

DYNAMIC THINNING OF ANTENNA ARRAY USING GENETIC ALGORITHM

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Abstract—Thinning is a technique by which the total number of active elements in an antenna array is reduced without causing major degradation in system performance. Dynamic thinning is the process of achieving this under real time conditions. Stochastic techniques have been useful in the design of thinned arrays. However while applying the technique to large 2-D arrays, under changing conditions problems arise due to the very large and rugged solution space. Also, evaluation of the objective function in such cases requires large computational resources, thus reducing the rate of convergence. This paper suggests a technique using Genetic Algorithm which is useful for overcoming these problems. After discussing the basic concept involving dynamic thinning and application methodology, simulation results of applying the technique to linear and planar arrays are presented.

1. INTRODUCTION

Thinning refers to strategic elimination of a subset of active elements within an antenna array, keeping the deviation of the resulting radiation pattern within limits. Dynamic thinning refers to achieving this purpose under changing conditions. This paper discusses certain issues related to dynamic thinning of large antenna arrays using Genetic Algorithm.

Major focus of earlier research on thinned arrays has been to consider the array as non-uniform/aperiodic and find antenna element positions/inter element spacings using various techniques [1–4]. Most of these methods used analytical techniques to design. But analytical

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design of thinned antenna arrays with thousands of radiating elements capable of scanning over large angular region is a difficult task. It involves solving of complex, non-linear, and non-differentiable functions demanding efficient numerical techniques for their solution. Some of the numerically intensive methods are based on perturbation technique, linear programming, dynamic programming and the Mini-Max approach [5–9].

In recent years, stochastic techniques such as Genetic Algorithms [10–12], Simulating Annealing [13–15], Particle swarm Optimization [16] and Ant colony technique [17–20] have been used for design of thinned arrays.

Above approaches have been useful in designing thinned linear and planar arrays [21–24] for keeping side lobe level below a desired value or to achieve a shaped beam pattern or to obtain a deep notch within a radiation pattern.

However, when the working condition changes, it has been observed that the thinned design does not yield the desired results and there is a need for *ab-initio* design for every changed condition. A simple example is the electronic scanning array where the thinning needs to be re-worked for different scan angles.

This paper investigates adapting the design based on Simple Genetic Algorithm (SGA) for changing or dynamic conditions, calling it ‘dynamic thinning’. The approach followed is based on,

- (a) reducing the design time, so as to facilitate real-time computation
- (b) system integration involving thinning

The paper is organized in six sections. Section 2 provides basic aspects related to requirements of array antenna thinning, which forms the basis for the following sections. Results of applying Simple Genetic Algorithm to thinning of a 100 element linear array are also presented here. Section 3 explains some prominent factors affecting Dynamic Thinning, which need special attention. Computational and system integration approaches for resolving the issues related to dynamic thinning are addressed in Section 4. Section 5 presents some simulation results on the computational aspects related to dynamic thinning as discussed in the previous section. The paper is summarized in Section 6 by drawing some conclusions.

All results presented in this paper are based on study of Array Factor of a set of isotropic radiators placed at a uniform spacing of half wavelength. No mutual coupling effects are considered. The work reported here is based on using Simple GA (SGA) for thinning, but can be adapted for any other design approach also.

2. REQUIREMENTS AND APPROACH FOR ARRAY THINNING

2.1. Least Square (LS) Requirement

The problem in a thinned array design is how to reduce the total number of elements optimally with respect to a desired objective. This can be described as follows. Let a linear array be composed of M identical antenna elements. The array factor is given

$$F_M(\theta) = \sum_{i=1}^M R_i e^{jk d_i \cos(\theta)} \quad (1)$$

where R_i is the complex excitation coefficient of the i th element located at $x = d_i$ along the linear array direction x , and k ($= 2\pi/\lambda$) is the spatial wave number. The objective is to synthesize a new linear antenna array that has the minimum number of elements, producing an array factor which differs from the array factor $F_M(\theta)$ by less than a prescribed tolerance ε , with all other conditions remaining the same. That is, we intend to find a solution to,

$$\left[\begin{array}{l} \min \{Q\} \\ Const. \end{array} \left\{ \min_{\{R_i, d_i\}_{i=1, \dots, Q}} \left\| F_M(\theta) - \sum_{i=1}^Q R_i e^{jk d_i \cos(\theta)} \right\| \right\} \leq \varepsilon \quad (2)$$

where R_i and d_i ($i = 1, \dots, Q \leq M$) are the complex excitations and locations for Q antenna elements respectively, for all angles of θ of interest; $L = 2$ if the least square error (LSE) is used.

2.2. Thinning Factor (TF) Requirement

By thinning, lesser number of antenna elements participate in the formation of the radiating beam in comparison to an un-thinned array. Thinning factor (TF) is defined as

$$TF = (N_{total} - N_{active})/N_{total} \quad (3)$$

$$N_{inactive} = N_{total} - N_{active} \quad (4)$$

Here N_{total} , N_{active} and $N_{inactive}$ are total number of elements in the array and total number of active and inactive elements in the thinned array respectively.

2.3. Objective Function Requirement

Objective Function (OF) requirement for thinning the antenna array can be of two types

- (a) It can be a cost function based on the variation between the envelopes of the desired radiation pattern and actual radiation pattern of the thinned array. In such a case, the purpose of design shall be to bring the variation below the set limit of ε .

$$\varepsilon(\theta) = \sum_1^M [\max\{(|Fm(\theta)| - |F \max d(\theta)|), 0\} + \max\{(|F \min d(\theta)| - |Fm(\theta)|), 0\}] \quad (5)$$

where, $F \min d(\theta)$ and $F \max d(\theta)$ are the desired minimum and maximum radiation respectively and $Fm(\theta)$ is the actual radiation in the direction of angle θ .

- (b) It can be any one of the following factors or a combination of them. It can also be a variation based on these factors.

$$\text{Gain Reduction Factor, GRF} = 20 \log_{10} \left(\frac{N \text{ total}}{N \text{ active}} \right) \quad (6)$$

$$\text{Side Lobe Variation Factor, SLVF} = |(SL)_M - (SL)_N| \quad (7)$$

$$\text{Beamwidth Variation Factor, BWVF} = |(BW)_M - (BW)_N| \quad (8)$$

Here $(SL)_M$, $(SL)_N$ refer to the side lobe levels (in dB) and $(BM)_M$, $(BM)_N$ refer to the beam widths of the full and thinned arrays respectively. Depending on the type of OF, the procedure for thinning would aim to either maximize or minimize it.

2.4. Approach Using Simple Genetic Algorithm

Simple Genetic Algorithm can be successful in synthesizing a thinned array by considering it as a combinatorial optimization problem. The goal is to strategically remove a subset of active elements in the array to achieve the OF. This problem is binary in nature when active elements are represented by a value of '1' and inactive elements are represented by a value of '0'. The binary string [111...1] represents an array without thinning; and the binary string [1000...1] an array with maximum thinning. Here the end elements of an array are presumed to remain always active. The problem then becomes how to choose a specific combination of 1's and 0's in order to satisfy the constraints.

For applying SGA, a set of random binary strings is taken as initial population, where a '1' represents an active element and a '0' an inactive element in each string. By following the standard SGA procedure involving objective function evaluation, sorting, natural selection and reproduction using crossover and mutation, the population can be refined iteratively till it meets the required objective.

at obtaining ‘an acceptable solution’ rather than finding ‘the optimum solution’

3.1. Computational Complexity

The size of the array affects computational effort involved in thinned array design in two ways,

- a) OF evaluation: For large arrays, requirements of computer resources for OF evaluation far exceed the functional requirements for implementation of SGA. In most cases, the evaluation of OF is computationally intensive involving lengthy procedures. All efforts to reduce the intensity of computation would help reduce overall time, helping in real-time implementation.
- b) Exploring solution space: Solution space is the total number of solutions, out of which one which meets the requirement is finally chosen as the solution. Table 1 depicts the relationship between solution space and N_{total} for symmetrical linear antenna arrays. For $TF = 0.25$, the solution space which is about 1.8×10^3 for an array of 32 elements increases to 10.51×10^6 when the total number of elements in the array is doubled. A practical phased array antenna may consist of hundreds or thousands of elements. In such cases, the solution space would be very large and also rugged. Exploring such a large solution space using SGA would not only require time but also may result in getting trapped in a local minimum leading to premature convergence. As the solution space increases, speed of convergence of SGA procedure gets affected drastically, which affects usage of SGA for dynamic conditions.

Table 1. Solution space for different N_{Total} and TF values.

N_{Total}	TF	$N_{Inactive}$	Solution Space
32	0.25	8	1820
64	0.25	16	10.51×10^6
256	0.125	32	9.33×10^{19}
256	0.25	64	1.47×10^{30}
1024	0.125	128	Too Large
1024	0.25	256	Too Large
4096	0.125	256	Too Large
4096	0.25	1024	Too Large

3.2. Concept of Acceptable Solution

Generally an optimization problem would strive at getting at the globally optimized solution within the set constraints. Thinning problem is an optimization problem, where we would like to minimize the number of active elements, N_{active} out of a total available N_{total} elements so as to meet the OF.

In a real-time scenario as in the case of dynamic thinning where time is at premium, a solution which gives a near-optimum value would be a more useful input than the absolute minimum value of N_{active} , as far as the OF is satisfied. The penalty paid in terms of extra time spent for obtaining the exact minimum solution may defeat the very purpose of the mission in such cases. Also, for practically applying the result, knowing all the elements of the set $\{N_{active}\}$ is more important than merely knowing the minimum value of N_{active} .

Thus, dynamic thinning should look for acceptable solution rather than the absolute minimum value of N_{active} . However, what is an acceptable solution would depend on the operational scenario. For example, obtaining a thinning solution which can meet the objectives fully with $N_{active} = 3000$, within a fraction of time would be much more valuable than getting a similar solution with $N_{active} = 2998$ (where $N_{Total} = 4096$). Similarly, obtaining a solution which can offer a side lobe value of say -39.8 dB within a fraction of time would be much more valuable than getting a similar solution which provides a side lobe value of -40 dB after a long time. Dynamic thinning can benefit by recognizing such acceptable solutions relevant to the operational scenario.

4. RESOLVING THE ISSUES OF DYNAMIC THINNING

Approaches suggested below are aimed at resolving above issues so that SGA can be used for dynamic thinning of an antenna system based on real time operational scenario.

- a) Bulk Array Computation
- b) Zoning techniques
- c) Dynamic Thinning Programmer

4.1. Bulk Array Computation

Implementation of GA requires evaluation of OF for every member of the population in each iteration. Each of the evaluation is based on the array factor calculation $F_M(\theta)$, using Equation (1), which is highly nonlinear and involves lengthy procedures. Thus, for large arrays,

requirements of computer resources for objective function evaluation would far exceed the functional requirements for SGA.

For dynamic thinning, special attention needs to be paid for the OF evaluation. The suggested method here is called as ‘Bulk Array computation’ (BAC) and is based on creating a table to store the data of the radiated fields of all elements in all directions as is done by Brill [25]. But, instead of adding contribution of radiations from the ‘active’ elements, $F_M(\theta)$ is calculated by subtracting the contributions from the ‘inactive’ elements.

Major steps involved in the computation are:

- a) Generate data for creating ‘Element Table’ which has all details about element location and its complex excitation coefficient.
- b) Generate data for creating ‘Angle Table’ which contains details of each angular direction in (θ, φ) coordinate.
- c) Compute and store the radiated field due to each element of the array in each direction of interest.
- d) Initial population of ‘inactive elements’ is generated.
- e) Effect of ‘inactive elements’ is then subtracted from the stored data of the radiated field of the array; radiation pattern of the thinned array corresponding to each member of the population over the required angular sector is then computed.
- f) Feedback parameter is extracted from the set of radiated patterns of the thinned arrays and is used in iterative manner to generate successive populations, using GA procedures.
- g) This is continued iteratively till the terminating criterion is obtained.

4.2. Zoning Technique

Zoning refers to partitioning the antenna array into convenient zones, so that the solution space can be usefully explored. Though there is no restriction in the total number of zones N_Z , each zone is expected to consist of at least 2 elements. $N_Z = 1$ refers to no partitioning of the array. Figure 2 shows typical zoning of linear and planar arrays, where $N_Z = 3$.

Generally, it is expected that number of inactive elements in the central portion (shown as zone 1 in Figure 2) of a thinned array would be much less than in other zones. Such a-prior knowledge can help in partitioning of the array into zones and better exploitation of the solution space. In general, zoning is based on dividend or return likely to yield while exploiting the zone [26]. Though it is possible

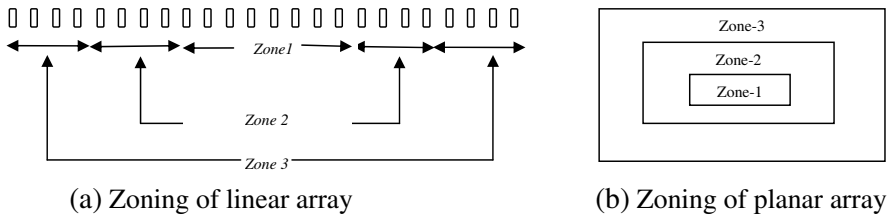


Figure 2. Zoning of antenna array.

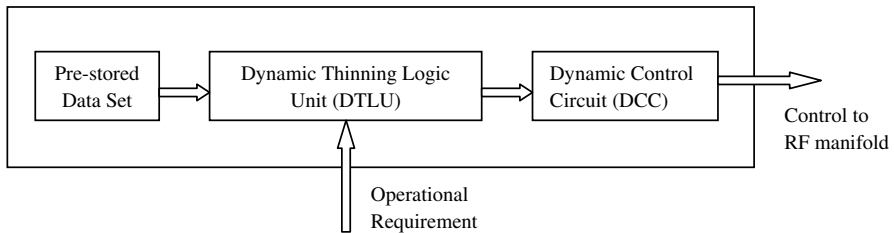


Figure 3. Dynamic thinning programmer.

to partition into any number of zones, it may not be advisable to consider more than 2 or 3 zones, as shown later.

Zoning can help in enhancing convergence rate, since the proportion of the exploring space to total solution space reduces drastically in case of large arrays.

4.3. Dynamic Thinning Programmer

Though BAC and Zoning technique described above can reduce the overall computation time and help in achieving fast convergence the time response may not be adequate in case of very large arrays having thousands of elements, due to operational constraints. In such cases, it is proposed that a Dynamic Thinning Programmer (DTP) as shown in Figure 3 be used for system integration. DTP consists of a Pre-stored Data Set, a Dynamic Thinning Logic Unit (DTLU), and a Dynamic Control Circuit (DCC).

The Pre-stored Data Set contains information about the elements of the set $\{N_{active}\}$ relevant for various conditions in the form of look-up tables. Based on operational requirements, appropriate trigger signals would be sent to the DTLU, which would retrieve information about the relevant $\{N_{active}\}$ set. For instance, in case of a scanning phased array the trigger signal would be the scan direction information, which will enable DTLU to retrieve information about

the on/off requirements of the array elements relevant for the required scanning conditions. In case of multi-mode arrays integrated with weapon systems, the trigger requirements would be derived from the operational requirements of the weapon system. These requirements would then be translated to appropriate control signals by the DCC and sent to the RF manifold for optimum thinning. By this process dynamic thinning can be achieved based on pre-stored data.

5. SIMULATION RESULTS AND DISCUSSIONS

This section deals with some of the simulation results obtained using the methods suggested for dynamic thinning. In all cases, symmetrical arrays with isotropic radiators placed at a uniform spacing of half wavelength are considered. No mutual coupling effects are considered.

5.1. Simulation Results on Zoning

Three studies were carried out

- a) To study effect of number of zones on speed of convergence
- b) To study effect of zoning for different types of objective functions
- c) To study different types of zoning

5.1.1. Study on Effect of Number of Zones

Though a number of different cases were studied, only typical results on a 200 element linear array are presented here. Zoning details for $N_Z = 1, 2, 3, 4$ are shown in Row 2 of Table 2. $TF = 0.24$ was considered, so that $N_{inactive} = 48$. This is distributed in increasing order from Zone 1 to Zone N , so that Zone 1 has least number of $N_{inactive}$ elements. It is ensured that symmetry is maintained in the thinned array.

Objective Function, OF was chosen as achieving -22 dB sidelobe level outside the sector of ± 0.01 radians. Maximum number of iterations $N_{it} = 500$. Terminating condition was to achieve OF or number of iterations = N_{it} , whichever was earlier. In order to remove any possible biases due to stochastic nature of the algorithms and obtain conclusive results, 100 runs were made in each case. Convergence behavior was studied by monitoring.

- a) Number of runs in which OF was achieved within N_{it}
- b) Number of iterations to achieve OF averaged over the 100 runs
- c) Sidelobe (dB) obtained while termination, averaged over the 100 runs

These are shown in Table 2.

Figure 4 shows the convergence trend of normalized sidelobe level for the four approaches used. For each curve at every iteration, an average value of normalized sidelobe level over 100 runs is calculated

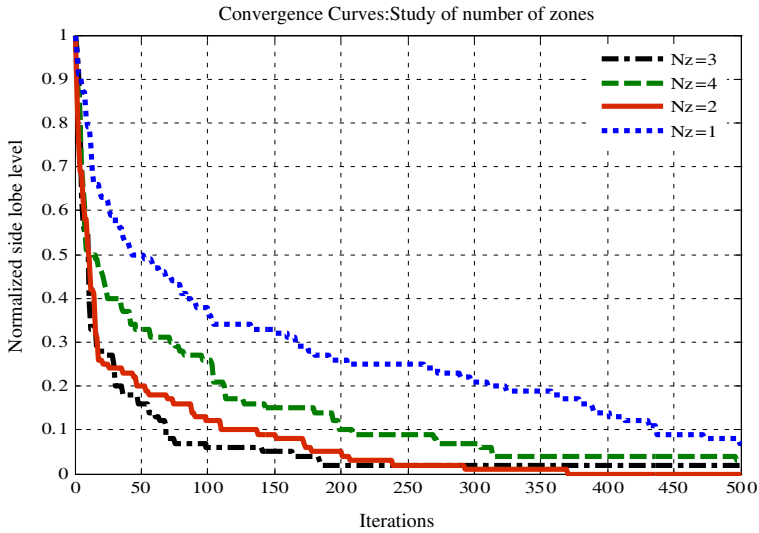


Figure 4. Convergence curves of study of number of zones.

Table 2. Study on number of zones.

	$N_Z = 1$	$N_Z = 2$	$N_Z = 3$	$N_Z = 4$
Zoning Details (Number of elements in half array)	Zone 1 : 100	Zone 1 : 40 Zone 2 : 60	Zone 1 : 40 Zone 2 : 30 Zone 3 : 30	Zone 1 : 40 Zone 2 : 20 Zone 3 : 20 Zone 4 : 20
Number of runs in which OF was achieved within N_{it}	1	90	40	3
Number of iterations to achieve OF averaged over the 100 runs	496	337	382	448
Sidelobe (dB) obtained while termination, averaged over the 100 runs	-20.35	-22.13	-22.03	-21.84

and is plotted.

Results indicate conclusively that zoning helps in convergence. For this particular case, $N_Z = 2$ is optimum.

5.1.2. Study on Effect of Zoning for Redefined Objective Function

The OF was redefined to achieve radiation pattern envelope with a main beam sector and a notch sector. The results discussed are for linear arrays with $N_{total} = 100$ and 200 . Main beam sector was taken as $\pm 0.02/\pm 0.01$ radians for the two arrays. The notch sector was from ± 0.48 to ± 0.52 radians, with notch depth of -36 dB/ -40 dB for the two arrays. Symmetrical notch is due to array symmetry.

Total number of iterations (N_{it}) and total number of runs (N_{run}) were taken as 500 and 100, as before. Only $N_Z = 1$ and $N_Z = 2$ were considered. Cost function (CF) was defined based on the difference between the desired radiation envelope and actual radiation pattern as per Equation (5).

Figure 5 shows a typical radiation pattern of the thinned array for $N_{total} = 200$ when the cost function reaches zero value, along with the desired radiation pattern envelope.

Convergence behavior was studied by monitoring

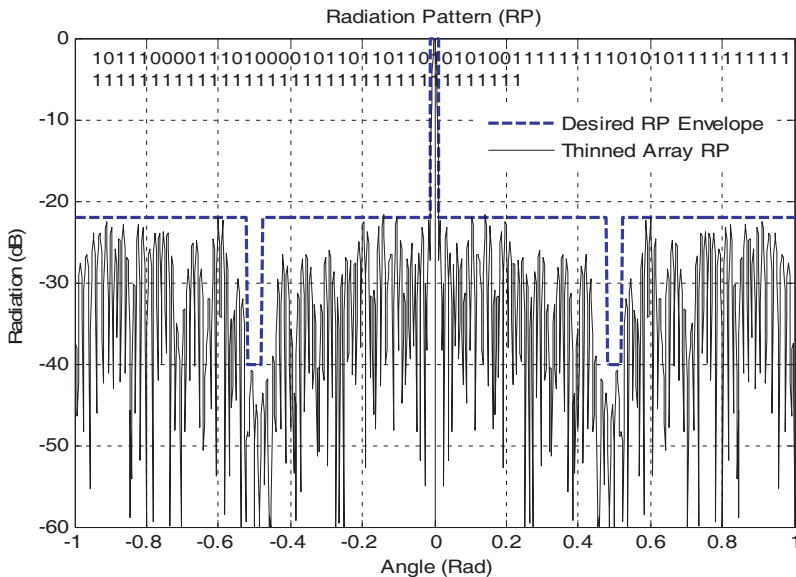


Figure 5. Typical radiation pattern of the thinned array for $N_{total} = 200$ (Inset shows the thinned binary string of Half Array).

Table 3. Simulation results for redefined OF.

	$N_{total} = 100$	$N_{total} = 100$	$N_{total} = 200$	$N_{total} = 200$
Zoning Details (Number of elements in half array)	$N_Z = 1$	$N_Z = 2$ Zone 1 : 20 Zone 2 : 30	$N_Z = 1$	$N_Z = 2$ Zone 1 : 40 Zone 2 : 60
Cost function at the end of N_{it} averaged over N_{run}	20	5.5	60	5.74
Number of runs when OF could be achieved	0	0	0	10

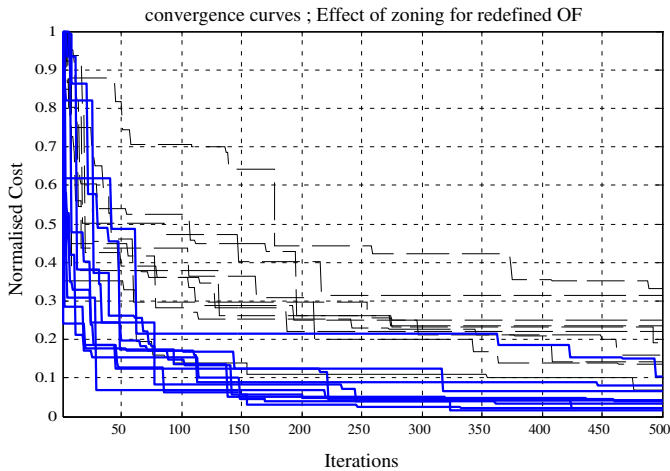


Figure 6. Convergence curves for 10 simulation runs ($N_{total} = 200$)
 - - - Black: $N_Z = 1$; — Blue: $N_Z = 2$.

- a) Cost function at the end of N_{it} , averaged over N_{run}
- b) Number of runs when OF could be achieved

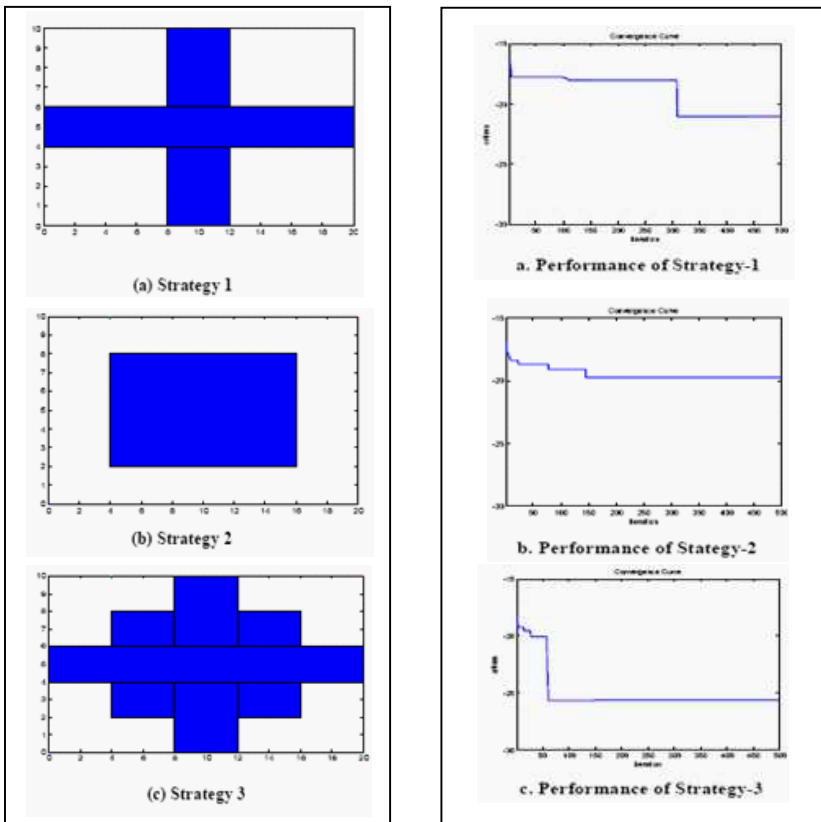
Results are given in Table 3.

Average cost as given in Row 3 of the Table 3 shows that the cost reduces by a factor of 3.6 (5.5 from 20) for $N_{total} = 100$ and by a factor of 10.4 (5.74 from 60) for $N_{total} = 200$ due to zoning. This indicates that zoning helps in faster convergence. Better convergence occurs when array size is larger.

It is interesting to note that in spite of the good cost reduction factor due to zoning, the ultimate success rates due to zoning are low for both $N_{total} = 100$ and 200 (0 and 10% respectively). This is shown in Row 4 of the same table. This indicates that zoning can help when we search for an ‘acceptable solution’ rather than when we seek an absolute reduction of cost function.

Corresponding convergence curves for 10 typical simulation runs for $N_{total} = 200$ are shown in Figure 6, which further confirms better convergence rate with zoning.

Although the discussion has been limited to an objective function with a main beam and a notch sector requirement, the approach can



(a) Zoning - Blue: Zone 1; White: Zone 2

(b) Convergence curves

Figure 7. Zoning for a planar array and corresponding convergence curves.

be extended to other similar types of OF.

5.1.3. Study on Different Types of Zoning

For this study, a planar array with 20×10 elements with $N_z = 2$ was considered. Three types of zoning were considered as shown in Figure 7(a). The objective was to obtain best sidelobe levels (better than -20 dB) in both the planes. Convergence graphs for the three zoning strategies are given in Figure 7(b). For this particular case, it was observed that strategy 3 provides better convergence.

Convergence occurs, in all the three cases, thus showing that different types of zoning can be employed depending on the need. As discussed earlier, a-priori knowledge can help in choosing the type of zoning most suitable based on the need.

Table 4. Reduction in computational operations.

Type of array	Total no. of operations by conventional method	Total no. of operations by suggested method	Reduction in no. of operations
<u>Symmetric Linear Array</u> $N_{total} = 200$ $TF = 0.22$ Ang. directions = 1000 Population = 100 No. of iterations = 50	3.9×10^8	1.1×10^8	2.8×10^8
<u>Symmetric Planar Array</u> $N_{total} = 50 \times 50$ $TF = 0.383$ Ang. directions = 150 Population = 50 No. of iterations = 300	12.9×10^{10}	8.07×10^{10}	4.83×10^{10}
<u>Symmetric Planar Array</u> $N_{total} = 64 \times 64$ $TF = 0.32$ Ang. directions = 1025×1025 Population = 20 No. of iterations = 300	4.38×10^{12}	2.06×10^{12}	2.32×10^{12}

5.2. Simulation Results on BAC Technique

As discussed earlier, Bulk Array computation' (BAC) is based on creating a table to store the data of the radiated fields of all elements in all directions and then subtracting the contributions from the 'inactive' elements to obtain $F_M(\theta)$. Table 4 compares the number of computational operations of this approach to that of conventional method of adding contributions of 'active' elements. There is an advantage by a factor of $(1-TF)/TF$ for each calculation of the array factor by following the suggested method.

A computational time comparative study conducted in this regard has shown that the conventional method of evaluating an objective function for a single member of a population took 14.6 seconds of CPU time whereas BAC took only 2.7 seconds. This simulation was carried out using MATLAB 7 on a hp-Compaq machine with Intel®Pentium®nx6120 1.73 GHz as the CPU and 512 MB of RAM. Thus there is a saving of 11.9 seconds per member of population per iteration.

5.3. Simulation Results on Scanning Array

Techniques discussed above were used to simulate thinning of a 64×64 element planar array, scanned to different angles in the two planes. Typical Radiation patterns for two different scan conditions are given in Figure 8.

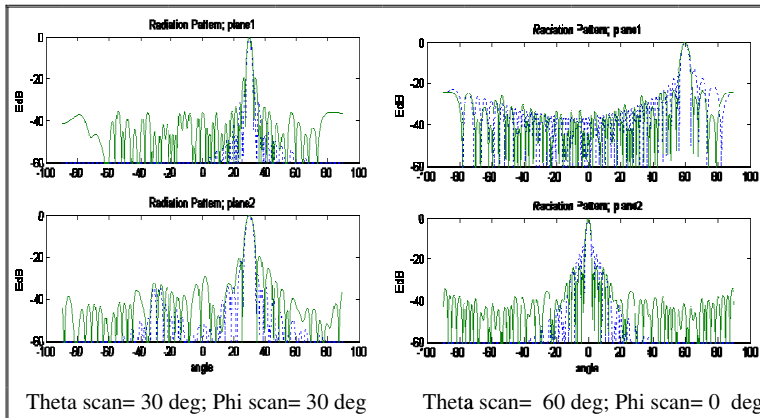


Figure 8. Radiation pattern for different scan conditions of thinned 4096 element planar array (Blue - - - Full Array; — Green: Thinned Array).

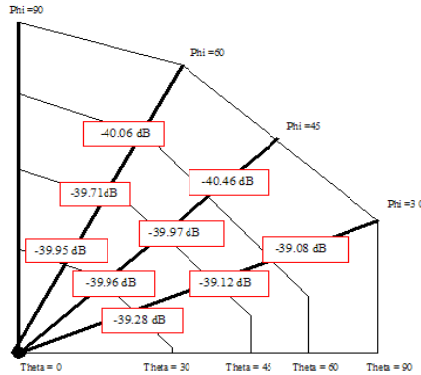


Figure 9. Typical scan diagram for a 4096 element array.

Further studies were taken up for obtaining an ‘acceptable solution’ for a thinned 64×64 array with -40 dB side lobe level for all scan conditions. Figure 9 shows typical scan diagram for one of such results. The diagram gives the sidelobe level for nine different scan conditions. In all these cases, only 2784 elements out of $N_{total} = 4096$ elements were active. It can be observed that the results are acceptable solutions. Thus zoning and BAC when used together has helped in achieving an acceptable solution even when the array size is as large as $N_{total} = 4096$ elements.

The details for Figure 8:

Total Number of Elements in the Planar Array = $N_{total} = 64 \times 64 = 4096$;

No. of elements in one quadrant of the array: = $N_{total}/4 = 1024$;

No. of elements switched off in one quadrant: 328.

6. SUMMARY AND CONCLUSIONS

The concept of dynamic array thinning is useful for real time thinning of antenna arrays in varying conditions. The problems in using Genetic Algorithm for dynamic thinning in the context of computational complexity, system integration and implementation were analyzed. Four different ideas were thrown up to resolve the issues connected with dynamic thinning — Bulk array Computation, Zoning, concept of acceptable solution and Dynamic thinning Programmer. Bulk array computation and zoning help in reducing computational intensity essential for achieving the required convergence rate. Simulation studies on combining these ideas with the concept of an acceptable

solution, give very encouraging results. In case of larger arrays, the time response can be further reduced by using a Dynamic Thinning Programmer for system integration.

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