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Dynamical behavior of a pest management model with impulsive effect and nonlinear incidence rate*

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Abstract. In this paper, we consider the pest management model with spraying microbial pesticide and releasing the infected pests, and the infected pests have the function similar to the microbial pesticide and can infect the healthy pests, further weaken or disable their prey function till death. By using the Floquet theory for impulsive differential equations, we show that there exists a globally asymptotically stable pest eradication periodic solution when the impulsive period $\tau < \tau_{\max}$, we further prove that the system is uniformly permanent if the impulsive period $\tau > \tau_{\max}$. Finally, by means of numerical simulation, we show that with the increase of impulsive period, the system displays complicated behaviors.

Mathematical subject classification: 34C05, 92D25.

Key words: pest-management model, impulsive effect, extinction, permanence.

1 Introduction

From the reports of Food and Agriculture Organization of the United Nations, the warfare between man and pests has lasted for thousands of years. With the

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development of society and progress of science and technology, there are many ways to control agricultural pests, for instance biological pesticides, chemical pesticides, remote sensing and measuring and so on. A great deal of pesticides were used to control pests. Generally speaking, pesticides are useful because they can quickly kill a significant portion of a pest population and sometimes provide the only feasible method for preventing economic loss. However, pesticides pollution is also recognized as a major health hazard to human beings and to natural enemies. Hence, many scholars put forward Integrated Pest Management (IPM) (see [1, 3, 4, 11]), IPM is a pest management system that in the context of the associated environment and the population dynamics of the pest species, utilizes all suitable techniques and methods in as a compatible manner as possible and maintains the pest populations at levels below those causing economic injury.

Recently, the models for pest control were studied by some authors ([2-6]) and some results were obtained. As we all know, most of the research literature on these epidemic models assumed that the disease incubation is negligible, so that, once infected, each susceptible individual (S) instantaneously becomes infectious (I) and later recovers (R) with a permanent or temporary acquired immunity. A model based on these assumptions is often called an SIR or SIRS model. The SIR epidemiological model was studied in [7], they assumed that the susceptible satisfied the logistic equation and the incidence rate was of the form kIS^q and the total population was not constant.

However, it is inevitable that IPM may cause pollution to the environment more or less due to the use of chemical pesticide. Therefore, in this paper we propose a biological control strategy-controlling the pest by introducing microbial pesticide and infected pests simultaneously. Compared with the chemical pesticide, the application shows that the microbial pesticide is an effective, highly infectious and safe bio-pesticide which can be used in both short-term and long-term controls and plays an important role in pest management.

The model we consider is based on the following SI model:

$$\begin{cases} \dot{S} = -\beta S(t)I(t), \\ \dot{I} = \beta S(t)I(t) - dI(t), \end{cases} \tag{1.1}$$

where $S(t)$ and $I(t)$ are densities of the susceptible and infectious, respectively,

$\beta > 0$ is called the transmission coefficient, $d > 0$ is the death rate of the infectious pests.

For IPM strategy, we combine the biological control and chemical control. The infectious pests are released periodically every time period τ, meanwhile periodic spraying the microbial pesticide for susceptible pests. Based on biological control strategy in pest management, we construct a pest-epidemic model with impulsive control. Impulsive differential equations found in almost every domain of applied science and have been studied in many investigations ([8-13]). But to our knowledge there are only a few papers and books on mathematical model of the dynamics of microbial diseases in pest control. Li et al. [14], Anderson et al. [15] and Jong et al. [16] pointed out that standard incidence is more suitable than bilinear incidence. Levin et al. [17] have adopted a incidence form like $\beta S^l I^h$ or $\frac{\beta S^h I^h}{N}$, $l > 0$, $h > 0$ which depends on different infective diseases and environment. So we develop (1.1) by introducing a constant periodic releasing of the infective pests and spraying microbial pesticides at fixed moment. That is, we consider the following impulsive differential equations:

$$\begin{cases} \left.\begin{aligned} \dot{S}(t) &= rS\left(1 - \frac{S+\theta I}{K}\right) - \beta S I^q, \\ \dot{I}(t) &= \beta S I^q - dI, \end{aligned}\right\} t \neq n\tau,\ n \in \mathrm{N}, \\ \left.\begin{aligned} \Delta S &= -(\mu_1 + \mu_2)S, \\ \Delta I &= \mu_1 S - \mu_3 I + p, \end{aligned}\right\} t = n\tau,\ n \in \mathrm{N}, \end{cases} \tag{1.2}$$

where $\Delta S(t) = S(t^+) - S(t)$, $\Delta I(t) = I(t^+) - I(t)$. $S(t)$ is in the absence of $I(t)$ grows logistically with carrying capacity K, and with an intrinsic birth rate constant r, the nonlinear incidence rate was of the form $\beta S I^q$, $q > 1$; $0 \leq \mu_1 < 1$ represents the fraction from susceptible to infectious due to spraying the microbial pesticide at $t = n\tau$; $0 \leq \mu_2 < 1, 0 \leq \mu_3 < 1$ which represent the traction of susceptible and infective pests due to spraying pesticides at $t = n\tau$, respectively; and $0 < \theta < 1$, $\mu_1 + \mu_2 < 1$; $p > 0$ is the release amount of the infected pests at $t = n\tau$, $n \in \mathrm{N}, \mathrm{N} = \{0, 1, 2, \cdots\}$, τ is the period of the impulsive effect. That is, we can use a combination of biological and chemical tactics to eradicate pests or keep the pest population below the damage level.

2 Notations and definitions

In this section, we give some notations which will prove useful and give some definitions.

Let $R_+ = [0, \infty)$, $R_+^2 = \{x \in R^2 \colon x > 0\}$, $\Omega = \text{int}\, R_+^2$, N be the set of all nonnegative integers. Denote $f = (f_1, f_2)$, the map defined by the right hand side of the first two equations of system (1.2). Let $V_0 = \{V \colon R_+ \times R_+^2 \mapsto R_+\}$, continuous on $(n\tau, (n+1)\tau] \times R_+^2$, and $\lim_{(t,y)\to(n\tau^+,x)} V(t, y) = V(n\tau^+, x)$ exists.

Definition 2.1. *$V \in V_0$, then for $(t, x) \in (n\tau, (n+1)\tau] \times R_+^2$, the upper right derivative of $V(t, x)$ with respect to the impulsive differential system (1.2) is defined as*

$$D^+V(t, x) = \lim_{h\to 0} \sup \frac{1}{h}\Big[V(t + h, x + hf(t, x)) - V(t, x)\Big].$$

The solution of system (1.2) is a piecewise continuous function $x \colon R_+ \mapsto R_+^2$, $x(t)$ is continuous on $(n\tau, (n+1)\tau]$, $n \in$ N and $x(n\tau^+) = \lim_{t\to n\tau^+} x(t)$ exists. Obviously the smoothness properties of f guarantee the global existence and uniqueness of solution of system (1.2), for details (see [18]).

We will use a basic comparison result from impulsive differential equations. For convenience, we state it in our notations.

Suppose $g \colon R_+ \times R_+ \mapsto R$ satisfies:

(H) *g is continuous in $(n\tau, (n+1)\tau] \times R_+$ and for $x \in R^+$, $n \in$ N, $\lim_{(t,y)\to(n\tau^+,x)} g(t, y) = g(n\tau^+, x)$ exists.*

Lemma 2.2. *Let $V \in V_0$, assume that*

$$\begin{cases} D^+V(t, x) \le g(t, V(t, x)), & t \ne n\tau, \\ V(t, x(t^+)) \le \psi_n(V(t, x(t))), & t = n\tau, \end{cases} \tag{2.1}$$

where $g \colon R_+ \times R_+ \mapsto R$ satisfies (H) *and $\psi_n \colon R_+ \mapsto R_+$ is nondecreasing. Let $h(t)$ be the maximal solution of the scalar impulsive differential equation*

$$\begin{cases} \dot u(t) = g(t, u(t)), & t \ne n\tau, \\ u(t^+) = \psi_n(u(t)), & t = n\tau, \\ u(0^+) = u_0 \end{cases} \tag{2.2}$$

existing on $[0, \infty)$*. Then* $V(0^+, x_0) \leq u_0$ *implies that* $V(t, x(t)) \leq h(t), t \geq 0$*, where* $x(t)$ *is any solution of* (1.2)*, similar result can be obtained when all the directions of the inequalities in the lemma are revered and* ψ_n *is nonincreasing. Note that if we have some smoothness conditions of* g *to guarantee the existence and uniqueness of solutions for* (2.2)*, then* $h(t)$ *is exactly the unique solution of* (2.2).

Lemma 2.3. *Suppose that* $x(t)$ *is a solution of system* (1.2) *with* $x(0^+) \geq 0$*, then* $x(t) \geq 0$ *for all* $t \geq 0$*. Further, if* $x(0^+) > 0$*, then* $x(t) > 0$ *for all* $t > 0$*.*

For convenience, we give some basic properties of the following system

$$\begin{cases} \dot{I} = -dI, & t \neq n\tau, \\ \Delta I = -\mu_3 I + p, & t = n\tau. \end{cases} \tag{2.3}$$

Then we have the following lemma:

Lemma 2.4. *System* (2.3) *has a unique positive periodic solution* $\widetilde{I}(t)$ *with period* τ *and for every solution* $I(t)$ *of* (2.3) *such that* $|I(t) - \widetilde{I}(t)| \to 0$ *as* $t \to \infty$*, where*

$$\widetilde{I}(t) = \frac{p\exp(-d(t - n\tau))}{1 - (1 - \mu_3)\exp(-d\tau)}, \ \widetilde{I}(0^+) = \frac{p}{1 - (1 - \mu_3)\exp(-d\tau)}$$

and $\widetilde{I}(t)$ *is globally asymptotically stable. Hence the solution of* (2.3) *is*

$$I(t) = (1 - \mu_3)(\widetilde{I}(0^+) - \frac{p\exp(-d(t - n\tau))}{1 - (1 - \mu_3)\exp(-d\tau)})\exp(-dt) + \widetilde{I}(t).$$

Lemma 2.5. *There exists a constant* $M > 0$ *such that* $S(t) \leq M, I(t) \leq M$ *for each positive solution* $x(t) = (S(t), I(t))$ *of* (1.2) *with all* t *large enough.*

Proof. Define $V(t, x(t)) = S(t) + I(t)$. Then $V(t, x(t)) \in V_0$ and the upper right derivative of $V(t, x(t))$ along solution of (1.2) is described as

$$\begin{aligned} D^+V(t, x(t)) + dV(t, x(t)) &= (r + d)S(t) - \frac{rS^2(t)}{K} - \frac{r\theta S(t)I(t)}{K} \\ &\leq (r + d)S(t) - \frac{rS^2(t)}{K} \leq L_0, \end{aligned}$$

where

$$L_0 = \frac{K(r+d)^2}{4r},$$

when $t = n\tau$, we obtain

$$V(n\tau^+) = (1-\mu_2)S(n\tau) + (1-\mu_3)I(n\tau) + p \le V(n\tau) + p.$$

According to Lemma 2.2, for $t \in (n\tau, (n+1)\tau)$, we have

$$\begin{aligned} V(t, x(t)) &\le V(0^+)\exp(-dt) + \int_0^t L_0 \exp(-d(t-s))\mathrm{d}s \\ &\quad + \sum_{0<n\tau<t} p\exp\left(\int_{n\tau}^t (-d)\mathrm{d}s\right) \\ &\le V(0^+)\exp(-dt) + \frac{L_0}{d}(1-\exp(-dt)) \\ &\quad + \frac{p\exp(-d(t-\tau))}{1-\exp(d\tau)} + \frac{p\exp(\mu\tau)}{\exp(d\tau)-1} \\ &\to \frac{L_0}{d} + \frac{p\exp(d\tau)}{\exp(d\tau)-1}, \; t \to \infty. \end{aligned}$$

Definition 2.6. *System* (1.2) *is said to be permanent if there exists positive constants* m, M *such that each positive solution* $(S(t), I(t))$ *of system* (1.2) *satisfies* $m \le S(t) \le M, m \le I(t) \le M$ *for all* t *sufficiently large.*

3 Stability of the pest-eradication periodic solution

In this section, we study the stability of the pest-eradication periodic solution of system (1.2).

Theorem 3.1. *The pest-eradication periodic solution* $(0, \widetilde{I}(t))$ *of system* (1.2) *is globally asymptotically stable provided*

$$\begin{aligned} r\tau - \frac{pr\theta(1-\exp(-d\tau))}{dK[1-(1-\mu_3)\exp(-d\tau)]} - \frac{p^q\beta(1-\exp(-qd\tau))}{2d[1-(1-\mu_3)\exp(-d\tau)]} \\ < -\ln(1-\mu_1-\mu_2). \end{aligned} \tag{3.1}$$

Proof. Firstly, we prove the local stability of a τ-period solution $(0, \widetilde{I}(t))$ may be determined by considering the behavior of small-amplitude perturbations $(u(t), v(t))$ of the solution.

Define

$$S(t) = u(t), \quad I(t) = v(t) + \widetilde{I}(t),$$

where $u(t)$, $v(t)$ are small perturbations, there may be written as

$$\begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \end{pmatrix}$$

where $\Phi(t)$ satisfy

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} r - \frac{r\theta}{k}\widetilde{I}(t) - \beta\widetilde{I}^q(t) & 0 \\ \beta\widetilde{I}^q(t) & -d \end{pmatrix} \Phi(t),$$

where $\Phi(0)$ is the identity matrix. The resetting impulsive conditions of (1.2) becomes

$$\begin{pmatrix} u(n\tau^+) \\ v(n\tau^+) \end{pmatrix} = \begin{pmatrix} 1 - \mu_1 - \mu_2 & 0 \\ \mu_1 & 1 - \mu_3 \end{pmatrix} \begin{pmatrix} u(n\tau) \\ v(n\tau) \end{pmatrix}.$$

Hence, if absolute values of all eigenvalues of

$$M = \begin{pmatrix} 1 - \mu_1 - \mu_2 & 0 \\ \mu_1 & 1 - \mu_3 \end{pmatrix} \Phi(\tau),$$

are less than one, the τ-periodic solution is locally stable. By calculating, we have

$$\Phi(\tau) = \begin{pmatrix} 1 - \mu_1 - \mu_2 & 0 \\ \mu_1 & 1 - \mu_3 \end{pmatrix}$$
$$\times \begin{pmatrix} \exp\left(\int_0^\tau \left(r - \frac{r\theta}{K}\widetilde{I}(t) - \beta\widetilde{I}^q(t)\right) \mathrm{d}t\right) & 0 \\ * & \exp(-d\tau) \end{pmatrix},$$

there is no need to calculate the exact form of $(*)$ as it is not required in the analysis that follows. Then the eigenvalues of M denoted by λ_1, λ_2 are the following:

$$\lambda_1 = (1-\mu_1-\mu_2)\exp\left(\int_0^\tau \left(r - \frac{r\theta}{K}\widetilde{I}(t) - \beta\widetilde{I}^q(t)\right)\mathrm{d}t\right),$$

$$\lambda_2 = (1-\mu_3)\exp(-d\tau),$$

$\lambda_1 < 1$ if (3.1) holds true. According to Floquet theory, the pest-eradication solution $(0, \widetilde{I}(t))$ is locally asymptotically stable.

In the following, we prove the global attractivity. Choose a sufficiently small $\varepsilon > 0$ such that

$$\delta = (1-\mu_1-\mu_2)\times \exp\left(\int_0^\tau \left(r - \frac{r\theta}{K}(\widetilde{I}(t)-\varepsilon) - \beta(\widetilde{I}(t)-\varepsilon)^q\right)\mathrm{d}t\right) < 1.$$

Noting that $\dot{I}(t) \geq -dI(t)$ as $t \neq n\tau$ and $\Delta I(t) \leq -\mu_3 I(t) + p$ as $t = n\tau$, consider the following impulsive differential equation:

$$\begin{cases} \dot{x}(t) = -dx(t), & t \neq n\tau, \\ \Delta x(t) = -\mu_3 x(t) + p, & t = n\tau, \end{cases} \tag{3.2}$$

by Lemma 2.4, system (3.2) has a globally asymptotically stable positive periodic solution

$$\widetilde{x}(t) = \frac{p\exp(-d(t-n\tau))}{1-(1-\mu_3)\exp(-d\tau)}.$$

So by Lemmas 2.2 and 2.4, we get

$$I(t) \geq x(t) > \widetilde{I}(t) - \varepsilon. \tag{3.3}$$

From system (1.2), we obtain that

$$\begin{cases} \dot{S}(t) \leq S(t)\left(r - \frac{r\theta}{K}(\widetilde{I}(t)-\varepsilon) - \beta(\widetilde{I}(t)-\varepsilon)^q\right), & t \neq n\tau, \\ \Delta S(t) = -(\mu_1+\mu_2)S(t), & t = n\tau. \end{cases} \tag{3.4}$$

Integrating (3.4) on $(n\tau, (n+1)\tau]$, which yields

$$\begin{aligned} S((n+1)\tau) &= S(n\tau^{+})\times \\ &\quad \exp\left(\int_{n\tau}^{(n+1)\tau}\left(r - \frac{r\theta}{K}(\widetilde{I}(t)-\varepsilon) - \beta(\widetilde{I}(t)-\varepsilon)^{q}\right)\mathrm{d}t\right) \\ &= (1-\mu_1-\mu_2)S(n\tau)\times \\ &\quad \exp\left(\int_{n\tau}^{(n+1)\tau}\left(r - \frac{r\theta}{K}(\widetilde{I}(t)-\varepsilon) - \beta(\widetilde{I}(t)-\varepsilon)^{q}\right)\mathrm{d}t\right) \\ &= S(n\tau)\delta. \end{aligned} \tag{3.5}$$

Thus, $S(n\tau) \leq S(0)\delta^n$ and $S(n\tau) \to 0$ as $n \to \infty$. Therefore, $S(t) \to 0$ as $t \to \infty$, since $0 < S(t) < (1-\mu_1-\mu_2)S(n\tau)\exp(r\tau)$ for $n\tau < t \leq (n+1)\tau$.

Next, we prove that $I(t) \to \widetilde{I}(t)$ as $t \to \infty$, for a sufficiently small $0 < \varepsilon < \dfrac{d}{\beta M^{q-1}}$, there exists a $T_1 > 0$ such that $0 < S(t) < \varepsilon$ for all $t > T_1$. From system (1.2), we have

$$\begin{cases} \dot{I}(t) \leq (\beta\varepsilon M^{q-1} - d)I(t), & t \neq n\tau, \\ \Delta I(t) \leq \mu_1\varepsilon - \mu_3 I(t) + p, & t = n\tau, \end{cases} \tag{3.6}$$

considering the following comparison system

$$\begin{cases} \dot{y}(t) = (\beta\varepsilon M^{q-1} - d)y(t), & t \neq n\tau, \\ \Delta y(t) = \mu_1\varepsilon - \mu_3 y(t) + p, & t = n\tau. \end{cases} \tag{3.7}$$

By Lemma 2.4, system (3.7) has a positive periodic solution

$$\widetilde{y}(t) = \frac{(\mu_1\varepsilon + p)\exp(-(d-\beta\varepsilon M^{q-1})(t-n\tau))}{1-(1-\mu_3)\exp(-(d-\beta\varepsilon M^{q-1})\tau)}, \quad n\tau < t \leq (n+1)\tau,$$

which is globally asymptotically stable. Thus, for a sufficiently small ε_1, there exists a $T_2 > T_1 > 0$ such that $t > T_2$

$$I(t) \leq y(t) < \widetilde{y}(t) + \varepsilon_1. \tag{3.8}$$

Combining (3.3) and (3.8), we obtain $\widetilde{I}(t) - \varepsilon < I(t) < \widetilde{y}(t) + \varepsilon_1$ for t large enough, let $\varepsilon, \varepsilon_1 \to 0$, we get $\widetilde{y}(t) \to \widetilde{I}(t)$, then $I(t) \to \widetilde{I}(t)$ as $t \to \infty$. This completes the proof.

4 Permanence

Theorem 4.1. *System* (1.2) *is uniformly permanent if*

$$r\tau - \frac{pr\theta(1-\exp(-d\tau))}{dK[1-(1-\mu_3)\exp(-d\tau)]} - \frac{p^q\beta(1-\exp(-qd\tau))}{2d[1-(1-\mu_3)\exp(-d\tau)]} > -\ln(1-\mu_1-\mu_2). \tag{4.1}$$

Proof. Suppose $x(t) = (S(t), I(t))$ is a solution of (1.2) with $x(0) > 0$, from Lemma 2.5, we may assume $S(t) \leq M, I(t) \leq M$ and $M > (r/\beta)^{\frac{1}{q}}$, for t large enough.

Let $m_2 = \dfrac{p\exp(-d\tau)}{1-(1-\mu_3)\exp(-d\tau)} - \varepsilon_2$, where $\varepsilon_2 > 0$ sufficiently small.

According to Lemmas 2.2 and 2.4, we have $I(t) > m_2$ for t large enough. So, if we can find positive number $\overline{m}_1 > 0$, such that $S(t) > \overline{m}_1$ for t large enough, then our aim is obtained.

Next, we will do it in the following two steps for convenience.

Step I: If (4.1) holds true, we can choose $0 < m_1 < \dfrac{d}{\beta M^{q-1}}$ and ε_3 small enough such that

$$\delta_1 = (1-\mu_1-\mu_2)\times$$
$$\exp\left(\int_{n\tau}^{(n+1)\tau}\left(r - \frac{rm_1}{k} - \frac{r\theta}{k}(\widetilde{I}(t)+\varepsilon_3) - \beta(\widetilde{I}(t)+\varepsilon_3)^q\right)dt\right) > 1,$$

we will prove there exist a $t_1 \in (0,\infty)$, such that $S(t_1) \geq m_1$. Otherwise $S(t) < m_1$ for all $t > 0$. From system (1.2), we obtain that

$$\begin{cases} \dot{I}(t) \leq (\beta m_1 M^{q-1} - d)I(t), & t \neq n\tau, \\ \Delta I(t) \leq \mu_1 m_1 - \mu_3 I(t) + p, & t = n\tau, \end{cases} \tag{4.2}$$

consider the following comparison system

$$\begin{cases} \dot{z}(t) = (\beta m_1 M^{q-1} - d)z(t), & t \neq n\tau, \\ \Delta z(t) = \mu_1 m_1 - \mu_3 z(t) + p, & t = n\tau, \end{cases} \tag{4.3}$$

by Lemmas 2.2 and 2.4 on (4.3), we have $z(t) \to \tilde{z}(t)$ as $t \to \infty$, where

$$\tilde{z}(t) = \frac{(\mu_1 m_1 + p)\exp(-(d - \beta m_1 M^{q-1})(t - n\tau))}{1 - (1 - \mu_3)\exp(-(d - \beta m_1 M^{q-1}))}, \ n\tau < t \le (n+1)\tau.$$

Therefore, there exists a $T_3 > 0$ such that

$$I(t) \le z(t) < \tilde{z}(t) + \varepsilon,$$

for $t > T_3$. Thus

$$\begin{cases} \dot{S}(t) \ge S(t)(r - \frac{rm_1}{K} - \frac{r\theta}{K}(\tilde{I}(t) + \varepsilon) - \beta(\tilde{I}(t) + \varepsilon)^q), & t \ne n\tau, \\ \Delta S(t) = -(\mu_1 + \mu_2)S(t), & t = n\tau, \end{cases} \tag{4.4}$$

for $t > T_3$, integrating (4.4) on $(n\tau, (n+1)\tau]$, $n \ge N_1$, here N_1 is a nonnegative integer and $N_1\tau \ge T_3$, then we obtain

$$\begin{aligned} S((n+1)\tau) &\ge S(n\tau)(1 - \mu_1 - \mu_2) \\ &\quad \times \exp\left(\int_{n\tau}^{(n+1)\tau} \left(r - \frac{rm_1}{K} - \frac{r\theta}{K}(\tilde{I}(t) + \varepsilon) - \beta(\tilde{I}(t) + \varepsilon)^q\right) \mathrm{d}t\right) \\ &= S(n\tau)\delta_1. \end{aligned}$$

Then $S((N_1 + k)\tau) \ge S(N_1\tau)\delta_1^k \to \infty$, $k \to \infty$, which is a contradiction to $S(t) < m_1$ for all $t > 0$. Hence there exists a t_1 such that $S(t_1) \ge m_1$.

Step II: If $S(t) \ge m_1$ for all $t \ge t_1$, then our aim is obtained. Otherwise $S(t) < m_1$ for some $t \ge t_1$, setting $t^* = \inf_{t>t_1}\{S(t) < m_1\}$, there are the following two cases for t^*:

Case (a): If $t^* = n_1\tau, n_1$ is some positive integer. In this case $S(t) \ge m_1$ for $t \in [t_1, t^*)$ and $(1 - \mu_1 - \mu_2)S(t^{*^+}) = (1 - \mu_1 - \mu_2)S(t^*) < m_1$. Let $T_4 = n_2\tau + n_3\tau$, where $n_2 = n_2' + n_2''$, n_2', n_2'' and n_3 satisfy the following inequalities:

$$n_2'\tau > \frac{1}{\beta m_1 M^{q-1} - d} \ln \frac{\varepsilon_3}{(\mu_1 m_1 + p + M)(1 - \mu_3)},$$

$$(1 - \mu_1 - \mu_2)^{n_2} \exp(\eta n_2 \tau)\delta_1^{n_3} > 1,$$

where

$$\eta = r - \frac{rm_1}{K} - \frac{r\theta}{K}M - \beta M^q < 0.$$

We claim that there must be a time $t_2 \in (t^*, t^* + T_4)$ such that $S(t_2) \geq m_1$, if it is not true, i.e., $S(t) < m_1, t \in (t^*, t^* + T_4)$, similar to the analysis before, we consider system (4.3) with initial value $z(t^{*^+}) = I(t^{*^+}) \geq 0$, by Lemma 2.4, we have

$$\begin{aligned} z(t) &= (1-\mu_3)(z(t^{*^+}) - \frac{p+\mu_1 m_1}{1-(1-\mu_3)\exp(-(d-\beta m_1 M^{q-1})\tau)}) \\ &\quad \times \exp(-(d-\beta m_1 M^{q-1})(t-t^*)) + \tilde{z}(t) \end{aligned}$$

for $t \in (n\tau, (n+1)\tau],\ n_1 \leq n \leq n_1 + n_2 + n_3$. Then

$$|z(t) - \tilde{z}(t)| < (1-\mu_3)(M+p+\mu_1 m_1)\exp(-(d-\beta m_1 M^{q-1})(t-n_1\tau)) < \varepsilon_3,$$

and $I(t) \leq z(t) < \tilde{z}(t) + \varepsilon_3$ for $t^* + n_2' \leq t \leq t^* + T_4$, which implies that system (4.4) holds for $[t^* + n_2\tau, t^* + T_4]$, integrating system (4.4) on this interval, we have

$$S((n_1+n_2+n_3)\tau) \geq S((n_1+n_2)\tau)\delta_1^{n_3}. \tag{4.5}$$

In addition, we have

$$\begin{cases} \dot{S}(t) \geq S(t)\left(r - \dfrac{rm_1}{K} - \dfrac{r\theta}{K}M - \beta M^q\right) = \eta S(t), \\ \Delta x_1(t) = (1-\mu_1-\mu_2)x_1(t). \end{cases} \tag{4.6}$$

Integrating system (4.6) on the interval $[t^*, (n_1+n_2)\tau]$, which yields

$$S((n_1+n_2)\tau) \geq m_1(1-\mu_1-\mu_2)^{n_2}\exp(\eta n_2\tau), \tag{4.7}$$

combining (4.5) and (4.7), we have

$$S((n_1+n_2+n_3)\tau) \geq m_1(1-\mu_1-\mu_2)^{n_2}\exp(\eta n_2\tau)\delta_1^{n_3} > m_1,$$

which is a contradiction, so there exists a time $t_2 \in [t^*, t^* + T_4]$ such that $S(t_2) > m_1$, let $\hat{t} = \inf_{t \geq t^*}\{S(t) \geq m_1\}$, since $0 < \mu_1 + \mu_2 < 1$, $S(n\tau^+) = (1-\mu_1-\mu_2)S(n\tau) < S(n\tau)$ and $S(t) < m_1,\ t \in (t^*, \hat{t})$. Thus, $S(\hat{t}) = m_1$, suppose $t \in (t^* + (l-1)\tau, t^* + l\tau] \subset (t^*, \hat{t}]$, l is a positive integer and $l \leq n_2 + n_3$, from system (4.6), we have

$$\begin{aligned} S(t) &\geq (1-\mu_1-\mu_2)^l m_1 \exp(l\eta\tau) \\ &\geq (1-\mu_1-\mu_2)^{n_2+n_3} \exp((n_2+n_3)\tau) \triangleq \overline{m}_1 \end{aligned}$$

for $t > \hat{t}$. The same arguments can be continued since $S(\hat{t}) \geq m_1$. Hence $S(t) \geq \overline{m}_1$ for all $t > t_1$.

Case (b): If $t^* \neq n\tau$, then $S(t^*) = m_1$ and $S(t) \geq m_1,\ t \in [t_1, t^*]$, suppose $t^* \in (n_1'\tau, (n_1'+1)\tau]$, we also have two subcases for $t \in [t^*, (n_1'+1)\tau]$ as follows:

Case (i): $S(t) \leq m_1,\ t \in [t^*, (n_1'+1)\tau]$, we claim that there exists a $t_2' \in [n_1'\tau, (n_1'+1)\tau + T_4]$ such that $S(t_2') > m_1$. Otherwise integrating system (4.6) on the interval $[(n_1'+1+n_2)\tau, (n_1'+1+n_2+n_3)\tau]$ produces

$$S\big((n_1'+1+n_2+n_3)\tau\big) \geq S\big((n_1'+1+n_2)\tau\big)\delta_1^{n_3}.$$

Since $S(t) \leq m_1, t \in [t^*, (n_1'+1)\tau]$, system (4.7) holds on $[t^*, (n_1'+1+n_2+n_3)\tau]$, thus

$$\begin{aligned} S\big((n_1'+1+n_2)\tau\big) &= S(t^*) \exp\Big(\eta\big((n_1'+1+n_2)\tau - t^*\big)(1-\mu_1-\mu_3)^{n_2}\Big) \\ &\geq m_1(1-\mu_1-\mu_3)^{n_2} \exp(\eta n_2\tau) \end{aligned}$$

and

$$S\big((n_1'+1+n_2+n_3)\tau\big) \geq m_1(1-\mu_1-\mu_3)^{n_2+n_3} \exp(\eta n_2\tau)\delta_1^{n_3} > m_1,$$

which is a contradiction. Let $\check{t} = \inf_{t>t^*}\{S(t) \geq m_1\}$, then $S(\check{t}) = m_1$ and $S(t) < m_1$ for $t \in (t^*, \check{t})$. Choose $t \in (n_1'\tau + (l'-1)\tau, n_1' + l'\tau] \subset (t^*, \check{t})$, l' is a positive integer and $l' < 1 + n_2 + n_3$, we have

$$\begin{aligned} S(t) &\geq ((n_1'+l'-1)\tau^+) \exp(\eta(t-(n_1'+l'-1)\tau)) \\ &\geq (1-\mu_1-\mu_3)^{l'-1} S(t^*) \exp(\eta(t-t^*)) \\ &\geq (1-\mu_1-\mu_3)^{n_2+n_3} \exp((n_2+n_3+1)\eta\tau), \end{aligned}$$

so we have $S(t) \geq \overline{m}_1$ for $t \in (t^*, \check{t})$. For $t > \check{t}$, the same argument can be continued since $S(\check{t}) \geq m_1$.

Case (ii): If there exists a $t \in (t^*, (n_1' + 1)\tau]$ such that $S(t) \geq m_1$. Let $\bar{t} = \inf_{t>t^*}\{S(t) \geq m_1\}$, then $S(t) < m_1$ for $t \in [t^*, \bar{t})$ and $S(\bar{t}) = m_1$. For $t \in [t^*, \bar{t})$, (4.6) holds and integrating (4.6) on $[t^*, \bar{t})$, we have

$$S(t) \geq S(t^*)\exp(\eta(t - t^*)) \geq m_1 \exp(\eta\tau) > \overline{m}_1.$$

Since $S(\bar{t}) \geq m_1$ for $t > \bar{t}$, the same argument can be continued. Hence, we have $S(t) > \overline{m}_1$ for all $t > t_1$. Thus in both cases, we conclude $S(t) \geq m_1$ for all $t \geq t_1$. The proof is complete.

Remark 1. Let

$$\begin{aligned} f(\tau) &= r\tau - \frac{pr\theta(1-\exp(-d\tau))}{dK[1-(1-\mu_3)\exp(-d\tau)]} - \frac{p^q\beta(1-\exp(-qd\tau))}{qd[1-(1-\mu_3)\exp(-d\tau)]^q} \\ &\quad - \ln(1-\mu_1-\mu_2). \end{aligned}$$

Since $f(0) = -\ln\frac{1}{1-\mu_1-\mu_2}$, $f(\tau) \to +\infty$ as $\tau \to \infty$ and $f'(\tau) > 0$, so $f(\tau) = 0$ has a unique positive root, denoted by $\tau_{\max}$. From Theorem 3.1 and Theorem 4.1, we know that the pest-eradication periodic $(0, \widetilde{I}(t))$ is globally asymptotically stable when $\tau < \tau_{\max}$. If $\tau > \tau_{\max}$, the system (1.2) is permanence.

Remark 2. If $\mu_1 = \mu_2 = \mu_3 = 0$, that is, we only choose the biological control, we can obtain that τ_0 is the threshold and $\tau_{\max} > \tau_0$, which implies that we must release more infected pest to eradicate the pests. If $p = 0$, that is there is no periodic releasing infective pests, so we can easily obtain that $\tau_1 = -\frac{1}{r}\ln(1-\mu_1-\mu_2)$ is the threshold and $\tau_{\max} > \tau_1$, it is obviously, impulsive releasing pests may lengthen the period of spraying pesticides and therefore reduce the cost of pests control.

5 Numerical analysis and conclusion

In this paper, we have investigated dynamical behaviors of an SI model with impulsive transmitting infected pests and spraying pesticides at fixed moment. The purpose of this paper is the behavior of an impulsively controlled integrated pest management model. To limit the damaging potential of the pest population, a biological control, consisting in the release of infective pests, and a

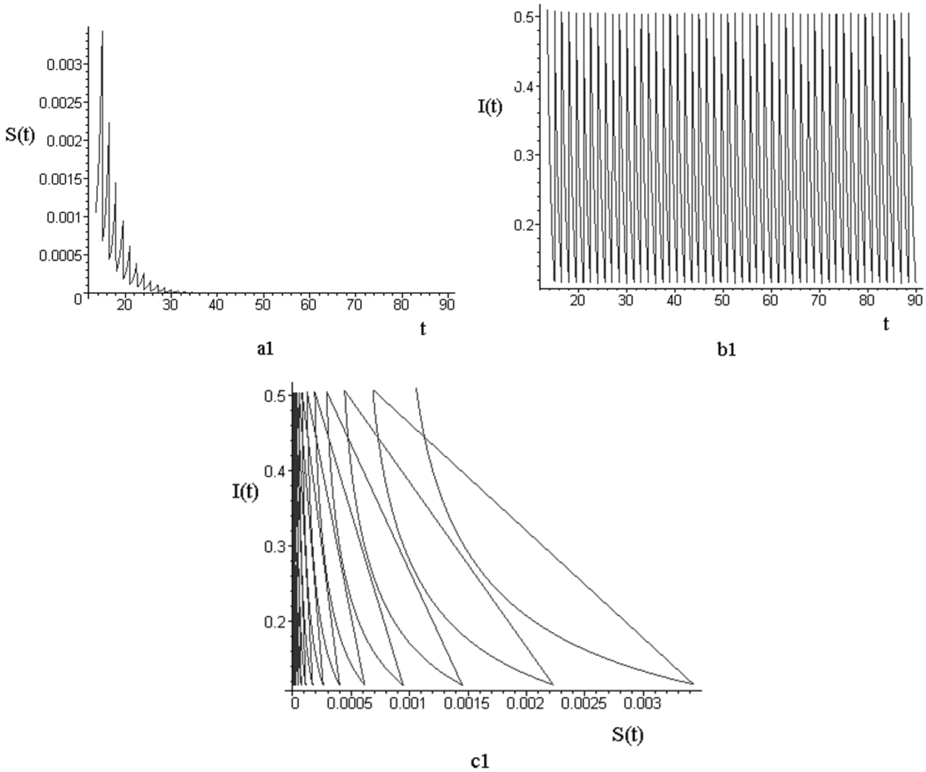


Figure 1 – Pest-eradication solution of system (1.2) which is globally asymptotically stable when $\tau = 1.5 < \tau_{\max}$. (a1) Time series of $S(t)$ in (1.2) with initial value (0.1,0.1), (b1) Time series of $I(t)$ in (1.2) with initial value (0.1,0.1), (c1) Phase portraits in (1.2) with initial value (0.1,0.1).

chemical control, consisting in pesticide spraying, are applied in pest management. An unspecified nonlinear force of infection is assumed to describe the transmission of the disease which is spread through the release of infected individuals, and it is assumed that the infective pest population neither damages the crops, nor reproduces. We have shown that there exists an asymptotically stable the susceptible pest-eradication periodic solution if impulsive period is less than some threshold. When the stability of pest-eradication periodic solution is lost, system (1.2) is permanent, which is in line with reality from a biological point of view. Numerical results show that system (1.2) can take on various kinds of periodic fluctuations, which implies that the presence of pulse makes the dynamic behavior more complex (see Fig. 3).

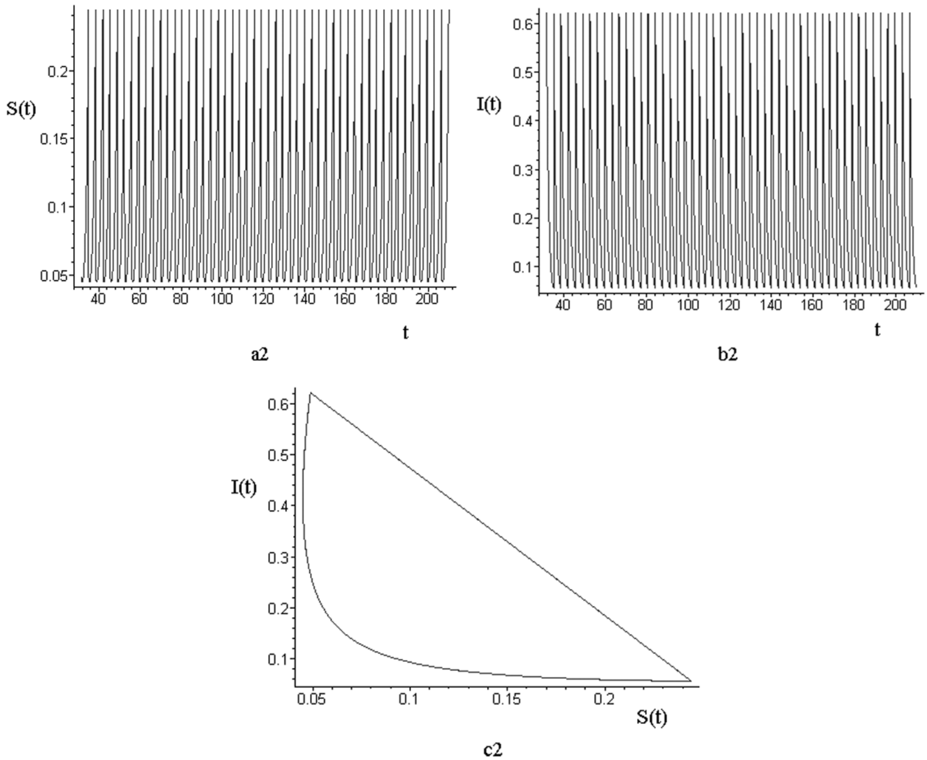
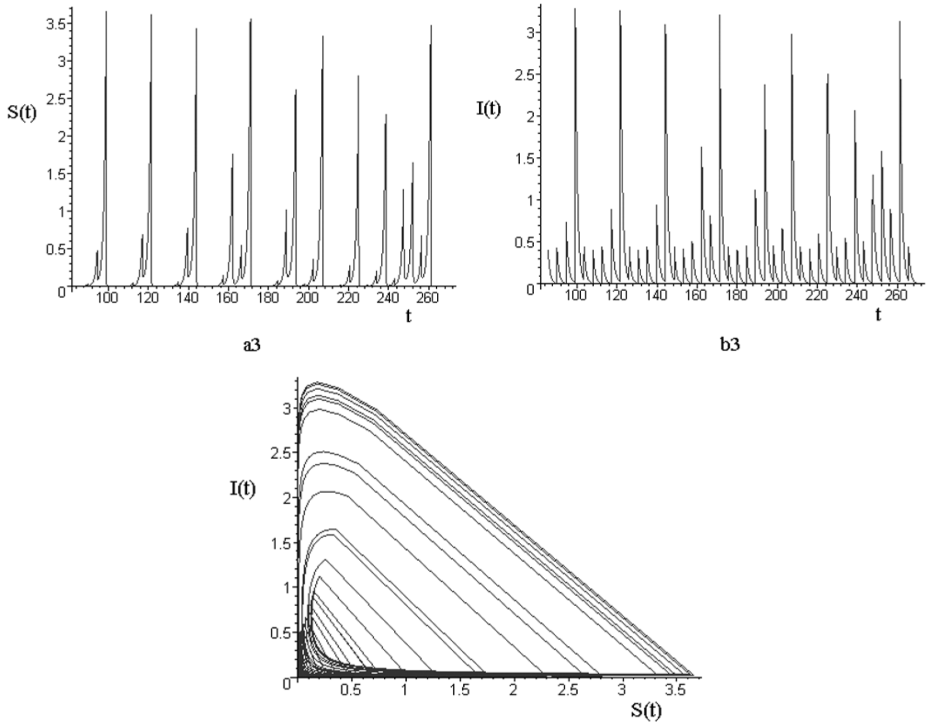


Figure 2 – Dynamical behavior of system (1.2) with $\tau = 3.5 > \tau_{\max}$. (a2) Time series of $S(t)$ in (1.2) with initial value (0.1,0.1), (b2) Time series of $I(t)$ in (1.2) with initial value (0.1,0.1), (c2) A τ-periodic solution.

It is observed that, theoretically speaking, the control strategy can be always made to succeed by the use of proper pesticides, while as far as the biological control is concerned, its sufficient effectiveness can also be reached provided that the numbers μ_i $(i = 1, 2, 3)$ of infected pests released each time or the period τ is proper, that is, from Theorem 3.1 and Theorem 4.1, we know that the pest-eradication periodic $(0, \widetilde{I}(t))$ is globally asymptotically stable when $\tau < \tau_{\max}$ (see Fig. 1). If $\tau > \tau_{\max}$, the system (1.2) is permanence (see Fig. 2). Any of these features alone can ensure the global success of our control strategy, although in concrete situations these may or may not be biologically feasible or may require a large amount of resources.

To facilitate the interpretation of our mathematical findings by numerical analysis, we consider the hypothetical set of parameter values as $r = 1$,



c3

Figure 3 – Dynamical behavior of system (1.2) with $\tau = 4.5 > 2\tau_{\max}$, (a3) Time series of $S(t)$ in (1.2) with initial value (0.1,0.1), (b3) Time series of $I(t)$ in (1.2) with initial value (0.1,0.1), (c3) Phase portraits in (1.2) with initial value (0.1,0.1).

$\theta = 0.91,\ \beta = 2,\ d = 0.98,\ q = 1.8,\ p = 0.4,\ \mu_1 = 0.7,\ \mu_2 = 0.1,\ \mu_3 = 0.1,\ \tau_{\max} = 1.838045.$

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