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Dynamical Properties of Euclidean Solutions in a Multidimensional Cosmological Model

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In the framework of the Hartle-Hawking no-boundary proposal, we investigated quantum creation of the multidimensional universe with a cosmological constant (Λ) but without matter fields. We have found that the classical solutions of the Euclidean Einstein equations in this model have “quasi-attractors”, i.e., most trajectories on the a-b plane, where a and b are the scale factors of external and internal spaces, go around a point. It is presumed that the wave function of the universe has a hump near this quasi-attractor point. In the case that both the curvatures of external and internal spaces are positive, and $\Lambda > 0$, there exist Lorentzian solutions which start near the quasi-attractor, the internal space remains microscopic, and the external space evolves into our macroscopic universe.

§1. Introduction

In modern theories of unified physical interactions, spacetime has more than four dimensions. It is well known that Kaluza-Klein theory [1),2)] created interest in investigations in spacetime which has more than four dimensions. Superstring theories (e.g. Ref. 3)) are at the moment the most promising candidates for a unified description of the basic physical interactions. There are five anomaly-free, perturbative superstring theories. The critical dimensions of spacetime are ten for these theories. There is now evidence that the five superstring theories are related by duality symmetries. Furthermore, they are related to $N = 1$, $D = 11$ supergravity theory. It is conjectured that these theories are the limits of one theory, M-theory, in which spacetime has eleven dimensions.

Since spacetime has four large dimensions and some additional number of small and highly curved spatial dimensions, we can see only the four large dimensions. How does compactification of internal space take place? The first answer is through multidimensional cosmology. In general relativity, the geometry of spacetime is dynamical. The three-dimensional space we observe was once as small as the internal space, and expanded during evolution of the universe, while internal space contracted or has remained small during evolution of the universe. Therefore, internal space is microscopic and is not observable. This explanation is called dynamical compactification. The second explanation involves spontaneous symmetry breaking. Though

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spacetime would have $SO(D-1,1)$ for $D > 4$, after symmetry breaking only $SO(3,1)$ is visible, and internal space is static and at the Planck scale. This explanation is called spontaneous compactification. Lately, another possible mechanism of compactication by branes in string theory has been discussed.[4),5)] It is possible that the Standard Model gauge fields exist on branes rather than in the bulk of spacetime. If this is the case, the situation in which gravity exists in the bulk of the 10-dimensional spacetime, while the Standard Model particles exist on a 3-brane is possible. It remains unknown which mechanism of compactification explains our universe. In this paper we consider dynamical compactification only in the view of multidimensional cosmology.

The simplest multidimensional cosmological model that is useful for understanding compactification is the type of extended Friedmann-Robertson-Walker universe. The pioneering work for this model was done by Chodos and Detweiler.[6)] They found that the 5-dimensional vacuum Einstein equations possess the Kasner solution, which describes a universe in which the internal space shrinks, while the external space expands. Freund investigated the model in the case of $D = 11$ supergravity.[7)] Various multidimensional models were investigated, and the manner in which the existence of internal space might impact cosmological issues such as entropy production and inflation (e.g. Refs. 8)–10)) was discussed.

On the other hand, the creation of the universe is a problem of great interest. In the standard big-bang cosmology, the universe appeared from an initial singularity. When the density is greater than the Planck density, classical general relativity breaks down and quantum gravitational effects are large. Based on quantum gravitational theory, the scenario of quantum creation of the universe was suggested. Vilenkin asserted that the inflationary universe appears through quantum tunneling from nothing.[11)] Later Hartle and Hawking suggested the scenario of the creation of the universe through a different method.[12)] That method is called the Hartle-Hawking no-boundary proposal. According to this proposal the universe in the quantum era is Euclidean manifold without "boundary", and through analytic continuation to the Lorentzian manifold, the classical universe is created. The condition of analytic continuation gives the initial condition of the universe. Recently, a pre-big-bang scenario has been suggested.[13)] This scenario is based on string theory and has attractive features. However, it has a graceful exit problem. In this paper we consider quantum cosmology from the viewpoint of Hartle and Hawking.

The concept of quantum creation is extended to the multidimensional universe with consistency. Hu and Wu[14),15)] discussed the vacuum model in which the spacetime metric has the form $R \times S^3 \times S^n$ in the framework of the Hartle-Hawking no-boundary proposal, where R is the time, S^3 is the external space, and S^n is the internal space. They showed that the universe most probably evolves with an exponentially expanding external space and a static internal space. This solution implies that we observe 4-dimensional spacetime because internal spaces are compact and static at Planckian scales. More complicated multidimensional quantum cosmological models have been investigated (e.g. Refs. 16)–20), 25)).

Models based on unified theories, such as supergravity theories, which are the low energy limits of superstring theories, would be more realistic than the simplest vacuum model.[16),18),20)] However, it is thought that the essence of the creation of the universe may be included in the vacuum model, and it is useful for investigating more realistic models. Detailed analysis of the vacuum model is, therefore, important. Hu and Wu discussed only the instanton that nucleates the most probable universe, but the instanton solutions are unstable with respect to perturbations, and the Euclidean classical solutions near the instanton have effects on the wave function of the universe. Therefore we analyse instantons in more detail, and also investigate dynamical properties of the Euclidean Einstein equations of this model by numerical calculations. As shown in §3, the dynamical properties of the system are of interest, and it is found that the system has a "quasi-attractor".

The plan of the paper is as follows. In §2, we introduce the multidimensional cosmological model with cosmological constant (Λ) but without matter fields. In §3, we give the constraints on the signatures of the spatial curvature and the cosmological constant and analyze the Euclidean classical solutions. In §4, we investigate continuation from the Euclidean classical solutions to Lorentzian solutions, and discuss the scenario quantum creation of the universe. The last section is devoted to conclusions and remarks.

§2. The model

We consider a $D(=1+m+n)$-dimensional vacuum universe with a cosmological constant (Λ) but without matter fields. For cosmological purposes, we assume a metric of the form

$$\tilde{g}_{AB} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & a^2(t)g_{ij} & 0 \\ 0 & 0 & b^2(t)g_{IJ} \end{pmatrix}, \tag{2·1}$$

where $i,\ j = 1,\ 2,\ ...,\ m$ and $I,\ J = 1,\ 2,\ ...,\ n$. The variables a and b are the scale factors of m-dimensional and n-dimensional spaces, respectively. The gravitational action is described as

$$S = \frac{1}{16\pi\tilde{G}}\int d^Dx\sqrt{-\tilde{g}}[\tilde{R} - 2\Lambda] + S_{\rm boundary}, \tag{2·2}$$

where $\tilde{G}$ is the D-dimensional gravitational constant, and Λ is the cosmological constant. The last term $S_{\rm boundary}$ is the York-Gibbons-Hawking boundary term.[21),22)] Substituting the metric, the action is given by

$$\begin{aligned} S = \frac{1}{16\pi\tilde{G}}\int & dt d^mx d^nx\sqrt{g_m g_n}a^m b^n \\ & \times\left[-m(m-1)\left(\frac{\dot{a}}{a}\right)^2 - 2mn\frac{\dot{a}\dot{b}}{ab} - n(n-1)\left(\frac{\dot{b}}{b}\right)^2\right. \\ & \left. +m(m-1)\frac{k_m}{a^2} + n(n-1)\frac{k_n}{b^2} - 2\Lambda\right], \end{aligned} \tag{2·3}$$

where the dots denote derivatives with respect to time t, g_m and g_n are the determinants of the metrics g_{ij} and g_{IJ}, and k_m and k_n denote the signs of the curvature of external and internal space (1, 0 or -1), respectively. The momenta conjugate to the scale factors a and b are given by

$$\pi_a = \frac{\partial \mathcal{L}}{\partial \dot{a}} = -\frac{\sqrt{g_i g_e}}{16\pi\tilde{G}} a^m b^n \left[2m(m-1)\frac{\dot{a}}{a^2} + 2mn\frac{\dot{b}}{ab} \right], \tag{2·4}$$

$$\pi_b = \frac{\partial \mathcal{L}}{\partial \dot{b}} = -\frac{\sqrt{g_i g_e}}{16\pi\tilde{G}} a^m b^n \left[2mn\frac{\dot{a}}{ab} + 2n(n-1)\frac{\dot{b}}{b^2} \right]. \tag{2·5}$$

The Hamiltonian $\mathcal{H}$ is described as

$$\mathcal{H} = \pi_a \dot{a} + \pi_b \dot{b} - \mathcal{L} \tag{2·6}$$

$$= -\frac{\sqrt{g_i g_e}}{16\pi\tilde{G}} a^m b^n \left[m(m-1)\left(\frac{\dot{a}}{a}\right)^2 + 2mn\frac{\dot{a}\dot{b}}{ab} + n(n-1)\left(\frac{\dot{b}}{b}\right)^2 + V \right]. \tag{2·7}$$

The potential energy V is defined by

$$V = m(m-1)\frac{k_m}{a^2} + n(n-1)\frac{k_n}{b^2} - 2\Lambda. \tag{2·8}$$

The Einstein equations are given by

$$m\frac{\ddot{a}}{a} + n\frac{\ddot{b}}{b} - \alpha = 0, \tag{2·9}$$

$$\frac{\ddot{a}}{a} + (m-1)\left(\frac{\dot{a}}{a}\right)^2 + n\frac{\dot{a}\dot{b}}{ab} + \frac{(m-1)k_m}{a^2} - \alpha = 0, \tag{2·10}$$

$$\frac{\ddot{b}}{b} + (n-1)\left(\frac{\dot{b}}{b}\right)^2 + m\frac{\dot{a}\dot{b}}{ab} + \frac{(n-1)k_n}{b^2} - \alpha = 0, \tag{2·11}$$

where

$$\alpha = \frac{2\Lambda}{m+n-1}. \tag{2·12}$$

These equations imply the Hamiltonian constraint

$$m(m-1)\left(\frac{\dot{a}}{a}\right)^2 + 2mn\frac{\dot{a}\dot{b}}{ab} + n(n-1)\left(\frac{\dot{b}}{b}\right)^2 + V = 0. \tag{2·13}$$

§3. Euclidean solutions

We consider the quantum creation of the universe in the framework of the Hartle-Hawking no-boundary proposal. In the WKB approximation, the Hartle-Hawking wave function can be expressed in the form

$$\Psi[h_{ij}, \Phi] \sim \Sigma_k A_k \exp(-B_k), \tag{3·1}$$

where B_k is the Euclidean action for the solutions of the Euclidean field equations, which are compact and have the given 3-metric h_{ij} and matter field Φ on the boundary. The prefactor A_k represents the fluctuations around these solutions. Since the factor $\exp(-B_k)$ dominates wave function behavior, we analyse the classical solutions of the Euclidean Einstein equations.

The Euclidean Einstein equations are given by

$$m\frac{a''}{a}+n\frac{b''}{b}+\alpha=0, \tag{3·2}$$

$$\frac{a''}{a}+(m-1)\left(\frac{a'}{a}\right)^2+n\frac{a'b'}{ab}-\frac{(m-1)k_m}{a^2}+\alpha=0, \tag{3·3}$$

$$\frac{b''}{b}+(n-1)\left(\frac{b'}{b}\right)^2+m\frac{a'b'}{ab}-\frac{(n-1)k_n}{b^2}+\alpha=0, \tag{3·4}$$

where primes denote derivatives with respect to the imaginary time τ. The Hamiltonian constraint in the Euclidean version is described as

$$m(m-1)\left(\frac{a'}{a}\right)^2+2mn\frac{a'b'}{ab}+n(n-1)\left(\frac{b'}{b}\right)^2-V=0. \tag{3·5}$$

We assume that $a'=0$ and/or $b=0$ at $\tau=0$. The Hartle-Hawking no-boundary proposal gives the boundary condition for the Euclidean Einstein equations. According to this proposal, the Euclidean solutions must be analytic. At $\tau=0$, therefore, a and b can be expanded in Taylor series. This determines the boundary conditions of the Euclidean solutions. In addition, the external space of the present universe is macroscopic and the scale factor of the internal space is related to the gauge coupling constant. Therefore we do not consider trivial solutions, $a(\tau)=0$ and/or $b(\tau)=0$ for any τ. These constraints lead to the following boundary conditions:

(a) Boundary Condition 1 (BC1)

$$k_m=1, \tag{3·6}$$

$$a(0)=0, \qquad a'(0)=1, \tag{3·7}$$

$$b(0)=b_0, \qquad b'(0)=0. \tag{3·8}$$

(b) Boundary Condition 2 (BC2)

$$k_n=1, \tag{3·9}$$

$$a(0)=a_0, \qquad a'(0)=0, \tag{3·10}$$

$$b(0)=0, \qquad b'(0)=1. \tag{3·11}$$

The boundary condition BC2 is the symmetric version of the boundary condition BC1. These boundary conditions imply that either internal space or external space has positive curvature ($k_m=1$ or $k_n=1$). We can solve the Euclidean Einstein equations numerically under these boundary conditions. Below, the families of Euclidean solutions parameterized by b_0 and a_0 are calculated.

We assume that the Euclidean solutions are connected to classical Lorentzian solutions at a finite τ, where $a'=b'=0$. Due to the Hamiltonian constraint (2·13), the value of the potential at the connection surface must be

$$V=\frac{m(m-1)k_m}{a^2}+\frac{n(n-1)k_n}{b^2}-2\Lambda=0. \tag{3·12}$$

Table I. The signatures of k_m, k_n and Λ under the condition that the surface $V=0$ exists.

$(m-1)k_m$	$+$	$+$	$+$	0	0	0	$-$	$-$	$-$
$(n-1)k_n$	$+$	0	$-$	$+$	0	$-$	$+$	0	$-$
$\Lambda=0$	no	no	yes	no	yes	no	yes	no	no
$\Lambda>0$	yes	yes	yes	yes	no	no	yes	no	no
$\Lambda<0$	no	no	yes	no	no	yes	yes	yes	yes

Now we discuss the constraints on the signature of the spatial curvatures and the cosmological constant. First, it is easily shown that either internal space or external space must have positive curvature ($k_m=1$ or $k_n=1$) from the boundary conditions BC1 and BC2. Second, the condition that the connection surface $V=0$ exists gives constraints on the signatures of k_m, k_n and Λ. For example, when each signature of k_m and k_n is the same, and $\Lambda=0$, the potential $V(a,b)$ cannot be zero anywhere. The combinations of signatures of k_m, k_n and Λ for which the surface $V=0$ exists are shown in Table I.

Even if these conditions are satisfied, however, the Euclidean solutions cannot be necessarily connected to Lorentzian solutions, since in some classes of solutions, a and b increase monotonically and have no velocity-zero points, i.e. points for which $\dot{a}=\dot{b}=0$. We calculated Euclidean solutions numerically, changing the initial values, and investigated whether these Euclidean solutions can be connected to Lorentzian solutions or not. Except for the case in which $(m-1)k_m>0$, $(n-1)k_n>0$ and $\Lambda>0$, the scale factors a and b vary monotonically, and the condition $a'=b'=0$ cannot be satisfied at any τ. Then, we found that Euclidean solutions can be connected to the Lorentzian region only for the case in which $(m-1)k_m>0$, $(n-1)k_n>0$ and $\Lambda>0$. After all, only the case in which $k_m=k_n=1$ and $\Lambda>0$ is possible in our model. This implies that the internal space cannot be 1-dimensional, because the curvature term of the Einstein equations is zero for 1-dimensional space. Therefore the dimensions of space m and n must be greater than 2.

We now discuss the Euclidean solutions in the case that $k_m=k_n=1$ and $\Lambda>0$ in detail. In this case, there are the following three exact solutions:

(1)

$$a=\sqrt{\frac{m(m+n-1)}{2\Lambda}}\sin\left(\sqrt{\frac{2\Lambda}{m(m+n-1)}}\tau\right), \tag{3·13}$$

$$b=\left[\frac{(n-1)(n+m-1)}{2\Lambda}\right]^{1/2}\equiv b_\Lambda. \tag{3·14}$$

(2)

$$a=\sqrt{\frac{(m+n-1)(m-1)}{2\Lambda}}\sin\left(\sqrt{\frac{2\Lambda}{(m+n)(m+n-1)}}\tau\right), \tag{3·15}$$

$$b=\sqrt{\frac{(m+n)(n-1)}{2\Lambda}}\sin\left(\sqrt{\frac{2\Lambda}{(m+n)(m+n-1)}}\tau\right). \tag{3·16}$$

(3)

$$a = \left[\frac{(m-1)(n+m-1)}{2\Lambda}\right]^{1/2} \equiv a_A, \tag{3·17}$$

$$b = \sqrt{\frac{n(m+n-1)}{2\Lambda}} \sin\left(\sqrt{\frac{2\Lambda}{n(m+n-1)}}\tau\right). \tag{3·18}$$

The instanton solutions (1) and (3) were discussed by Hu and Wu.[14] In addition we have found the instanton solution (2).

In Figs. 1–3, the paths in the a-b plane are shown. Here we set $\Lambda = 1$ since we can always set $\Lambda = 1$ by rescaling the cosmic time. Figures 1 and 2 are for the case of the boundary condition BC1 (Eqs. (3·3) and (3·4)). Figure 3 is for the case of the boundary condition BC2 (Eqs. (3·6) and (3·7)). The essential property of the dynamical system is independent of the dimension of the g_{ij} or g_{IJ} spaces, as shown in Figs. 1 and 2. Note that this dynamical system has a characteristic point A:

$$(a_A, b_A) = \left(\left[\frac{(m-1)(n+m-1)}{2\Lambda}\right]^{1/2}, \left[\frac{(n-1)(n+m-1)}{2\Lambda}\right]^{1/2}\right). \tag{3·19}$$

The trajectories of the Euclidean solutions on the scale factor plane a-b can be classified into the following three classes. First, if b_0 (or a_0) is greater than a critical value $b_{\rm cr}$ (or $a_{\rm cr}$),

$$b_{\rm cr}, a_{\rm cr} = \sqrt{\frac{(D-1)(D-2)}{2\Lambda}}, \tag{3·20}$$

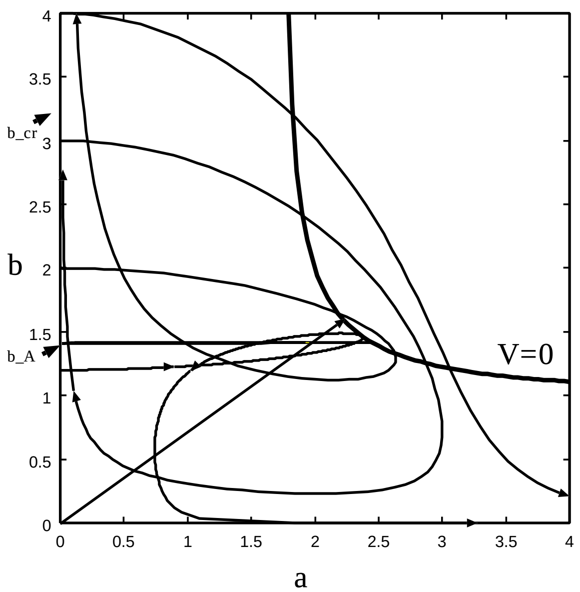

Fig. 1. Euclidean solutions in the case that $m = 3$, $n = 2$, $k_m = k_n = 1$, $\Lambda = 1$ and with the boundary condition BC1 (Eqs. (3·3) and (3·4)). The critical value $b_{\rm cr}$ is 3.16.

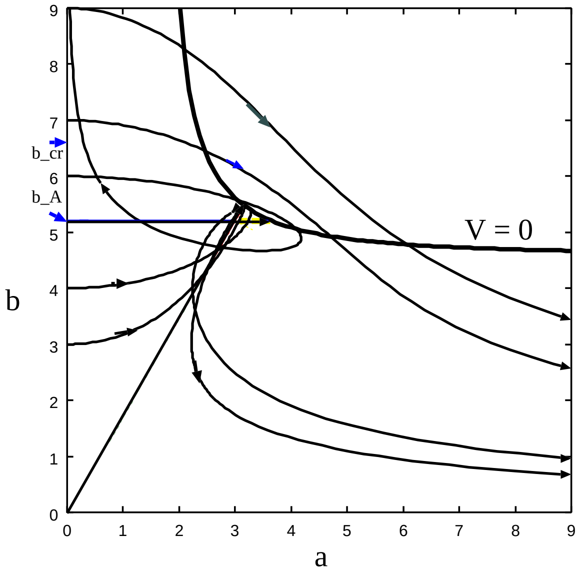

Fig. 2. Euclidean solutions in the case that $m = 3$, $n = 7$, $k_m = k_n = 1$, $\Lambda = 1$ and with the boundary condition BC1 (Eqs. (3·3) and (3·4)). The critical value $b_{\rm cr}$ is 6.71.

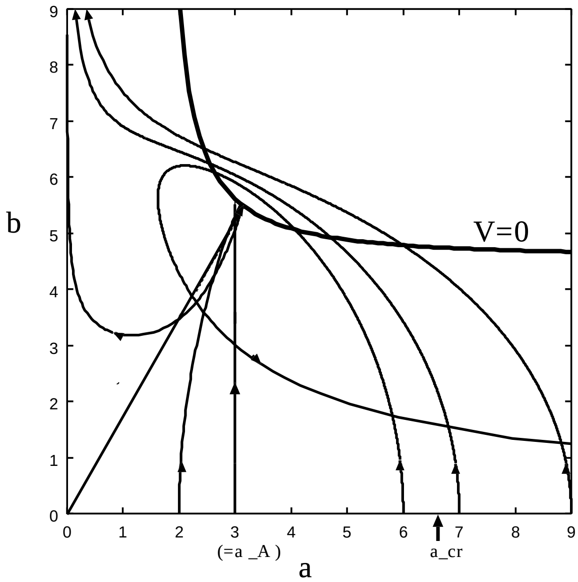

Fig. 3. Euclidean solutions in the case that $m = 3$, $n = 7$, $k_m = k_n = 1$, $\Lambda = 1$ and with the boundary condition BC2 (Eqs. (3·6) and (3·7)). The critical value $a_{\rm cr}$ is 6.71.

the trajectories flow to $(a, b) \to (\infty, 0)$ (or $(0,\infty)$) directly. Second, if b_0 (or a_0) is less than a critical value $b_{\rm cr}$ (or $a_{\rm cr}$) but greater than b_A (or a_A), the trajectories go around the point (a_A, b_A) and flow to $(a, b) \to (0, \infty)$ (or $(\infty,0)$). When b_0 (or a_0) is just equal to b_A (or a_A), b (or a) is constant (the solution (1) (or (3))) and the trajectory is a straight line. Third, if b_0 (or a_0) is lower than b_A (or a_A), the trajectories go around the point (a_A, b_A) and flow to $(a,b) \to (\infty, 0)$ (or $(0,\infty)$).

Note that the critical values $b_{\rm cr}$ and $a_{\rm cr}$ depend only on the dimension of the spacetime D as described by (3·20). We call the point (a_A, b_A), around which the trajectories circulate, the "quasi-attractor." The "quasi-attractor" plays a role in gathering the trajectories of the classical Euclidean solutions to the instanton solutions (Eqs. (3·9)–(3·14)), and approximately connecting to the Lorentzian solutions. In the next section we discuss the properties of the Lorentzian solutions.

§4. Lorentzian solutions

In this section we investigate the Lorentzian solutions that are continued from the Euclidean solutions we have presented in the preceding section in the case that the curvatures of both external and internal spaces are positive and $\Lambda > 0$. The Euclidean manifold is connected to the Lorentzian manifold, where the velocities of the scale factors are vanishing, and it can be shown that at the connection surface, the value of the potential V must be vanishing.

There are the three following exact solutions which are analytically continued from instantons:

(1)

$$a = \sqrt{\frac{m(m+n-1)}{2\Lambda}} \cosh\left(\sqrt{\frac{2\Lambda}{m(m+n-1)}} t\right), \tag{4·1}$$

$$b = \left[\frac{(n-1)(n+m-1)}{2\Lambda}\right]^{1/2}. \tag{4·2}$$

(2)

$$a = \sqrt{\frac{(m+n)(m-1)}{2\Lambda}} \cosh\left(\sqrt{\frac{2\Lambda}{(m+n)(m+n-1)}} t\right), \tag{4·3}$$

$$b = \sqrt{\frac{(m+n)(n-1)}{2\Lambda}} \cosh\left(\sqrt{\frac{2\Lambda}{(m+n)(m+n-1)}} t\right). \tag{4·4}$$

(3)

$$a = \left[\frac{(m-1)(n+m-1)}{2\Lambda}\right]^{1/2}, \tag{4·5}$$

$$b = \sqrt{\frac{n(m+n-1)}{2\Lambda}} \cosh\left(\sqrt{\frac{2\Lambda}{n(m+n-1)}} t\right). \tag{4·6}$$

In the preceding section we found that the "quasi-attractor" plays a role in gathering the paths of the classical Euclidean solutions to the instanton solutions (Eqs. (3·9)–(3·14)), and approximately connecting to Lorentzian solutions.

Strictly speaking, the Euclidean solutions near these instanton solutions cannot analytically continue to Lorentzian solutions. Since these nearby solutions exist

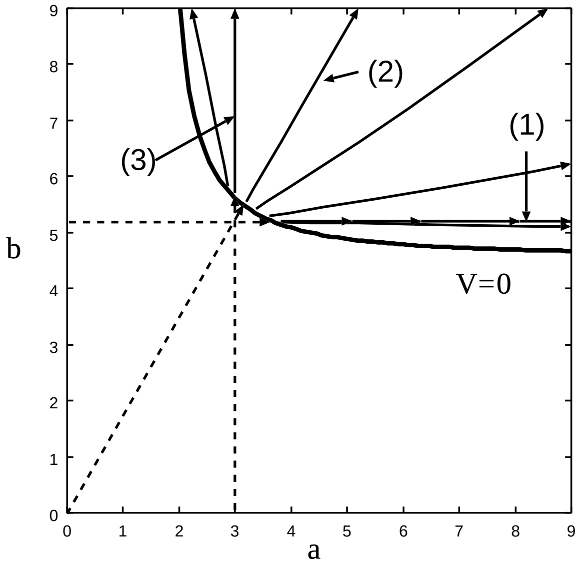

Fig. 4. Lorentzian solutions in the case $m = 3$, $n = 7$, $k_m = k_n = 1$ and $\Lambda = 1$. The number in this figure represents that of the Lorentzian analytic solutions (Eqs. (4·2)∼(4·7)). The dotted lines represent the instantons (Eqs. (3·9)∼(3·14)).

very densely, their effects are necessary for estimating the wave function of the universe. Therefore we make detailed analysis of these solutions. The Lorentzian solutions under the initial conditions that the velocities of the scale factors on the connection surface $V(a, b) = 0$ are zero are approximately continued from these solutions. Thus we calculate the Lorentzian solutions continued approximately from these solutions. In Fig. 4 the Lorentzian solutions in the a-b plane are shown. The solutions between the solutions (1) and (3) show that both external and internal spaces expand monotonically. On the other hand, the outer solutions of the exact solution (3) show that internal space expands and external space contracts. The

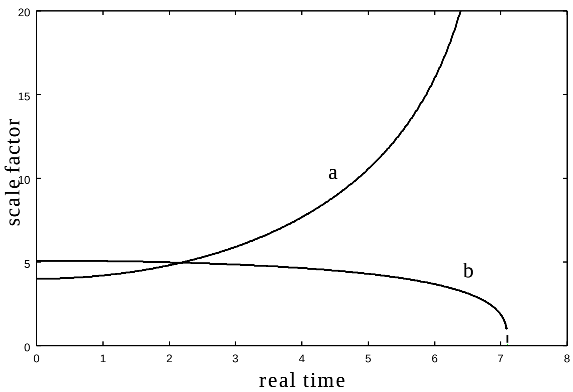

Fig. 5. Lorentzian solution in the case $m = 3$, $n = 7$, $k_m = k_n = 1$, $\Lambda = 1$, and the initial conditions are $a = 4$, $\dot{a} = 0$, $b = [21/(1 - 3/4^2)]^{1/2}$, $\dot{b} = 0$.

outer solutions of the exact solution (1), which are nothing but the symmetric version of the former solutions, show that external space expands and the internal space contracts (Fig. 5). The last solutions imply that external space of the observed universe is macroscopic and internal space of the observed universe is microscopic.

§5. Summary and discussion

In this paper we have investigated the quantum creation of a multi-dimensional universe with a cosmological constant but without matter fields in the framework of the Hartle-Hawking no-boundary proposal. The most interesting result is that there are "quasi-attractor," i.e., most trajectories of the classical solutions of the Euclidean Einstein equation go around on the a-b plane, independently of the initial values, provided that the initial values of the scale factors are smaller than a critical value in the case that the curvatures of both the external and internal spaces are positive and $\Lambda > 0$. This characteristic behavior of the evolution of the scale factors in Euclidean time is essentially independent of the numbers of external and internal spaces dimensions, provided that $m, n \geq 2$.

Since the trajectories approach the quasi-attractor at the first step, it plays a role in gathering the paths of the classical Euclidean solutions to the instantons ((3·9) –(3·14)) independently of the initial values. Then, it is very natural to assume that the wave function has a hump near this quasi-attractor point. There Lorentzian solutions which start from the points on the continuation surface near the quasi-attractor give a dominant contribution to the wave function in the Lorentzian region. As the example in Fig. 5 is shown, there are solutions for which we can interpret that dynamical compactification takes place; i.e. the external space evolves to the macroscopic realm and the internal space remains microscopic.

In order to understand the role of this quasi-attractor in the wave function and the creation of the universe more deeply, and to confirm the above conjecture, it is necessary to analyze the Wheeler-De Witt equations and to calculate the wave function of the universe. The Wheeler-De Witt equations of the multi-dimensional vacuum universe have been analysed by Chmielowski,[23)] but only for models with zero cosmological constant. According to his result, if neither of the subspaces has negative curvature, then there exists no Lorentzian solution. Obviously, interesting cases are those of the positive cosmological constant, as we have discussed in the present paper. Analysis of the Wheeler-De Witt equations in this model is under progress.

In order to make more realistic models, we must consider more complicated models on the basis of fundamental theories. As shown in Fig. 5, the solutions which show that the external space expands and the internal space collapses in finite time are adequate models for our universe, provided that the collapse of inner space is halted. Furthermore, because the volume of the internal space is related to the constants of nature, the present internal space must be stable at the order of the Planck scale. Fortunately, it is considered that the collapse of the internal space is halted by the pressure of the gauge fields or by a quantum effects, and the internal space is stabilized. It was shown that in $D = 6$, $N = 2$ supergravity theories, the solution of (the 4-dimensional Friedmann universe) $\times$ (a constant S^2) is the attractor; i.e., all cosmological solutions starting from arbitrary initial conditions approach the above spacetime asymptotically.[24)] Moreover, stable compactification of the extra dimensions by quantum effects was studied. The quantum corrections to the effective potential are attributed to the Casimir effects in many works (e.g. Refs. 26), 25) and references therein). In Ezawa et al.,[27)] the quantum effects of higher curvature gravity theories are investigated. From these investigations, it is known that an effective potential of the model obtained under dimensional reduction to a 4-dimensional effective theory has minima at the Planck scale for the scale factors of internal spaces. Therefore the internal spaces can be stable.

Our work is simply based on canonical quantum gravity in general relativity. It is, however, thought that general relativity is a low-energy limit of the ultimate unified theory. Although the Einstein equation would be necessarily modified at high energy, it must include essential points of the ultimate theory. It is natural to conjecture that the ultimate theory may include the present simple model as an approximate description. It seems that analysis of the simple model given in the present paper may be useful for further investigations of the basis of fundamental theories such as superstring theories and supergravity theories.

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