Dynamical interpretation of conditional patterns

By R. J. Adrian[1], & R. D. Moser[2], P. Moin[2,3]

1. Background

While great progress is being made in characterizing the three-dimensional structure of organized turbulent motions using conditional averaging analysis, there is a lack of theoretical guidance regarding the interpretation and utilization of such information. Questions concerning the significance of the structures, their contributions to various transport properties, and their dynamics cannot be answered without recourse to appropriate dynamical governing equations. One approach which addresses some of these questions uses the conditional fields as initial conditions and calculates their evolution from the Navier-Stokes equations, yielding valuable information about stability, growth, and longevity of the mean structure. To interpret *statistical* aspects of the structures, we need a different type of theory which deals with the structures in the context of their contributions to the statistics of the flow. As a first step toward this end, an effort has been made to integrate the structural information from the study of organized structures with a suitable statistical theory. This is done along the lines of Adrian (1977) by stochastically estimating the two-point conditional averages that appear in the equation for the one-point probability density function, and relating the structures to the conditional stresses. Salient features of the estimates are identified, and the structure of the one-point estimates in channel flow is defined.

2. One-point probability density

The equation governing f, the probability density function defined by

$$f_1(\mathbf{v},\mathbf{x},t)d\mathbf{v} = \text{Prob} \quad \mathbf{v} \leq \tilde{\mathbf{u}}(\mathbf{x},t) < \mathbf{v} + d\mathbf{v} \tag{1}$$

is

$$\frac{\partial f_1}{\partial t} = -\frac{\partial}{\partial v_i}\left\{ \left\langle \frac{\partial \tilde{u}_i}{\partial t}\middle|\mathbf{v}\right\rangle f_1(\mathbf{v},\mathbf{x},t)\right\} \tag{2}$$

where

$$\tilde{\mathbf{u}} = \mathbf{U} + \mathbf{u}, \quad <\tilde{\mathbf{u}}> = \mathbf{U} \tag{3}$$

1 University of Illinois
2 NASA/Ames Research Center
3 Stanford University

and u are respectively the total, mean, and fluctuating velocities, and v is a dummy variable for the p.d.f. (Lundgren 1967). The fluid dynamics is contained entirely within the conditional average of the time derivative which is given in terms of the Navier-Stokes equations by

$$\left\langle \frac{\partial \tilde{u}_i}{\partial t} \middle| \mathbf{v} \right\rangle = -\left\langle \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} \middle| \mathbf{v} \right\rangle - \left\langle \frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x_i} \middle| \mathbf{v} \right\rangle + \nu \left\langle \frac{\partial^2 \tilde{u}_i}{\partial x_l \partial x_l} \middle| \mathbf{v} \right\rangle \tag{4}$$

We can relate the p.d.f. equation to organized structures by observing that the two-point conditional average

$$< \tilde{\mathbf{u}}(x',t) | \tilde{\mathbf{u}}(x,t) = \mathbf{v} > = < \mathbf{u}' | \mathbf{v} > \tag{5}$$

is a good descriptor of organized structure, coinciding in most cases with structures found by other methods of conditional analysis and flow visualization (Adrian and Moin 1988), and that we can rewrite (4) in terms of $< \mathbf{u}' | \mathbf{v} >$ as follows:

$$\left\langle \frac{\partial \tilde{u}_i}{\partial t} \middle| \mathbf{v} \right\rangle = \lim_{x' \to x} \left\{ -v_j \frac{\partial}{\partial x_{j'}} < u_i' | \mathbf{v} > -\frac{1}{\rho} \frac{\partial}{\partial x_{i'}} < \tilde{p}' | \mathbf{v} > +\nu \frac{\partial^2}{\partial x_l' \partial x_l'} < \tilde{u}_i' | \mathbf{v} > \right\} \tag{6}$$

where $< \tilde{p}' | v >$ satisfies a Poisson equation:

$$\nabla'^2 < \tilde{p}' | \mathbf{v} > = -\rho \frac{\partial^2}{\partial x_i' \partial x_j'} < \tilde{u}_i' \tilde{u}_j' | \mathbf{v} >$$

$$= -\rho \frac{\partial^2}{\partial x_i' \partial x_j'} \{ U_i' U_j' + U_i' < u_j' | \mathbf{c} > + U_j' < u_i' | \mathbf{c} > + < u_i' u_j' | \mathbf{c} > \} \tag{7}$$

In (6) $< \mathbf{u}' | \mathbf{v} >$ has been decomposed into the mean plus the conditional average of the fluctuation:

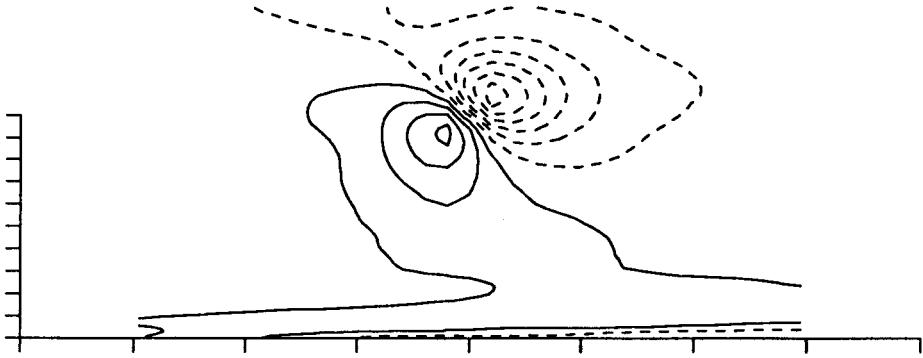$$< \tilde{\mathbf{u}}' | \mathbf{v} > = \mathbf{U}' + < \mathbf{u}' | \mathbf{c} > \tag{8}$$

where $\mathbf{c} = \mathbf{v} - \mathbf{U}$ is a dummy fluctuation variable. Equations (6) and (7) show that the conditional eddy $< \mathbf{u}' | \mathbf{c} >$ enters each stress term of the Navier-Stokes equations and the convective term.

Let us approximate $< \mathbf{u}' | \mathbf{c} >$ by a linear stochastic estimate (Adrian 1977):

$$< u_i' | \mathbf{c} > = L_{ij}(\mathbf{x}, \mathbf{x}') c_j \tag{9}$$

where L_{ij} is found from (Adrian, Moin & Moser 1987)

$$< u_l u_j > L_{ij} = < u_i' u_l >, \quad l = 1, 2, 3 \tag{10}$$

(a) FIGURE 1. Spanwise vorticity in the x-y plane of the stochastic estimate given a Q2 Reynolds stress event at $y^+ = 180$ (centerline) of the turbulent channel flow. The vertical axis is y and the end of the axis corresponds to the channel centerline. The contour increment is 1, and negative contours are dashed.

Substitution of (9) into the convective and viscous terms of (6) shows that the structure enters the dynamical statement in (6) through the following quantities:

$$L_{ij,k}(\mathbf{x},\mathbf{x}) = \lim_{\mathbf{x'}\to\mathbf{x}} \frac{\partial}{\partial x'_k} L_{ij}(\mathbf{x},\mathbf{x'}) \tag{11}$$

$$L_{ij,kk}(\mathbf{x},\mathbf{x}) = \lim_{\mathbf{x'}\to\mathbf{x}} \frac{\partial^2}{\partial x'_k \partial x'_k} L_{ij}(\mathbf{x},\mathbf{x'}) \tag{12}$$

These variables depend upon gradients of the two-point spatial correlation, $< u'_i u_l >$ which are *local* in nature ($\mathbf{x'} \to \mathbf{x}$). Thus, the only important feature of the coherent structure, in this approximation, is the gradient close to the center of the structure. We propose in future to study the behavior of $L_{ij,k}$ and $L_{ij,kk}$ for channel flow.

The *non-local* structure of $< \mathbf{u'}|\mathbf{c} >$ appears in (7), showing that the geometry and size of coherent structures will be important in determining closure approximation of the fast pressure-strain terms. The conditional Reynolds stress term in (7) determines the slow pressure component. It constitutes a separate closure problem for the p.d.f. equation.

3. Structure of the one-point conditional eddy

The structure of the linearly estimated conditional eddy $\breve{u}'_i = L_{ij}u_j(\mathbf{x})$ has been determined from the channel flow data base of Kim, Moin & Moser (1987), using the R_{ij} tensor computer by Moin & Moser (1989). The conditional velocity, $\mathbf{u}(\mathbf{x})$, was prescribed at $y^+ = 5, 20, 30, 49, 103$, and 180 for the maximum Reynolds stress events defined by Moin, Adrian, and Kim (1987). The flow patterns for the second quadrant event at three levels are shown in Figures 1-3. Note that the mean velocity and vorticity are *not* included in these figures. For the conditions prescribed far
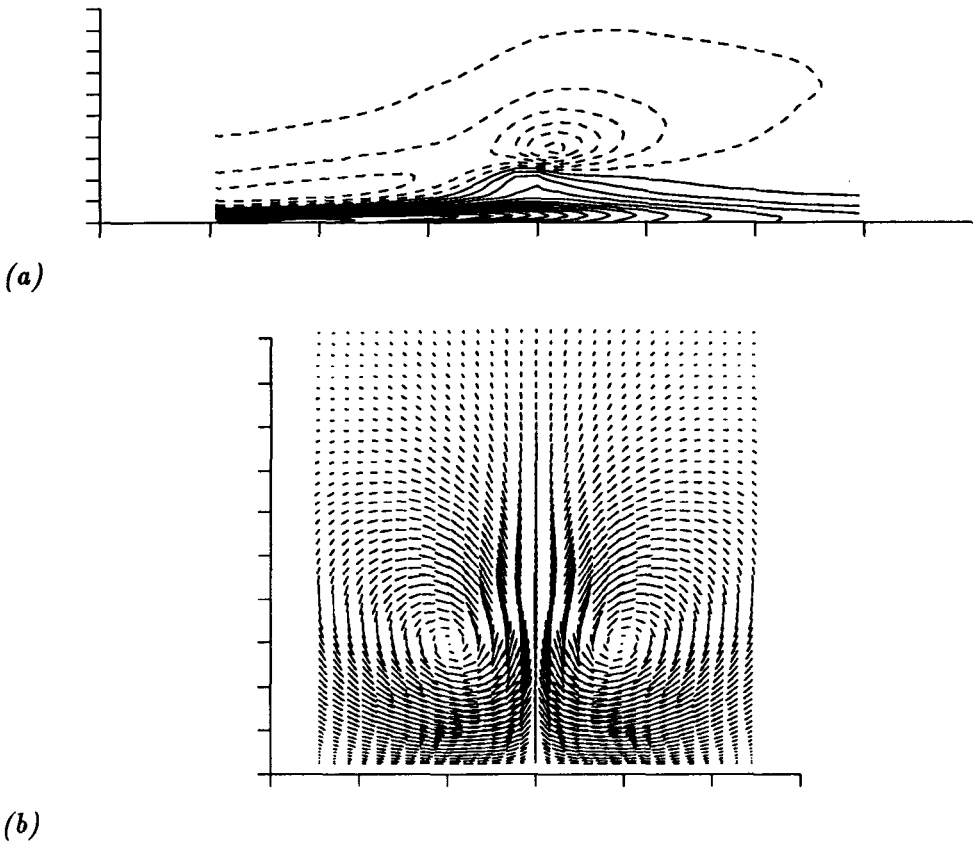
(a)

(b)

FIGURE 2.   Spanwise vorticity (*a*) in the *x*-*y* plane and velocity vectors (*b*) in the *z*-*y* plane of the stochastic estimate given a Q2 Reynolds stress event at $y^+ = 49$ of the turbulent channel flow. The vertical axis is *y* and the end of the axis corresponds to the channel centerline. In (*a*) the contour increment is 2, and negative contours are dashed.

above the wall ($y^+ = 180$) the eddy is nearly a vortex ring inclined at an angle of approximately 45° to the flow, as determined by the direction of u(x). The ring is centered on x and it pumps fluid through its center so as to create large positive *v* and negative *u*, as explained in Adrian and Moin (1988). Note that addition of the mean velocity to the fluctuating field in Figure 1 may result in a structure resembling a hairpin vortex rather than a ring.

When the conditions are prescribed closer to the wall, $y^+ = 49$, the fluctuating flow pattern looks more like a hairpin vortex (Figure 2). The head of this hairpin is similar to the top half of the vortex ring in Figure 1, but the bottom looks like two streamwise oriented vortices. Their circulations are shown clearly in Figure 2(b), whose plane passes through the conditional point x.

When the conditions are prescribed very close to the wall, $y^+ = 5$, the v-component of velocity is strongly suppressed, and the flow of the conditional eddy is essentially two streamwise vortices, c.f. Figure 3(b). These vortices lift low speed fluid up from the wall and bring it back down toward the wall, as shown by the v-contours in Figure 3(c). They are elongated in the x-direction, but not so much as to be two-dimensional.

It must be emphasized that the structure described above should not be construed as the structure of the instantaneous flow. Certainly it possesses many features of the organized structures in the flow field, but it is a smoothed picture which is also symmetrized due to the symmetries of the spatial correlation function. There is no implication, for example, that the instantaneous structures always occur with pairs of streamwise vortices, and, in fact, observations of instantaneous events suggest that double vortices are more rare than singles. However, the symmetry and smoothness of the conditional eddy is acceptable if it is interpreted in the context of the f_1-equation, since this equation deals, inherently, with the conditional field, and it does not require that they be perfectly faithful reproductions of the instantaneous realizations.

4. Conclusions

Coherent structures, as described by the conditional averages $< \mathbf{u}'|\mathbf{c} >$, contain structural information needed to close the p.d.f. equations. The form of the structure is expected to influence the behavior of the p.d.f., and hence the behavior of moment equation models that are derivable, in principle, from the solutions of the p.d.f. equation. Ultimately, this structure must be manifested in the values of coefficients that appear in closure models of the moment equations. More work is needed to assess the details of the way that structure feeds into the p.d.f. equations.

Acknowledgement

REFERENCES

ADRIAN, R. J. 1979 Conditional Eddies in Isotropic Turbulence. *Phys. Fluids.* **22**, 2065-2070.

ADRIAN, R. J., & MOIN, P. 1988 Stochastic Estimation of Organized turbulence Structure: Homogeneous Shear Flow. *J. Fluid Mech.* **190**, 531-559.

MOIN, P., ADRIAN, R.J., & KIM, J. 1987 Stochastic Estimation of Organized Structures in Turbulent Channel Flow. *6th Turbulent Shear Flow Symposium, Toulouse, France, Sept. 7-9, 1987.*
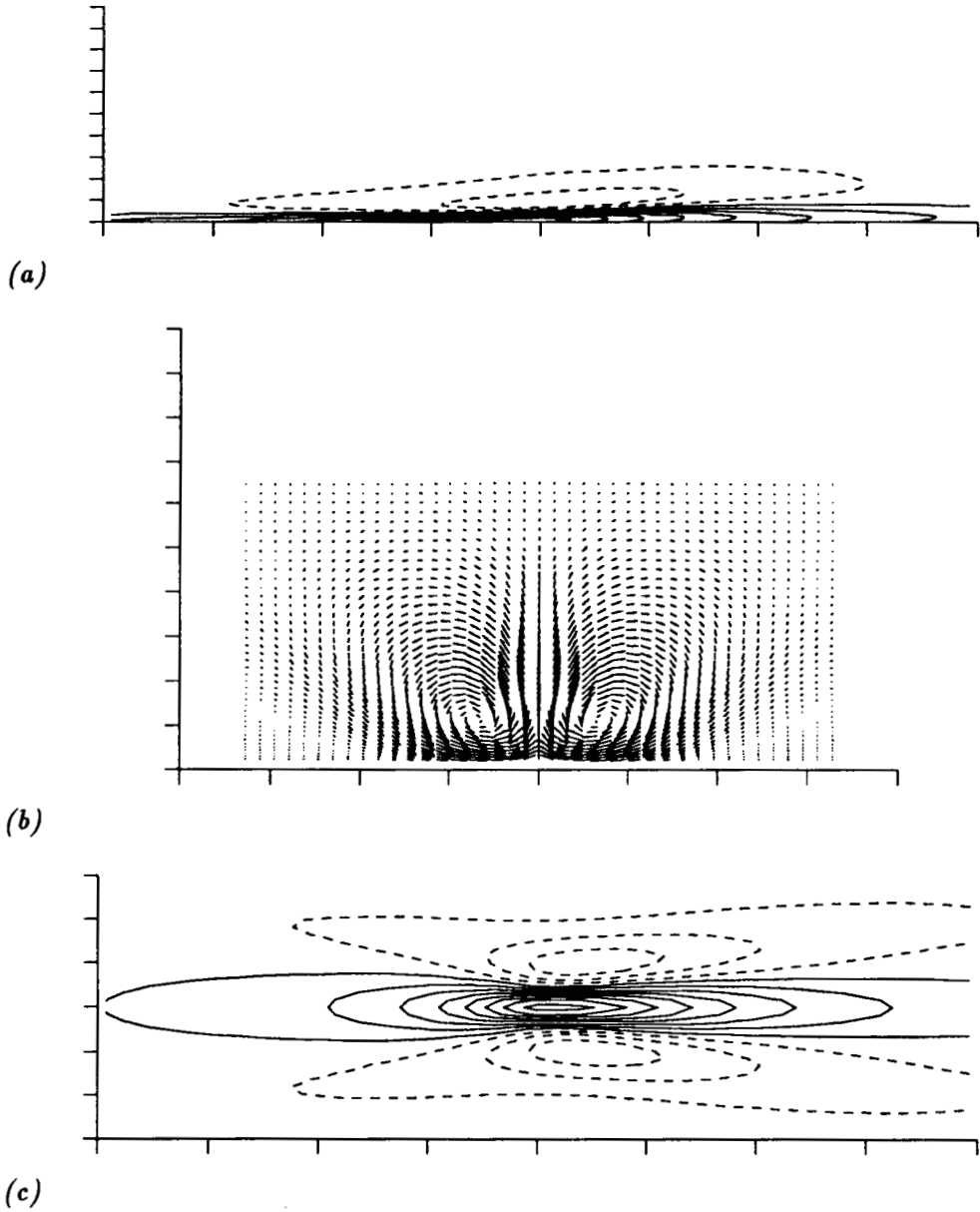
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(a)

(b)

(c)

FIGURE 3.   Spanwise vorticity (a) in the x-y plane, velocity vectors (b) in the z-y plane and normal velocity in the x-z plane of the stochastic estimate given a Q2 Reynolds stress event at $y^+ = 5$ of the turbulent channel flow. The vertical axis is y in (a) and (b), and the end of the axis corresponds to the channel centerline. The vertical axis is z in (c). The contour increment is 10 in (a) and 0.01 in (c), and negative contours are dashed.

MOIN, P. & MOSER, R. D. 1989 Characteristic eddy decomposition of turbulence. To appear in *J. Fluid Mech.*

ADRIAN, R. J., MOSER, R. D., & MOIN, P 1987 Stochastic Estimation of Conditional Eddies, Center for Turbulence Research Report, *CTR-S87*.

LUNDGREN, T. S. 1967 Distribution Functions in the Statistical Theory of Turbulence. *Phys. Fluids.* **10**, 969.