used an inverted model approach. The study was conducted at a scale 1:200, in which only the body of water above the thermocline was considered, giving the physical model a very shallow depth.

Results of their experiments agreed with those of a mathematical model. Because of the concerns expressed above there is no guarantee that the results also represent prototype conditions without further verification.

In the study by Jirka et al. (1977) the time averaged fields of temperature and velocities were measured and some pertinent aspects of these fields were determined.

In order to study the quantitative aspects of the phenomena in a physical model the authors believe that additional measurements are necessary.

It is recommended that the results of a small scale model (e.g. 1:50) be compared with the results of another, several times larger physical model (e.g. 1:10) and subsequently with the full scale (prototype).

The prototype in question could be the Mini-OTEC plant, when it will be back in operation for prototype testing, such as planned.

It is furthermore suggested to measure in the model the fields of some fluctuating properties of turbulence (viz of the root mean square values of the fluctuating velocities u', v' and w' and also the Reynolds stresses $-\rho\overline{u'v'}$.).

The overall picture of the phenomenon is characterized by its time average properties and therefore it is only reasonable to start the investigation of similarity by using the time average properties. It is perfectly possible that the rejection of the Reynolds criterion may not affect noticeably the similarity of the time average velocity fields in model and prototype but it may affect the similarity of the fluctuating velocities and thus the similarity of the internal structure of turbulence and its consequences (such as energy dissipation, diffusion, etc.).

3.2. Facilities For Proposed Research

Because of the large water depth involved model facilties need to be of large size in order to allow testing the phenomenon at a desirable scale.

Such facilities are limited on a worldwide basis. The new ocean tank of the Norwegian Hydrodynamic Laboratories in Trondheim, Norway (size 50 x 50 x 10m^3) would be an excellent facility for such testing.

At the University of Hawaii's JKK Look Laboratory, the large steel circular model tank would be a suitable facility. The tank has an opening at the top and on the side.

The tank is 9.14m in diameter and 12.34m high. (See Figure 2.)

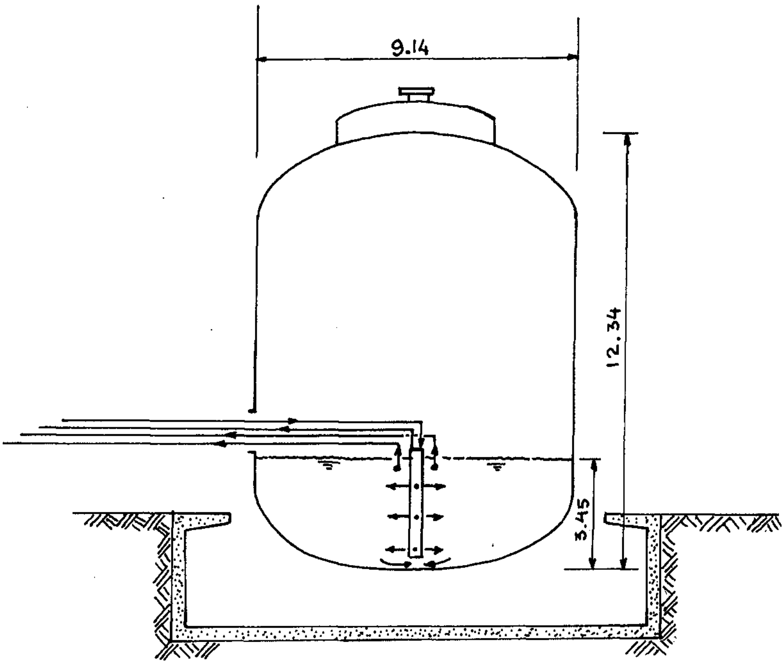

Figure 2. University of Hawaii's Large Circular Testing Tank

An analysis was made to determine if this tank could be made suitable for the testing of OTEC plants. The results were promising (Lee, 1978). Experiments can be carried out by using either the lower portion of the tank or by utilizing its full water depth.

The facility is considered suitable because it allows the simulation of ocean depths at a reasonable scale.

Heaters and coolers will have to be added for the establishment of a vertical temperature gradient in the tank and for the heating and cooling of the circulating fluid.

The circuits that provide the water circulation will be able to simulate the following conditions:

1. Skimming of "hot" water from the upper layers.
2. Discharging warm water at varying levels.
3. Pumping cold water from the deep portion of the tank.
4. Discharging cool water at varying levels.
5. Combining the warm water and cool water effluent into a single discharge arrangement at various levels. (Mini-OTEC arrangement)

4. THEORETICAL BASIS FOR MODEL SCALE DETERMINATION

From the pioneer work of Jirka et al., carried out in MIT in 1977, it is clear that the most relevant dimensionless variables determining the flow phenomenon around an OTEC plant are the densiometric Froude number Fr and the Reynolds number Re. If the geometry of the OTEC plant is specified then the velocity U which appears in both of these numbers can be any "typical velocity" of the system. If the undistorted model operates with the prototype fluid, that is with water having the same temperature, then, as is well known, a simultaneous fulfillment of both Froude and Reynolds criteria is impossible. Thus Jirka et al. made an attempt to achieve the dynamic similarity by identifying the model and prototype values of the Froude number only ($Fr'' \approx Fr'$), i.e. by identifying the scale of the Froude number with unity ($\lambda_{Fr} = 1$) and disregarding the Reynolds number completely (Re). This method of modelling is yet another example of the contemporary modelling convention, where a dimensionless variable, such as the Reynolds number, is rejected on the ground that its "influence on the phenomenon is negligible". This statement often is more a convenient justification of our conventional modelling methods than a reflection of truth. Indeed a physical phenomenon has an unlimited number of various properties which are different functions of the dimensionless variables defining that phenomenon. But if so then the influence of a certain variable, such as the Reynolds number, may indeed be irrelevant with regard to one set of the properties and yet it may turn out to be quite relevant with regard to another.

Notations.

For the analysis the following notations are utilized.

1. Model values are specified by ("), prototype values by ('). For example if α is a quantity to be studied then

α'' is the model value of α
α' is the prototype value of α

and $\frac{\alpha''}{\alpha'} = \lambda_\alpha$ is the scale of α . (Yalin, 1971).

2. The ambient values (i.e. the values undisturbed by the OTEC plant) of any value of α is denoted by $\bar{\alpha}$. (See Figure 3.)

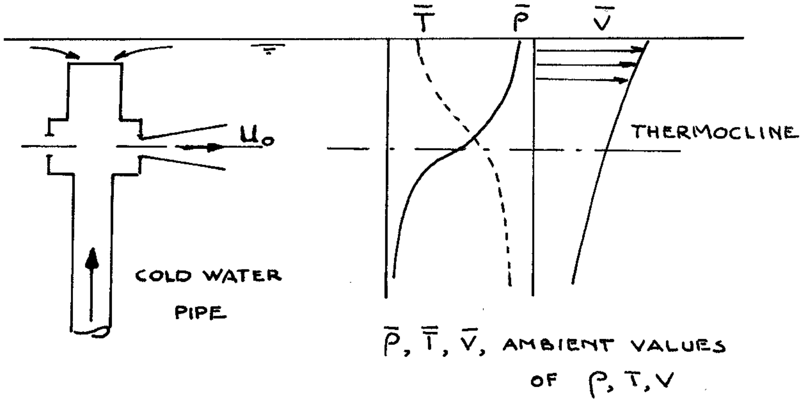

Figure 3. Schematics of OTEC Circulation and Ambient Ocean Conditions

Characteristic Parameters.

It is assumed that the model is undistorted and that it operates with the prototype fluid.

Since

$$\rho = f_1 (T, S) \quad \text{and} \quad \mu = f_2 (T, S) \qquad (1)$$

the geometrically similar distribution of the ambient temperature and salinity in the model will yield the geometrically similar distributions of $\bar{\rho}''$ (= $\bar{\rho}'$) and $\bar{\mu}''$ (= $\bar{\mu}'$) [the same applies for ρ_0 and μ_0]. Accordingly, it is assumed, that the necessary distribution of the ambient temperature and thus of $\bar{\rho}''$ and $\bar{\mu}''$ is provided. Similarly, we assume that the model has the geometrically similar distribution of $\bar{V}'$ (even if the scale $\lambda_{\bar{V}'}$ will be revealed later on).

If the ambient conditions are specified then the flow phenomenon in the region under investigation will be dependent only on the geometry of the plant and on the nature of its functioning, i.e. it will be dependent

a) on the geometric parameters (lengths and angles) L_1, L_2, L_3,... θ_1, θ_2, ...

b) on the kinematic parameters (which in the steady state case under consideration are velocities only)
U_1, U_2, U_3, U_4,....

c) on the physical properties (ρ, μ) of "in" and "out" flows*
(ρ_1, μ_1), (ρ_2, μ_2), (ρ_3, μ_3),....

and

d) on the acceleration due to gravity g.

Hence any quantitative property A of the present phenomenon must be expected to vary as a certain function of the position in space (x,y,z) and of the parameters above:

$$A = f_A (x, y, z, L_i, \theta_j, U_k, \rho_l, \mu_m)^{**} \tag{2}$$

The subscript A in f_A indicates that the form of the function (2) depends on the quantity A under consideration: different properties A of a phenomenon are different functions of the same parameters (describing that phenomenon).

Let L, U, ρ, μ be some (single) parameters selected from the groups L_i, U_k, ρ_ℓ, μ_m (as their "representatives"), e.g. one can identify L, U, ρ, μ with L_4, U_3, ρ_3, μ_3 respectively. In this case the (dimensional) relation (2) can be brought into the dimensionless form

$$\Pi_A = \phi_A \left[\frac{x}{L}, \frac{y}{L}, \frac{z}{L}, \frac{L_i}{L}, \frac{U_k}{U}, \frac{\rho_l}{\rho}, \frac{\mu_m}{\mu}, \frac{UL\rho}{\mu}, \frac{U}{\sqrt{gL}}, \theta_j\right] \tag{3}$$

Here Π_A is the "dimensionless version" of the property A under investigation

$$\Pi_A = \rho^\alpha L^\beta U^\gamma A \tag{4}$$

where depending on the dimension of A the exponents α, μ, γ must be determined so that the power product (4) becomes dimensionless.

Dynamic Similarity, Model Scales

If all Π_A are identical in model and prototype, i.e. if

$$\Pi_A' = \Pi_A'' \quad \text{or} \quad \lambda_{\Pi_A} \equiv 1 \quad \text{(for any A)} \tag{5}$$

is valid then model and prototype are dynamically similar (and the predictions obtained from the model are reliable).

Since Π_A is given by the dimensionless variables shown in (3) the model and prototype identity of all Π_A [as implied by (5)] can certainly be achieved if the identify of all variables [on the right of (3)] is provided.

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* T&S need not be included

** i=1,2,..., N_L, j=1,2,..., N_θ,...etc. where N_L, N_θ,... are the numbers of all the pertinent lengths, angles,...etc.

Now, the identities

$$\left(\frac{L_i}{L}\right)' \equiv \left(\frac{L_i}{L}\right)'' \quad \text{and} \quad \theta_j' \equiv \theta_j'' \qquad (6)$$

are provided: because model is geometrically similar.

The identities

$$\left(\frac{\rho_1}{\rho}\right)' \equiv \left(\frac{\rho_1}{\rho}\right)'' \quad \text{and} \quad \left(\frac{\mu_m}{\mu}\right) \equiv \left(\frac{\mu_m}{\mu}\right) \qquad (7)$$

are also provided, because model is supposed to operate with the prototype fluid and the similarity in the distribution of the ambient temperature and salinity is provided.

The identities

$$\left(\frac{x}{L}\right)' = \left(\frac{x}{L}\right)'' , \left(\frac{y}{L}\right)' = \left(\frac{y}{L}\right)'' , \left(\frac{z}{L}\right)' = \left(\frac{z}{L}\right)'' \qquad (8)$$

are also valid, since the measurements and predictions will be carried out for corresponding points (of the space occupied by flow).

Since all velocities will be scaled down in the same proportion λ_U (whatever its value may be!) the identity

$$\left(\frac{U_k}{U}\right)' = \left(\frac{U_k}{U}\right)'' \qquad (9)$$

is also satisfied. Thus the identity of all Π_A, i.e. the dynamic similarity depends entirely on the model and prototype identity of the Reynolds and Froude numbers

$$X = \frac{UL\rho}{\mu} \quad \text{and} \quad Y = \frac{U}{\sqrt{gL}} \qquad (10)$$

At this state it should be mentioned that in the present phenomenon the free surface is not disturbed significantly and therefore the main role of the acceleration due to gravity g is not so much to reflect the influence of the free surface, but to reflect the influence of the gravity difference of the fluid layers having different densities. In other words the role of g consists of acting on various ρ_ℓ and generate various specific weights $g\rho_\ell$ and their differences such as

$$g\rho - g\rho_\ell = g(\rho - \rho_\ell) \qquad (11)$$

Since g will thus appear in the expression of any Π_A in the form (11), one could consider the form (11) in the first place, i.e. one could take, say, $g(\rho-\rho_1)$, rather than simply g, (when writing d). In this physically more meaningful (and yet mathematically equivalent approach) we would have

$$Y_* = \frac{U}{\sqrt{gWL}} \quad ^{*)} \qquad (\text{with } W = 1 - \frac{\rho_1}{\rho}) \tag{12}$$

[rather than $Y = U/\sqrt{gL}$ in Eqs. (3) and (10)].

The mathematical equivalence with regard to model tests follows from the fact that

$$Y_* = Y \cdot (1 - \frac{\rho_1}{\rho})^{-\frac{1}{2}} \tag{13}$$

since in the present case the model and prototype values of ρ_1/ρ are identical. The model and prototype identity of Y-numbers implies automatically the model and prototype identity of Y_*-numbers and vice versa.

Hence in accordance with convention established in the field we replace consideration of (10) with that of

$$X = \frac{UL}{\nu} \qquad Y_* = \frac{U}{\sqrt{gWL}} \tag{14}$$

As is well known the simultaneous identity of model and prototype values of X and Y_* (i.e. $\lambda_X \equiv 1$ and $\lambda_{Y_*} \equiv 1$) cannot be achieved (if the small scale model operates with the prototype fluid). Indeed

$$\lambda_X = 1 \quad \text{yields} \quad \lambda_U = \frac{1}{\lambda_L} \; , \tag{15}$$

whereas

$$\lambda_{Y_*} = 1 \; (\text{or } \lambda_Y \equiv 1) \quad \text{gives} \quad \lambda_U = \sqrt{\lambda_L} \tag{16}$$

(e.g. if λ_L = 1/16 then Reynolds criterion requires λ_U = 16 whereas Froudian criterion demands λ_U = 1/4).

In the situations such as above the conventional approach is to ignore one of the criteria. Accordingly in the present field, one usually finds some justifications and states that "the influence of the Reynolds number X is negligible with regard to the present phenomenon and thus that it can be ignored". Accordingly, the condition $\lambda_X = 1$ and thus $\lambda_U = 1/\lambda_L$ are excluded and the flow velocities are scaled down according to $\lambda_Y = \lambda_{Y_*} = 1$, i.e. as $\lambda_U = \sqrt{\lambda_L}$. The weakness of this seemingly reasonable statement lies in the word "phenomen", which is used as if it were a single entity. In fact, however, the term "phenomenon" stands for a multitude of the quantitative properties A

$$A_1, A_2, A_3, \ldots\ldots\ldots A_k \ldots\ldots \tag{17}$$

Indeed it is the sum total of these unlimited number of properties that constitute a physical phenomenon, e.g. A_1 may be the temperature at the space point m_1, A_2 the temperature of a point m_2, A_3 - root mean square value of the vertical component of the fluctuating velocity of turbulence at a point m_3, A_4 the rate of energy dissipation at yet another location...etc. So, when saying that "X is unimportant"...which

* Referred to as "densimetric Froude Number"

property do we have in mind? A_1 or A_3...or which one?... Each of these properties A_n are different functions ϕ_{A_n} (of the same variables) and therefore some of them may vary indeed with X only in a feeble manner, but some others may vary with X strongly... It follows that the statement "X is unimportant" is rather sweeping and shallow. Being a different function of X, Y_*,...etc. every property A has "its own degree of importance" with regard to the variables X, Y_*...etc., the "measure" of the degree of importance of X, say, on any A_k can be reflected by the partial derivative of A_k with respect to X viz by

$$\frac{\partial A_k}{\partial X} \tag{18}$$

If $\frac{\partial A_k}{\partial X} \to \approx 0$ then X can be neglected with respect to that A_k. If however $\left|\frac{\partial A_k}{\partial X}\right| >> 0$ then the influence of X cannot be neglected (for that A_k).

Alternative Approach to Dynamic Similarity*

In the modelling of the present phenomenon all dimensionless variables, other than X and Y_* are identical in model and prototype. Thus any A_k can be regarded as a function of X and Y_* only:

$$\Pi_{A_k} = \phi_{A_k}(X, Y_*) \tag{19}$$

Consider the total differential $\delta\Pi_{A_k}$ implying the difference between the model and prototype values of Π_{A_k}:

$$\delta\Pi_{A_k} = \Pi_{A_k}'' - \Pi_{A_k}' = \Pi_{A_k}'(\lambda_{\Pi_{A_k}} - 1) \tag{20}$$

On the other hand

$$\delta\Pi_{A_k} = \frac{\partial\Pi_{A_k}}{\partial X}\cdot\delta X + \frac{\partial\Pi_{A_k}}{\partial Y_*}\cdot\delta Y_* \tag{21}$$

where

$$\left.\begin{aligned} \delta X &= X'' - X' = X'(\lambda_X - 1) \\ \text{and}\quad \delta Y_* &= Y_*'' - Y' = Y_*'(\lambda_{Y_*} - 1) \end{aligned}\right\} \tag{22}$$

Hence

$$\Pi_{A_k}'' - \Pi_{A_k}' = \frac{\partial\Pi_{A_k}}{\partial X}X'(\lambda_X - 1) + \frac{\partial\Pi_{A_k}}{\partial Y_*}Y_*'(\lambda_{Y_*} - 1) \tag{23}$$

What is actually required is the identity of the model and prototype values of Π_{A_k}; i.e. that $\Pi_{A_k}' \equiv \Pi_{A_k}''$ or that

* Outlined in the "invited opening lecture" by M.S. Yalin in the 15th International Congress of the IAHR-BadenBaden, Germany W., Aug. 1977

$$\Pi_{A_k}'' - \Pi_{A_k}' \equiv 0 \tag{24}$$

As seen from (23), the requirement (24) can be achieved if

$$\frac{\partial \Pi_{A_k}}{\partial X} \cdot X' (\lambda_X - 1) + \frac{\partial \Pi_{A_k}}{\partial Y_*} \cdot Y_*' (\lambda_{Y_*} - 1) = 0 \tag{25}$$

is provided.

Clearly if $\lambda_X = 1$ and $\lambda_{Y_*} = 1$ (i.e. if the model and prototype values of X and Y_* were identical) then (25) and thus (24) are satisfied and the dynamic similarity is achieved for any A_k. If, however, accordance with the usual practice $Y_*' = Y_*''$, i.e. $\lambda_{Y_*} = 1$ is provided but the equality of X numbers is ignored (i.e. if $X' \neq X''$ and $\lambda_X \neq 1$) then

$$\delta\Pi_{A_k} = \Pi_{A_k}'' - \Pi_{A_k}' = \frac{\partial \Pi_{A_k}}{\partial X} \cdot X' (\lambda_X - 1) \neq 0 \tag{26}$$

and

$$\Pi_{A_k}'' \neq \Pi_{A_k}' \tag{27}$$

In this case the dynamic similarity is violated, for the model and prototype values of Π_{A_k} are not identical and as seen from (26) the difference between Π_{A_k}'' and Π_{A_k}' (i.e. the error) increases in proportion to

$\dfrac{\partial \Pi_{A_k}}{\partial X}$, X' and the deviation of λ_X from unity.

But why the requirement (24), reflecting the dynamic similarity of the property A_k, should necessarily be achieved by identifying one of the scales of the dimensionless variables (in our case λ_{Y_*}) with unity? If the purpose of the exercise is to fulfill the requirement (24) then this can be achieved by any pair $\lambda_{Y_*} = \alpha$ and $\lambda_X = \beta$ which constitute the solution of the equation (25):

$$\frac{\partial \Pi_{A_k}}{\partial X} X' (\beta - 1) + \frac{\partial \Pi_{A_k}}{\partial Y_*} Y' (\alpha - 1) = 0 \tag{28}$$

Clearly, in this case neither α nor β may be equal to unity ($\alpha \neq 1$, $\beta \neq 1$).

Denoting for brevity

$$\frac{\partial \Pi_{A_k}}{\partial X} = \Pi_X \quad \text{and} \quad \frac{\partial \Pi_{A_k}}{\partial Y} = \Pi_Y \tag{29}$$

we determine from (28)

$$\frac{\beta - 1}{\alpha - 1} = \frac{\Pi_y . Y'}{\Pi_x . X'} \tag{30}$$

where X' and Y' are always positive (X' > 0, Y' > 0). It follows then

(i) if Π_y and Π_X have different signs (i.e. if ΠA_k is an increasing function of Y_* but is a decreasing function of X or vice versa) then (β-1) and (α-1) have also different signs: i.e. one among α and β is larger than unity whereas another is smaller than unity, i.e.

$$\text{if} \quad \Pi_y = -\Pi_x \quad \text{then} \quad \left.\begin{array}{l} \text{either } \alpha>1 \text{ while } \beta<1 \\ \text{or } \quad \alpha<1 \text{ while } \beta>1 \end{array}\right\} \tag{31}$$

(ii) The Eq. (30) (which is in fact (28) and which thus implies (24) reflecting the similarity of the property A_k) does not have a unique solution. Indeed for the given values of Π_X, Π_y, X' and Y' and thus for the given value (C, say) of its right hand side, the Eq. (30) can be satisfied for an infinite number of pairs α and β:

$$\frac{\beta - 1}{\alpha - 1} = C \tag{32}$$

(as it is one equation for two unknowns). e.g. if C = -2 and thus (32) is

$$\frac{\beta - 1}{\alpha - 1} = -2$$

then any of the pairs:

$$\left.\begin{array}{l} \alpha_1 = 1/2, \ \beta_1 = 2 \\ \alpha_2 = 1/4, \ \beta_2 = 2.5 \\ \alpha_3 = 0.7, \ \beta_3 = 1.6 \\ \alpha_4 = 1.2, \ \beta_4 = 0.6....\text{etc.} \end{array}\right\} \tag{33}$$

are the solutions [note, that since C < 0 (which means that $\Pi_y = -\Pi_X$), then when one of the scales (α&β) is larger than unity the other is smaller than unity, as predicted by (31)].

(iii) Clearly the solution of (32) would be most elegant if α and β would deviate from unity as little as possible. This can be achieved with the aid of the straight-line diagram of α_i versus β_i e.g. the values (33), which correspond to C = -2, are forming the straight line shown in Figure 4. From this diagram it is clear that the solutions α_3, β_3 and α_4, β_4 should be regarded as more preferable than α_1, β_1 and α_2, β_2.

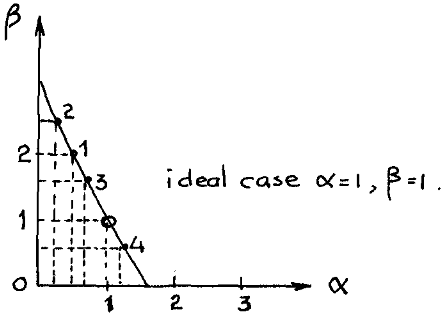

Figure 4. Combinations of Model Scales

(iv) The selection of scales can be based on the simultaneous consideration of several properties A_1, A_2,...A_N. Consider e.g. only two A_1 and A_2:

$$\frac{(\Pi_y)_1(Y')_1}{(\Pi_x)_1(X')_1} = C_1 \quad \text{and} \quad \frac{(\Pi_y)_2(Y')_2}{(\Pi_x)_2(X')_2} = C_2 \tag{34}$$

in this case

$$\frac{\beta - 1}{\alpha - 1} = C_1 \qquad \frac{\beta - 1}{\alpha - 1} = C_2 \tag{35}$$

and we have two lines (intersecting at α=1; β=1) which can be shown e.g. as in Figure 5

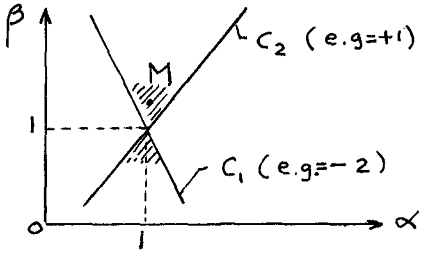

Figure 5. Intersecting Model Scales

As has been mentioned earlier each A_j requires "its own" scales α and β for its dynamically similar modelling. Hence A_1 and A_2 will require some pairs of α and β lying on their respective straight lines (C_1 and C_2 in the figure above). On the other hand if the lines do not diverge from each other substantailly and if the scales are selected in the neighborhood of the point $\alpha=1$, $\beta=1$ (shaded regions in figure above) then one can consider the α and β values of a point such as M somewhere in between. Admittedly, in this case neither A_1 nor A_2 will be rigorously reproduced. On the other hand the deviation from the rigor may be insignificant while the sacrifice of the rigor may be regarded as compensated by the advantage of studying both properties (A_1 & A_2) in the same model [or with regard to OTEC models "by the advantage of studying both A_1 and A_2 for the same model velocity u" (i.e. during the same run)"]

Clearly the idea above can be generalized to overall properties provided that their respective straight lines (e.g. C_1, C_2, C_3, C_4) do not diverge much from each other and/or the point M corresponding to common solution is reasonably close to the point $\alpha=1$; $\beta=1$

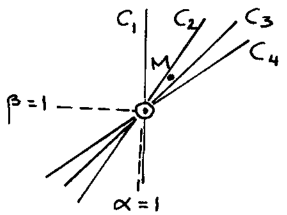

Figure 6. Intersecting Model Scales for Several Properties

(v) All the considerations above rest on the assumption that the values of

$$X', Y', \frac{\partial \Pi_{A_k}}{\partial X}, \frac{\partial \Pi_{A_k}}{\partial Y}$$

corresponding to the property A_k are known (or estimated from some preliminary experiments) in the region X', Y' under investigation. If however this is not so then an "experimental model" should be adopted which is outlined in Figure 7.

1. Take prototype values X' and Y_*' and find the point M' (on the X, Y_* plane)
2. Let λ be the model scale. It is assumed that model is undistorted and that $\lambda_\rho = \lambda_\mu = 1$.

   (i) if the model were Froudian: $\lambda_{Y_*} = 1 \rightarrow Y_*'' = Y'$ and $X'' << X'$ (point M_y)

   (ii) if it were a Reynolds model $\lambda_X = 1 \rightarrow X'' = X'$ and $Y_*'' >> Y_*'$ (point M_x)

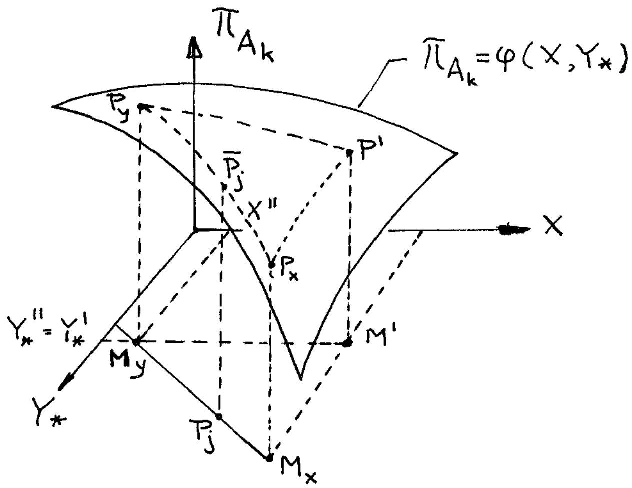

Figure 7. Graphical Presentation of Π_{A_k} as Function of X and Y_*

3. For a Froudian model $\lambda_u = \sqrt{\lambda}$ and thus $u'' = u'(\lambda)^{1/2}$
For a Reynolds model $\lambda_u = \lambda^{-1}$ and thus $u'' = u'(\lambda)^{-1}$

i.e. the exponent ω of λ is distributed along the straight line interval M_xM_y as in Figure 8

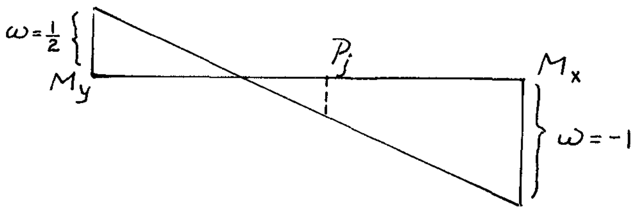

Figure 8. Variation of Exponent ω of λ

4. Now if $\frac{\partial \Pi_{A_k}}{\partial X}$ and $\frac{\partial \Pi_{A_k}}{\partial Y}$ have different signs, e.g. if Π_{A_k} increases with Y_* but decreases with X (which is likely to be the case) then

$$\overline{P_yM_y} > \overline{P'M'} \quad \text{while} \quad \overline{P_xM_x} < \overline{P'M'}$$

where $\overline{P'M'}$ signifies the prototype value Π_A'.

5. Now alter the model velocity u" in the in interval

$$(u'\lambda^{1/2}) < u'' < (u'\lambda^{-1}) \qquad (36)$$

(and thus move along the straight line $\overline{M_yM_x}$) until such a point P_j is found where the ordinate $P_j\bar{P}_j$, implying Π_A'', becomes approximately equal to $\overline{P'M'}$ implying (beforehand measured) prototype value Π_A'. Knowing thus "the solution point P_j" determine the corresponding exponent ω from Figure 8. The coordinates of P_j are model values X_j'' and Y_{*j}''. Form the ratios X_j''/X' and Y_{*j}''/Y'; these ratios are the scales β and α sought:

$$\alpha = \frac{Y_{*j}''}{Y'} \quad ; \quad \beta = \frac{X_j''}{X'} \tag{37}$$

5. CONCLUSIONS AND RECOMMENDATIONS

1) The potential effect of one or more large OTEC plants on the ocean environment requires further investigations.
2) In these investigations, field studies, laboratory investigations (physical modelling) and mathematical (numerical) modelling support each other and are all required to provide satisfactory answers.
3) In the physical modelling of OTEC plants special consideration has to be given to the similarity criteria to be used to convert model data to prototype values.
4) In the past the similarity criteria utilized can hardly be regarded as satisfactory. Since the simultaneous fulfillment of both Reynolds and Froude criteria is impossible (if the model is operated with prototype fluid), contemporary practice often uses one of these criteria as dominant, the other criterion being ignored. This practice may lead to erroneous results in model interpretation.
5) The present paper is intended as a step in improving the modelling technique. Rather than supposing that one of the parameters (e.g. the Reynolds number) is irrelevant and that similarity is based on the other parameter (the Froude number) a method is proposed in which the influence of both numbers is compromised.

6. REFERENCES

1) Bathen, K.H. (1975). "An evaluation of oceanographic and socio-economic aspects of nearshore ocean thermal energy conversion pilot plant in subtropical Hawaiian waters". Final report for NSF/RANN, ERDA, April 1975.
2) Jirka, G.H., Johnson, R.P., Fry, D.J. and Harleman, D.R.F. (1977). "Ocean Thermal Energy Conversion Plants" experimental and analytical study of mixing and recirculation". Ralph M. Parsons Laboratory to Water Resources & Hydrodynamics, Dept. of Civil Engineering, M.I.T., Report No. 231.
3) Lee, T.T. (1978). "Assessment of the impact of OTEC power plant on the near-field environment". Proposal to University of Hawaii Sea Grant College, 1978.
4) Richards, William E. and Vadus, Joseph R. (1980). "Ocean Thermal Energy Conversion: Technology Development". Journal M.T.S. Feb.-Mar. 1980. Vol. 14-No. 1, pp. 3-14.
5) Yalin, M.S. (1971). "Theory of Hydraulic Models". London MacMillan.
6) Yamashita, Yoshihiko (1979). "Three-dimensional mathematical model for Ocean Thermal Energy Conversion system operations". PhD dissertation, Univ. of Hawaii, Dept. of Ocean Engineering.