# Dynamic Stability Improvement of a Power System Based on a PSO-Tuned H<sub>2</sub> Controller

M. Mohseni Mirabadi, N. R. Abjadi, S. Hoghoughi-Isfahani, and S. Shojaeian

Abstract—To supply power demands reliably, power system should cope with various disturbances and faults and its stability should be retained. A power system may be failed due insufficient damping of synchronizing torque. In this paper, to improve the dynamic stability of a power system a feedback control based on H2 method is designed. To formulate the problem appropriately, linear matrix inequality (LMI) theory is employed. To optimize the overall closed-loop system response, the parameters of controller is optimized using particle swarm optimization (PSO) algorithm. Simulation results represent the effectiveness and validity of the proposed controller and its superiority over conventional power system stabilizer (PSS).

*Index Terms*—Dynamic Stability, Robust Control, Linear Matrix Inequality (LMI), Single Machine Infinite Bus (SMIB), Power System Stabilizer (PSS), H<sub>2</sub>Control.

#### I. INTRODUCTION

WITH the growth of power networks, low frequency oscillations appear in power system. Small and sudden disturbances cause such oscillations. In more cases, these oscillations are damped quickly and the amplitude of the oscillation is under a certain value; but depending on the operating point conditions and system parameters values, these oscillations may become continuing for a long time and in the worst case, their amplitudes are increased. The dynamic stability of the power system is an important factor in development power networks. In [1], using fuzzy logic laws, a controller is designed for STATCOM and the improvement of power system dynamic stability is studied. In [2], a robust controller is proposed for SVC control to improve the damping of synchronous machine oscillations. The achieved results in this work are compared to the ones from a conventional power system stabilizer (PSS). In [3], to increase the dynamic stability a UPFC is employed and two control methods are proposed. In this work the effect of UPFC capacitance value on dynamic stability is investigated. There are various PSS structures; but conventional PSS is still interesting because of its simple structure and good flexibility

M. Mohseni-Mirabadi, N.R. Abjadi and S. Hoghoughi-Esfahani are with the Deparetment of Engineering, Shahrekord University, Shahrekord, Iran. (e-mail: Mohsenimehdi.sku@gmail.com, navidabjadi@yahoo.com, said\_hoghoughi@yahoo.com)

S. Shojaeian is with the Department of Electrical Engineering, Khomeinishahr Branch, Islamic Azad University, Isfahan, Iran (e-mail: shojaeian@iaukhsh.ac.ir) and feasibility. However the performance of conventional PSS is sensitive to operating point of the system which is changed by load variation; thus the conventional PSS may be failed capability [4]. In [5], the effect of the injected reactive power of STATCOM on grid voltage and the damping of synchronous machine oscillations is investigated. Most of the controllers proposed for this purpose need a perfect model of the power system with good precision. It is worthwhile to note the power system is a nonlinear coupled system. Most of the models used in controller design are a linear approximation around the operating point. Usually the design of the controller is based on the worst operating point and simply the damping torque is increased. With a change in load or system parameters, the good performance of the system is not guaranteed. In this paper, a robust H2 state feedback control to improve dynamic stability of power system in the presence of parametric uncertainties is introduced. This controller overcomes the mentioned difficultly in power system. To achieve the best controller tuning, the particle swarm optimization (PSO) is employed. The obtained results are compared to the results with a conventional PSS.

### II. SELECTED POWER SYSTEM AND IT'S MODELING

The power system under study in this paper is presented in Fig. 1. In this figure,  $V_t$  and  $V_o$  are terminal voltage and infinite bus voltage respectively. A local load with Y=G+jB admittance is on generator bus and the transmission line is presented with Z=R+jX total impedance [6].



Fig. 1. Selected power system

Considering Heffron-Philips model for a synchronous generator, the model of the system is given by the following equations:

$$\dot{\delta} = \omega_0 \omega \tag{1}$$

$$\dot{\omega} = \frac{1}{M} (T_m - T_e - D\omega) \tag{2}$$

$$\dot{E}'_{q} = \frac{1}{T'_{d0}} (E_{fd} - E'_{q} - (X_{d} - X'_{d})i_{d})$$
(3)

$$T_e = C_3 E'_q \sin \delta + C_4 \sin 2\delta \tag{4}$$

$$i_d = \frac{E'_q - V_o \cos \delta}{X'_d} \tag{5}$$

$$C_3 = \frac{V_o}{X'_d} \tag{6}$$

$$C_4 = \frac{V_o^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d'} \right)$$
(7)

After linearization, considering state variables as  $X_1 = \delta$ ,  $X_2 = \omega$ ,  $X_3 = E'_q$  and input variables as  $u_1 = E_{fd}$  and  $u_2 = T_m$  these equations can be written as:

$$\Delta \dot{X}_1 = \omega_0 \Delta X_2 \tag{8}$$

$$\Delta \dot{X}_2 = \frac{1}{M} (\Delta u_1 - \Delta T_e - D\Delta X_2) \tag{9}$$

$$\Delta X_{3} = \frac{1}{T_{d0}'} (\Delta u_{2} - \Delta X_{3} - (X_{d} - X_{d}')\Delta i_{d})$$
(10)

$$\Delta i_{d} = \frac{\Delta X_{3}}{X'_{d}} + \frac{V_{o} \sin X_{10} \Delta X_{2}}{X'_{d}} = Y_{d} \Delta X_{3} + F_{d} \Delta X_{1}$$
(11)

$$\Delta T_e = (C_3 \sin X_{10}) \Delta X_3 + (C_3 X_{30} \cos X_{10} + 2C_4 \cos 2X_{10}) \Delta X_1$$
(12)

Substituting (11) and (12) in (9) and (10) the following linear state equation is obtained.

$$\begin{bmatrix} \Delta \dot{X}_{1} \\ \Delta \dot{X}_{2} \\ \Delta \dot{X}_{3} \end{bmatrix} = \begin{bmatrix} 0 & \omega_{0} & 0 \\ \frac{-k_{1}}{M} & \frac{-D}{M} & \frac{-k_{2}}{M} \\ \frac{-k_{4}}{T_{d0}'} & 0 & \frac{-1}{k_{3}T_{d0}'} \end{bmatrix} \cdot \begin{bmatrix} \Delta X_{1} \\ \Delta X_{2} \\ \Delta X_{3} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{M} & 0 \\ 0 & \frac{1}{T_{d0}'} \end{bmatrix} \cdot \begin{bmatrix} u_{1} \\ u_{2} \end{bmatrix}$$
(13)

Where

$$k_{1} = C_{3}X_{30}\cos X_{10} + 2C_{4}\cos 2X_{10}$$

$$k_{2} = C_{3}\sin X_{10}$$

$$k_{3} = \frac{1}{1 + (X_{d} - X'_{d})Y_{d}}$$

$$k_{4} = (X_{d} - X'_{d})F_{d}$$

$$Y_{d} = \frac{1}{X'_{d}}$$

$$F_d = \frac{V_o \sin X_{10}}{X'_d}$$

 $T_m$  is mechanical input torque;  $T_e$  is electromagnetic torque of machine; M and D are inertia constant and damping coefficient of machine respectively;  $\omega_0$  is synchronous speed;  $E_{fd}$  is excitation voltage;  $X_d$  and  $X'_d$  are synchronous and transient d-axis machine reactance respectively;  $X_q$  is synchronous q-axis machine reactance;  $\delta$  is power angle;  $T'_{d0}$  is open circuit time constant of the machine and  $X_{10}$  is the initial value of power angle.

#### III. ROBUST CONTROL

Using a suitable robust control, the closed-loop system remains stable even in the presence of system uncertainties. Most of these uncertainties are due the approximation in modeling the system. Usually in system modeling small time constants and some nonlinear and time varying terms are neglected. To retain the good performance of the closed-loop system despite these approximations, a robust controller is design for introduced power system [6].

Considering model uncertainties, the state equation

$$\dot{X}(t) = AX(t) + BU(t)$$
<sup>(14)</sup>

Is written as

$$\dot{X}(t) = (A + D\Delta(t)E_1)X(t) + (B + D\Delta(t)E_2)U(t)$$
 (15)

Where  $\Delta(t)$  represents an scalar or matrix including uncertainties which is satisfied  $\Delta^T(t)\Delta(t) \le I$ , D,  $E_1$  and  $E_2$ are scalars or matrices relating to uncertainties coefficients. In control theory, the main aim is the obtaining of a stabilizing

Feedback gain (state feedback) as follow:

$$U = KX$$
 (16)  
To solve this control problem the following matrix inequalities  
are used:

$$\begin{bmatrix} G_{1} & G_{2}^{T} & QR_{1}^{1/2} & Y^{T}R_{2}^{1/2} \\ G_{2} & -\varepsilon I & 0 & 0 \\ R_{1}^{1/2}Q^{T} & 0 & -I & 0 \\ R_{2}^{1/2}Y & 0 & 0 & -I \end{bmatrix} < 0$$
(17)  
$$\begin{bmatrix} G_{1} = QA^{T} + AQ + BY + Y^{T}B^{T} + DMD \\ G_{2} = E_{1}Q + E_{2}Y \end{bmatrix}$$

 Where:
 (18)
 (19)

 Q > 0 (19)
 (19)

In these LMIs, Y is a variable without sign;  $R_1$  and  $R_2$  are positive definite constants relating to the following  $H_2$  cost function:

$$J = \int_0^\infty (X^T R_1 X + U^T R_2 U) dt$$

Using MATLAB coding the following state feedback gain can be obtained:

$$K = YQ^{-1} \tag{20}$$

#### IV. TUNING BY PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO) was introduced in 1995 by Kennedy and Eberhart [7].

In PSO algorithm, a random population of points is generated. Each point represents a member of the population. In PSO algorithm there is no sudden jump or confusion; each point is a solution. Considering X and V as particle position and velocity respectively, the position of n<sup>th</sup> particle in a space with m dime is represented with  $X_n = [X_{n1}, X_{n2}, ..., X_{nm}]$ .

The position of each particle is changed in next stage and it reaches a new position. The best position of n<sup>th</sup> particle which is corresponding with the lowest cost function for that particle is saved in  $P_{best_n}$ . In addition,  $P_{best}$  of all particles are compared and the position of particle which has the lowest cost function is saved in  $G_{best}$ . The next vector of each particle is depending on its position and its distance to its  $P_{best}$  and its distance to  $G_{best}$ . The relations of particles movements are as follow:

$$V_{nm}^{i+1} = w \times V_{nm}^{i} + C_1 \times rand() \times (P_{best_{nm}} - X_{nm}^{i} + C_2 \times rand() \times (G_{best} - X_{nm}^{i}))$$
(21)

$$X_{nm}^{i+1} = X_{nm}^{i} + CV_{nm}^{i+1}$$
(22)

$$\left|V_{nm}^{i+1}\right| \le V_{\max} \tag{23}$$

Where  $V_{\text{max}}$  is a parameter that prevents to go out of suitable search space which causes the solution to be in acceptable region;  $C_1$  and  $C_2$  are constants which represent the speed of learning or pulling to  $P_{best}$  and  $G_{best}$ ; the weighing function w is given by:

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{iter_{\max}} \times iter$$
(24)

Here  $w_{\min}$  and  $w_{\max}$  are minimum and maximum of weighing function; *iter* is the number of iterations.

In order to optimize the parameters of the controller with PSO, the following cost function is used

$$CostFunction = \int_0^{t_1} t_s |e(t)| dt$$
<sup>(25)</sup>

Where  $t_1$  is the final time of simulation; e is the error signal

and  $t_s$  is the settling time of the system.

# V. SIMULATION AND RESULTS

To show the effectiveness of the proposed controller, simulation results are demonstrated in this section. The simulation results are obtained for two cases: without considering uncertainties and with considering uncertainties. Using the model and system parameters the following state space and control matrices are obtained,

$$A = \begin{bmatrix} 0 & 376.9911 & 0 \\ -0.0286 & 0 & -0.0748 \\ -0.1683 & 0 & -0.4371 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 0 \\ 0.0893 & 0 \\ 0 & 0.1874 \end{bmatrix}$$

To obtain the coefficients and parameters of (15) two cases are considered.

# A. Without considering uncertainties

In this case these coefficients and parameters are given by

$$E_1 = E_2 = 0$$
$$D = 0$$

The parameters of the proposed controller  $R_1$  and  $R_2$  should be positive definite. In first step, they selected as unitary matrices. Using these matrices, some machine input variables become out of reasonable range. With PSO algorithm the optimal matrices are obtained as

$$R_{1} = \begin{bmatrix} 3.19 & 0 & 0 \\ 0 & 47.53 & 0 \\ 0 & 0 & 0.000001 \end{bmatrix}$$
$$R_{2} = \begin{bmatrix} 58.81 & 0 \\ 0 & 142.1 \end{bmatrix}$$

With MATLAB coding the following state feedback gain is obtained

$$K = \begin{bmatrix} -0.0378 & -27.9272 & 0.165 \\ -0.0055 & 0.1430 & -0.0394 \end{bmatrix}$$

The obtained results for sudden change in load are represented as follow.

Assuming a 500MW load is omitted from the power grid at t=0.1 sec and it is returned after t=1.1 sec. This test illustrates the dynamic stability of the system clearly. The obtained results are shown in Figs. (2)-(4). As can be seen from these figures, again the oscillations are damped rapidly and the

maximum overshoot is small. The machine power angle has little oscillations. After the load variations, machine return to steady-state conditions.



Fig. 4. Machine phase 'a' current for sudden variations of load

# B. Power system with parametric uncertainties

In this section, it is assumed that there are uncertainties in damping coefficient (D) and inertia constant (H) of synchronous machine. The uncertainty in D is modeled as

$$\frac{D+\delta_D}{M} = \frac{D}{M} + \frac{\delta_D}{M}$$
(26)

Using the following geometric series

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \dots$$
(27)

The uncertainty in H is modeled as follow

$$\frac{-k_1}{2(H+\delta_H)} = \frac{-k_1}{2H(1+\frac{\delta_H}{H})} = \frac{-k_1}{2H} + \frac{k_1}{2H^2}\delta_H$$
(28)

In (28) and (30),  $\delta_D$  and  $\delta_H$  represent the uncertainties in D and H respectively. The metrics of (17) is given by

The matrices of (17) is given by,

$$E_{1} = \begin{bmatrix} 0 & \frac{-1}{2H} & 0 \\ \frac{-k_{1}}{2H^{2}} & \frac{D}{2H} & \frac{-k_{2}}{2H^{2}} \end{bmatrix}$$
$$E_{2} = \begin{bmatrix} 0 & 0 \\ \frac{-1}{2H^{2}} & 0 \end{bmatrix}$$
$$D = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

The matrices  $R_1$  and  $R_2$  using PSO algorithm after 100 iterations are obtained as,

$$R_{1} = \begin{bmatrix} 6.6907 & 0 & 0\\ 0 & 45.2011 & 0\\ 0 & 0 & 0.1026 \end{bmatrix}$$
$$R_{2} = \begin{bmatrix} 78.0133 & 0\\ 0 & 35.3664 \end{bmatrix}$$

Using MATLAB software, the state feedback gain is obtained as,

$$K = \begin{bmatrix} -0.1549 & -57.0393 & 0.0819 \\ -0.0781 & 1.0032 & -0.3686 \end{bmatrix}$$

Assuming a 500MW load is omitted from the power grid at t=0.1sec and it is returned after t=1.1sec in the presence of uncertainties. The obtained results are shown in Figs. (5)-(7). As can be seen from these figures, again the oscillations are damped rapidly and the maximum overshoot is small. The machine power angle has little oscillations. Conventional PSS operates very weak in this test.



Fig. 5. Machine rotor speed for sudden variations of load



Fig. 6. Machine torque angle for sudden variations of load



Fig. 7. Machine phase 'a' current for sudden variations of load

# VI. CONCLUSION

Considering the capability of robust control in the presence of uncertainties, in this paper, a robust  $H_2$  state feedback controller is designed for a power system. Parameters of the controller are tuned using PSO algorithm. The performance of the closed-loop system is investigated without and with uncertainties in the mechanical parameters with sudden load variation.

The obtained simulation results show the superiority of the proposed controller over conventional PSS. Using the proposed conventional the oscillations have a small amplitude and they are damped rapidly.

# VII. APPENDIX

System data for single machine infinite bus power system Generator [8]:

$$H = 5.6 X_d = 1.8 pu X_q = 1.8 pu D = 1 pu X'_d = 0.32 pu$$
$$T'_{d0} = 5.3371 \text{sec} f = 60 Hz$$

Transmission Line and Load:

$$R = 0.1273 pu \ X = 0.85 pu \ G = 0.27027 pu$$
  
 $B = 0$ 

Exciter (simplified IEEE type-ST1):

$$K_A = 10 T_A = 0.01 \text{sec}$$

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#### BIOGRAPHIES

**M. Mohseni Mirabadi** received the B.Sc. degree in electrical engineering from Khomeinishahr Branch, Islamic Azad University, Isfahan, Iran, in 2010 and the M.Sc. degree in electrical engineering from the Shahrekord University, Shahrekord, Iran, in 2013. His research interests are robust and nonlinear control, power system control and stability and linear matrix inequality.



**N. R. Abjadi** received the B.Sc., M.S., and Ph.D. degree all in electrical engineering from Isfahan University of Technology, Isfahan, Iran, in 1999, 2002, and 2010 respectively. He is currently an assistant professor in the electrical engineering in the faculty of engineering, Shahrekord University, Shahrekord, IRAN. His main research interests are application of nonlinear control and electric motor drives in general.



**S. Hoghoughi-Isfahani** received the B.Sc. degree from Sharif University of Technology, Tehran, Iran in 1989, the M.S. degree from Amir-Kabir University of Technology, Tehran, Iran, in 1993 and the Ph.D. degree from University of New South Wales, Canberra, Australia, in 1999. He is currently an assistant professor in the electrical engineering in the faculty of engineering,

Shahrekord University, Shahrekord, Iran. His research interests are robust control, linear matrix inequality, and guaranteed cost function control.



**S. Shojaeian** received the B.Sc. and M.S. degree from Isfahan University of Technology, Isfahan, Iran, in 1997 and 2001 respectively and the Ph.D. from Islamic Azad University, science and research branch, Tehran, Iran in 2012. He is currently an assistant professor in the department of electrical engineering Khomeinishahr branch, Islamic Azad University, Isfahan, IRAN. His

research interests are nonlinear control application in power system and power system stability and reliability.