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DYNAMIC STABILITY OF A PARACHUTE POINT-MASS LOAD SYSTEM

TECHNICAL DOCUMENTARY REPORT No. FDL-TDR-64-126

JUNE 1965

AF FLIGHT DYNAMICS LABORATORY
RESEARCH AND TECHNOLOGY DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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Project No. 6065, Task No. 606503

(Prepared under Contract No. AF 33(657)-11184 by the Department of Aeronautics and Engineering Mechanics, University of Minnesota, Minneapolis, Minnesota; Helmut G. Heinrich and Lawrence W. Rust, Jr., Authors)

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FOREWORD

This report was prepared by the Department of Aeronautics and Engineering Mechanics of the University of Minnesota in compliance with US Air Force Contract Nos. AF 33(616)-6372 and AF 33(616)-8310, Project No. 6065, Task No. 606503, "Theoretical Parachute Investigations."

The work accomplished under these contracts was sponsored jointly by QM Research and Engineering Command, Department of the Army; Bureau of Aeronautics and Bureau of Ordnance, Department of the Navy; and Air Force Systems Command, Department of the Air Force and was directed by a Tri-Service Steering Committee concerned with Aerodynamic Retardation. The work was administered under the direction of the Recovery and Crew Station Branch, AF Flight Dynamics Laboratory, Research and Technology Division. Mr. Rudi J. Berndt and Mr. James H. DeWeese were the project engineers.

The authors wish to pay tribute to the late Mr. Toma Riabokin who contributed much to this objective by making a thorough literature survey and by establishing fundamental methods. Unfortunately he left us before this study had been completed. Also, the efforts of the students of the University of Minnesota in support of the various phases of this study are acknowledged and appreciated.

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ABSTRACT

The dynamic stability of a parachute-load system has been analytically investigated for a point-mass load and a statically stable parachute. A typical system consisting of a relatively large suspended load mass and small ribless guide surface parachute has been numerically calculated. Utilizing the apparent mass and apparent moment of inertia, as well as the aerodynamic coefficients of the parachute canopy, the equations of motion for the system have been solved. The influence of several design parameters upon the dynamic stability characteristics of the system has been discussed.

This technical documentary report has been reviewed and is approved.

Theron J. Baker
THERON J. BAKER
Vehicle Equipment Division
AF Flight Dynamics Laboratory

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SYMBOLS

$A_1, A_2, A_3, B_1, B_2, B_3$	constants of integration
C_N	coefficient of normal force
C_T	coefficient of tangent force
$\left(\frac{\partial C_N}{\partial \alpha}\right)_s$	slope of the normal force coefficient versus α for static conditions
g	acceleration of gravity = 32.17 ft/sec^2
$\mathbf{I}$	dimensionless moment of inertia = $I/\pi \rho r^5$
I_a	apparent moment of inertia of the parachute canopy and inertia effects of the enclosed air about the center of mass of the system (slug-ft^2)
$\mathbf{L}$	dimensionless length = L/r
L_1	distance between the center of mass of the system and the canopy center of pressure
L_2	distance between the center of mass of the system and the center of volume of the canopy
L_3	distance between the center of mass of the system and the point load
L_4	distance between the center of mass of the system and the center of mass of the canopy material
$\mathbf{m}$	dimensionless mass = $m/\pi \rho r^3$
m_l	mass of the suspended point mass load (slug)
m_p	mass of the parachute material (slug)
m_{a_x}	apparent mass, including the inertia effect of the enclosed air of the parachute in the x-direction (slug)
m_{a_y}	apparent mass, including the inertia effect of the enclosed air of the parachute in the y-direction (slug)
N	normal force acting on the canopy
r	characteristic radius of the canopy

SYMBOLS (cont.)

T	tangent force acting on the canopy
$\bar{v}$	dimensionless velocity = $\tilde{v}/v_o$
$\vec{v}$	velocity vector of the center of mass of the system (ft/sec) = $v_x\hat{i} + v_y\hat{j}$
v_o	equilibrium velocity of the system (ft/sec)
W_ℓ	weight of the suspended load
α	angle of velocity vector of the center of mass with respect to axis of the canopy
α_p	angle of velocity vector of the center of volume of the canopy with respect to axis of the canopy $= \alpha + \frac{\omega L_2}{v_o}$
β	angle between the direction of the velocity of the center of mass and the vertical
ρ	air density (slugs/ft^3)
τ	dimensionless time = $\frac{tv_o}{r}$
ω	angular velocity of the parachute axis

I. INTRODUCTION

When considering the complete pattern of motion of a parachute-load system, the dynamic stability of the system is a prime consideration.

For such a consideration, one thinks of a freely moving system as having six degrees of freedom, consisting of linear and angular velocities about three mutually perpendicular axes. The angular velocities result in an oscillating motion, with the mode of oscillation generally referred to as the stability characteristic of the system.

It has been noted (Ref 1) that such characteristic motions depend on the aerodynamic coefficients, their derivatives, and on the mass and moments of inertia of the mechanical system as well as on the apparent mass and apparent moment of inertia.

Combining an aerodynamically neutral load with a static aerodynamically stable parachute, a closed form solution is possible provided that the initial oscillations are small and that the parachute has a constant stability derivative $\left(\frac{\partial C_N}{\partial \alpha}\right)_s$ in the range of oscillations. This process is valuable for a certain group of applications and has the advantage of being acceptable for engineering calculations.

For a more general type of problem, where the oscillations may be large and the stability derivative is not constant, solutions can be obtained merely for specific cases and in a numerical way by means of analog or digital computers. Several of these more general as well as specific cases have been investigated and solved in a series of publications by R. Ludwig and W. Heins (Refs 2, 3 and 4). These studies discuss also the influence of several design parameters such as suspension line length, effective porosity of the canopy, apparent mass and weight of the load for the particular application.

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II. EQUATIONS OF MOTION

For the analysis of the dynamics of a parachute-load system, one assumes the case of two falling bodies connected by rigid lines. Figure 1 presents the load-parachute system, the body fixed coordinate axes, x-y-z, as well as the system of external forces. The x-axis coincides with the parachute axis of symmetry, the y-axis is perpendicular to the x-axis and in the plane of motion of the system with the z-axis defined by a right hand coordinate system. The origin, (0,0,0), is at the center of mass of the entire system.

In deriving the equations of motion of the system, the following assumptions will be used:

1. The entire system constitutes a rigid body.
2. The load is considered as a point mass and does not possess aerodynamic characteristics or a moment of inertia.
3. The mass and the aerodynamic forces of the suspension lines can be neglected.
4. The effect of the apparent mass acts at the center of volume of the canopy.
5. The motion is restricted to the x-y plane.

The velocity of the center of mass of the entire system can be presented as:

$$\vec{v} = v_x \hat{i} + v_y \hat{j} \tag{1}$$

where: v_x = velocity of the center of mass in the direction of the parachute axis
v_y = velocity of the center of mass in the direction perpendicular to the parachute axis.

The corresponding momentum, $\vec{M}$, of the parachute-load system, excluding the effects of the apparent and enclosed mass, is:

$$\vec{M} = (m_p + m_\ell)\ (v_x \hat{i} + v_y \hat{j}) \tag{2}$$

where: m_p = mass of parachute material
m_ℓ = mass of the suspended load.

The equations of motion of the system, lateral and rotational, can be written in accordance with Newton's Law as:

$$\text{a)} \quad \sum \vec{F} = \frac{d^{(1)}\vec{M}}{dt} \qquad \text{b)} \quad \sum \vec{R} \times \vec{F} = \frac{d^{(1)}\vec{H}}{dt} \tag{3}$$

in which the following notation is used:

$\vec{F}$ = external force

$\vec{R}$ = radius vector

$\vec{H}$ = angular momentum.

The derivative $\frac{d^{(1)}(\ )}{dt}$ represents differentiation with respect to a space fixed (inertial) reference frame (x', y', z') (Fig 2). A derivative with respect to a particular reference frame can be expressed with respect to another frame which rotates relative to the first one by means of the relationship (Ref 5, pp. 53-55):

$$\frac{d^{(1)}\vec{A}}{dt} = \frac{d^{(2)}\vec{A}}{dt} + \vec{\omega} \times \vec{A} \tag{4}$$

where: $\frac{d^{(1)}\vec{A}}{dt}$ = rate of change of $\vec{A}$ with respect to reference frame 1

$\frac{d^{(2)}\vec{A}}{dt}$ = rate of change of $\vec{A}$ with respect to reference frame 2

$\vec{\omega}$ = angular velocity of reference frame 2 with respect to reference frame 1.

In this case, $\vec{A} = \vec{M}$, reference frame 1 is the space (x', y', z') frame, and reference frame 2 is the body fixed reference frame (x, y, $\dot{z}$).

Therefore, one may write:

$$\frac{d^{(1)}\vec{M}}{dt} = \frac{d^{(2)}\vec{M}}{dt} + \vec{\omega} \times \vec{M} \tag{5}$$

where $\vec{\omega} = \omega\ \hat{k} = \omega\ \hat{k}'$ = angular velocity of the parachute axis with respect to the z-axis.

Performing the operations indicated in this equation, one obtains:

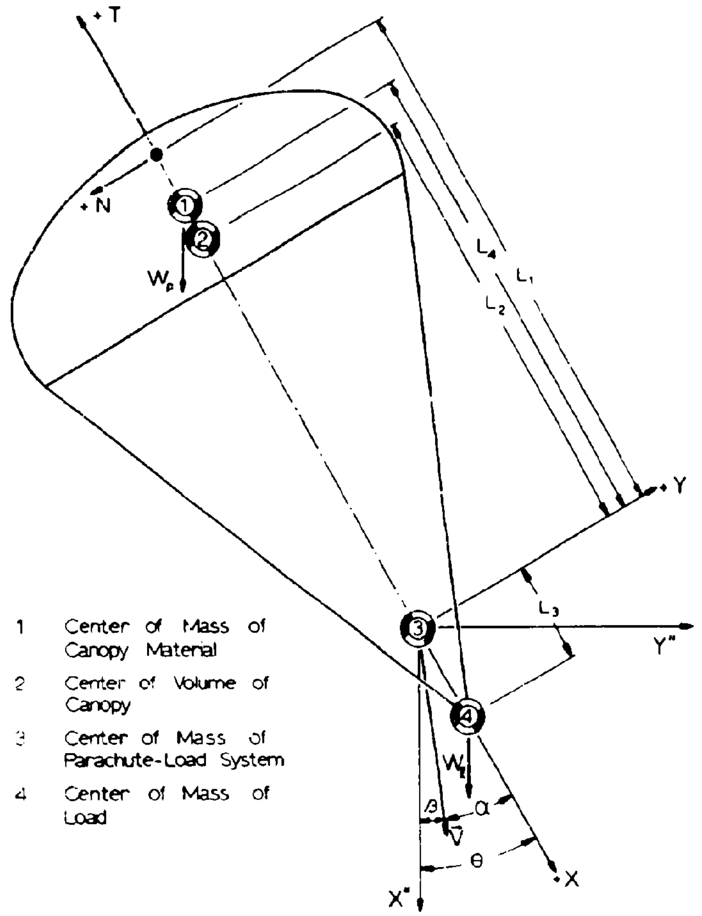
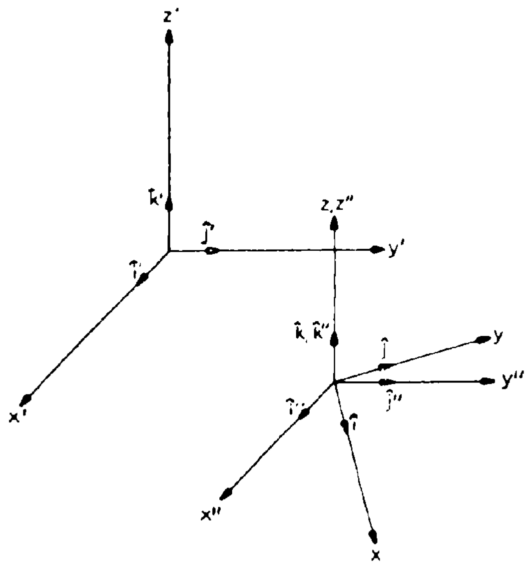


Fig 1. System of Reference for Parachute Load System in Motion



1) x-y-z: body fixed coordinates rotate and translate with the parachute system	3) x"-y"-z": coordinates moving with the origin at center of mass of system and remaining parallel to x'-y'-z' system
2) x'-y'-z': coordinates fixed in space	4) $\hat{i}$, $\hat{j}$, $\hat{k}$; $\hat{i}'$, $\hat{j}'$, $\hat{k}'$; $\hat{i}''$, $\hat{j}''$, $\hat{k}''$: unit vectors

Since the system can only move in x-y plane, z and z' axes are parallel.

Fig. 2. Stationary and Moving Coordinate Systems

$$\frac{d^{(1)}\vec{M}}{dt} = (m_p + m_\ell)\left[(\dot{v}_x - v_y\omega)\hat{\imath} + (\dot{v}_y + v_x\omega)\hat{\jmath}\right]. \qquad (6)$$

The aerodynamic forces are, in accordance to Fig 1:

$$\vec{F}_a = -T\hat{\imath} - N\hat{\jmath} \qquad (7)$$

and the gravity forces are given by:

$$\vec{F}_g = (W_\ell + W_p)\cos\theta\ \hat{\imath} - (W_\ell + W_p)\sin\theta\ \hat{\jmath}. \qquad (8)$$

Note: the gravity force on the enclosed mass is balanced by the bouyancy force on the canopy.

The effects of the apparent and the enclosed mass are given by:

$$\vec{F}_{am} = -m_{a_x} a_{x_{c.v.}}\hat{\imath} - m_{a_y} a_{y_{c.v.}}\hat{\jmath} \qquad (9)$$

where: m_{a_x} = apparent mass* in the x-direction

m_{a_y} = apparent mass* in the y-direction

$a_{x_{c.v.}}$ = acceleration of the center of volume of the canopy in the x-direction

$a_{y_{c.v.}}$ = acceleration of the center of volume of the canopy in the y-direction.

The acceleration of the center of volume of the canopy is given by:

$$\frac{d^{(1)}\vec{v}_{c.v.}}{dt} = \vec{a}_{c.v.} = a_{x_{c.v.}}\hat{\imath} + a_{y_{c.v.}}\hat{\jmath} \qquad (10)$$

where: $\vec{v}_{cv}$ = velocity of the center of volume of the canopy with respect to the space fixed frame

$$= v_x\hat{\imath} + v_y\hat{\jmath} + \vec{\omega}\times(-L_2\hat{\imath})$$

$$= v_x\hat{\imath} + (v_y - \omega L_2)\hat{\jmath}. \qquad (11)$$

Therefore, utilizing relation 4 one obtains the total acceleration:

*See definitions of symbols.

$$\vec{a}_{c.v.} = \left[\dot{v}_x - \omega(v_y - \omega L_2)\right]\hat{\imath} + \left[\dot{v}_y - \dot{\omega} L_2 + \omega v_x\right]\hat{\jmath} \; . \qquad (12)$$

Consequently, the acceleration in the x- and y-directions is:

$$\begin{aligned} a_{x_{c.v.}} &= \dot{v}_x - \omega\,(v_y - \omega L_2) \\ a_{y_{c.v.}} &= \dot{v}_y - \dot{\omega} L_2 + \omega v_x \; . \end{aligned} \qquad (13)$$

Utilizing these relations in Eqn 9, one finds the apparent mass force as:

$$\vec{F}_{am} = -m_{a_x}\left[\dot{v}_x - \omega v_y + \omega^2 L_2\right]\hat{\imath} - m_{a_y}\left[\dot{v}_y - \dot{\omega} L_2 + \omega v_x\right]\hat{\jmath} . \qquad (14)$$

If one now substitutes all of these forces and relation 6 into Newton's Law (Eqn 3a), two scalar equations representing the motion in the x- and y- directions are obtained.

$$(m_p + m_{a_x} + m_\ell)(\dot{v}_x - v_y\omega) + m_{a_x} L_2 \omega^2 + T - (W_\ell + W_p)\cos\theta = 0 \qquad (15)$$

$$(m_p + m_{a_y} + m_\ell)(\dot{v}_y + v_x\omega) - m_{a_y} L_2 \dot{\omega} + N + (W_\ell + W_p)\sin\theta = 0. \qquad (16)$$

Newton's second law, which governs the rotational motion (Eqn 3b), can be expressed as follows:

$$\frac{d^{(1)}\vec{H}}{dt} = \sum \vec{R} \times \vec{F} \qquad (17)$$

with:

$$\vec{H} = (I_{cm} + I_a)\,\omega\,\hat{k} \qquad (18)$$

where:

$\vec{H}$ = angular momentum of the system with respect to a nonrotating reference frame with its origin at the center of mass of the system

$\vec{R}$ = position vector from the center of mass of the system to the point of application of an external force $\vec{F}$

$\vec{F}$ = external force

I_a = apparent moment of inertia of the parachute canopy and inertia effects of the enclosed air about the center of mass of the system

I_{cm} = moment of inertia of the load and parachute material about the center of mass of the system

The external moments ($\sum \vec{R} \times \vec{F}$) are given by:

$$\sum \vec{R} \times \vec{F} = (-L_1\hat{i})x(-N\hat{j}) + (L_3\hat{i})x(-W_\ell \sin\theta\,\hat{j}) + (-L_4\hat{i})x(-W_p \sin\theta\,\hat{j})$$

or

$$\sum \vec{R} \times \vec{F} = \hat{k}\left[+ L_1N - W_\ell L_3 \sin\theta + W_p L_4 \sin\theta\right]$$

and since the last two terms $\left[- W_\ell L_3 \sin\theta + W_p L_4 \sin\theta\right]$ cancel each other because of the definition of the center of mass, one obtains:

$$\sum \vec{R} \times \vec{F} = \hat{k} L_1 N \tag{19}$$

The moments due to the apparent mass effect which, in this discussion, encompasses the effect of the enclosed air mass have not been identified separately since their contribution to the moment equation is included in the experimentally determined value of I_a.

Introducing relations 18 and 19 into Eqn 17 yields:

$$\frac{d^{(1)}}{dt}\left[(I_{cm} + I_a)\omega\hat{k}\right] = + L_1N\hat{k} \tag{20}$$

and after rearranging:

$$\left[I_{cm} + I_a\right]\dot{\omega} - L_1N = 0 \; . \tag{21}$$

Three differential equations for the motion of the parachute-load system have now been obtained. They are reproduced here for future reference:

momentum equation for the x-direction

$$(m_p+m_{a_x}+m_\ell)(\dot{v}_x - v_y\omega) + m_{a_x}L_2\omega^2 + T - (W_\ell + W_p)\cos\theta = 0 \tag{22}$$

momentum equation in the y-direction

$$(m_p+m_{a_y}+m_\ell)(\dot{v}_y + v_x\omega) - m_{a_y}L_2\dot{\omega} + N + (W_\ell + W_p)\sin\theta = 0 \tag{23}$$

angular momentum equation

$$(I_{cm} + I_a)\dot{\omega} - L_1N = 0 \; . \tag{24}$$

III. LINEARIZED EQUATIONS

Equations 22 through 24 constitute the general equations of motion of a parachute-load system and apply equally well to large or small oscillations. It should be noted, however, that they are nonlinear, i.e., containing terms of the form $v_y \omega$. Thus, it becomes exceedingly difficult, if not impossible, to find a closed form solution of these equations without reasonable restrictive assumptions.

The simplest, but perhaps also the most important case of the dynamic stability problem, is given by a parachute-load system in which a parachute, which is statically stable about zero angle of attack, oscillates over a relatively small angle range whereby its stability derivative $\left(\frac{\partial C_N}{\partial \alpha}\right)_s$ assumes a constant value. Furthermore, on- -' assume that the system descends approximately vertically w--h its equilibrium velocity where the aerodynamic drag equals the weight. These assumptions may be expressed as:

$$\begin{aligned}
\sin\theta &= \theta & \cos\theta &= 1 \\
\sin\alpha &= \alpha & \cos\alpha &= 1 \\
\sin\beta &= \beta & \cos\beta &= 1 \\
v &= v_o + \tilde{v} & [\tilde{v} &\ll v_o]
\end{aligned} \tag{25}$$

where v_o = equilibrium velocity

$\tilde{v}$ = deviation of v from the equilibrium velocity.

From the geometry of Fig 1:

$$\begin{aligned}
v_x &= v\cos\alpha = (v_o + \tilde{v})\cos\alpha \\
v_y &= -v\sin\alpha = -(v_o + \tilde{v})\sin\alpha \\
\dot{v}_x &= -(v_o + \tilde{v})\sin\alpha\,\dot{\alpha} + \dot{\tilde{v}}\cos\alpha \\
\dot{v}_y &= -(v_o + \tilde{v})\cos\alpha\,\dot{\alpha} - \dot{\tilde{v}}\sin\alpha
\end{aligned} \tag{26}$$

Using the small angle assumptions, and neglecting second order terms, i.e., terms of the form $\alpha\dot{\alpha}$, one obtains:

$$v_x = v_o + \tilde{v} \qquad \dot{v}_x = \dot{\tilde{v}}$$

$$v_y = -v_o\,\alpha \qquad \dot{v}_y = -v_o\,\dot{\alpha} \; . \tag{26a}$$

Substituting Eqn 26a into relations 22 through 24 yields, after neglecting second order terms and recognizing that $\theta = \alpha + \beta$,:

$$(m_p + m_{a_x} + m_\ell)\,\dot{\tilde{v}} + T - (W_\ell + W_p) = 0 \tag{22a}$$

$$(m_p + m_{a_y} + m_\ell)v_o\dot{\beta} - m_{a_y}L_2(\ddot{\alpha}+\ddot{\beta})+N+(W_\ell + W_p)(\alpha+\beta) = 0 \tag{23a}$$

$$(I_{cm} + I_a)(\ddot{\alpha}+\ddot{\beta}) - L_1 N = 0 \tag{24a}$$

The normal and tangent forces (N and T) are conventionally represented as:

$$N = C_N \tfrac{1}{2}\rho v_p{}^2 S$$

$$T = C_T \tfrac{1}{2}\rho v_p{}^2 S$$

where: $v_p = \sqrt{v_x{}^2 + (v_y - \omega L_2)^2}$, the absolute velocity of the center of volume of the canopy.

Following the assumption of small oscillations, one obtains:

$$v_p{}^2 = v_o{}^2 + 2\,v_o\,\tilde{v}$$

and therefore:

$$N = C_N\,\tfrac{1}{2}\,\rho\,(v_o{}^2 + 2\,v_o\,\tilde{v})\pi r^2 \tag{27}$$

$$T = C_T\,\tfrac{1}{2}\,\rho\,(v_o{}^2 + 2\,v_o\,\tilde{v})\pi r^2 \tag{28}$$

where: r = characteristic radius of the canopy.
For future convenience, the related area of the coefficients has been introduced as $\pi\,r^2$.

Experiments (Ref 6) have shown that, for the parachutes under consideration, C_T is approximately constant over a relatively large range of α . Substituting relation 28 into Eqn 22a yields:

$$(m_p + m_{a_x} + m_\ell)\dot{\tilde{v}} + C_T\tfrac{1}{2}\rho(v_o{}^2 + 2v_o\tilde{v})\pi r^2 - (W_\ell + W_p) = 0 \tag{22b}$$

Reference 6 also shows that the normal coefficient, in the range of interest, is proportional to the angle of attack. This may be expressed as:

$$C_N = \left(\frac{\partial C_N}{\partial \alpha}\right)_s \alpha_p \tag{29}$$

where: $\left(\frac{\partial C_N}{\partial \alpha}\right)_s$ = slope of C_N versus α for static conditions

α_p = instantaneous angle of attack of the canopy.

It should be noted that $\alpha_p \neq \alpha$. The angle of attack α_p of the canopy is measured relative to the local velocity vector at the center of volume of the canopy. Thus, when the canopy oscillates about the center of gravity of the system, the angle of attack of the canopy consists of the angle α (Fig 3) and a contribution $\Delta\alpha$, induced by the rotation of the canopy. From geometric considerations, it is apparent that:

$$\Delta\alpha \cong \frac{\omega L_2}{v} \cong \frac{\omega L_2}{v_o} \tag{30}$$

One finds α_p to be:

$$\alpha_p = \alpha + \Delta\alpha = \alpha + \frac{\omega L_2}{v_o} \tag{31}$$

and with Eqn 29:

$$C_N = \left(\frac{\partial C_N}{\partial \alpha}\right)_s \left[\alpha + \frac{L_2}{v_o}(\dot{\alpha} + \dot{\beta})\right] \tag{32}$$

Introducing Eqns 32 and 27 into Eqns 23a and 24a, one obtains:

$$(m_p + m_{a_y} + m_\ell) v_o\dot{\beta} - m_{a_y} L_2 (\ddot{\alpha} + \ddot{\beta}) + \left(\frac{\partial C_N}{\partial \alpha}\right)_s \left[\alpha + \frac{L_2}{v_o}(\dot{\alpha} + \dot{\beta})\right] \tfrac{1}{2}\rho v_o^2 \pi r^2 + (W_\ell + W_p)(\alpha + \beta) = 0 \tag{33}$$

$$(I_{cm} + I_a)(\ddot{\alpha} + \ddot{\beta}) - L_1 \left(\frac{\partial C_N}{\partial \alpha}\right)_s \left[\alpha + \frac{L_2}{v_o}(\dot{\alpha} + \dot{\beta})\right] \tfrac{1}{2}\rho v_o^2 \pi r^2 = 0 \tag{34}$$

Equations 22b, 33, and 34 now represent a set of linear differential equations governing the motion of the parachute-load system.

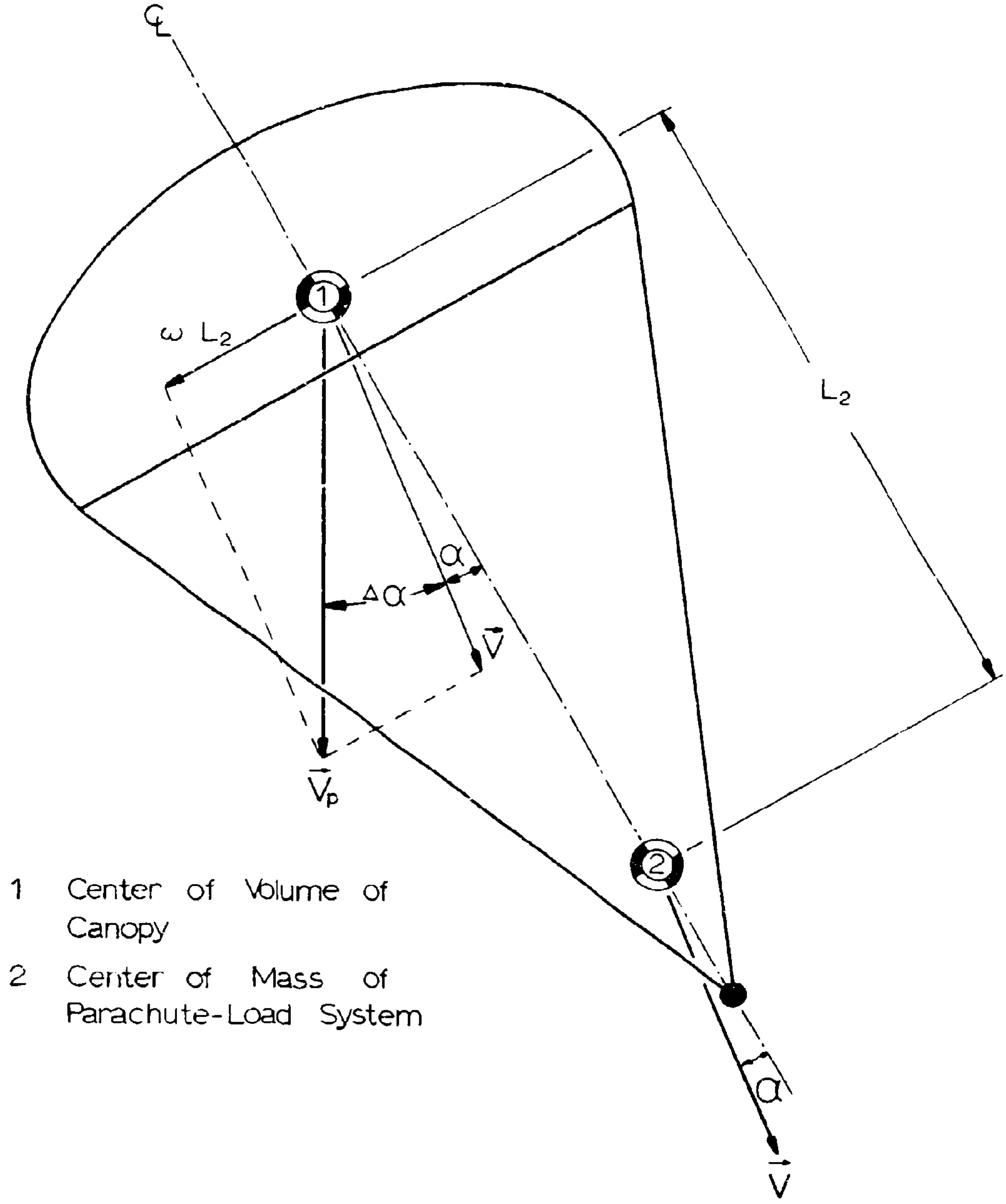


Fig 3. Geometric Representation of $\Delta\alpha$

IV. DIMENSIONLESS EQUATIONS OF MOTION

It is convenient to express the linearized equations of motion in a dimensionless form by introducing a dimensionless time, distance, mass, moment of inertia and velocity.

$$
\tau = \frac{t\,v_o}{r} \qquad \overline{L} = \frac{L}{r}
$$

$$
\overline{m} = \frac{m}{\pi \rho r^3} \qquad \overline{v} = \frac{\tilde{v}}{v_o} \tag{35}
$$

$$
\overline{I} = \frac{I}{\pi \rho r^5}
$$

These definitions, substituted in Eqns 22b, 33, and 34, yield:

$$
(\overline{m}_p + \overline{m}_{\text{[illegible]}} + \overline{m}_\ell)\, \overline{v}' + \frac{C_T}{2}(1 + 2\overline{v}) - \frac{W_\ell + W_p}{\rho v_o^2 \pi r^2} = 0 \tag{22c}
$$

$$
\begin{aligned}
&(\overline{m}_p + \overline{m}_{a_y} + \overline{m}_\ell)\beta' - \overline{m}_{a_y}\overline{L}_2(\alpha'' + \beta'') \\
&\quad + \tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s \left[\alpha + \overline{L}_2(\alpha' + \beta')\right] + \frac{W_\ell + W_p}{\rho v_o^2 \pi r^2}(\alpha + \beta) = 0
\end{aligned} \tag{33a}
$$

$$
(\overline{I}_{cm} + \overline{I}_a)(\alpha'' + \beta'') - \tfrac{1}{2}\overline{L}_1\left(\frac{\partial C_N}{\partial \alpha}\right)_s \left[\alpha + \overline{L}_2(\alpha' + \beta')\right] = 0 \tag{34a}
$$

where the prime (') indicates differentiation with respect to the dimensionless time τ.

A further simplification is introduced by considering the definition of equilibrium velocity

$$
C_T \tfrac{1}{2} \rho\, v_o^2 \pi r^2 = W_\ell + W_p
$$

or

$$
\frac{W_\ell + W_p}{\rho v_o^2 \pi r^2} = \frac{C_T}{2} \; . \tag{36}
$$

Introducing this relation into Eqns 22c and 33a, one obtains:

$$(\overline{m}_p + \overline{m}_{a_x} + \overline{m}_\ell)\ \overline{v}' + C_T\ \overline{v} = 0 \tag{22d}$$

$$\begin{aligned}(\overline{m}_p + \overline{m}_{a_y} + \overline{m}_\ell)\beta' - \overline{m}_{a_y}\ \overline{L}_2\ (\alpha'' + \beta'') \\ + \tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s \left[\alpha + \overline{L}_2\ (\alpha' + \beta')\right] + \frac{C_T}{2}(\alpha + \beta) = 0.\end{aligned} \tag{33b}$$

Rearranging Eqns 33b and 34a, one finally obtains the new set of equations:

$$(\overline{m}_p + \overline{m}_{a_x} + \overline{m}_\ell)\ \overline{v}' + C_T\ \overline{v} = 0 \tag{37}$$

$$\begin{aligned}\overline{m}_{a_y}\ \overline{L}_2\ \alpha'' - \tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s \overline{L}_2\ \alpha' - \left[\tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s + \frac{C_T}{2}\right]\alpha \\ + \overline{m}_{a_y}\ \overline{L}_2\beta'' - \left[\tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s \overline{L}_2 + m_p + m_{a_y} + m_\ell\right]\beta' - \frac{C_T}{2}\beta = 0\end{aligned} \tag{38}$$

$$\begin{aligned}(\overline{I}_{cm} + \overline{I}_a)\ \alpha'' - \tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s \overline{L}_1\overline{L}_2\ \alpha' - \tfrac{1}{2}\ \overline{L}_1\ \left(\frac{\partial C_N}{\partial \alpha}\right)_s \alpha \\ + (\overline{I}_{cm} + \overline{I}_a)\ \beta'' - \tfrac{1}{2}\ \overline{L}_1\overline{L}_2\ \left(\frac{\partial C_N}{\partial \alpha}\right)_s \beta' = 0.\end{aligned} \tag{39}$$

V. FREQUENCY EQUATION

A. Solution of the Linearized, Dimensionless Differential Equations of Motion

Upon examination, it becomes evident that Eqn 37 can be integrated directly to give:

$$\bar{v} = \bar{v}_i \, e^{-\frac{C_T \tau}{\bar{m}_p + \bar{m}_{a_x} + \bar{m}_\ell}} \tag{40}$$

where: $\bar{v}_i = \bar{v}$ at $\tau = 0$.

One observes, however, that the remaining two equations cannot be solved as easily as they are coupled together. In order to obtain solutions for α and β as a function of τ, one may assume solutions of the form:

$$\alpha = Ae^{\lambda\tau} \qquad \beta = Be^{\lambda\tau} \tag{41}$$

where A, B and λ are constants. Substitution of these relations into Eqns 38 and 39 yields:

$$a_{11} A + a_{12} B = 0 \tag{42}$$

$$a_{21} A + a_{22} B = 0 \tag{43}$$

where:

$$a_{11} = \bar{m}_{a_y} \bar{L}_2 \lambda^2 - \tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s \bar{L}_2 \lambda - \left[\tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s + \frac{C_T}{2}\right]$$

$$a_{12} = \bar{m}_{a_y} \bar{L}_2 \lambda^2 - \left[\tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s \bar{L}_2 + \bar{m}_p + \bar{m}_{a_y} + \bar{m}_\ell\right]\lambda - \frac{C_T}{2}$$

$$a_{21} = \left[\bar{I}_{cm} + \bar{I}_a\right]\lambda^2 - \tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s \bar{L}_1 \bar{L}_2 \lambda - \tfrac{1}{2} \bar{L}_1 \left(\frac{\partial C_N}{\partial \alpha}\right)_s$$

$$a_{22} = \left[\bar{I}_{cm} + \bar{I}_a\right]\lambda^2 - \tfrac{1}{2}\left(\frac{\partial C_N}{\partial \alpha}\right)_s \bar{L}_1 \bar{L}_2 \lambda \; .$$

Equations 42 and 43 are homogeneous with respect to the constants A and B. Therefore, the system will have a nontrivial solution only if the determinant of the coefficients is zero. That is, if:

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0. \qquad (44)$$

Solving for D and setting it equal to zero, the frequency equation of the system is obtained as:

$$\begin{aligned} &(\overline{I}_{cm} + \overline{I}_a)(\overline{m}_p + \overline{m}_{a_y} + \overline{m}_\ell)\ \lambda^3 \\ &- \tfrac{1}{2}\left[\overline{I}_{cm} + \overline{I}_a + (\overline{m}_p + \overline{m}_\ell)\ \overline{L}_1\ \overline{L}_2\right]\ \left(\frac{\partial C_N}{\partial \alpha}\right)_s \lambda^2 \\ &- \tfrac{1}{2}\ \overline{L}_1(\overline{m}_p + \overline{m}_{a_y} + \overline{m}_\ell)\left(\frac{\partial C_N}{\partial \alpha}\right)_s \lambda\ - \tfrac{1}{4}\ \overline{L}_1\ C_T\left(\frac{\partial C_N}{\partial \alpha}\right)_s = 0. \end{aligned} \qquad (45)$$

Equation 45 may be written symbolically as:

$$a\ \lambda^3 + b\ \lambda^2 + c\ \lambda\ + d\ =\ 0 \qquad (46)$$

where a, b, c and d are the coefficients of the frequency equation (45).

Before determining the dynamic stability characteristics of the system by means of its frequency equation, the approach to the complete solution shall be discussed. In any practical case, certain initial conditions will be known. For example, at $\tau = 0$, one has $\alpha = \alpha_o$, $\beta = \beta_o$, and $(\alpha' + \beta') = (\alpha' + \beta')_o$. In general, the cubic frequency equation has three roots, λ_1, λ_2 and λ_3, two of which will usually be complex numbers, i.e., $\lambda = n + im$.

With three values of λ, one would have a general solution given by:

$$\alpha = A_1 e^{\lambda_1 \tau} + A_2 e^{\lambda_2 \tau} + A_3 e^{\lambda_3 \tau} \qquad (47)$$

$$\beta = B_1 e^{\lambda_1 \tau} + B_2 e^{\lambda_2 \tau} + B_3 e^{\lambda_3 \tau} \qquad (48)$$

Applying the previous initial conditions, one obtains:

$$A_1 + A_2 + A_3 = \alpha_o \tag{49}$$

$$B_1 + B_2 + B_3 = \beta_o \tag{50}$$

$$(A_1 + B_1)\,\lambda_1 + (A_2 + B_2)\,\lambda_2 + (A_3 + B_3)\,\lambda_3 = (\,\alpha' + \beta')_o \tag{51}$$

So far, one has three equations and six unknowns, A_1, A_2, A_3, B_1, B_2 and B_3. One must therefore find three additional equations to completely determine the constants.

These relationships can be obtained by utilizing Eqn 43. Thus, one obtains three additional equations for the constants of the form:

$$\frac{A_i}{B_i} = -\left[\frac{(\bar{I}_{cm} + \bar{I}_a)\,\lambda_i^{\,2} - \frac{1}{2}\,\bar{L}_1\bar{L}_2\left(\frac{\partial C_N}{\partial \alpha}\right)_s \lambda_i}{(\bar{I}_{cm} + \bar{I}_a)\,\lambda_i^{\,2} - \frac{1}{2}\,\bar{L}_1\bar{L}_2\left(\frac{\partial C_N}{\partial \alpha}\right)_s \lambda_i - \frac{1}{2}\,\bar{L}_1\left(\frac{\partial C_N}{\partial \alpha}\right)_s}\right] \tag{52}$$

where i takes on the values of 1, 2, and 3.

One now has six equations for the six unknowns A_i and B_i. The system is therefore closed and solvable, an example of which is shown in Section VI.

B. Stability Criteria

Often, one is not confronted with the problem of solving the frequency equation but one only wishes to determine whether or not the system is dynamically stable, which can be accomplished by utilizing Routh's criteria (Refs 1 and 7). This criteria requires that for an oscillating system, whose oscillations should eventually approach zero, the coefficient of the frequency equation (45) or (46) must satisfy the following five conditions:

$$a > 0 \qquad b > 0 \qquad c > 0 \qquad d > 0 \qquad bc > d \tag{53}$$

Examination of the coefficient of λ^3 (Eqn 45) shows that, for all systems, "a" is a positive term. One next observes that if $\left(\frac{\partial C_N}{\partial \alpha}\right)_s$ is negative (i.e., statically stable parachute) the coefficient of λ^2 is positive. Similarly, the coefficient (c) of λ is positive if the parachute is statically stable and d > 0 if $\left(\frac{\partial C_N}{\partial \alpha}\right)_s < 0.$

It only remains to examine the term bc - d. By means of substitution, one may write:

$$bc-d = \tfrac{1}{4}\bar{L}_1\left(\frac{\partial C_N}{\partial \alpha}\right)_s \left\langle (\bar{m}_p+\bar{m}_{a_y}+\bar{m}_\ell)\left[\bar{I}_{cm}+\bar{I}_a+(\bar{m}_p+\bar{m}_\ell)\bar{L}_1\bar{L}_2\right]\left(\frac{\partial C_N}{\partial \alpha}\right)_s + C_T \right\rangle . \tag{54}$$

The stability derivative on the right side of this equation shows again that a statically stable parachute is necessary for a dynamically stable system. Furthermore, the composition of the bracketed term indicates that the mass and inertia terms, the length dimensions and the aerodynamic terms $\left(\frac{\partial C_N}{\partial \alpha}\right)_s$ and C_T must be properly balanced in order to satisfy this condition required for dynamic stability.

VI. NUMERICAL DETERMINATION OF THE AMPLITUDE-TIME RELATIONSHIP OF A PARACHUTE STABILIZED LOAD HAVING NEUTRAL AERODYNAMIC STABILITY

Objects which possess almost neutral aerodynamic stability are frequently decelerated and stabilized by means of an aerodynamically stable parachute. In such cases, one desires to know whether or not the system will behave with dynamic stability and how fast the initial oscillations decay. Questions of this nature can be answered through the solution of the frequency equation (46).

For the purposes of such a numerical solution, one may choose a ribless guide surface parachute because of its suitable aerodynamic stability. For the determination of the apparent mass (m_a) one must know the enclosed mass. Similarly, the canopy surface area is required to determine the parachute canopy mass m_p. The idealized canopy consists of a spherical cap and a truncated cone base. From Ref 8, the volume and surface area of the cap amount to:

$$V_{cap} = \frac{\pi}{3} h_1^2 (3R - h_1) \tag{55a}$$

$$S_{cap} = 2 \pi R h_1 \quad . \tag{55b}$$

Similarly, the volume and surface area of the truncated cone is:

$$V_{cone} = \frac{\pi}{3} h_2 (r^2 + r_1 r + r_1^2) \tag{56a}$$

$$S_{cone} = \pi (r + r_1) \left[h_2^2 + (r - r_1)^2 \right]^{\frac{1}{2}} \tag{56b}$$

From the geometry of Fig 4 follows:

$$h_1 = \frac{3 - \sqrt{5}}{2} r \qquad r_1 = 0.7 r$$

$$h_2 = 0.3 r \qquad R = \frac{3}{2} r$$

Utilizing these values one obtains:

$$V_{cap} = \frac{27 - 11\sqrt{5}}{12} \pi r^3 \qquad S_{cap} = \frac{3}{2} (3 - \sqrt{5}) \pi r^2$$

$$V_{cone} = 0.219 \pi r^3 \qquad S_{cone} = 0.51 \sqrt{2} \pi r^2 \tag{57}$$

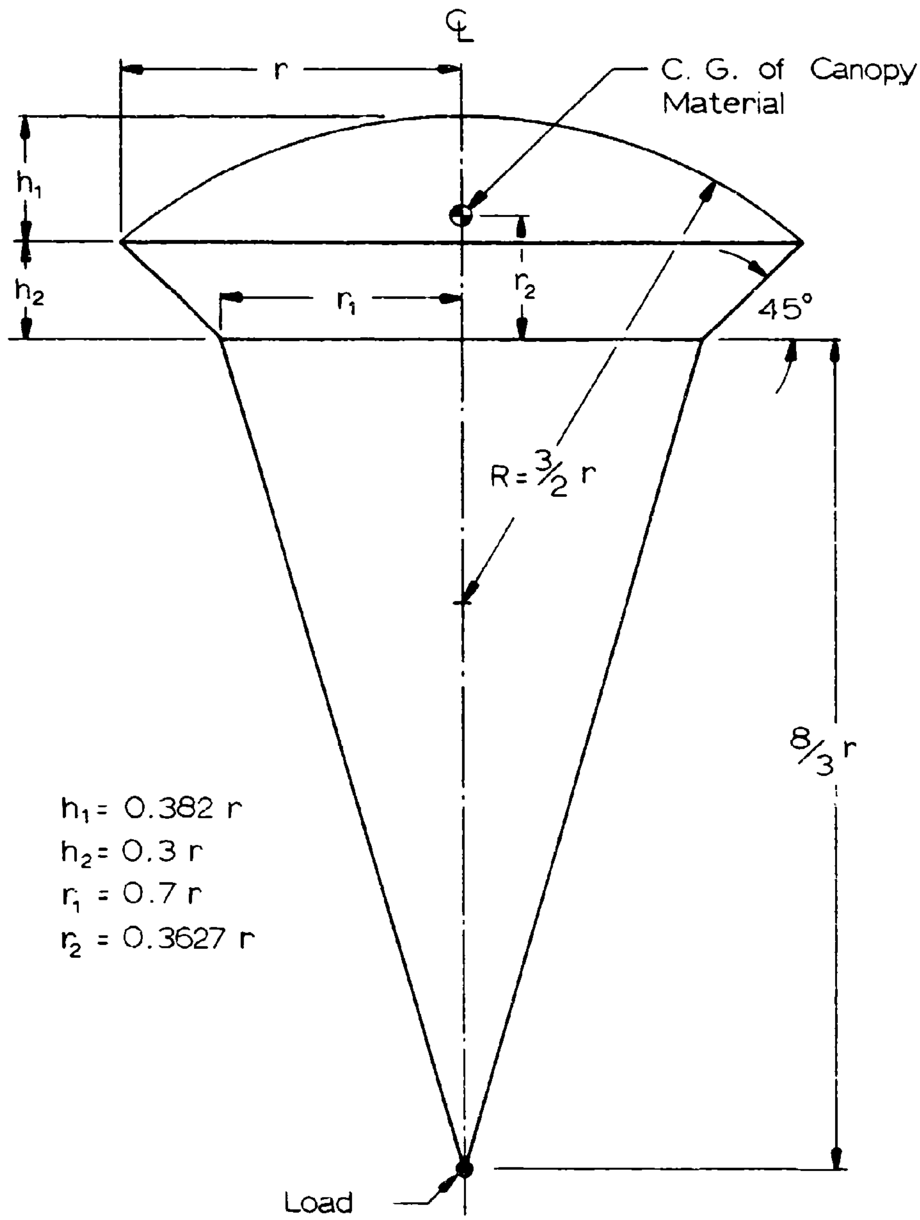


Fig 4. Geometry of the Ribless Guide Surface Parachute

The total volume and total surface area of the canopy amount to, respectively:

$$V = 0.419 \pi r^3$$

$$S = 1.867 \pi r^2$$

The enclosed mass (Ref 9) and parachute mass then become:

$$m_e = \rho V = 0.419 \rho \pi r^3$$

$$m_p = \sigma S = 1.867 \sigma \pi r^2$$

where: ρ = air density

σ = mass of cloth per square foot.

Choosing a nylon cloth with a weight of 7 oz/yd^2 for the canopy material, one obtains ($\sigma = 1.51107 \times 10^{-3}$ slugs/ft^2):

$$m_p = 2.821 \times 10^{-3} \pi r^2$$

From the definition of the dimensionless mass $\bar{m}$, one finally finds:

$$\bar{m}_p = \frac{2.821 \times 10^{-3}}{\rho r} \tag{58}$$

For $r = 2.5$ ft, $\rho = 2.378 \times 10^{-3}$ slugs/ft^3:

$$\bar{m}_p = 0.475 \quad . \tag{59}$$

Similarly, the dimensionless apparent mass $\bar{m}_a$ can be derived from:

$$\bar{m}_a = K\bar{m}_e \tag{60}$$

where K = 0.3 from Ref 9 for a nominal cloth porosity of 70 ft^3/ft^2-min., and

$$\bar{m}_e = \frac{m_e}{\pi \rho r^3} = 0.419 \tag{61}$$

The frequency equation (Eqn 46) requires the terms of the apparent mass in the x and y directions. As a first approximation, Ref 10 proposes to set:

$$m_{a_x} = m_{a_y}$$

which amounts to, in view of Eqns 60 and 61:

$$\bar{m}_{a_x} = \bar{m}_{a_y} = 0.1257$$

In the following, one must know the center of mass of the system, a part of which depends upon the center of mass of the canopy material. The calculation is cumbersome but straightforward and provides:

$$X_{m_p} = 0.9067 \text{ ft (for } r = 2.5 \text{ ft)}$$

Note: X_{m_p} is the location of the center of mass of the parachute material measured from the plane of the mouth of the canopy.

Choosing a 350 lb load, the center of mass of the system is located at a distance of 0.0385 ft above the center of mass of the suspended weight. The aerodynamic center of pressure of the parachute canopy can be determined from conventional three component measurements. However, in general and as a first approximation, one may assume that the center of pressure lies at the center of volume of the canopy. Therefore:

$$L_1 \cong L_2 = 7.34 \text{ ft}$$

or:

$$\bar{L}_1 = \bar{L}_2 = 2.94 .$$

With these dimensions and masses, the moment of inertia of the system is:

$$\bar{I}_{cm} = 4.36$$

The apparent moment of inertia follows from Ref 11, which gives I_a for various canopy shapes as determined by experiment. In this reference the apparent moment of inertia was measured about a point 2.66r upstream of the plane of the canopy inlet area. A dimensionless parameter A is defined as:

$$\frac{I_a}{I_R} = A$$

where: I_a = apparent moment of inertia about the reference point

I_R = moment of inertia about the same point of a point mass located at the center of volume of the canopy. The value of this mass is taken to be the mass of air enclosed in a sphere with the radius of the inflated parachute.

Thus, I_R can be expressed as:

$$I_R = \frac{4}{3} \pi r^3 \rho L^2$$

where: L = distance from reference point to center of volume of the canopy.

Therefore, the apparent moment of inertia is:

$$I_a = A \frac{4}{3} \pi r^3 \rho L^2 .$$

The distance L for the present problem is measured from the center of mass of the system as:

$$L = L_2 = 2.936r.$$

Thus:

$$I_a = A \frac{4}{3} \pi \rho (2.936)^2 r^5 .$$

From the definition of $\overline{I}_a$, one has:

$$\overline{I}_a = \frac{I_a}{\pi \rho r^5} = \frac{4}{3} (2.936)^2 A.$$

From Ref 11 the value of A for a ribless guide surface canopy is 0.187 and one finds:

$$\overline{I}_a = 2.13 .$$

Also, since W_ℓ = 350 lbs:

$$\overline{m}_\ell = 93.204$$

The aerodynamic coefficients C_T and $\left(\frac{\partial C_N}{\partial \alpha}\right)_s$ are given in Ref 6 as functions of effective porosity. For the aerodynamic coefficients chosen in this case, the effective porosity amounts to C = 0.025. Thus from Ref 6, one obtains:

$$C_T = 1.08$$

$$\left(\frac{\partial C_m}{\partial \alpha}\right)_s = 0.0144 \text{ per degree}$$

$$= 0.825 \text{ per radian.}$$

In this reference, C_m was defined as:

$$C_m = \frac{LN}{\frac{8}{3} qsr}$$

where L = distance from confluence point to the apex of the canopy

q = dynamic pressure.

With the definition $C_N = \frac{N}{qs}$ one finds:

$$C_m = \frac{3}{8} \frac{L}{r} C_N \quad .$$

From the geometry of Fig 4 follows:

$$C_N = 0.82\, C_m$$

or

$$\left(\frac{\partial C_N}{\partial \alpha}\right)_s = 0.82 \left(\frac{\partial C_m}{\partial \alpha}\right)_s$$

and finally:

$$\left(\frac{\partial C_N}{\partial \alpha}\right)_s = -0.676 \text{ per radian.}$$

The negative sign has been introduced because of the opposite sign convention utilized in Ref 12 and the present report.

Utilizing the previous value of C_T, one finds:

$$v_o = 117.8 \text{ ft/sec} \; .$$

Summarizing all of the results, one finds for a 5 ft diameter ribless guide surface parachute, constructed of 7 oz nylon material and having a porosity of 70 ft^3/ft^2-min., at sea level conditions with a 350 lb load:

$$\bar{I}_{cm} = 4.36$$

$$\bar{I}_a = 2.13 \qquad (62) \text{ (cont.)}$$

$$\bar{m}_p = 0.475$$

$$\bar{m}_e = 0.419$$

$$\bar{m}_{a_y} = \bar{m}_{a_x} = 0.1257$$

$$\bar{m}_\ell = 93.204 \qquad \text{(concl.)}$$

$$\bar{L}_1 = \bar{L}_2 = 2.94 \qquad (62)$$

$$\left(\frac{\partial c_N}{\partial \alpha}\right)_s = -0.676 \text{ per radian}$$

$$c_T = 1.08 .$$

Substitution of these values into the frequency equation (45) yields, after combining terms:

$$610 \lambda^3 + 276 \lambda^2 + 93.5 \lambda + 0.537 = 0 \qquad (63)$$

Certain terms are negligible in this particular case as can be seen by observing the contribution of each term. With this observation the original frequency equation can be simplified to the form:

$$\bar{m}_\ell (\bar{I}_{cm} + \bar{I}_a) \lambda^3 - \frac{\bar{m}_\ell \bar{L}_1 \bar{L}_2}{2} \left(\frac{\partial c_N}{\partial \alpha}\right)_s \lambda^2 - \frac{\bar{m}_\ell}{2} \bar{L}_1 \left(\frac{\partial c_N}{\partial \alpha}\right)_s \lambda - \tfrac{1}{4} \bar{L}_1 c_T \left(\frac{\partial c_N}{\partial \alpha}\right)_s = 0 \qquad (64)$$

which provides the following numerical result:

$$605 \lambda^3 + 272 \lambda^2 + 92.6 \lambda + 0.537 = 0 \qquad (65)$$

It is seen that the frequency equations (63) and (65) are almost identical. The simplification was justified for a small parachute with a relatively heavy load. Thus, if one were interested in solving such a problem, the simplified frequency equation (64) could be used with very good accuracy.

Returning now to the problem at hand, Eqn 63 becomes, on dividing by 610:

$$\lambda^3 + 0.4525 \lambda^2 + 0.1533 \lambda + 0.000881 = 0 . \qquad (63a)$$

One notices that all coefficients are positive and that bc - d = 0.0684 > 0 and thus, this is a dynamically stable system.

To completely specify the motion of the system, the frequency equation must be solved. Reference 8, page 295, gives a method of solving any cubic equation. Utilizing this method, one finds:

$$\begin{aligned} \lambda_1 &= -0.00553 \\ \lambda_2 &= -0.22348 + 0.31741\, i \\ \lambda_3 &= -0.22348 - 0.31741\, i \end{aligned} \tag{66}$$

Thus, from Eqns 47 and 48, the general solutions for α and β are:

$$\begin{aligned} \alpha = A_1 e^{-0.00553\tau} &+ A_2 e^{(-0.2235 + 0.3174\, i)\tau} \\ &+ A_3 e^{(-0.2235 - 0.3174\, i)\tau} \end{aligned} \tag{67}$$

$$\begin{aligned} \beta = B_1 e^{-0.00553\tau} &+ B_2 e^{(-0.2235 + 0.3174\, i)\tau} \\ &+ B_3 e^{(-0.2235 - 0.3174\, i)\tau} \end{aligned} \tag{68}$$

From Eqn 52 one obtains:

$$\frac{A_i}{B_i} = - \frac{6.49\, \lambda_i^2 + 2.92\, \lambda_i}{6.49\, \lambda_i^2 + 2.92\, \lambda_i + 0.994}$$

Using the value of λ_i from Eqn 66 yields:

$$\begin{aligned} \frac{A_1}{B_1} &= 0.016307 \\ \frac{A_2}{B_2} &= 65.7543 - 34.7973\, i \\ \frac{A_3}{B_3} &= 65.7543 + 34.7973\, i \end{aligned} \tag{69}$$

Using these relations in Eqn 67 one finds:

$$\alpha = 0.016307B_1e^{\lambda_1\tau} + (65.7543 - 34.7973i)B_2e^{\lambda_2\tau} + (65.7543 + 34.7973i)B_3e^{\lambda_3\tau} \qquad (67a)$$

As an initial condition, outlined in Eqns 49 through 51, one may choose:

$$\text{at } \tau = 0, \quad \alpha = 10° = 0.1745 \text{ radians}$$

$$\beta = 0$$

$$\alpha' + \beta' = 0$$

Thus, one obtains from Eqns 67a and 68:

$$0.016307B_1 + (65.7543 - 34.7973i)B_2 + (65.7543 + 34.7973i)B_3 = 0.1745$$

$$B_1 + B_2 + B_3 = 0$$

and

$$0.00562B_1 + (3.8749 - 28.9648i)B_2 + (3.8749 + 28.9648i)B_3 = 0.$$

Solving these simultaneous equations for B_1, B_2, and B_3 gives:

$$B_1 = -0.002856$$

$$B_2 = 0.001428 - 0.000191\,i$$

$$B_3 = 0.001428 + 0.000191\,i$$

and, from relations 69:

$$A_1 = -0.0000466$$

$$A_2 = 0.087251 - 0.062250\,i$$

$$A_3 = 0.087251 + 0.062250\,i$$

Using these relations, one obtains after a cumbersome calculation:

$$\alpha = -0.0000466e^{-0.00553\tau} + 0.21434e^{-0.22348\tau}\cos(0.31741\tau - 0.62)$$
$$\beta = -0.002856e^{-0.00553\tau} + 0.002882e^{-0.22348\tau}\cos(0.31741\tau - 0.133). \qquad (70)$$

The values of $\alpha + \beta$ are presented in Table 1. The amplitude-time relationship is shown in Fig 5.

Table 1. $\alpha + \beta$ as a Function of τ and t for the Ribless Guide Surface Parachute

τ	t (sec.)	$\alpha + \beta$ (deg.)
0	0	10.000
2	0.042	7.780
4	0.085	2.914
6	0.127	0.736
8	0.170	-0.884
10	0.212	-1.384
12	0.254	-1.002
14	0.297	-0.569
16	0.339	-0.237
18	0.382	-0.066
20	0.424	-0.036
22	0.466	-0.056
24	0.509	-0.102
26	0.551	-0.136
28	0.594	-0.152
30	0.636	-0.148

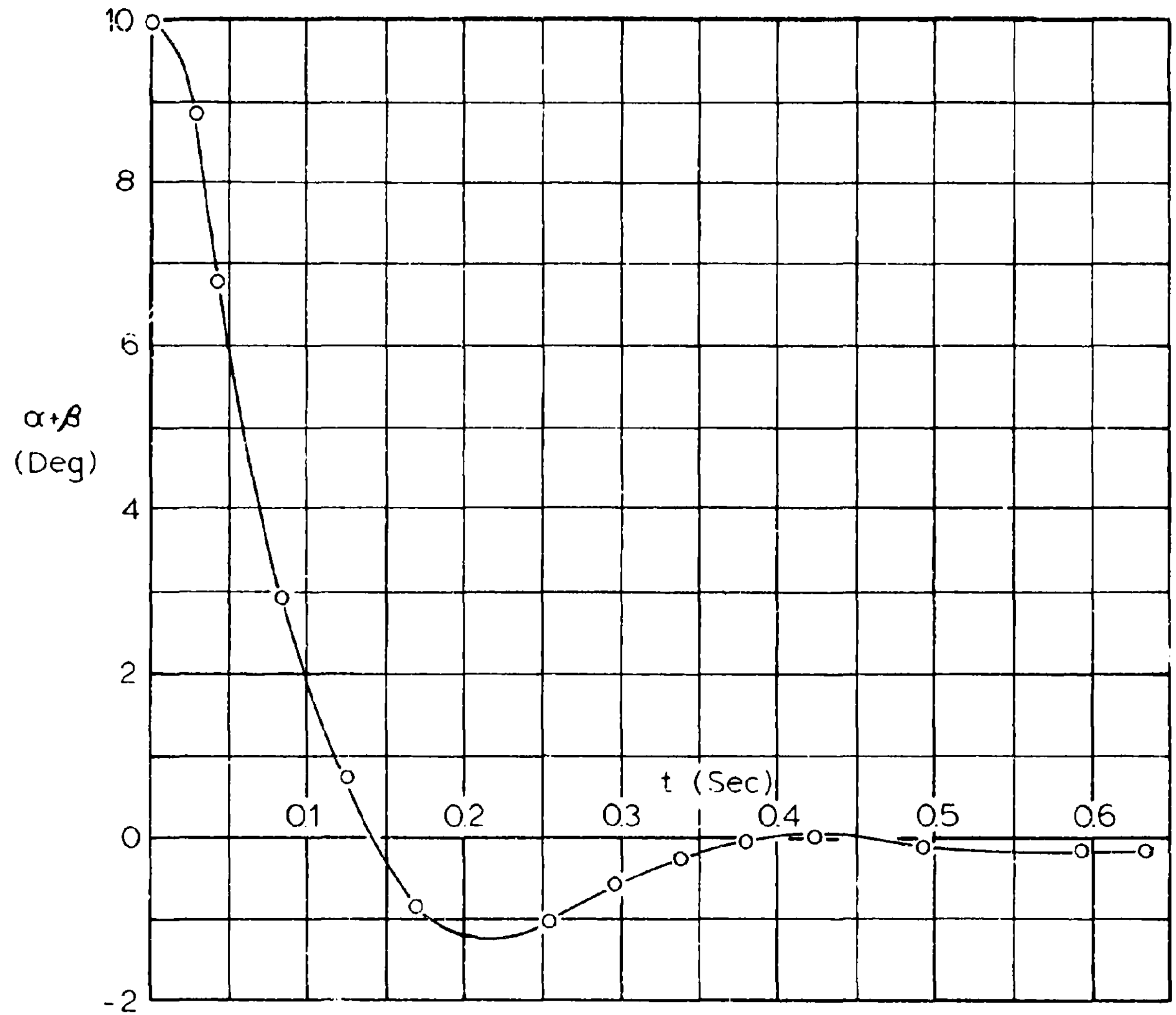


Fig 5. $\alpha + \beta$ as a Function of Time for the Ribless Guide Surface Parachute

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13. ABSTRACT

The dynamic stability of a parachute load system has been analytically investigated for a point-mass load and a statically stable parachute. A typical system consisting of a relatively large suspended load mass and small ribless guide surface parachute has been numerically calculated. Utilizing the apparent mass and apparent moment of inertia, as well as the aerodynamic coefficients of the parachute canopy, the equations of motion for the system have been solved. The influence of several design parameters upon the dynamic stability characteristics of the system has been discussed.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Dynamic stability Parachute-load system Point mass load Equations of motion						

INSTRUCTIONS

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