Title: Dynamical analysis in growth models: Blumberg's equation

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**Abstract:** We present a new dynamical approach to the Blumberg's equation, a family of unimodal maps. These maps are proportional to Beta(p, q) probability densities functions. Using the symmetry of the Beta(p, q) distribution and symbolic dynamics techniques, a new concept of mirror symmetry is defined for this family of maps. The kneading theory is used to analyze the effect of such symmetry in the presented models. The main result proves that two mirror symmetric unimodal maps have the same topological entropy. Different population dynamics regimes are identified, when the intrinsic growth rate is modified: extinctions, stabilities, bifurcations, chaos and Allee effect. To illustrate our results, we present a numerical analysis, where are demonstrated: monotonicity of the topological entropy with the variation of the intrinsic growth rate, existence of isentropic sets in the parameters space and mirror symmetry.

**Author Keywords:** Population dynamics; Blumberg's equation; Topological entropy; Kneading theory; Bifurcations and chaos; Symbolic dynamics; Beta(p, q) densities

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