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Dynamical symmetry breaking and space-time topology

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Vacuum polarization and spontaneous symmetry breaking in a two-dimensional massless fermion field theory with quartic interactions (the Gross-Neveu model) are studied in a cylindrical ($R^1 \times S^1$) space-time topology. In the case of untwisted fermion fields the symmetry-breaking behavior is similar to the model in Minkowski space-time except that the location of the effective potential minima depends upon the size of the space. In the case of a twisted fermion field there exists a critical size of the space such that the symmetry breaking occurs only for space larger than the critical one.

I. INTRODUCTION

Quantum field theories in a topologically nontrivial space-time have been studied by various authors. The Casimir effect is well known.[1] The effects of topology on the vacuum energy, mass generation, and spontaneous symmetry breaking in a ϕ^4 self-interacting scalar theory have been investigated.[2,3] Vacuum polarization for quantum electrodynamics in a $S^1 \times R^3$ space-time was studied by Ford.[4] Nontrivial topology may give rise to new field configurations, so-called twisted fields as introduced by Isham.[5] Free twisted fields in various cases have been discussed by Dewitt, Hart, and Isham.[6] Twisted scalar fields with ϕ^4 self-interaction[3] and twisted spinor fields in quantum electrodynamics[4] have also been studied.

In this paper we investigate the effects of topology in the model proposed by Gross and Neveu.[7] This model contains many of the desirable features of a realistic field theory, such as nontrivial scattering, renormalizability, asymptotic freedom, and dynamical mass generation. In addition, the model is well studied[8] and the phase structure at finite temperature[9,10] and density[11] have been analyzed in the large-N (fermion component) limit. In Sec. II we introduce the Gross-Neveu model in space-time with an $R^1 \times S^1$ topology and study vacuum polarization up to one-loop order. In Sec. III spontaneous symmetry breaking is investigated by evaluating the effective potential in the large-N limit. The divergence and renormalization of the potential is shown to be the same as the model in Minkowski space-time. The minimum of the potential depends on the size of the space coordinate L, and therefore dynamically generated mass and the vacuum energy are functions of L.

In Sec. IV we consider the twisted fermion field whose effective potential has a strong dependence upon the size L. There exists a critical length L_c such that only for $L > L_c$ does spontaneous symmetry breaking occur, and for a smaller size the symmetry remains unbroken. This is quite analogous to the symmetry restoration by the temperature effect.[9] In the final section we discuss the extremely small size limit ($L \to 0$) and its implication in connection with the Kaluza-Klein ansatz in dimensional compactification.

II. VACUUM POLARIZATION

The Gross-Neveu model[7] contains N-component fermion fields with quartic interactions in two space-time dimensions. Its Lagrangian density is

$$\mathcal{L}_\psi = \bar{\psi} i \partial\!\!\!/ \psi + \tfrac{1}{2} g^2 (\bar{\psi}\psi)^2 , \tag{1}$$

where ψ is the N-component massless fermion field. This model can be studied by using the equivalent Lagrangian

$$\mathcal{L}_\sigma = \bar{\psi} i \partial\!\!\!/ \psi - \tfrac{1}{2}\sigma^2 - g(\bar{\psi}\psi)\sigma , \tag{2}$$

which is more convenient for the analysis of spontaneous symmetry breaking. In the large-N limit with fixed $\lambda \equiv N g^2$ the dominant Feynman diagrams are those containing the maximal number of fermion loops. For example, the lowest-order σ self-energy graph is simply a one-fermion-loop contribution

$$\pi(p)=-Ng^2\int\frac{d^2k}{(2\pi)^2}\frac{\mathrm{Tr}\,\not k(\not k-\not p)}{k^2(k-p)^2}$$
$$=\frac{i\lambda}{2\pi}\int_0^1 d_\alpha\left[\ln\left[\frac{-\Lambda^2}{\alpha(1-\alpha)p^2}\right]-2\right], \quad (3)$$

where Λ is an ultraviolet cutoff. We renormalize by requiring that the σ propagator,

$$D(p)=\frac{-i}{1+i\pi(p)},$$

satisfy

$$D_R(p^2)=-i \quad \text{at } p^2=-\mu^2 . \quad (4)$$

The renormalized propagator and self-energy are

$$D_R(p^2,\mu^2)=-i\Big/\left[1+\frac{\lambda}{2\pi}\ln(-p^2/\mu^2)\right],$$
$$\pi_R(p^2,\mu^2)=\frac{-i\lambda}{2\pi}\ln(-p^2/\mu^2) . \quad (5)$$

In this section we study the σ self-energy graph when the space-time topology is $R^1\times S^1$ (the space coordinate is a circle with period L) instead of the original $R^1\times R^1$. The lowest-order graph in $1/N$ is

$$\tilde{\pi}(p)=-\frac{Ng^2}{2\pi L}\int_{-\infty}^{\infty}dk_0\sum_{m=-\infty}^{\infty}\frac{\mathrm{Tr}\,\not k(\not k-\not p)}{k^2(k-p)^2}$$
$$=\frac{2iNg^2}{2\pi L}\int_{-\infty}^{\infty}dk_4\sum_{m=-\infty}^{\infty}\frac{{k_4}^2+\left[\frac{2\pi}{L}\right]^2m^2-k_4p_4-\left[\frac{2\pi}{L}\right]^2mn}{\left[{k_4}^2+\left[\frac{2\pi}{L}\right]^2m^2\right]\left[(k_4-p_4)^2+\left[\frac{2\pi}{L}\right]^2(m-n)^2\right]}, \quad (6)$$

where $m=(L/2\pi)k_1$, $n=(L/2\pi)p_1$ are integers, and we take the usual Wick rotation $k_0=ik_4$, $p_0=ip_4$. We first perform the summation

$$S=\sum_{m=-\infty}^{\infty}\frac{ab+m(m-n)}{(a^2+m^2)[b^2+(m-n)^2]}$$
$$=\int_{-\infty}^{\infty}dx\frac{ab+x(x-n)}{(a^2+x^2)[b^2+(x-n)^2]}+\Delta , \quad (7)$$

where

$$\Delta=\frac{2(a+b)\pi}{(a+b)^2+n^2}\left[\frac{\mathrm{sgn}(a)}{e^{2\pi(a)}-1}+\frac{\mathrm{sgn}(b)}{e^{2\pi(b)}-1}\right], \quad (8)$$

and $a\equiv(L/2\pi)k_4$, $b\equiv(L/2\pi)(k_4-p_4)$. From (6) and (7) we find that

$$\tilde{\pi}(p)=\pi(p)+\pi_\Delta(p) , \quad (9)$$

where $\pi_\Delta(p)$ is the change in σ self-energy caused by the change of the space-time topology.

We can see that the ultraviolet divergence of $\tilde{\pi}(p)$ is contained only in $\pi(p)$, and $\pi_\Delta(p)$ is finite. By examining the integral

$$\pi_\Delta(p)=\frac{2i\lambda}{(2\pi)^2}\int_0^\infty dy\frac{4\pi}{e^{2\pi y}-1}\left[\frac{2y-\frac{L}{2\pi}p_4}{\left[2y-\frac{L}{2\pi}p_4\right]^2+n^2}+\frac{2y+\frac{L}{2\pi}p_4}{\left[2y+\frac{L}{2\pi}p_4\right]^2+n^2}\right], \quad (10)$$

we find that the factor $[\exp(2\pi y)-1]^{-1}$ renders the integral finite. Since the divergence is wholly contained in $\pi(p)$ and independent of L, the renormalization can be done just as in Minkowski space-time.

The pole at spacelike momenta of the propagator (5) signifies a spontaneous symmetry breaking. In our case we have a correction term $\pi_\Delta(p^2)-\pi_\Delta(\mu^2)$ in the propagator and it is not easy to see whether the pole persists in spite of such a change. We will study the symmetry breaking by examining the effective potential in the next section.

III. SPONTANEOUS SYMMETRY BREAKING

By examining the effective potential $V(\sigma_c)$ Gross and Neveu showed that their model exhibits a spontaneous symmetry breaking. They evaluated $V(\sigma_c)$ in leading order in $1/N$ by summing the tree graph and all one-loop graphs of the effective potential. It is

$$V(\sigma_c)=\tfrac{1}{2}{\sigma_c}^2-iN\sum_{n=1}^{\infty}\int^{\Lambda}\frac{d^2k}{(2\pi)^2}\frac{1}{n}\frac{(g^2{\sigma_c}^2)^n}{(k^2)^n}$$
$$=\tfrac{1}{2}{\sigma_c}^2-\frac{\lambda}{4\pi}{\sigma_c}^2(\ln\Lambda^2-\ln g^2{\sigma_c}^2+1) , \quad (11)$$

where σ_c is the classical field defined by $\sigma_c=\langle 0\,|\,\sigma\,|\,0\rangle$.

In our case we cannot introduce the Lorentz-invariant cutoff Λ because of the $R^1\times S^1$ topology. We will take the dimensional regularization as used by Ford and others.[3,4] The one-loop contribution to the effective potential (we denote it V_1) is

$$V_1=-\frac{iN}{(2\pi)^{\omega+1}L}\sum_{n=1}^{\infty}\int d^{\omega}k\,dk_0\sum_{m=-\infty}^{\infty}\frac{(g^2\sigma_c^2)^n}{n\left[k_0^2-k^2-\left[\frac{2\pi}{L}m\right]^2\right]^n}$$
$$=-\frac{N}{(2\pi)^{\omega+1}L}\int d^{\omega+1}k\sum_{m=-\infty}^{\infty}\ln\left[1+\frac{g^2\sigma_c^2}{k^2+\left[\frac{2\pi}{L}m\right]^2}\right], \tag{12}$$

where the two-dimensional potential is obtained in the limit $\omega\to 0$. For the summation we make use of the formula

$$\sum_{m=-\infty}^{\infty}\ln\left[1+\frac{b^2}{a^2+m^2}\right]=\int_{-\infty}^{\infty}\ln\left[1+\frac{b^2}{a^2+\tau^2}\right]d\tau+2\ln\left[\frac{1-e^{-2\pi(a^2+b^2)^{1/2}}}{1-e^{-2\pi|a|}}\right]. \tag{13}$$

Then V_1 can be expressed as

$$V_1=-\frac{N}{(2\pi)^{\omega+2}}\int d^{\omega+2}k\,\ln\left[1+\frac{g^2\sigma_c^2}{k^2}\right]+\Delta\ , \tag{14}$$

where the first term is just the effective potential in the Euclidean space-time which has the ultraviolet divergence in it. In the limit $\omega\to 0$ this term reproduces the Gross-Neveu case. The second term Δ is the difference of the effective potential between the Euclidean and the $R^{\omega+1}\times S^1$ space-time. It can be integrated as

$$\Delta=-\frac{2N}{(2\pi)^{\omega+1}L}\int d^{\omega+1}k\,\ln\frac{1-e^{-L(k^2+g^2\sigma_c^2)^{1/2}}}{1-e^{-L(k^2)^{1/2}}}$$
$$=-\frac{2N}{(2\pi)^{\omega+1}L}\frac{2\pi^{(\omega+1)/2}}{\Gamma\left[\frac{\omega+1}{2}\right]}\int_0^{\infty}dx\,x^{\omega}\ln\frac{1-e^{-L(x^2+g^2\sigma_c^2)^{1/2}}}{1-e^{-Lx}}\ . \tag{15}$$

Because Δ is finite at $\omega=0$ we take this limit, and we have

$$\Delta=-\frac{8\pi\lambda}{L^2g^2}\left[\frac{1}{24}-\frac{1}{4\pi^2}\int_{Lg\sigma_c}^{\infty}\frac{[x^2-(Lg\sigma_c)^2]^{1/2}}{e^x-1}dx\right]. \tag{16}$$

Since the divergent part of V_1 is only in the first term of V_1 and independent of the size L, the renormalization of $V(\sigma_c)$ can be done as in the Euclidean space-time case. To be explicit we perform the integration in (14) as

$$V_1=\frac{\lambda\sigma_c^2}{2\pi}\frac{\Gamma\left[1-\frac{\omega}{2}\right]}{\omega\left[1+\frac{\omega}{2}\right]}\frac{(g\sigma_c)^{\omega}}{2^{\omega}\pi^{\omega/2}}+\Delta$$
$$\underset{\omega\to 0}{\sim}\frac{\lambda\sigma_c^2}{4\pi}\left[\frac{2}{\omega}+\ln g^2\sigma_c^2+\text{const}\right]+\Delta\ , \tag{17}$$

and renormalize, following Coleman and Weinberg,[12] by demanding that (in the $L\to\infty$ limit)

$$\left.\frac{\partial^2 V}{\partial\sigma_c^2}\right|_{\sigma_0}=1\ . \tag{18}$$

Then we have

$$\frac{V}{\sigma_0^2}=\tfrac{1}{2}y^2+\frac{\lambda}{4\pi}y^2(\ln y^2-3)-\frac{8\pi\lambda}{B^2}\left[\frac{1}{24}-\frac{1}{4\pi^2}\int_{B|y|}^{\infty}\frac{[x^2-(By)^2]^{1/2}}{e^x-1}dx\right]$$
$$=\frac{V_{\rm GN}}{\sigma_0^2}+\Delta(y)\ , \tag{19}$$

where $V_{\rm GN}$ is the effective potential of the original Gross-Neveu model, $\Delta(y)=\Delta/\sigma_0^2$, $y=\sigma_c/\sigma_0$, and $B=Lg\sigma_0$.

For the analysis of the symmetry behavior we recall that $V_{\rm GN}$ has a minimum at $y=\pm\exp(1-\pi/\lambda)$, and a local maximum at $y=0$, and thus it gives rise to spontaneous symmetry breaking. The correction term $\Delta(y)$ is a monotonically decreasing even function such that $\Delta(0)=0$ and $\Delta(y)\to-\pi\lambda/(3B^2)$ as $|y|\to\infty$. Therefore $y=0$ remains a local maximum, and the qualitative feature of the potential does not change. Since $\Delta(y)\to 0$ as $L\to\infty$ as it should be, for a large L the symmetry behavior is almost identical to the original Gross-Neveu model.

For a small L (or B) we make an approximation of $\Delta(y)$, as

$$\Delta(y)\simeq-\frac{\lambda}{\pi}y^2\frac{1}{e^{B|y|/2}-1}\ . \tag{20}$$

In the extreme limit $B\to 0$, $\Delta(y)$ approaches $-2\lambda|y|/(\pi B)$ and the minimum of the potential occurs at

$$|y_M| = e^{1-\pi/\lambda} e^{2/B} , \tag{21}$$

which is larger than that of $V_{\rm GN}$ by the large factor $\exp(2/B)$. Therefore, the location of the minimum is much influenced by the size L. In the extremely small L, the position of the minimum moves far out so that the original value of the Gross-Neveu model is almost irrelevant. We confirmed this by a numerical integration of (19). The solid curves in Fig. 1 represent the potentials for three B values with $\lambda=\pi$. The original Gross-Neveu potential ($B=\infty$) is also given by the dashed curve in Fig. 1(a). We choose a relatively large B ($B=10$) [Fig. 1(a)] to show that the position of the minimum does not change substantially for large B. Figure 1(c) shows that for a very small B ($B=0.01$) the location is indeed far out from the original value $y=1$. We also include $B\simeq 1.76$ [Fig. 1(b)] for the purpose of comparison because this B is the critical value in the twisted fermion field model.

IV. TWISTED FERMION FIELD

Nontrivial space-time topology may give rise to twisted field configurations which was introduced by Isham,[5] and have been studied by various authors.[3,4,6] In our case ($R^1 \times S^1$) there is one twisted fermion field which is antiperiodic in the space coordinate. For the study of the effective potential, all we need is to replace the momentum $k_1=(2\pi/L)m$ by $k_1=(2\pi/L)(m+\frac{1}{2})$ in (12) so that we have

$$V_1^{\rm twist} = -\frac{N}{(2\pi)^{\omega+1}L} \int d^{\omega+1}k \sum_m \ln\left[1+\frac{g^2\sigma_c^2}{k^2+\left[\frac{2\pi}{L}\right]^2(m+\frac{1}{2})^2}\right] . \tag{22}$$

Making use of the relation

$$\sum_{m=-\infty}^{\infty} \ln\left[1+\frac{b^2}{a^2+(m+\frac{1}{2})^2}\right] = \sum_{m=-\infty}^{\infty}\left[\ln\left[1+\frac{(2b)^2}{(2a)^2+m^2}\right]-\ln\left[1+\frac{b^2}{a^2+m^2}\right]\right] , \tag{23}$$

we can express $V_1^{\rm twist}$ in terms of V_1 as

$$V_1^{\rm twist}(g,N) = V_1\left[2g,\frac{N}{2}\right] - V_1(g,N) , \tag{24}$$

where $V_1(g,N)$ is the one-loop portion of the effective potential of the ordinary fermion field. After renormalization we obtain

$$\begin{aligned}\frac{V^{\rm twist}}{\sigma_0^2} &= \tfrac{1}{2}y^2+\frac{\lambda}{4\pi}y^2(\ln y^2-3)-\frac{4\pi\lambda}{B^2}\left[\frac{1}{24}-\frac{1}{4\pi^2}\int_{2B|y|}^{\infty}\frac{(x^2-4B^2y^2)^{1/2}}{e^x-1}dx\right]\\ &\quad+\frac{8\pi\lambda}{B^2}\left[\frac{1}{24}-\frac{1}{4\pi^2}\int_{B|y|}^{\infty}\frac{(x^2-B^2y^2)^{1/2}}{e^x-1}dx\right]\\ &= \tfrac{1}{2}y^2+\frac{\lambda}{4\pi}y^2(\ln y^2-3)+\Delta^{\rm twist}(y) .\end{aligned} \tag{25}$$

It would be illuminating to rewrite $\Delta^{\rm twist}(y)$ in a form similar to $\Delta(y)$ as

$$\Delta^{\rm twist}(y) = \frac{\pi\lambda}{6B^2}-\frac{2\lambda}{\pi B^2}\int_{B|y|}^{\infty}\frac{(x^2-B^2y^2)^{1/2}}{e^x+1}dx \tag{26}$$

and

$$\Delta(y) = -\frac{\pi\lambda}{3B^2}+\frac{2\lambda}{\pi B^2}\int_{B|y|}^{\infty}\frac{(x^2-B^2y^2)^{1/2}}{e^x-1}dx . \tag{27}$$

We notice that $\Delta^{\rm twist}(y)$ is a monotonically increasing even function with $\Delta^{\rm twist}(0)=0$ and $\Delta^{\rm twist}(\infty)=\pi\lambda/6B^2$. The important difference between $\Delta^{\rm twist}(y)$ and $\Delta(y)$ is their opposite sign, which makes the symmetry-breaking pattern quite distinct. For the ordinary case the effective potential always has a minimum at $\sigma_c\neq 0$ irrespective of the size L. But for the twisted field there exists a critical length L_c such that only for $L>L_c$ the potential has a minimum at $\sigma_c\neq 0$, while for $L<L_c$ the origin ($\sigma_c=0$) becomes a minimum and the symmetry is not broken.

In order to estimate the critical length we examine the effective potential near the origin. The first derivative is

$$\begin{aligned}\frac{\partial V^{\rm twist}}{\partial y} &= y\left[1+\frac{\lambda}{2\pi}(\ln y^2-2)+\frac{2\lambda}{\pi}\int_0^{\infty}\frac{dx}{(x^2+B^2y^2)^{1/2}(e^{(x^2+B^2y^2)^{1/2}}+1)}\right]\\ &\simeq y\left\{1+\frac{\lambda}{2\pi}(\ln y^2-2)+\frac{2\lambda}{\pi}\left[-\frac{\gamma}{2}-\tfrac{1}{4}\ln\left[\frac{By}{\pi}\right]^2\right]\right\} ,\end{aligned} \tag{28}$$

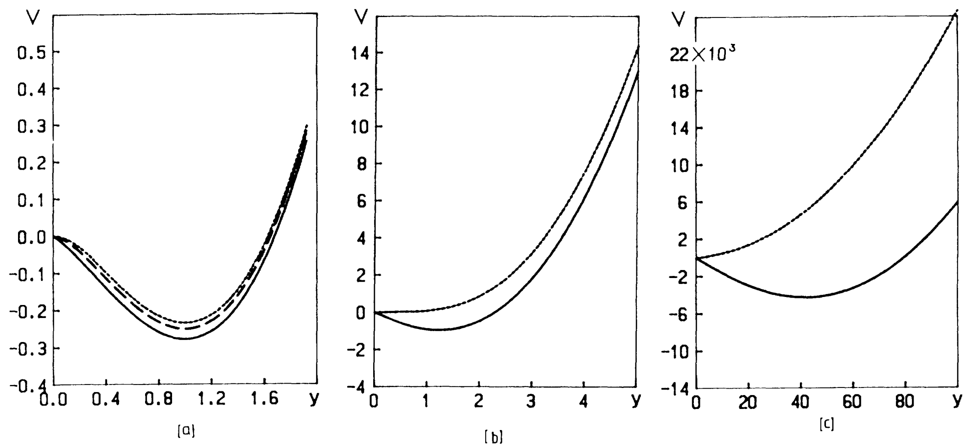

FIG. 1. The effective potential $V(y)$ for the untwisted fermion field (solid curve) and the twisted fermion field (dotted curve). Three typical values of the parameter B are treated: (a) $B=10$, (b) $B=1.76$, and (c) $B=0.01$. The potential of the original Gross-Neveu model is shown for comparison by the dashed curve in (a).

where $\gamma=0.577\ldots$ is Euler's constant, and we make an approximation of the integral for small y following Dolan and Jackiw.[13] The origin ($y=0$) is always an extremum and its nature is determined by the sign of the quantity inside the parentheses. The critical value is then

$$B_c=L_c g\sigma_0=\pi e^{\pi/\lambda-(1+\sigma)} . \tag{29}$$

The dotted lines in Fig. 1 show $V^{\text{twist}}/\sigma_0^2$ for three B's: namely, $B=10$ [Fig. 1(a)], $B=1.76=B_c$ [Fig. 1(b)], and $B=0.01$ [Fig. 1(c)], where we choose $\lambda=\pi$. This numerical computation confirms our analysis, and it also shows that the origin is a global minimum and there is no other local minimum in the case $B<B_c$.

Physical relevance of the twisted fermion field is not so clear to the authors. One may study electron fields in a closed one-dimensional system such as a loop in a laboratory or a cosmic string.[14] Another interesting view point arises if we exchange the space and the time coordinate, then we have the Gross-Neveu model at finite temperature.[8,9] Indeed our effective potential (22) is just the one of the finite-temperature model, and the critical length corresponds to the critical temperature. Naturally our analysis is similar to Ref. 9, but we emphasize the relations between the ordinary and the twisted fields.

As examined closely by Ford[4] and also by Avis and Isham[5] there is a problem with taking twisted and untwisted spinors as distinct theories. It is that under certain Lorentz transformations one type of spinor can be transformed into the other. They suggested that twisted and untwisted spinors be regarded as different topological sectors of the same theory; i.e., both would be summed over in virtual processes.

In our model we can easily evaluate the effective potential of the summed version by using the relation (23) and find that

$$V_1^{\text{sum}}(g,N)=V_1\left[2g,\frac{N}{2}\right] . \tag{30}$$

After renormalization we obtain

$$\frac{V_1^{\text{sum}}}{\sigma_0^2}=\tfrac{1}{2}y^2+\frac{\lambda}{2\pi}y^2(\ln y^2-3) -\frac{4\pi\lambda}{B^2}\left[\frac{1}{24}-\frac{1}{4\pi^2}\int_{2B|y|}^{\infty}\frac{(x^2-4B^2y^2)^{1/2}}{e^x-1}\right] . \tag{31}$$

Since the relation (30) clearly shows that the qualitative nature of the potential is very similar to the untwisted case, we do not draw them in the figures.

V. DISCUSSION

We have explicitly shown that the divergent parts of the vacuum polarization and effective potential of the Gross-Neveu model in $R^1\times S^1$ is independent of the size L and therefore the renormalization procedure is the same as the Minkowski space-time case. For the spontaneous symmetry breaking in the ordinary fermion field case we found that the existence of the potential minimum is independent of L but its position is strongly dependent on it. In the twisted field case there exists a critical length L_c, and only for $L>L_c$ the symmetry breaking occurs. For a smaller size the potential has minimum at the origin and no symmetry breaking occurs.

Finally we make a comment on the zero-mode approximation in the dimensional compactification of Kaluza-Klein theories. For example, in the five-dimensional space-time with $M_4\times S^1$ topology, a field $\phi(x,y)$ is expanded in a Fourier series and only the first term is retained as

$$\phi(x,y)=\sum_{n=0}^{\infty}\phi^{(n)}(x)e^{iny/L}\underset{L\to 0}{\sim}\phi^{(0)}(x) , \tag{32}$$

where $x\in M_4$, $y\in S^1$, and L is the size of the extra dimension. Whether this zero-mode approximation is valid and consistent was questioned by Duff and Toms.[15] They found that the zero-mode approximation is not valid if the theory is quantized. Our model is another example of the general features found by them.

In our simplified $R^1 \times S^1$ topology we can explicitly examine the effects of the neglected higher modes on the potential in the $L \to 0$ limit. In the zero-mode approximation we expect that

$$V_{KK} = \tfrac{1}{2}\sigma_c{}^2 - \frac{\lambda |\sigma_c|}{Lg} . \tag{33}$$

This should be compared with the V of the Gross-Neveu model in $R^1 \times S^1$ in the $L \to 0$ limit, which is

$$V \underset{L\to 0}{\sim} \tfrac{1}{2}\sigma_c{}^2 - \frac{\lambda}{4\pi}\sigma_c{}^2(\ln\Lambda^2 - \ln g^2\sigma_c{}^2 + 1) - \frac{\lambda}{Lg}|\sigma_c| , \tag{34}$$

where the last term is the approximation of Δ in the $Lg \to 0$ limit. Comparing these we found that the whole quantum correction (one-loop level) is missing in the zero-mode approximation, and therefore renormalization is quite altered. However, the symmetry breaking and minimum position of the potential is rather dominated by the term $\lambda |\sigma_c| /(Lg)$ (since $L \to 0$), and therefore the Kaluza-Klein ansatz is sensible here. Our simple model suggests that the renormalization parameters are influenced by the missing higher modes, but symmetry analysis remains valid when the zero-mode approximation is taken.

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