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Abstract Dynamical responses, such as motion and destruction of hyper-elastic cylindrical shells subject to periodic or suddenly applied constant load on the inner surface, are studied within a framework of finite elasto-dynamics. By numerical computation and dynamic qualitative analysis of the nonlinear differential equation, it is shown that there exists a certain critical value for the internal load describing motion of the inner surface of the shell. Motion of the shell is nonlinear periodic or quasi-periodic oscillation when the average load of the periodic load or the constant load is less than its critical value. However, the shell will be destroyed when the load exceeds the critical value. Solution to the static equilibrium problem is a fixed point for the dynamical response of the corresponding system under a suddenly applied constant load. The property of fixed point is related to the property of the dynamical solution and motion of the shell. The effects of thickness and load parameters on the critical value and oscillation of the shell are discussed.

Key words hyper-elastic cylindrical shells, nonlinear differential equation, periodic oscillation, quasi-periodic oscillation, critical load

Introduction

In recent years, hyper-elastic materials, such as rubber, synthetic elastomers and polymeric materials, have been used in a broader and broader range of engineering fields due to their unique and non-replaceable properties. Therefore, the nonlinear problems, such as the instability of hyper-elastic materials and structures, have attracted much attention in the world as they play a fundamental role on the failure of materials[1−3]. The well-known examples are the inflation of spherical hyper-elastic balloons[4−5] and the deformation of the hyper-elastic spherical shells or cylindrical shells[6−8]. Most of the literatures on such mechanics deal with elasto-statics which have been extensively studied. For example, when a cylindrical shell is inflated, it maintains a uniform inflation at the initial stage until a certain maximum of pressure is attained. After the maximum, the shell will undergo a non-uniform inflation. One part of the shell is highly stretched as a bubble while the remainder becomes lightly stretched. However, many physical problems are inherently dynamic, so a stability analysis of elasto-dynamics is important. As an example, intracranial aneurysms are typically subject to periodic or nearly periodic internal pressures[9]. Because of its property of large deformation, high elasticity and the application of hyper-elastic theory, the mathematical model for hyper-elastic dynamics is an ignition-boundary value problem with high nonlinear differential equations. Therefore, the solution of the dynamical problem is more difficult than the quasi-static problem. The system is autonomous when the forcing internal pressure is constant. Techniques for studying such autonomous systems are well-known, and they have been extensively studied[10−11]. For example, when an incompressible cylindrical shell is inflated by a suddenly applied constant internal pressure, it will undergo a nonlinearly periodic oscillation. However, dynamical cases of a time varying internal pressure have not been well studied. The case of some kinds of biological soft tissues and rubber spherical membranes surrounded by a fluid under a periodic internal loading was studied by Haslach and Humphrey[12]. A periodic orbit in a phase space exists near a static equilibrium, and a jump from one periodic orbit to another is possible for rubber models.

The purpose of the present paper is to further investigate the dynamical response and the destruction of incompressible hyper-elastic cylindrical shells under a periodic or constant internal pressure within a framework of finite elasto-dynamics. At first, the instability problem and the destruction of incompressible hyper-elastic cylindrical shells under a statically uniform internal pressure are examined within a framework of finite elasto-statics. The solution of the static equilibrium problem is the fixed point for the dynamic response of the corresponding autonomous system under a suddenly applied constant internal pressure. Then the dynamical response and the destruction of cylindrical shells under the suddenly applied constant internal pressure or the periodic internal pressure are examined within the framework of finite elasto-dynamics. The second order differential relationship between the deformation of the internal boundary of the shell and the internal pressure is obtained from the basic formulations. The displacement response curves, the phase portrait and the Poincaré maps are given out by the numerical computation through the Runge-Kutta integrator for the transformed first order differential equations. The dynamical response along with the destruction of the shell is discussed with these results following the usual dynamics. There exists a critical value for the shell under a suddenly applied constant internal pressure or a periodic internal pressure. When the pressure is less than the critical value, the shell will undergo a nonlinear periodic oscillation or quasi-periodic oscillation. When the pressure is larger than the critical value, the shell will be ultimately destroyed with time.

1 Formulations

Consider the finite deformation dynamics and the destruction for an incompressible hyper-elastic cylindrical shell with an undeformed internal radius A and an outer radius B. Assume it is set into motion by a periodic internal pressure $p(t) = p_0 + p_1 \sin(\omega t)$ at the initial time t_0. A point (r, θ, z) at time t is assumed to occupy the point (R, Θ, Z) in the undeformed state. Then the deformation function of the shell is given as

$$r = r(R, t), \quad \theta = \Theta, \quad z = Z \quad (A \leq R \leq B). \tag{1}$$

From the incompressibility condition of the material, we have

$$r = r(R, t) = \left[(R^2 - A^2) + a^2\right]^{\frac{1}{2}}, \tag{2}$$

where $a = r\,(A, t)$ is the deformed internal surface. The principal stretches are

$$\lambda_1 = \frac{\partial r(R, t)}{\partial R} = \frac{R}{r}, \quad \lambda_2 = \frac{r(R, t)}{R} = \frac{r}{R}, \quad \lambda_3 = 1. \tag{3}$$

The strain energy function of the shell is given as the neo-Hookean material with the strain energy function as follows:

$$W = \frac{\mu}{2}(I_1 - 3). \tag{4}$$

Here, the first invariant $I_1 = (\lambda_1^2 + \lambda_2^2 + \lambda_3^2)$, μ is the shear modulus for infinitesimal deformations. The corresponding non-zero principal components of the Cauchy stress tensor are

$$\begin{cases} \tau_{rr}(r,t) = \lambda_1 \dfrac{\partial W}{\partial \lambda_1} - p(r,t) = \mu \lambda_1^2 - p(r,t), \\ \tau_{\theta\theta}(r,t) = \lambda_2 \dfrac{\partial W}{\partial \lambda_2} - p(r,t) = \mu \lambda_2^2 - p(r,t), \end{cases} \tag{5}$$

where $p(r,t)$ is the hydrostatic pressure to be determined.

The motion equation with the absence of body forces is

$$\frac{d\tau_{rr}}{dr} + \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta}) = \rho \ddot{r} \quad (t \geq 0), \tag{6}$$

where ρ is the constant mass density of the material. The boundary conditions of the shell are

$$\begin{cases} \tau_{rr}(a,t) = -p(t), \\ \tau_{rr}(b,t) = 0. \end{cases} \tag{7}$$

Here, $b = r(B,t)$ is the deformed outer surface.

The initial stress-free conditions for the shell at time $t = 0$ are

$$\begin{cases} r(R,0) = R, \\ \dot{r}(R,0) = 0. \end{cases} \tag{8}$$

2 The governing differential equations

From the incompressible condition (2),

$$\ddot{r}(t) = \frac{1}{r}\left[\left(1 - \frac{a^2}{r^2}\right)\dot{a}^2 + a\ddot{a}\right]. \tag{9}$$

Substituting stresses (5) and (9) into the motion equation (6), we have

$$\frac{d}{dr}\left[\mu\lambda_1^2 - p(r,t)\right] + \frac{1}{r}\left[\mu\left(\lambda_1^2 - \lambda_2^2\right)\right] = \frac{\rho}{r}\left[\left(1 - \frac{a^2}{r^2}\right)\dot{a}^2 + a\ddot{a}\right]. \tag{10}$$

Integrating it with respect to r, then

$$\begin{aligned} &\mu\lambda_1^2 - p(r,t) + p(a,t) + \int_a^r \mu\left(\lambda_1^2 - \lambda_2^2\right)\frac{ds}{s} \\ &= \rho\dot{a}^2\frac{a^2 - r^2 + 2r^2\ln r - 2r^2\ln a}{2r^2} + \rho a\ddot{a}(\ln r - \ln a). \end{aligned} \tag{11}$$

Introducing it into (5), we have

$$\tau_{rr}(r,t) = \rho\dot{a}^2\frac{a^2 - r^2 + 2r^2\ln r - 2r^2\ln a}{2r^2} + \rho a\ddot{a}(\ln r - \ln a) - \int_a^r \mu\left(\lambda_1^2 - \lambda_2^2\right)\frac{ds}{s} - p(a,t). \tag{12}$$

Introducing it into (7), we have $p(a,t)=p(t)$, and

$$p(t) =\rho\dot{a}^2\frac{a^2-b^2+2b^2\ln b-2b^2\ln a}{2b^2}+\rho\ddot{a}(\ln b-\ln a)-\int_a^b\mu(\lambda_1^2-\lambda_2^2)\frac{ds}{s}. \tag{13}$$

By defining new invariants

$$\frac{a}{A}=x(t),\quad \frac{B^2}{A^2}-1=\delta,$$

we have

$$\frac{b^2}{A^2}=x^2(t)+\delta,\quad \frac{b^2}{B^2}=\frac{x^2+\delta}{\delta+1},\quad \frac{b}{a}=\sqrt{1+\frac{\delta}{x^2}}.$$

Let $\xi=\frac{r}{R}$, then

$$\xi=\left(1-\frac{a^2-A^2}{r^2}\right)^{-\frac{1}{2}},\quad \frac{dr}{r}=\frac{1}{1-\xi^2}\frac{d\xi}{\xi}.$$

Equation (13) may be rewritten as

$$\begin{aligned}
p(t)&=\rho A^2\dot{x}^2\Big[\frac{-\delta}{2(x^2+\delta)}+\ln\sqrt{1+\frac{\delta}{x^2}}\Big]+\rho A^2x\ddot{x}\ln\sqrt{1+\frac{\delta}{x^2}}-\int_x^{\sqrt{\frac{x^2+\delta}{\delta+1}}}\Big[\frac{\mu\left(\xi^{-2}-\xi^2\right)}{\xi\left(1-\xi^2\right)}\Big]d\xi\\
&=\rho A^2\dot{x}^2\Big[\frac{-\delta}{2(x^2+\delta)}+\ln\sqrt{1+\frac{\delta}{x^2}}\Big]+\rho A^2x\ddot{x}\ln\sqrt{1+\frac{\delta}{x^2}}\\
&\quad+\frac{1}{2}\frac{1+\delta}{\delta+x^2}-\frac{1}{2x^2}-\ln x+\frac{1}{2}\ln\frac{\delta+x^2}{1+\delta}.
\end{aligned}\tag{14}$$

Here, $a=xA, b=A\sqrt{x^2+\delta}$.

The initial conditions are

$$x(0)=1,\quad \dot{x}(0)=0. \tag{15}$$

By putting $x_1=x$ and $x_2=\dot{x}$, a system of first order equations are obtained from (14):

$$\begin{cases}
\dot{x}_1=x_2,\\
\dot{x}_2=\dfrac{1}{x_1\ln\sqrt{1+\frac{\delta}{x_1^2}}}\left[\dfrac{1}{\rho A^2}\left(-\dfrac{1}{2}\dfrac{1+\delta}{\delta+x_1^2}+\dfrac{1}{2x_1^2}+\ln x_1-\dfrac{1}{2}\ln\dfrac{\delta+x_1^2}{1+\delta}\right)\right]\\
\qquad+\dfrac{1}{x_1\ln\sqrt{1+\frac{\delta}{x_1^2}}}\left[\dfrac{p(t)}{\rho A^2}-x_2^2\left(\dfrac{-\delta}{2(x_1^2+\delta)}+\ln\sqrt{1+\dfrac{\delta}{x_1^2}}\right)\right].
\end{cases}\tag{16}$$

The corresponding initial conditions are

$$x_1(0)=1,\quad x_2(0)=0. \tag{17}$$

3 Static equilibrium solutions

The solution of the static equilibrium problem is the fixed point for the dynamic response of the corresponding autonomous system under a constant internal pressure. The property of the fixed point is related to the property of the dynamical solution and the motion of the shell. The equilibrium pressure function is the core of the static equilibrium problem. Let the right side of the motion equation (6) equal zero, the equilibrium equation for the cylindrical shell under a statically internal uniform pressure p_0 is obtained. Following a similar analogy as above, we have

$$p_0 = \int_{v(B)}^{v(A)} \frac{\mu(v^{-3} - v)}{v^2 - 1} dv. \tag{18}$$

Here,

$$v = v(R) = \frac{r(R)}{R} = \left(1 + \frac{a^2 - A^2}{R^2}\right)^{\frac{1}{2}}, \quad v(B) = \left(1 + \frac{a^2 - A^2}{B^2}\right)^{\frac{1}{32}}, \quad v(A) = \frac{a}{A} = x$$

describe the motion of the internal boundary of the shell.

Numerical results of (18) for the shell with different thickness are shown in Fig. 1. Material constants are taken as $\mu = 2.63$ MPa, $\rho = 950\ \mathrm{kg \cdot m^{-3}}$. As shown in Fig. 1, the deformation of the shell increases with the pressure, and there exists a horizontal asymptote. When the pressure is close to the one corresponding to the horizontal asymptote, the deformation of the shell may have a quick increase, which means the destruction of the shell. Therefore, this pressure may be taken as the critical value p_{cr} for the shell under the internal uniform pressure p_0. Also, the critical value p_{cr} increases with the thickness of the shell. For example, the values are 0.28 MPa, 1.45 MPa and 4.12 MPa for the shell with thickness $\delta = 0.234, 2.0, 4.0$, respectively. Therefore, it is more difficult to destroy a thicker shell.

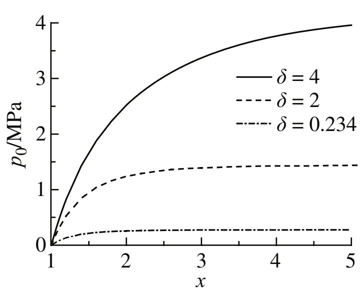

Fig. 1 The equilibrium pressure function

4 Dynamic response under a constant internal pressure

The system is autonomous when the forcing internal pressure is constant. Letting $p(t) = p_0$ and integrating (14) with respect to x, we obtain

$$x^2 \ln \sqrt{1 + \frac{\delta}{x^2}} \dot{x}^2 - \frac{2}{\rho A^2} \int_1^x x \left(\frac{1}{2}\frac{1+\delta}{\delta + x^2} - \frac{1}{2x^2} - \ln x + \frac{1}{2} \ln \frac{\delta + x^2}{1+\delta}\right) dx - \frac{p_0}{\rho A^2}(x^2 - 1) = 0. \tag{19}$$

From the theory of vibrations, the motion $x(t)$ is periodic if and only if the curve of x vs. $\dot{x} = V$ in the phase diagram is closed and owns a finite period $T = \oint \frac{dx}{V}$. For a given load p_0, the period motion $x(t)$ will occur if there is a root $x > 0$ for (19) when $V = 0$. Letting $V = \dot{x} = 0$ in (19) leads to

$$2 \int_1^x x \left(\frac{1}{2}\frac{1+\delta}{\delta + x^2} - \frac{1}{2x^2} - \ln x + \frac{1}{2} \ln \frac{\delta + x^2}{1+\delta}\right) dx - \frac{p_0}{\rho A^2}(x^2 - 1) = 0. \tag{20}$$

For a given load p_0, if there is a root $x > 0$ for (20), then it is the maximum radius of the internal surface of the shell in the oscillation process and denoted by $x_{\max}$. The curves between $x_{\max}$ and p_0 for the shell with different thickness are shown in Fig. 2. Then the curves computed from (19) between the velocity $V = \dot{x}$ and x are shown in Fig. 3 corresponding to different values of $x_{\max}$.

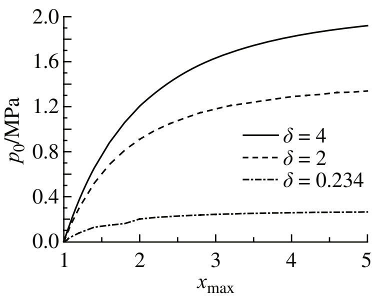

Fig. 2 $x_{\max}$ vs. p_0 curves

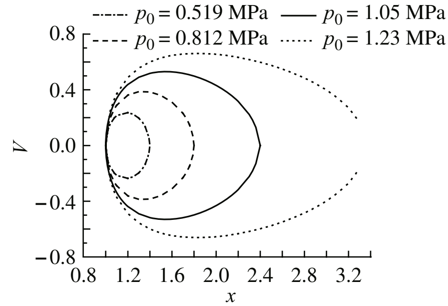

Fig. 3 Phase diagrams for $\delta = 2$

It is seen that there exists a critical value $p_{\rm cr}$ for the pressure of the shell (for example, when $\delta = 2$, $p_{\rm cr} = 1.204$ MPa). When $p_0 < p_{\rm cr}$, $x_{\max}$ increases with the pressure, and the corresponding phase curves in the phase plane are closed curves. Thus the shell undergoes a nonlinearly periodic oscillation. However, when $p_0 \geq p_{\rm cr}$, the phase curves in the phase plane are not closed. So the shell will be destroyed ultimately with time. At the same time, the critical value $p_{\rm cr}$ for the shell increases with the thickness of the shell, i.e., it is easier to destroy a shell with a thinner thickness.

The system is autonomous when the forcing internal pressure is constant. The fixed point for the system is $(x_1, x_2) = (x_{\rm s}, 0)$. Here, $x_{\rm s}$ is the corresponding static equilibrium of the system (the static deformation under the same load). The property of the fixed point is related to the property of the dynamical solution and the motion of the shell. The type of the fixed point should be determined from the Jacobian at the fixed point as usual. The Jacobian at the fixed point of equation (16) is

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}_{(x_{\rm s},0)} = \begin{pmatrix} 0 & 1 \\ J_{21} & 0 \end{pmatrix}, \tag{21}$$

where,

$$\begin{aligned} J_{21} = {} & \frac{1}{x_{\rm s} \ln\left(1 + \frac{\delta}{x_{\rm s}^2}\right)} \left[\frac{2\mu}{\rho A^2} \left(-\frac{1}{x_{\rm s}^3} - \frac{2x_{\rm s}}{\delta + x_{\rm s}^2} + \frac{1}{x_{\rm s}} + \frac{(1+\delta)\, x_{\rm s}}{(\delta + x_{\rm s}^2)^2} \right) \right] \\ & + \frac{2\delta}{x_{\rm s}^4 \left(1 + \frac{\delta}{x_{\rm s}^2}\right) \ln\left(1 + \frac{\delta}{x_{\rm s}^2}\right)} \left[\frac{p_0}{\rho A^2} + \frac{\mu}{\rho A^2} \left(\frac{1}{2x_{\rm s}^2} - \frac{1+\delta}{2\,(\delta + x_{\rm s}^2)} + \ln x_{\rm s} - \ln \frac{\delta + x_{\rm s}^2}{1+\delta} \right) \right] \\ & + \frac{1}{x_{\rm s}^2 \ln\left(1 + \frac{\delta}{x_{\rm s}^2}\right)} \left[\frac{2p_0}{\rho A^2} + \frac{2\mu}{\rho A^2} \left(\frac{1}{2x_{\rm s}^2} - \frac{1+\delta}{2\,(\delta + x_{\rm s}^2)} + \ln x_{\rm s} - \ln \frac{\delta + x_{\rm s}^2}{1+\delta} \right) \right]. \end{aligned} \tag{22}$$

The trace of the Jacobian is $\operatorname{tr} J = J_{11} + J_{22} = 0$, and the determinant of the Jacobian is $D = \det |J| = -J_{21}$. Numerical results for the determinant of the Jacobian for the shell in the case of $\delta = 2$ from (22) are shown in Table 1.

Table 1 Determinant of the Jacobian ($\delta = 2$)

$x_{\rm s}$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$D/10^6$	6.6	3.46	1.51	0.78	0.66	0.48	0.42	0.19	−0.16

As usual, if $D < 0$, the fixed point is a saddle and it is unstable. If $\operatorname{tr} J = 0$, $D > 0$, the fixed point is a center and it is stable. It is shown that if $p_0 < p_{\mathrm{cr}} = 1.204$ MPa ($x_{\mathrm{s}} < 1.9$), the fixed point is a center; and if $p_0 \geq p_{\mathrm{cr}} = 1.204$ MPa, the fixed point is an unstable saddle in the case of $\delta = 2$. And as usual, each center is surrounded by a homoclinic or heteroclinic orbit. As an example, for the shell in the case of $\delta = 2$ under the constant pressure $p_0 = 1.04$ MPa, $(x_{\mathrm{s}}, 0) = (1.6, 0)$ is the center of the closed curve (homoclinic orbit) with $(x(0), 0) = (1.0, 0)$ and $(x_{\max}, 0) = (2.4, 0)$ in the phase space shown as the solid curve in Fig. 3.

5 Dynamic response under a periodic internal pressure

In practice, rubber hoses or biological structures must endure periodic or near periodic forcing. The response to the periodic forcing must be examined for its stability and for the destruction of the structure. Therefore, the dynamic response and the destruction of the shell for a periodic forcing, such as $p(t) = p_1 + p_2 \sin(\omega t)$, are examined.

In the case of $p(t) = p_1 + p_2 \sin(3\pi t)$, numerical results, such as the displacement response curves, the phase portraits and the Poincaré maps, are given by numerical computation through the Runge-Kutta integrator of the first order differential equations (16). One of the displacement response curves is shown in Fig. 4. Five of the phase portraits and two of the Poincaré maps are shown in Figs. 5–9 and Figs. 10–11, respectively.

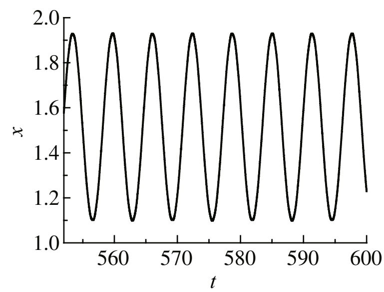

Fig. 4 Displacement response curve with $\delta = 2$, $p_1 = 0.3$ MPa and $p_2 = 0.1$ MPa

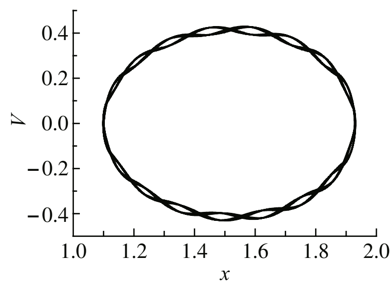

Fig. 5 Phase portrait with $\delta = 2$, $p_1 = 0.3$ MPa and $p_2 = 0.1$ MPa

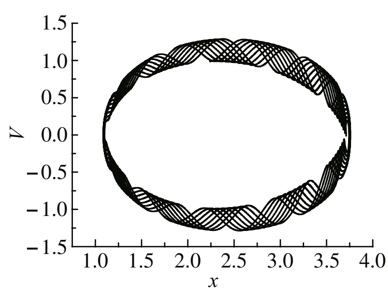

Fig. 6 Phase portrait with $\delta = 2$, $p_1 = 0.8$ MPa and $p_2 = 0.7$ MPa

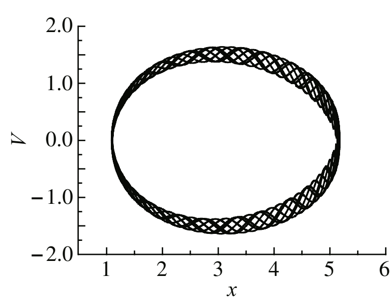

Fig. 7 Phase portrait with $\delta = 2$, $p_1 = 1.0$ MPa and $p_2 = 0.5$ MPa

As usual, for the internal periodic forcing $p(t) = p_1 + p_2 \sin(\omega t)$, the mean pressure is $p_{\mathrm{m}} = p_1 + p_2$. It is found that there exists a critical value $p_{\mathrm{cr}}^{\mathrm{m}}$ for the mean pressure for a given load amplitude p_2 and a given frequency ω. When the mean pressure $p_{\mathrm{m}} < p_{\mathrm{cr}}^{\mathrm{m}}$, the displacement response curves, the phase portrait and the Poincaré maps as shown above may be obtained. It

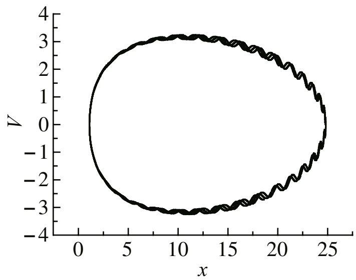

Fig. 8 Phase portrait with $\delta = 2$, $p_1 =$ 1.4 MPa and $p_2 = 0.1$ MPa

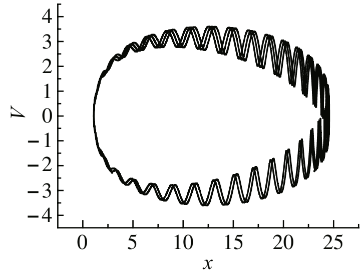

Fig. 9 Phase portrait with $\delta = 2$, $p_1 =$ 1.4 MPa and $p_2 = 0.5$ MPa

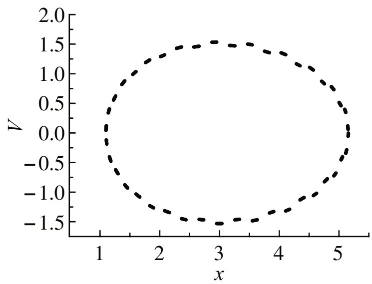

Fig. 10 Poincaré map with $\delta = 2$, $p_1 =$ 1.0 MPa and $p_2 = 0.1$ MPa

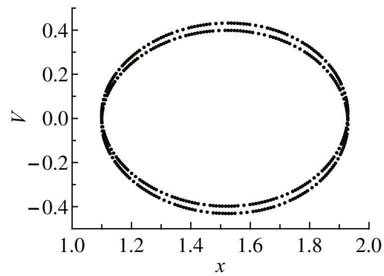

Fig. 11 Poincaré map with $\delta = 2$, $p_1 =$ 1.4 MPa and $p_2 = 0.1$ MPa

is shown by the figures that the motion of the shell presents nonlinear quasi-periodic oscillations. However, when the mean pressure $p_{\rm m} \geq p_{\rm cr}^{\rm m}$, the figures as shown above cannot be obtained. That is to say the shell may be destroyed ultimately with time.

At the same time, the effect of the load amplitude p_2 and the frequency ω on the critical mean pressures $p_{\rm cr}^{\rm m}$ may be ignored. For example, the value is always 1.4 MPa for the shell in the case of $\delta = 2$.

6 Conclusions

Dynamical response and the destruction of internally periodic pressurized or constant pressurized incompressible hyper-elastic cylindrical shells are examined within the framework of finite elasto-dynamics. There exists a critical value for the pressure of the shell under static equilibrium. When the pressure approaches this critical value, the shell expands quickly and will be destroyed. The solution of the static equilibrium problem is the fixed point for the dynamical response of the corresponding system under a suddenly applied constant load. The property of the fixed point is related to the property of the dynamical solution and the motion of the shell. There also exists a critical value for the pressure of the shell under a suddenly applied constant internal pressure. When the pressure is less than this critical value, the fixed point is a center surrounded by a homoclinic orbit, and the shell will undergo nonlinear periodic oscillation. But when the pressure is larger than the critical value, the fixed point is an unstable saddle, the phase portrait is a non-closed curve, and the shell will be destroyed ultimately with time. There also exists a critical value for the pressure of the shell under a periodic internal pressure by numerical computation of the first order differential equations. When the pressure is less than this critical value, the displacement response curves, the phase portrait and the

Poincaré maps may be given, and the shell will undergo a nonlinear quasi-periodic oscillation. But when the pressure is larger than this critical value, the figures cannot be obtained, and the shell will be destroyed ultimately with time. The critical value demanded for the destruction of the shell under a periodic internal pressure or a constant pressure is less than that under the static equilibrium, and the value under the constant pressure is the lowest.

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