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Dynamic Tire Force Control by Semi-active Suspensions†

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Abstract

This paper presents a semi-active suspension control algorithm to reduce dynamic tire forces including the development and application of observers for bilinear systems with unknown disturbances. The peak dynamic tire forces, which are greatly in excess of static tire forces, are highly dependent on the dynamic characteristics of vehicle suspensions. One way to reduce dynamic tire forces is to use advanced suspension systems such as semi-active suspensions.

Semi-active control laws to reduce dynamic tire forces are investigated and a bilinear observer structure for bilinear systems with unknown disturbances is formulated such that the estimation error is independent of the unknown external disturbances and the error dynamics are stable for bounded inputs. The motivation for the development of a disturbance decoupled bilinear observer comes from the state estimation problem in semi-active suspensions.

An experimental study on the performance of a semi-active suspension to reduce the dynamic tire forces is made via a laboratory vehicle test rig. The semi-active suspension has been implemented by the use of a modulable damper, accelerometers and a personal computer. Experimental studies using the laboratory test rig show that the performance of the semi-active suspension is close to that of the best passive suspension for all frequency ranges in the sense of minimizing the dynamic tire forces and that the dynamic tire force can be replaced by the estimated one. The dynamic tire forces for both passive and semi-active control test cases are compared to show the potential of a semi-active suspension to reduce the dynamic tire forces.

1. Introduction

Active and semi-active suspensions for ground vehicles have been a very active subject of research in the past years due to their potential to improve vehicle performance[1,2,3], and they have recently been commercialized on high performance automobiles. Development of active suspensions had been started in the 1930's, but most of the significant developmental work has been done since 1950. Semi-active suspensions were proposed in the early 1970's, showing that performance

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comparable to that of fully active suspensions can be achieved by the use of semi-active suspensions[5]. Many analytical and experimental studies on active and semi-active suspensions to improve ride quality and handling performance have been recently performed. The conclusion is that active and semi-active suspensions can provide substantial performance improvements over optimized passive suspensions in general and semi-active suspensions can be nearly as effective as fully active suspensions in improving ride quality using state variable feedback[4,5,6,7].

Although an active suspension provides better performance than semi-active suspensions, it has major drawbacks such as the need for a large external power source, increased complexity and cost, and decreased reliability. A semi-active suspension combines the advantages of both active and passive suspensions, i.e., it provides good performance compared to passive suspensions and is economical, safe and does not require either higher power actuators or a large power supply.

While considerable research on active and semi-active suspensions has been concentrated on the improvement of ride quality, little research has been made on the reduction by active/semi-active suspensions of the dynamic tire force of heavy vehicles to reduce pavement damage[8,9,10]. Semi-active suspensions to reduce dynamic tire force will be presented in this paper.

The motivation for the theory on the disturbance decoupled bilinear observer proposed in this paper comes from the state estimation problem in semi-active suspensions. A bilinear observer structure for bilinear systems with unknown disturbances is developed such that the estimation error is independent of the unknown external disturbances and the proposed observer is applied to estimate the tire force in a semi-active suspension.

2. A Semi-active Suspension Model and Control Laws

2.1 Bilinear Model of a Semi-active Suspension

This section describes a bilinear model of a semi-active suspension. A bilinear model of a semi-active suspension was introduced by Kimbrough[11] in 1986. Bilinear systems have structural properties that are useful for modeling semi-active suspensions.

Consider the quarter car semi-active suspension model shown in Fig.1. The equations of motion of this system can be written as follows:

$$m_s \ddot{z}_s = f_p + f_s \tag{2.1}$$

$$m_u \ddot{z}_u = -f_p - f_s + f_t \tag{2.2}$$

where

$$f_p \equiv -k_s (z_s - z_u),$$

$$f_t \equiv -k_t (z_u - z_r),$$

$$f_s \equiv a\ semi\text{-}active\ force$$

and k_s and k_t are the stiffness of the spring and the tire respectively.

The semi-active force, f_s, may be represented as a nonlinear function of the damping valve area of the semi-active damper, suspension velocity and the material properties of the fluid in the damper.

$$f_s = f_s(c, a_d, (\dot{z}_s - \dot{z}_u)) \tag{2.3}$$

where c is a constant dependent on the properties of the fluid, a_d the damping valve area and $(\dot{z}_s - \dot{z}_u)$ the suspension velocity. Since the damping valve area is controlled by an electromagnetic device such as a stepper motor, the equation of motion may be be written as follows:

$$\frac{d}{dt} a_d = f_a (v) \tag{2.4}$$

where v is the control input for the electromagnetic device.

By defining state variables for this system as follows:

$$\begin{aligned} x_1 &= z_s - z_u && \textit{suspension deflection} \\ x_2 &= \dot{z}_s && \textit{sprung mass velocity} \\ x_3 &= z_u - z_r && \textit{tire deflection} \\ x_4 &= \dot{z}_u && \textit{unsprung mass velocity} \\ x_5 &= a_d && \textit{damping valve area} \end{aligned}$$

we can rewrite the equation of motion as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 - x_4 \\ \dot{x}_2 &= -\frac{1}{m_s} k_s x_1 + \frac{1}{m_s} f_s(x_5, (x_2 - x_4)) \\ \dot{x}_3 &= x_4 - w \\ \dot{x}_4 &= \frac{1}{m_u} k_s x_1 - \frac{1}{m_u} k_t x_3 - \frac{1}{m_u} f_s(x_5, (x_2 - x_4)) \\ \dot{x}_5 &= f_a(v) \end{aligned} \tag{2.5}$$

where the unknown disturbance, w $(=\dot{z}_r)$, is the rate of change of road elevation.

Since the electromagnetic device has much a faster response than mechanical systems, the differential equation for the damping valve area dynamics may be replaced by an algebraic equation as follows:

$$x_5(t) = c_1 v(t) \tag{2.6}$$

where c_1 is a constant.

Fig. 2 shows the time response of the semi-active damper for a step input with a constant suspension velocity of 0.18 m/sec. This experimental result was obtained for the semi-active damper with twenty different damping rate settings* using the half car test rig at the Vehicle Dynamics Laboratory at UC Berkeley. A step command was used to modulate the damping rate of the semi-active damper by

*The semi-active dampers were provided by the Lord Corp. of Erie, Pa.

a stepper motor and a load cell was used to measure the force generated by the damper. Fig.2 illustrates that the response of the semi-active force may be approximated for constant suspension velocity as a first order dynamic equation as follows:

$$\frac{d}{dt} f_s = \frac{1}{T_{fs}} \left(b_1 \, v(t) - f_s \right) \tag{2.7}$$

where T_{fs} is 0.005 second.

Since the time constant, T_{fs}, is small enough, the relation between the semi-active force, $f_s(t)$, and the control input, $v(t)$, may be represented by an algebraic equation for constant suspension velocity as follows:

$$f_s(t) \approx k \, v(t) \tag{2.8}$$

where k is a constant dependent on the suspension velocity. This validates the equation (2.6). Thus the semi-active force may be written as a function of the input, $v(t)$, and the suspension velocity, $(x_2 - x_4)$, as follows:

$$f_s(t) = f_s \left(v(t), (x_2 - x_4) \right)$$

Typical force-velocity curves and their linear approximations for the different control inputs, v, are shown in Fig.3. This experimental data is also for the semi-active damper with twenty states. It is illustrated that the semi-active force-suspension velocity curves can be represented by a bilinear equation, i.e.,

$$f_s(t) = \alpha \, v(t) \, (x_2 - x_4)$$

Since αv is equivalent to the damping rate for the control input, v, for the electromagnetic device, we can define new input, $u(t)$, as follows:

$$u(t) = \alpha v(t) - b_s$$

where b_s is the passive damping rate and the state equation (2.5) can be rewritten as the following bilinear state equation:

$$\dot{x} = A\,x + D\,x\,u + F\,w \tag{2.9}$$

where the unknown disturbance w ($= \dot{z}_r$) is the rate of change of road elevation and

$$x = [\, z_s - z_u \;\; \dot{z}_s \;\; z_u - z_r \;\; \dot{z}_u \,]^T ,$$

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\dfrac{k_s}{m_s} & -\dfrac{b_s}{m_s} & 0 & \dfrac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & \dfrac{b_s}{m_u} & -\dfrac{k_t}{m_u} & -\dfrac{b_s}{m_u} \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\dfrac{1}{m_s} & 0 & \dfrac{1}{m_s} \\ 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{m_u} & 0 & -\dfrac{1}{m_u} \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} .$$

2.2 Semi-active Control Laws to Reduce Dynamic Tire Force

Consider the quarter semi-active suspension model shown in Fig.1. The equation of motion of this system is represented by a bilinear form:

$$\dot{x} = A\,x + B\,f_s + F\,\dot{z}_r$$
$$= A\,x + D\,x\,u + F\,w$$

The desired force, f_s, is found for the deterministic case by solving a typical LQ problem with the following performance index:

$$J = \lim_{T \to \infty} \int_0^T \left[\rho_1\, \ddot{z}_s^2 + \rho_2\,(z_s - z_u)^2 + \rho_3\, \dot{z}_s^2 + \rho_4\,(z_u - z_r)^2 + \rho_5\, \dot{z}_u^2 + r\,f_s^2 \right] dt \qquad (2.10)$$

where ρ_1 is weighting factors for sprung mass acceleration, ρ_2 through ρ_5 for states of the suspension system, and r for the input, i.e., control force. The performance index given by equation (2.10) can be rewritten as follows:

$$J = \lim_{T \to \infty} \int_0^T [\, x^T Q\, x + 2\, x^T M\, f_s + r\, f_s^2 \,]\, dt$$

$$Q \geq 0\,,\ \ Q - M\,r^{-1}\,M^T \geq 0\,,\ \ r > 0\,.$$

The optimal force which minimizes the performance index is given as following constant gain state feedback control law:

$$f_{s,opt} = -\,r^{-1}\,(\,B^T H + M^T\,)\,x \qquad (2.11)$$

where H is determined by solving the following algebraic Riccati equation:

$$-(A - Br^{-1}M^T)^T H - H(A - Br^{-1}M^T) - (Q - Mr^{-1}M^T) + HBr^{-1}B^T H = 0$$

The frequency responses for passive and active suspensions were computed to show the improvement over the passive suspensions and to compare the difference between the full state feedback case, the sprung mass velocity feedback case and the tire force feedback case. A comparison of frequency responses between passive, active suspension with state feedback (State fdbk) and active suspension with tire force feedback (TF fdbk) cases is shown in Fig.4. The tire force feedback control law is implemented by setting all feedback gains in the state feedback control law (2.11) equal to zero except the tire force feedback gain. The tire force and sprung mass acceleration are shown in Fig.4. There are two modal frequencies in the tire force case, i.e., a 2 Hz body mode and a 10 Hz axle mode. It shows that the peaks at all the modal frequencies are significantly reduced by the active suspensions and the performance of the tire force feedback case is close to that of the full state feedback case. This shows the intuitive result that the most important state variable in the implementation of the active suspension control law to reduce the tire force, i.e., the axle load, is the tire force. It is illustrated that the higher frequency components in the sprung mass acceleration are increased in the active suspension cases.

Fig.5 shows the comparison between the tire force feedback (TF fdbk), the sprung mass velocity feedback (SMV fdbk) and both the sprung mass velocity and tire force (TF&SMV fdbk) feedback cases. Looking at the tire force it can be seen that the peak at the body mode is significantly decreased for all cases and the peak in the sprung mass velocity feedback case at the axle mode are similar to that of the passive case. It shows that the frequency response of the sprung mass velocity feedback case is similar to that of the passive suspension at the higher frequency ranges and TF&SMV feedback case provides significant reductions of peaks at both body and axle modes.

Useful insight on the selection of a control law can be obtained from the above results. Since the full state feedback control law is very difficult to implement in real systems, the tire force feedback control law may be practical. Since measurements of the tire forces are very difficult to make, the bilinear observer is proposed to estimate the dynamic tire force from accelerometers in section 3.

A reasonable semi-active control law applicable to a suspension with continuously modulable dampers can be obtained from the active control law, i.e., it is given as follows[6]:

$$u(t) = \begin{cases} u_{\min} & \text{if } u^*(t) \le u_{\min} \\ u^*(t) & \text{if } u_{\min} \le u^*(t) \le u_{\max} \\ u_{\max} & \text{if } u_{\max} \le u^*(t) \end{cases} \tag{2.12}$$

where $u(t)$ is the damping rate of the modulable damper and

$$u^*(t) = -\frac{f_{s,opt}}{(\,suspension\ velocity\,)}$$

$f_{s,opt}$ is the desired control force which can be determined by some active control law, i.e., state feedback or the tire force feedback, or the sprung mass velocity feedback.

3. Disturbance Decoupled Bilinear Observers

An observer structure for bilinear systems is formulated such that the estimation error is independent of unknown external disturbances. The proposed observer is applied to estimate the tire force in a vehicle semi-active suspension problem.

The motivation for this study on bilinear observers was to estimate important states in a vehicle suspension application such as tire deflection, spring deflection rate and/or sprung mass velocity. These states are essential in semi-active suspension controls to improve ride quality or to reduce dynamic tire forces and are difficult to measure. A number of studies on active and semi-active suspension control laws have been recently performed assuming that all states are available, showing that the performance of the vehicle can be significantly improved when compared to passive suspensions by using either active or semi-active suspensions [4,12]. Also, it has been shown that the most important state variable for the suspension controls to improve ride quality is absolute sprung mass velocity and the most important one to reduce pavement damage is the dynamic tire force [4,12]. Although measurements of the sprung mass velocity may be made by integrating the output of an accelerometer, measurement of the tire forces is very difficult to make for real time control because of the unknown road input. Thus it is necessary in semi-active suspension control to design an observer which estimates necessary states, whose estimation error due to initial conditions converges to zero sufficiently quickly and whose error is independent of the unknown road input.

3.1 Disturbance Decoupled Bilinear Observers

Consider an n-dimensional bilinear system with unknown disturbance $w \in R^q$ represented by

$$\dot{x} = A\,x + \sum_{i=1}^{m} D^i\,x\,u_i + F\,w \tag{3.1}$$

$$y = C\,x + \sum_{i=1}^{m} E^i\,x\,u_i + F_y\,w \tag{3.2}$$

where x is an n-state vector, $u = [\,u_{1,}\ u_2\,,\,\ldots.\,,\,u_m\,]^T$ is an m-input vector, y is a p-output vector and all matrices are constant and have proper dimensions.

An intuitive approach to design of a state observer is to copy its state equation (3.1) plus a feedback term which utilizes the information contained in the measurement, y. The problems associated with this intuitive approach are due to the fact that error dynamics of the observer designed by this method depend on the unknown disturbance, w, in addition to the input, u. An observer structure for bilinear systems with unknown disturbances is formulated to overcome these problems.

For the bilinear system (3.1)-(3.2), a bilinear observer with the following structure is proposed to obtain the state estimate without the estimation error, which is independent of input, initial estimation error and the unknown disturbance:

$$\dot{z} = A_z\,z + \sum_{i=1}^{m} D_z^{\,i}\,z\,u_i + L\,y + H\,[\,P_2\,y - y_z\,] \tag{3.3}$$

$$y_z = C_z\,z + \sum_{i=1}^{m} E_z^{\,i}\,z\,u_i \tag{3.4}$$

$$v = B_z\,z + \sum_{i=1}^{m} B_u^{\,i}\,z\,u_i + P_1\,y \tag{3.5}$$

where $v \in R^q$, $z \in R^r$, $(\,r + q\,) \le n$, $y_z \in R^s$, $s \le p$ and all matrices are constant with proper dimensions. The bilinear observer represented by (3.3)-(3.5) is said to be an observer for $K\,x$ $(\,K \in R^{(q+r)\times n}\,)$ of the bilinear system (3.1)-(3.2), if

$$\lim_{t\to\infty} \frac{d^i}{dt^i} e(t) = 0, \qquad (\,i = 0,\,1,\,2,\,\cdots\,) \tag{3.6}$$

$$e(t) = \begin{bmatrix} v(t) \\ z(t) \end{bmatrix} - \begin{bmatrix} K\,x(t) \end{bmatrix} \tag{3.7}$$

independent of the input u, initial state x_0 and z_0 and unknown disturbance [13]. If $K = I_n$, it is said to be a state observer. It should be noted that the proposed bilinear observer requires no knowledge of the unknown disturbance w.

The necessary and sufficient conditions for the existence of a stable bilinear observer is given by Theorem 1 and Theorem 2.

THEOREM 1. z converges to $W\,x$ if and only if the following conditions are satisfied:

$$W\,A - A_z\,W - L\,C = \underline{0} \tag{3.8}$$

$$W D^i - D_z^i W - L E^i = \underline{0}, \quad i = 1, \cdots, m \tag{3.9}$$

$$P_2 C - C_z W = \underline{0} \tag{3.10}$$

$$P_2 E^i - E_z^i W = \underline{0} \quad i = 1, \cdots, m \tag{3.11}$$

$$W F - L F_y = \underline{0} \tag{3.12}$$

$$P_2 F_y = \underline{0} \tag{3.13}$$

and

$$\textit{the bilinear system } \dot{z} = A_e z \textit{ is asymptotically stable} \tag{3.14}$$

where W is defined by design procedure and

$$A_e = (A_z - H C_z + \sum_{i=1}^{m} D_z^i u_i - \sum_{i=1}^{m} H E_z^i u_i)$$

Proof: See Ref.[3,14].

THEOREM 2. v converges to $U x$ ($U \in R^{q \times n}$) as z converges to $W x$ if and only if :

$$P_1 F_y = \underline{0} \tag{3.15}$$

$$P_1 C = U - B_z W \tag{3.16}$$

$$P_1 E^i = - B_u^i W , \quad i = 1, \cdots, m \tag{3.17}$$

where U is to be chosen in the design procedure.

Proof: See Ref.[3,14].

The observer matrices are determined from the system equations (3.1)-(3.2) and only the observer feedback matrix H is the design parameter. The synthesis of the disturbance decoupled bilinear observer can be summarized as follows.

(i) Pick v as disturbance related states.

(ii) Obtain relations between v , the remaining states and the measurement y , i.e., find B_z , B_u^i and P_1 from the system equation.

(iii) Pick z from the relations obtained in (ii) such that

$$\begin{aligned} v &= f_1 (x, y) \\ &= f_2 (z, y) \end{aligned} \tag{3.18}$$

and obtain state equation for z, i.e., A_z , D_z^i, P_2 , C_z , E_z^i and L from the bilinear system equation.

(iv) Select the observer feedback matrix H to guarantee the stability of the homogeneous part of z dynamics.

Remark 1. The number of measurements should be greater than the number of unknown disturbances, i.e., $dim(y) = p > dim(w) = q$.

Remark 2. q outputs should be functions of the disturbance related states in order that step (ii) is possible.

In the design of the bilinear observer the stability of the observer depends on the selection of the observer feedback matrix H. H should be designed in order that the bilinear dynamic system

$$\begin{aligned} \dot{z} &= A_e(u)\, z \\ &= (A_z + H\, C_z)\, z + \sum_{i=1}^{m} (D_z^i - H\, E_z^i)\, u_i\, z \end{aligned} \tag{3.19}$$

is asymptotically stable. Design of the observer feedback matrix H that guarantees the stability of the bilinear system represented by equation (3.19) is presented in [3,14].

The stability guaranteed region of the gain H may be found either analytically or numerically. The problem of finding the stability guaranteed region of the observer gain H can be handled separately for each application. There is no simple way to do this systematically for any bilinear system.

The procedure for the gain H is summarized as follows:

(i) Choose a positive definite matrix P and find the eigenvalues of $Q(u)$. They are expressed as functions of the observer gain H for a positive definite matrix P.

(ii) Solve k inequalities simultaneously which are obtained from the condition (3.23) where k is the dimension of the observer state z. The solution of the k inequalities gives us the stability guaranteed region of the gain H for a given positive definite matrix P.

(iii) Find the stability guaranteed region of H for all possible positive definite matrices P.

(iv) The sum of the regions for H determined in the above procedure is the stability guaranteed region of H. This implies that the gain H chosen in this region guarantees the asymptotic convergence of the estimation error to zero.

As an application, the design of a disturbance decoupled bilinear observer for a semi-active suspension is explained in the next section.

3.2 The Design of a Bilinear Observer for a Semi-active Suspension

Based on the bilinear observer proposed in section 3.1 an observer is designed to estimate the dynamic tire force, which is difficult to measure in real time control. Assume that axle acceleration and sprung mass acceleration are measured. Thus the measurement y is

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dot{x}_2 \\ \dot{x}_4 \end{bmatrix} \tag{3.27}$$

--

and matrices C , E^i and F_y are determined by the state equation (2.9). Select tire deflection as v, which is the disturbance related state. i.e.,

$$v = \hat{x}_3 = U\hat{x} \tag{3.28}$$

$$U = [\,0\,0\,1\,0\,]$$

From the relation between $v\ (= \hat{x}_3)$ and the measurement y_2 , z is determined. i.e.,

$$\begin{aligned} v = \hat{x}_3 &= f_1(\,y_2, \hat{x}_1, (\,x_2 \hat{-} x_4\,)\,) \\ &= f_2(\,y_2, z\,) \end{aligned} \tag{3.29}$$

and

$$z = W\hat{x} = \begin{bmatrix} \hat{x}_1 \\ (\,x_2 \hat{-} x_4\,) \end{bmatrix} \tag{3.30}$$

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

In this case a bilinear observer for a semi-active suspension is expressed as

$$v = \hat{x_3} = [\,\frac{k}{k_t} \quad \frac{b}{k_t}\,]\,z + [\,0 \quad \frac{1}{k_t}\,]\,z\,u + [\,0 \quad -\frac{m_u}{k_t}\,]\,y \tag{3.31}$$

$$\dot{z} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m_s} & -\dfrac{b}{m_s} \end{bmatrix} z + \begin{bmatrix} 0 & 0 \\ 0 & -\dfrac{1}{m_s} \end{bmatrix} z\,u + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} [\,y_1 - y_{1z}\,] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y_2$$

where

$$y_{1z} = -\frac{k}{m_s} z_1 - \frac{b}{m_s} z_2 - \frac{1}{m_s} z_2\,u \tag{3.32}$$

Define the estimation error, e_z , as follows:

$$e_z = \begin{bmatrix} x_1 - z_1 \\ (\,x_2 - x_4\,) - z_2 \end{bmatrix} \tag{3.33}$$

Then, the error dynamics are expressed as

$$\dot{e}_z = \begin{bmatrix} h_1 \dfrac{k}{m_s} & 1 + h_1 \dfrac{(\,b+u\,)}{m_s} \\ (\,h_2 - 1\,)\dfrac{k}{m_s} & (\,h_2 - 1\,)\dfrac{(\,b+u\,)}{m_s} \end{bmatrix} e_z = A_e(u)\,e_z \tag{3.34}$$

It is straightforward to verify that the bilinear observer to estimate the dynamic tire force for a semi-active suspension satisfies the conditions (3.8)-(3.17) except the condition (3.14). Therefore if the error dynamics are stable, the dynamic tire force estimation error tends to zero by Theorem 1 and Theorem 2. The stability of the bilinear observer depends on the observer feedback gains (h_1 , h_2). The stability region of the gains (h_1 ,h_2) can be found by applying LEMMA 1 in [14].

LEMMA 1. The bilinear system (3.34) is asymptotically stable if the observer feedback gains (h_1 ,h_2) satisfy the following conditions:

$$- \frac{4\, m_s}{u_{max}} \left[1 + 2\, \frac{(b + u_{min})}{u_{max}} + 2 \sqrt{\frac{(b + u_{min})}{u_{max}} \left(1 + \frac{(b + u_{min})}{u_{max}} \right)} \right] < h_1 < 0$$

$$h_2 < 1$$

The stability guaranteed region for the bilinear observer gain is shown in Fig.6.

Proof: See Ref. [3,14].

The bilinear observer discussed in this section for a semi-active suspension estimates the tire deflection, i.e., the dynamic tire force, the spring deflection and the spring deflection rate with the axle acceleration and the sprung mass acceleration measurements. The suspension velocity should be known in order to determine the damping rate of the modulable shock absorber. The measurements of acceleration may be made with ease compared to velocity or deflection measurements. As mentioned in the introduction, this study has been motivated by a state estimation problem in semi-active suspension control to reduce the dynamic axle load where the dynamic tire force and the spring deflection rate are the most important states in the control law [4,8]. Therefore the bilinear observer designed in this section may be very effective in semi-active suspension control to reduce the dynamic tire force.

4. Laboratory Experiments

Experimental studies using a half car test rig were conducted to test the proposed semi-active suspension with the bilinear observer-controller. The objectives of this experiment were:

(*i*) to determine the potential of the semi-active suspension to reduce dynamic tire loading.

(*ii*) to verify the performance of the proposed disturbance decoupled bilinear observer under a realistic semi-active suspension system where real implementation problems such as nonlinearity of the semi-active dampers, parameter uncertainty and the effect of unmodeled dynamics etc. may arise.

(*iii*) to investigate the feasibility of the observer-controller from a real time perspective.

The semi-active suspension was implemented on a half car model. Though the half car model is different from the tractor/semi-trailer heavy truck model, experimental verification of the performance of the semi-active suspension using the half car model is important and may be helpful in real vehicle implementation.

4.1 Half Car Test Rig

The laboratory half car model used in experimental study is shown in Fig.7. The experimental setup of the U.C. Berkeley Active/Semi-active Suspension consists of a hydraulic power system, a road profile generating system, a vehicle dynamics simulating system , sensors and an electronic control system. The simulated vehicle, i.e., laboratory half car model, consists of four parts:

- Sprung mass(Vehicle body/chassis)
- Unsprung mass(tires and axle)
- Suspensions, and
- Guide rails.

The vehicle parameters for the half car test rig are given in Table 1.

A schematic drawing for experimental setup of the half car test rig is shown in Fig.8. The hydraulic power system consists of a Vickers vane pump, two electrohydraulic servo actuators (6 inch stroke, double action) and an accumulator. The actuators have a bandwidth of 20 Hz and generate road input for the test rig. The hydraulic actuators are controlled by digital closed-loop servo controllers (DCL). The DCL servo controller is integral to a high-performance flapper nozzle two stage servo valve. It contains a built-in PID algorithm. The DCL servo controllers are controlled by TS&S (Test System and Simulation, Inc.) X8700 computer software. Haversines, ramps, sinusoidal and some complex road profiles such as a superpositioned six sine wave form, pseudo random road inputs can be generated by the X8700 software.

The electronic control system consists of a personal computer, power supplies, amplifiers, D/A termination panels, etc. It provides power for all the sensors, hydraulic servo valves and the stepper motor of the semi-active damper. A 20 MHZ IBM compatible AT serves as the controller for the semi-active suspension. This is equipped with a flexible data acquisition system.

The half car test rig is equipped with various sensors for the measurement of important states of the system such as sprung mass and unsprung mass accelerations, suspension and tire deflections, tire and semi-active forces, sprung mass and suspension velocities. The suspension deflections are measured by Linear Variable Differentiable Transformers (LVDT). Accelerometers with ranges of ± 2g are stationed on the sprung mass and unsprung mass. The accelerometers are solid-state piezoresistive devices with built-in amplification and temperature compensation.

Linear optical encoders are used to measure the tire deflection, sprung mass displacement and the road input on the test rig. The encoders provide high resolution motion detection at a low cost.

Load cells are used to measure the tire force and the force provided by the semi-active damper. The suspension load cells have a range of 4,448 N and the tire force load cells can measure up to 22,240 N.

A semi-active damper with 20 states has been used to generate the desired semi-active forces. The semi-active damper mounted on the half car test rig is shown in Fig.9. The semi-active damper force versus suspension velocity curves are shown in Fig.10.

The damper position is controlled by a stepper motor. The stepping time of this stepper motor is 1 msec and the force response of the semi-active damper for one step change of the damper position from position 10 to 11 for constant suspension velocity is shown in Fig.2. There exist oscillations in the damper force after a step change due to the vibration of the stepper motor axle.

4.2 Experimental Results

A continuous semi-active control law was implemented using a continuously modulable damper with 20 damping rate settings. The damping rate was modulated between setting 1 and setting 14 which is approximately equivalent to 1058 *N/(m/sec)* to 5436 *N/(m/sec)* range.

Firstly, dynamic tire force measured from a load cell was used to implement a semi-active control law to reduce dynamic tire force. Frequency responses for passive and semi-active suspensions were obtained between 0.5 to 18 Hz road inputs. In addition, responses for superpositioned six sine wave road inputs were compared for passive and semi-active cases.

The dynamic tire forces were estimated using sprung and unsprung mass acceleration measurements by a disturbance decoupled bilinear observer. Then the estimated dynamic tire force was used for the semi-active control law.

4.2.1 Dynamic Tire Force Feedback

Fig.11 shows a comparison of the frequency response of the passive and semi-active suspensions for dynamic tire force and sprung mass acceleration. The control law was designed to minimize the dynamic tire force. For small dynamic tire force, there are three frequency ranges:

0.5 - 1.5 Hz : the hard passive is better than the soft passive

1.5 - 9.0 Hz : the soft passive is better than the hard passive

9.0 - 17. Hz : the hard passive is better than the soft passive

The comparison of the frequency responses indicates that the performance of the semi-active suspension is close to the best passive suspension for all frequency ranges in the sense of minimizing the dynamic tire force.

At 1.2 Hz, the sprung mass acceleration is lower than that of the soft passive case, whereas between 1.5 and 9 Hz, the sprung mass acceleration is close to the soft passive case. Sprung mass acceleration increases at the axle bounce mode frequency ,i.e., at 13 Hz to 15 Hz range.

Fig.11 illustrates that the semi-active suspension with the dynamic tire force minimizing control law improves both the dynamic tire force and the sprung mass acceleration within a 0.8 to 7 Hz range and aggravates both the sprung mass acceleration and the suspension deflection at the axle bounce mode frequency, i.e., at 13 to 15 Hz.

In order to compare the performance of the semi-active and passive suspensions for a more realistic road input case, experiments were executed for a sum of six sine waves road input. The amplitude of each sine wave was chosen to generate a similar spectral density to that of a real road. The amplitudes of the six sine waves used are shown in Fig.12. Comparison of the passive and semi-active cases are shown in Fig.13. It can be seen that peak dynamic tire force is reduced by 40 % in this case.

4.2.2 Dynamic Tire Force Estimation and Estimated Dynamic Tire Force Feedback

It was shown in section 4.2.1 that the dynamic tire force feedback semi-active control law provides good performance compared to passive suspensions at all frequency ranges and knowledge of the dynamic tire force and suspension velocity is essential in semi-active control to reduce the tire force variation. Although measurements of the tire force are very difficult to make for real time control, they can be estimated from accelerometers by the disturbance decoupled bilinear observer proposed in section 3.

The dynamic tire force was estimated from two acceleration measurements , i.e., sprung and unsprung mass accelerations, by a bilinear observer, and then the estimated dynamic tire force was used to implement a semi-active control law without the measurement of the tire force.

Fig.14 shows a comparison of the dynamic tire forces measured by the load cell and estimated by the observer for the realistic road input case described in section 4.2.1. The observer started to work after 0.2 seconds and the estimated dynamic tire force is very close to the real one. It was shown that the estimation error quickly dies out. The estimation error is due to parametric errors such as sprung mass error, spring stiffness error, and tire stiffness error and modeling error such as nonlinear damping characteristics and ignored friction.

Frequency characteristics of the semi-active suspension with the estimated DTF feedback are compared to those of the semi-active suspension with the measured DTF feedback and those of passive suspensions in Fig.15. While the dynamic tire force of the observer-controller case is similar to that of the DTF feedback case in a 2.0 to 15 Hz range, it was indicated that the dynamic tire force of the observer-controller case is greater than that of the measured DTF feedback case at body mode frequency range, i.e., 1.0 to 1.5 Hz range.

6. Conclusions

A bilinear model of a semi-active suspension was formulated and it was shown via experimental data that a bilinear model does represent a semi-active suspension with sufficient accuracy. The performance of tire force feedback and the sprung mass velocity feedback cases are compared to determine which state is most important in dynamic tire force control. It has been shown that the performance of the tire force feedback with optimal passive damping case is similar to that of the full state feedback case in the dynamic tire force control.

A bilinear observer whose estimation error is independent of the unknown disturbance has been developed and stability conditions for the observer were investigated. The necessary conditions for the measurements are relaxed compared to that of the disturbance decoupled bilinear observer that cancels the effect of the input and the disturbances in the observer internal model. The proposed bilinear observer was shown to be robust to parametric/ modeling errors.

The proposed disturbance decoupled bilinear observer is applied to a semi-active suspension system. The stability guaranteed region for the observer feedback gain in this case was provided. The dynamic tire force, the spring deflection and the spring deflection rate were estimated without estimation error using the axle acceleration and the sprung mass acceleration measurements which may be made with ease.

Experimental studies performed using a half car test rig were presented. The semi-active control law to reduce the dynamic tire force was implemented by the use of a semi-active damper with twenty states. The performances of passive and semi-active suspensions were compared for sinusoidal road and superpositioned sinusoidal road input cases. The superpositioned input was created to simulate a more realistic road input.

The dynamic tire force was estimated using sprung and unsprung mass acceleration measurements and the performances of the dynamic tire force feedback and the estimated tire force feedback cases have been compared to those of the passive suspensions.

Experimental results have shown that:

(i) the continuous semi-active control law can be implemented by a semi-active damper with twenty states.

(ii) the disturbance decoupled bilinear observer proposed in section 3 is effective for the estimation of the dynamic tire force.

(iii) the dynamic tire force can be reduced by the semi-active suspensions with dynamic tire force feedback and the dynamic tire force can be replaced by the estimated one.

Although the semi-active damper used in these experimental studies has a nonlinear force-velocity characteristic, the semi-active suspension can be described by a bilinear model and the bilinear observer-controller designed based on this bilinear model has shown good performance.

Experimental studies via the half car test rig have shown that the introduction of semi-active suspensions has the potential to dramatically reduce the dynamic tire force. The semi-active suspension has been implemented in the experimental studies by the use of a modulable damper, accelerometers and a personal computer.

This research will be useful in a variety of vehicle suspension situations. In addition to reduction of peak dynamic tire force, the control algorithm could be optimized to reduce the RMS tire force to improve vehicle handling characteristics or to reduce RMS sprung mass acceleration to improve vehicle ride quality. The disturbance decoupled bilinear observer developed in this research for tire force estimation could be useful in many automotive applications.

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Table 1 Half Car Parameters

Specifications	Value	Unit
Total Mass of the Sprung Mass	574.7	Kg
Pitch Moment of Inertia of the Sprung Mass	768.9	$Kg \cdot m^2$
Unsprung Mass	59.5	Kg
Spring Constant	16812.	N/m
Tire Stiffness	190000.	N/m
Wheel Base	2.74	m
Distance from the C.G. to the Front Suspension	1.38	m
Distance from the C.G. to the Rear Suspension	1.36	m
Equivalent Sprung Mass		
Front	285.3	Kg
Rear	289.4	Kg

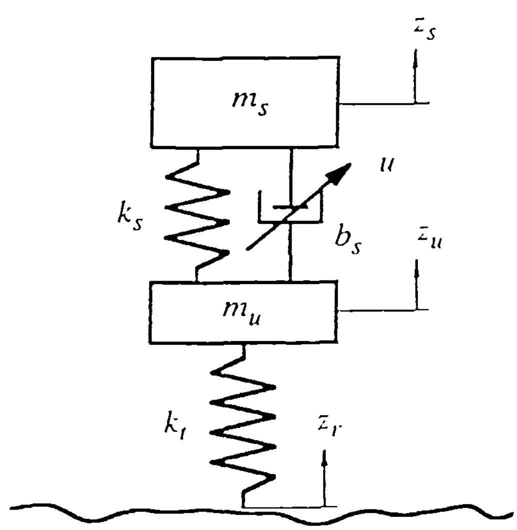


Fig.1 Quarter Car Semi-active Suspension Model

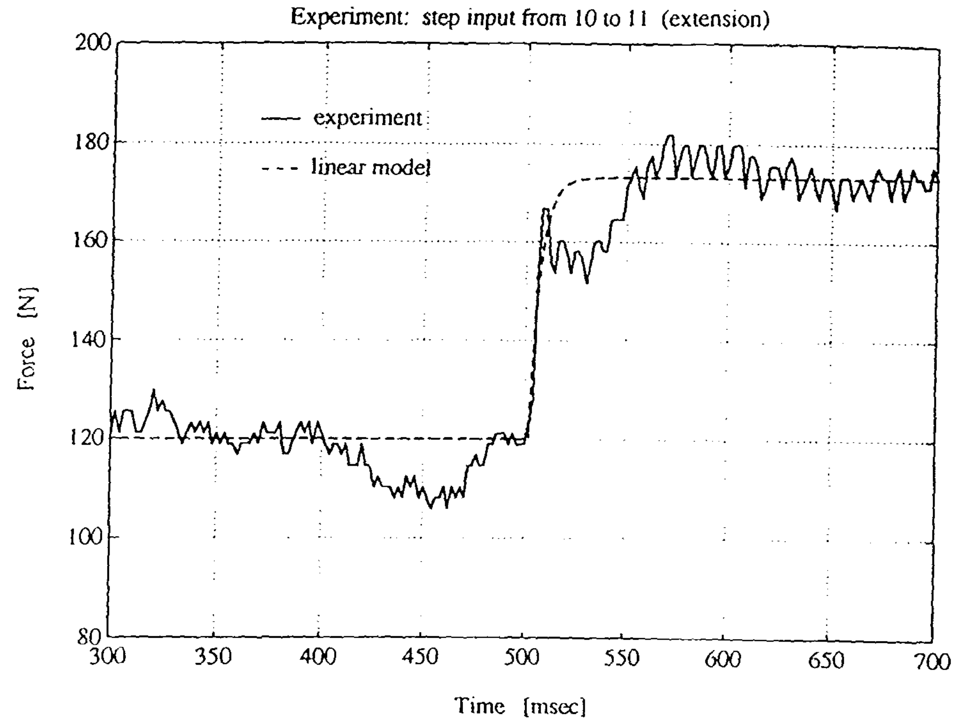


Fig.2 **Step Input Response of the Semi-active Damper for Constant Suspension Velocity of 0.18 *m/sec* and its Linear Approximation**

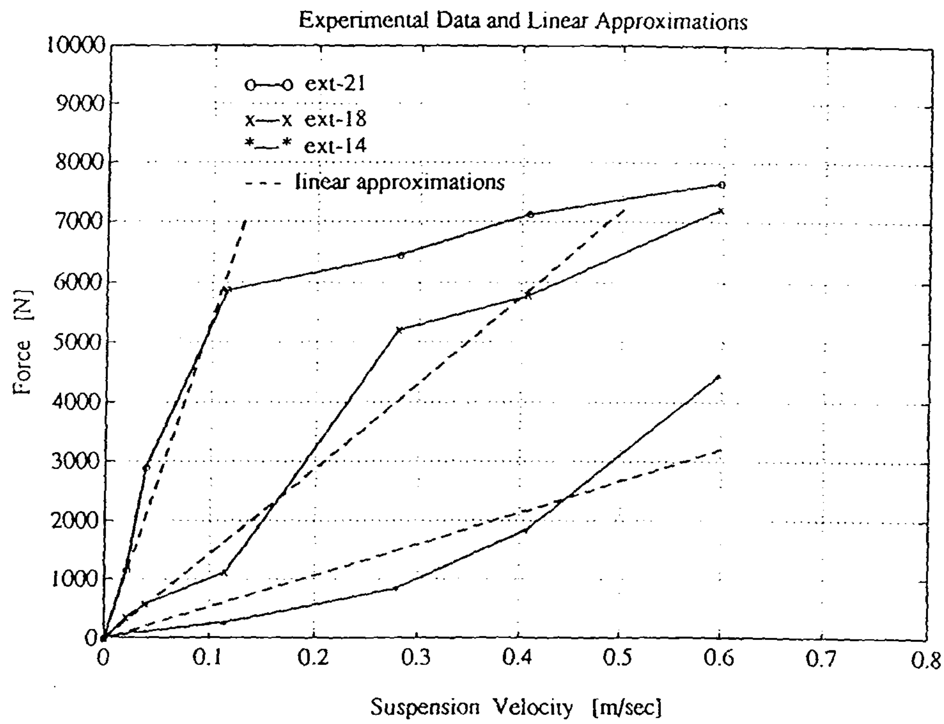


Fig.3 **Force-Velocity Relations and Their Linear Approximations (experimental data for a semi-active damper with 21 settings)**

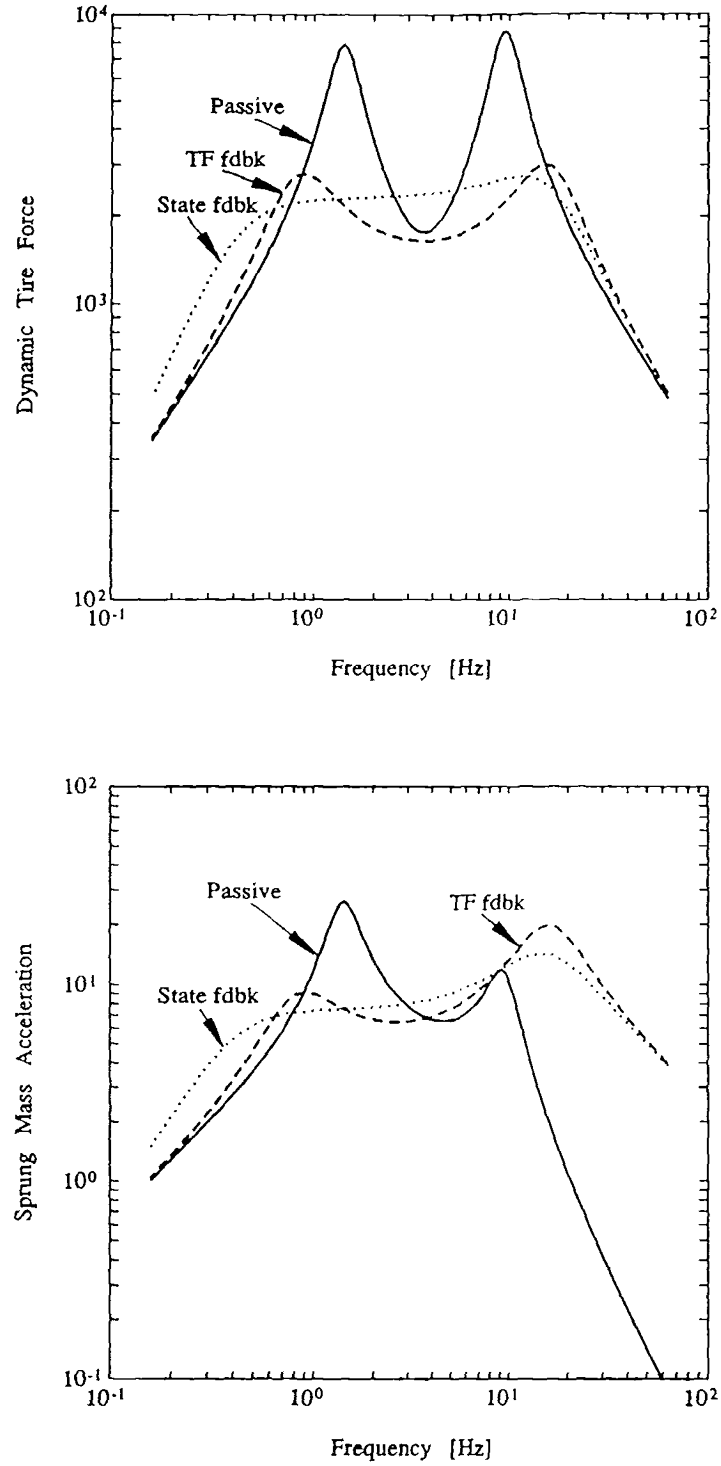


Fig.4 Frequency Responses of Quarter Car Model (A Comparison between State Feedback and Tire Force Feedback Cases)

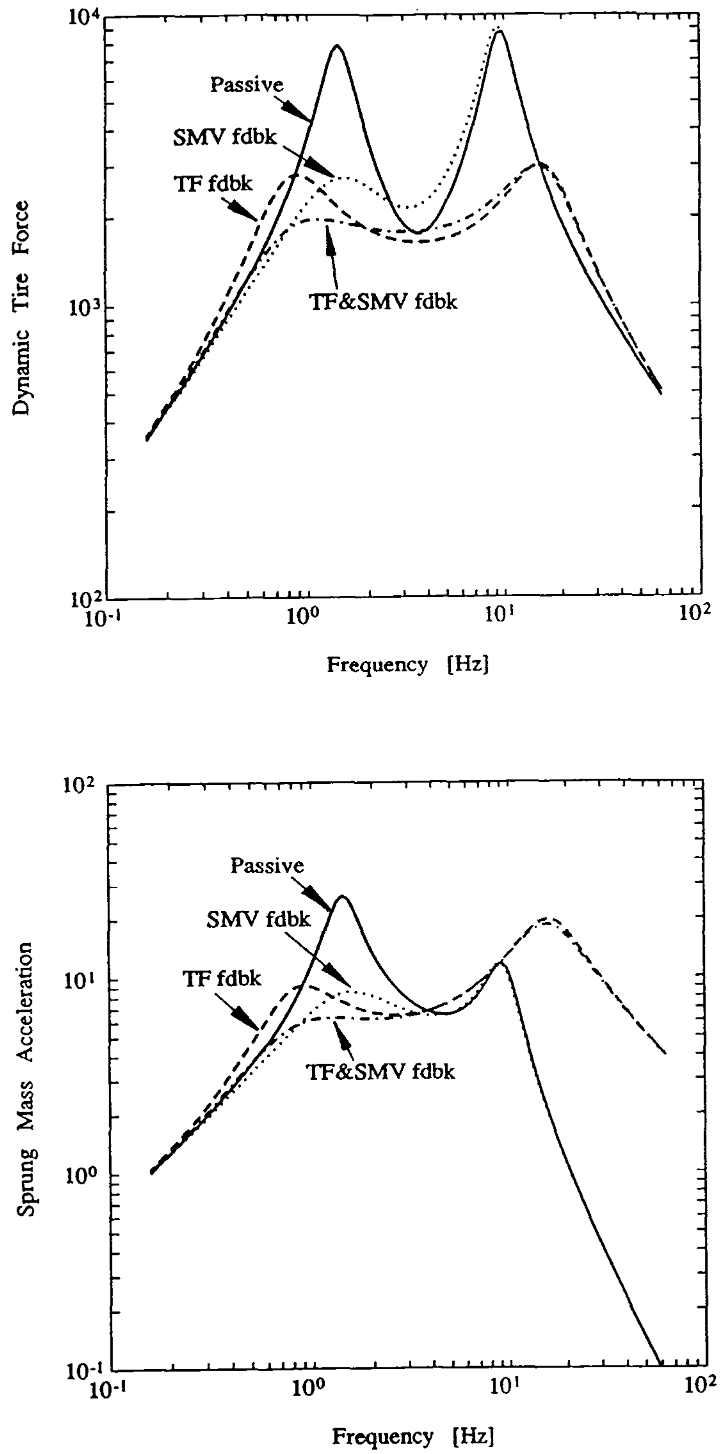
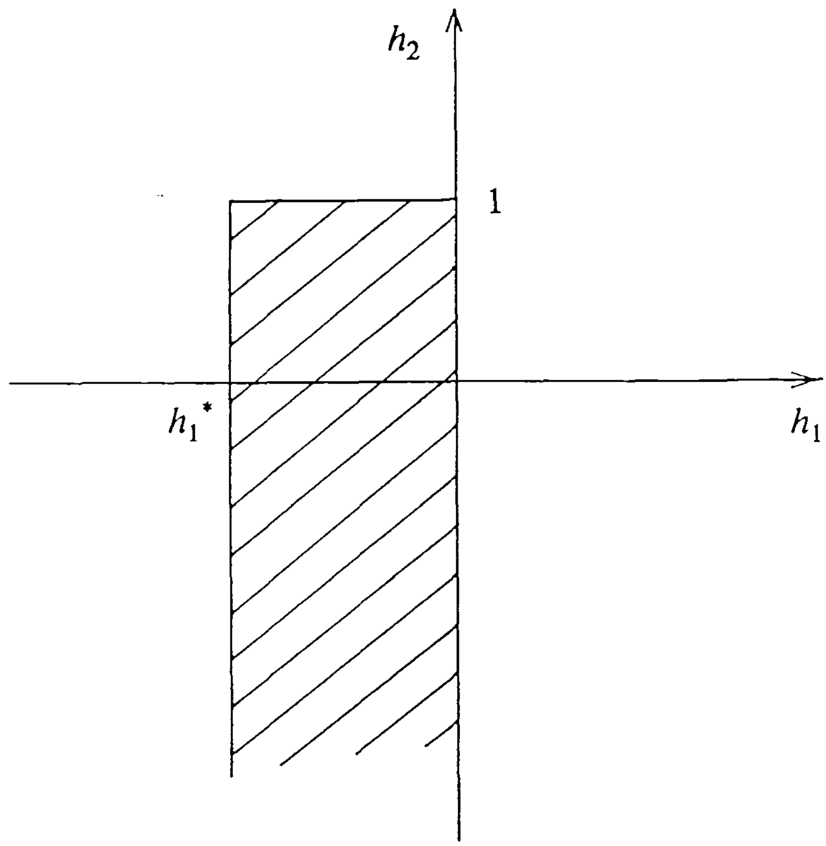


Fig.5 Frequency Responses of Quarter Car Axle Model (A Comparison between Tire Force Feedback and Sprung Mass Velocity Feedback Cases)



$$h_1^* = -\frac{4\,m_s}{u_{max}}\left[1 + 2\,\frac{(b+u_{min})}{u_{max}} + 2\sqrt{\frac{(b+u_{min})}{u_{max}}\left(1+\frac{(b+u_{min})}{u_{max}}\right)}\,\right]$$

Fig.6 The Stability Guaranteed Region for the Bilinear Observer Gain

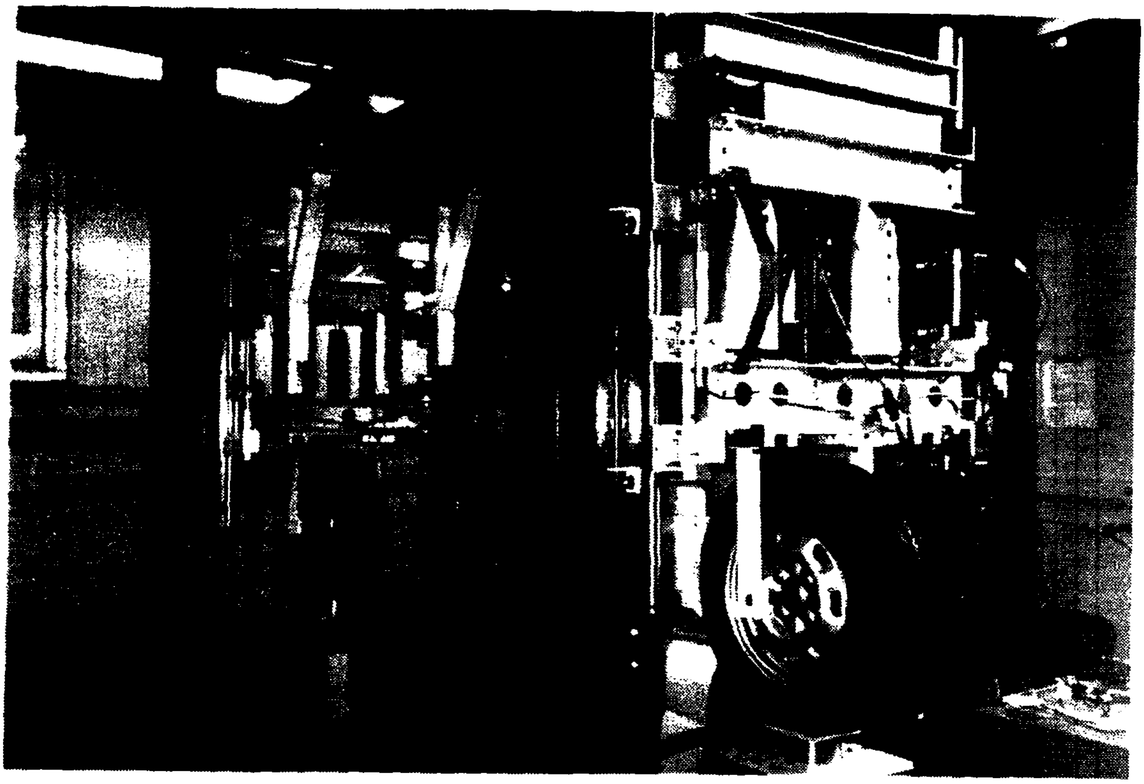


Fig. 7 Berkeley Half Car Suspension Test Laboratory

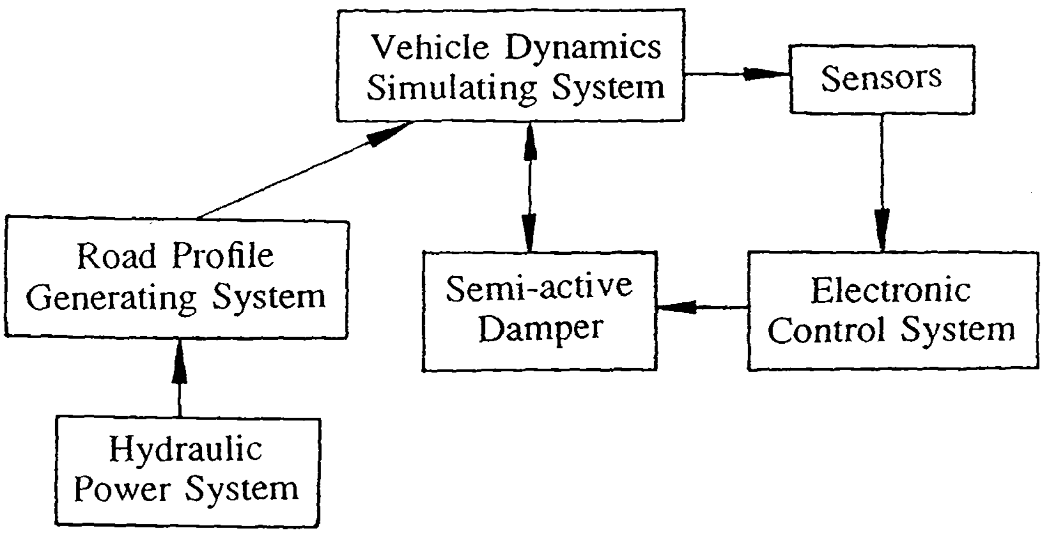


Fig. 8 Schematic Drawing for Experimental Setup of the Half Car Test Rig

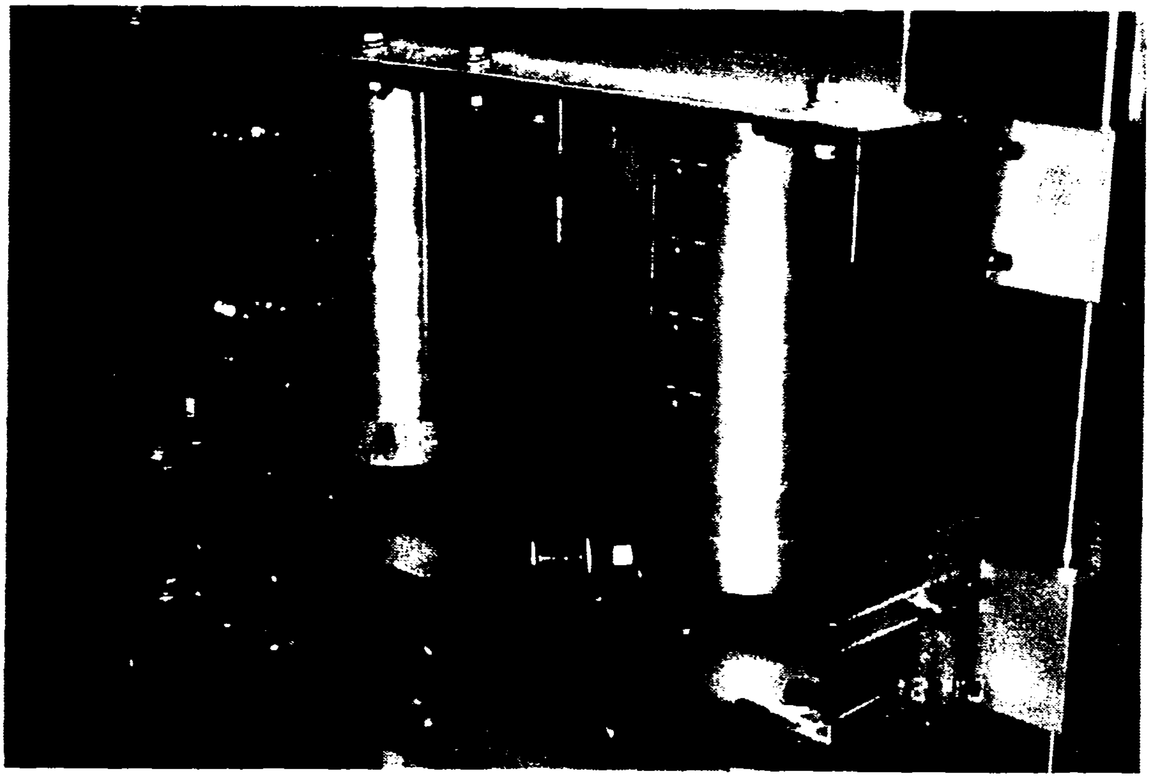


Fig. 9 Semi-active Suspension

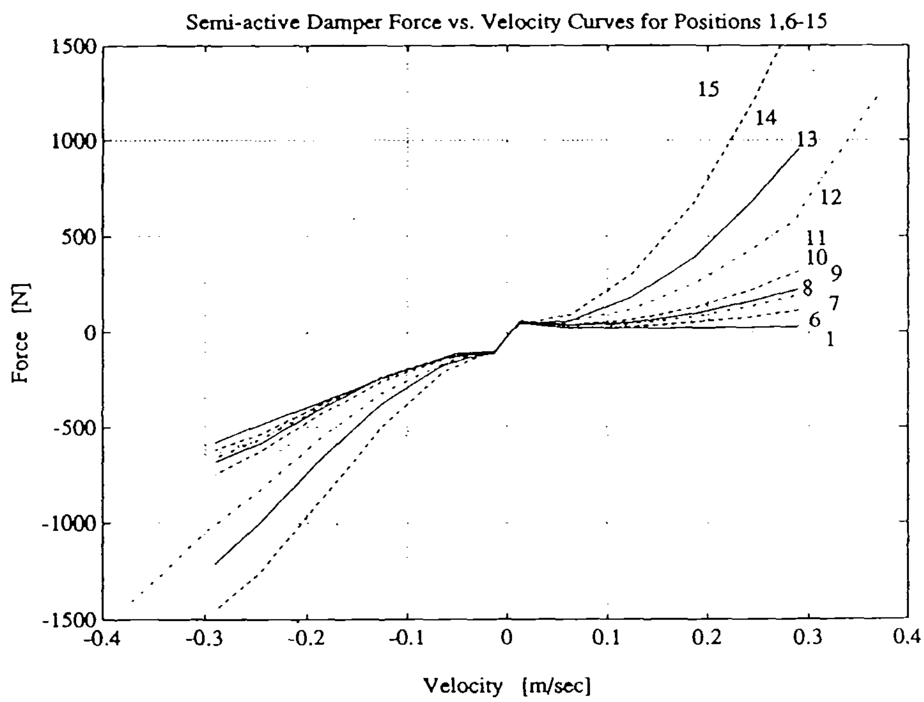
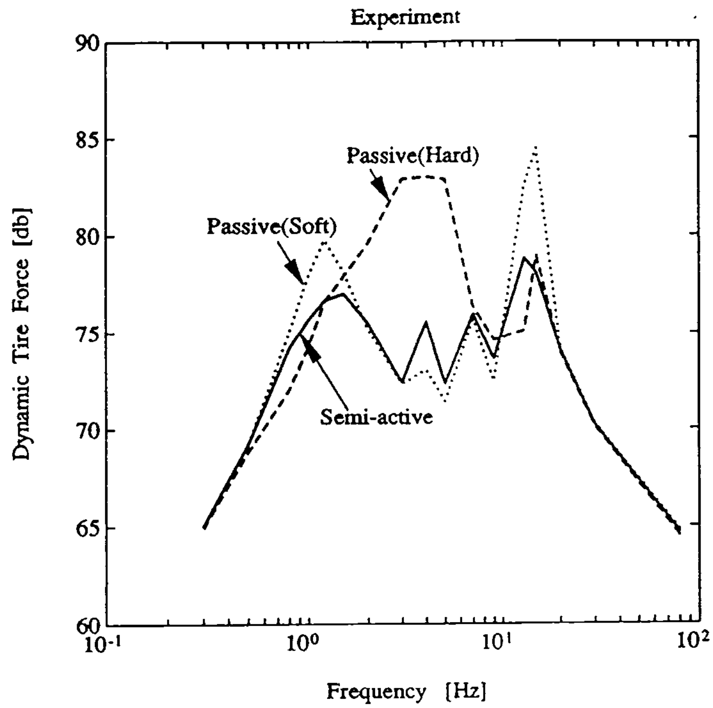
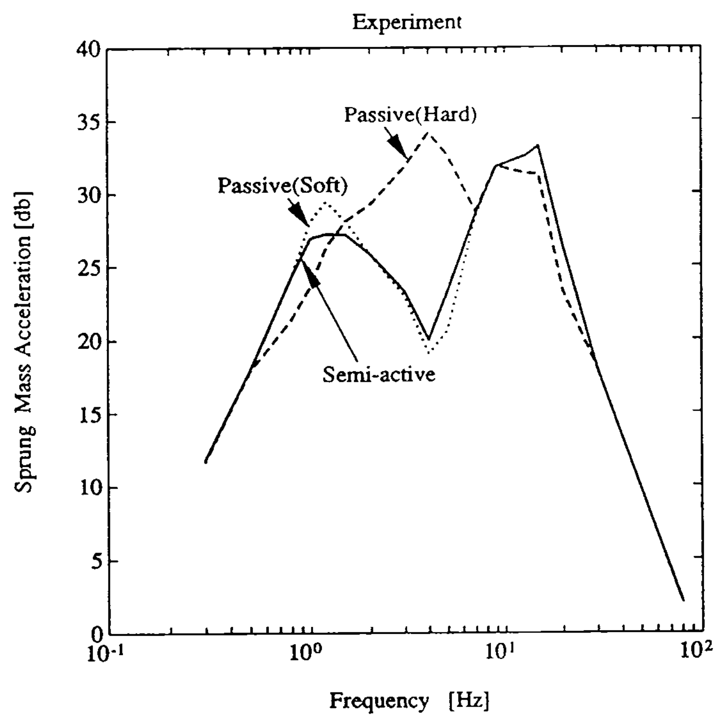


Fig. 10 Semi-active Damper force vs. Velocity Curves



(a) Dynamic Tire Force



(b) Sprung Mass Acceleration

Fig. 11 Comparison of Frequency Responses of Passive and Semi-active Suspensions

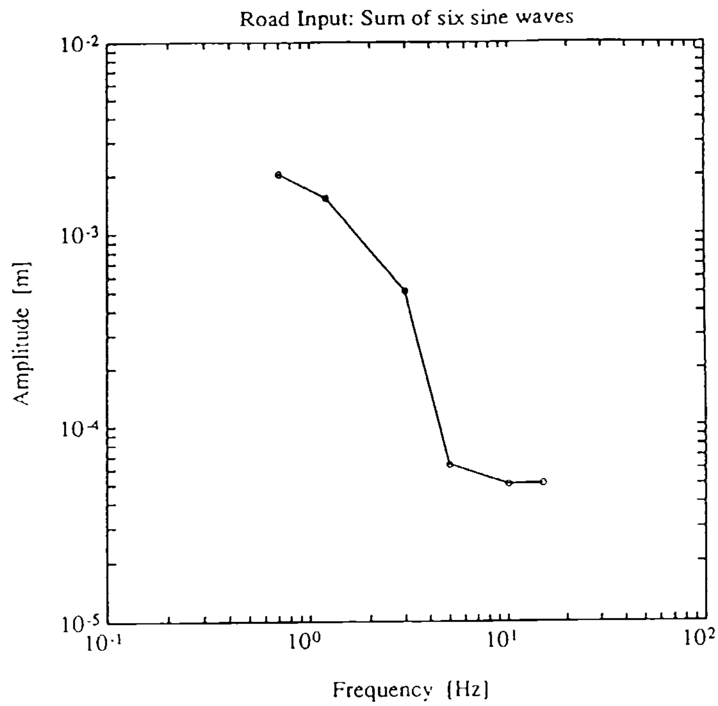


Fig. 12 Amplitude versus Frequency of Six Sine Waves

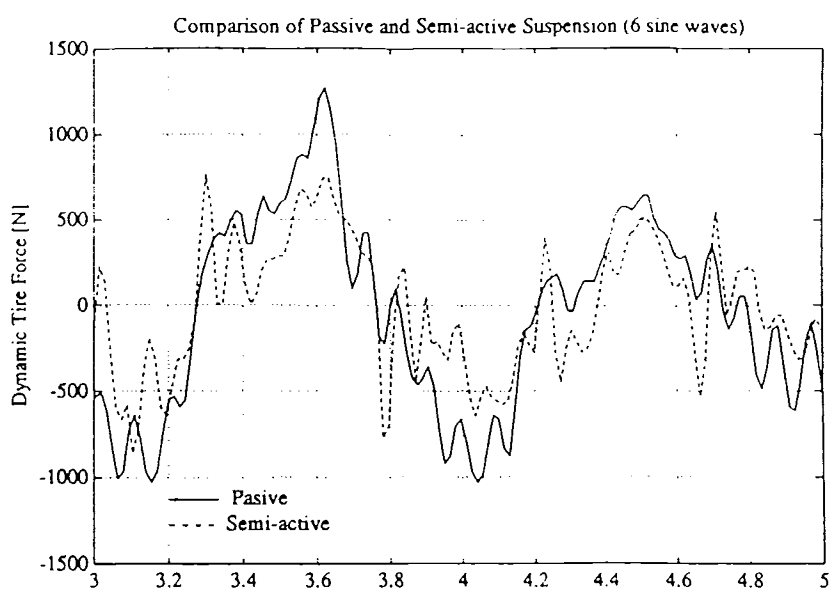


Fig. 13 Comparison of Dynamic Tire Force for Sum of Six Sine Waves Road Input Case

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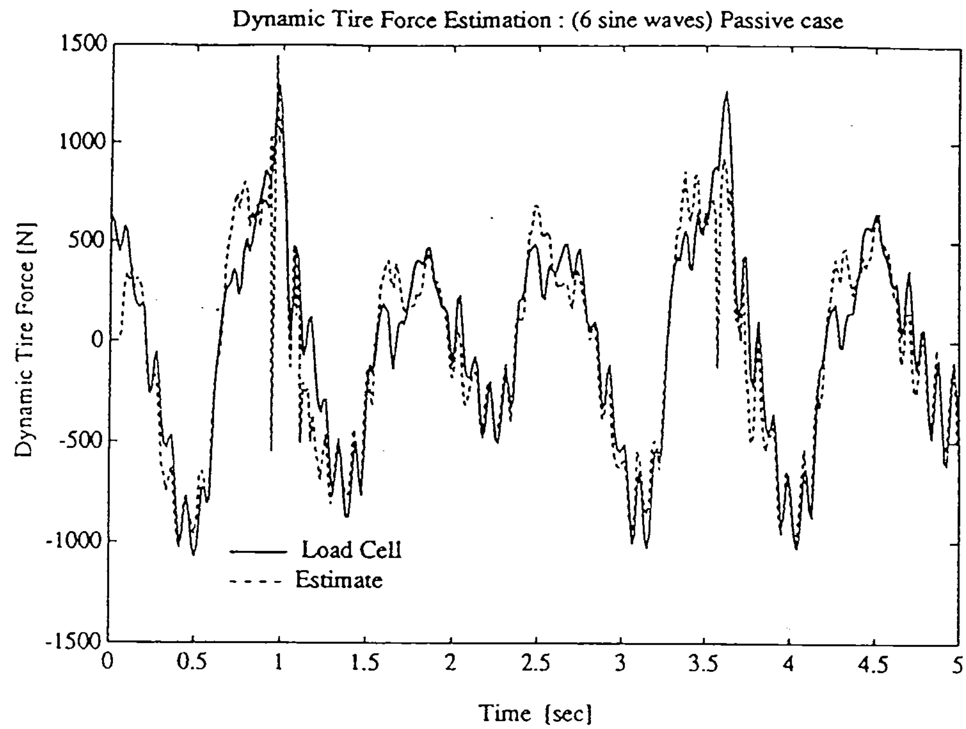
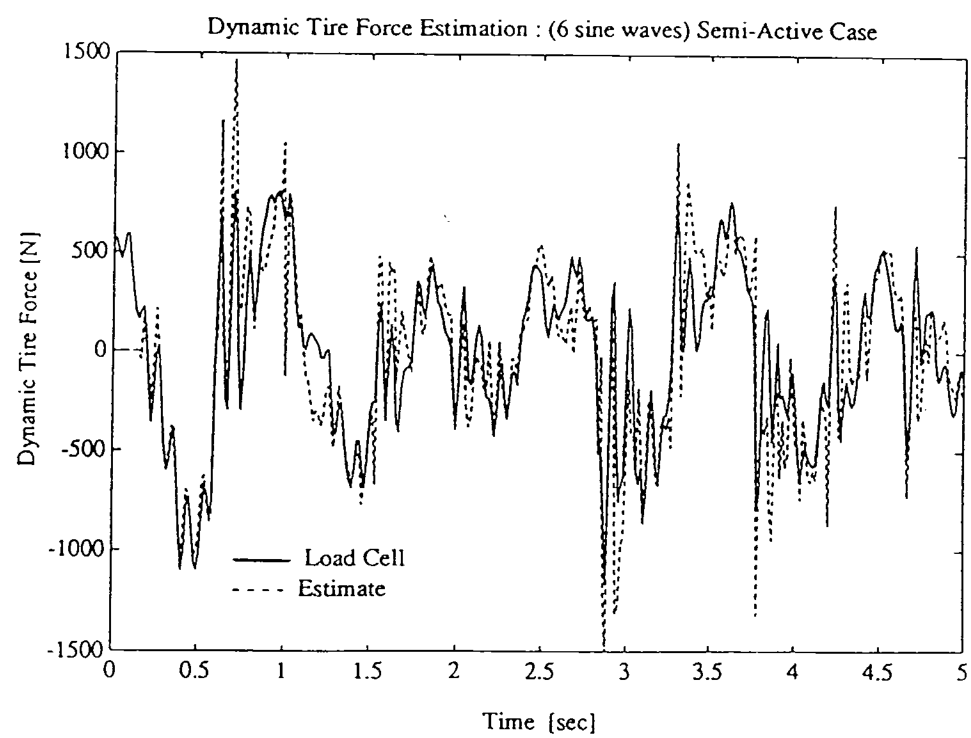


(a) Passive Case



(b) Semi-active case

Fig. 14 Comparison of Measured and Estimated Dynamic Tire Force (sum of six sine wave input case)

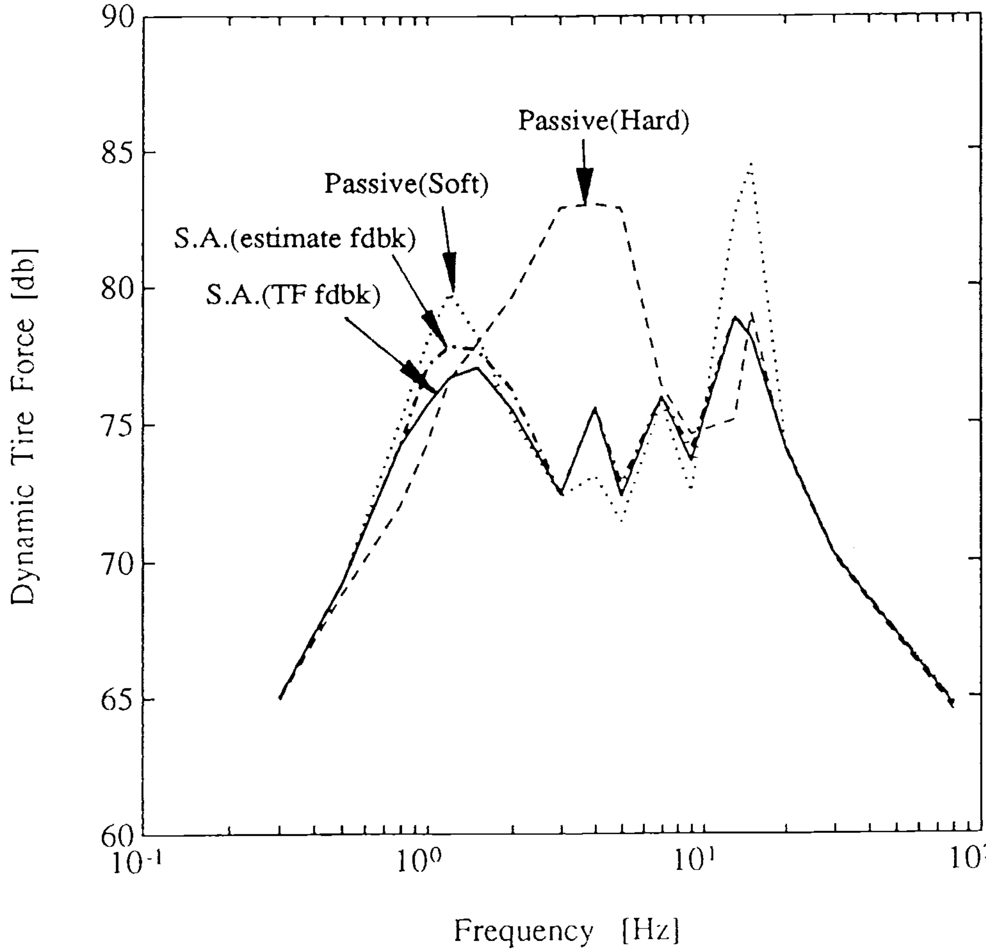


Fig. 15 Comparison of Frequency Responses of Passive and semi-active suspensions