

Dynamical Systems and Ergodic Theory

Mark Pollicott
University of Manchester

Michiko Yuri
University of Sapporo



CAMBRIDGE
UNIVERSITY PRESS

CONTENTS

Introduction	ix
Preliminaries	xi
1. Conventions	xi
2. Notation	xi
3. Pre-requisites in point set topology (Chapters 1-6)	xi
4. Pre-requisites in measure theory (Chapters 7-12)	xii
5. Subadditive sequences	xiii
6. References	xiii
Chapter 1. Examples and basic properties	1
1.1. Examples	1
1.2. Transitivity	2
1.3. Other characterizations of transitivity	4
1.4. Transitivity for subshifts of finite type	5
1.5. Minimality and the Birkhoff recurrence theorem	6
1.6. Commuting homeomorphisms	8
1.7. Comments and references	9
Chapter 2. An application of recurrence to arithmetic progressions	11
2.1. Van der Waerden's theorem	11
2.2. A dynamical proof	12
2.3. The proofs of Sulemma 2.2.2 and Sublemma 2.2.3	15
2.4. Comments and references	17
Chapter 3. Topological entropy	19
3.1. Definitions	19
3.2. The Perron-Frobenius theorem and subshifts of finite type	23
3.3. Other definitions and examples	26
3.4. Conjugacy	30
3.5. Comments and references	32
Chapter 4. Interval maps	33
4.1. Fixed points and periodic points	33
4.2. Topological entropy of interval maps	37
4.3. Markov maps	39

4.4. Comments and references	44
Chapter 5. Hyperbolic toral automorphisms	47
5.1. Definitions	47
5.2. Entropy for Hyperbolic Toral Automorphisms	49
5.3. Shadowing and semi-conjugacy	52
5.4. Comments and references	55
Chapter 6. Rotation numbers	57
6.1. Homeomorphisms of the circle and rotation numbers	57
6.2. Denjoy's theorem	60
6.3. Comments and references	64
Chapter 7. Invariant measures	65
7.1. Definitions and characterization of invariant measures	65
7.2. Borel sigma-algebras for compact metric spaces	65
7.3. Examples of invariant measures	67
7.4. Invariant measures for other actions	69
7.5. Comments and references	71
Chapter 8. Measure theoretic entropy	73
8.1. Partitions and conditional expectations	73
8.2. The entropy of a partition	76
8.3. The entropy of a transformation	79
8.4. The increasing martingale theorem	82
8.5. Entropy and sigma algebras	84
8.6. Conditional entropy	86
8.7. Proofs of Lemma 8.7 and Lemma 8.8	87
8.8. Isomorphism	88
8.9. Comments and references	89
Chapter 9. Ergodic measures	91
9.1. Definitions and characterization of ergodic measures	91
9.2. Poincaré recurrence and Kac's theorem	91
9.3. Existence of ergodic measures	93
9.4. Some basic constructions in ergodic theory	94
9.4.1. Skew products	95
9.4.2. Induced transformations and Rohlin towers	95
9.4.3. Natural extensions	96
9.5. Comments and references	97
Chapter 10. Ergodic theorems	99
10.1. The Von Neumann ergodic theorem	99
10.2. The Birkhoff theorem (for ergodic measures)	102
10.3. Applications of the ergodic theorems	106
10.4. The Birkhoff theorem (for invariant measures)	111
10.5. Comments and references	112

Chapter 11. Mixing Properties	113
11.1. Weak mixing	113
11.2. A density one convergence characterization of weak mixing	114
11.3. A generalization of the Von Neumann ergodic theorem	116
11.4. The spectral viewpoint	118
11.5. Spectral characterization of weak mixing	120
11.6. Strong mixing	122
11.7. Comments and reference	123
Chapter 12. Statistical properties in ergodic theory	125
12.1. Exact endomorphisms	125
12.2. Statistical properties of piecewise expanding Markov maps	126
12.3. Rohlin's entropy formula	133
12.4. The Shannon-McMillan-Brieman theorem	134
12.5. Comments and references	137
Chapter 13. Fixed points for homeomorphisms of the annulus	139
13.1. Fixed points for the annulus	139
13.2. Outline proof of Brouwer's theorem	144
13.3. Comments and references	146
Chapter 14. The variational principle	147
14.1. The variational principle for entropy	147
14.2. The proof of the variational principle	147
14.3. Comments and reference	152
Chapter 15. Invariant measures for commuting transformations	153
15.1. Furstenberg's conjecture and Rudolph's theorem	153
15.2. The proof of Rudolph's theorem	153
15.3. Comments and references	159
Chapter 16. Multiple recurrence and Szemerédi's theorem	161
16.1. Szemerédi's theorem on arithmetic progressions	161
16.2. An ergodic proof of Szemerédi's theorem	162
16.3. The proof of Theorem 16.2	163
16.3.1.(UMR) for weak-mixing systems, weak-mixing extensions and compact systems	163
16.3.2.The non-weak-mixing case	165
16.3.3.(UMR) for compact extensions	165
16.3.4.The last step	165
16.4. Appendix to section 16.3	166
16.4.1.The proofs of Propositions 16.3 and 16.4	166

16.4.2. The proof of Proposition 16.5	171
16.4.3. The proof of Proposition 16.6	171
16.4.4. The proof of Proposition 16.7	172
16.4.5. The proof of Proposition 16.8	173
16.4.6. The proof of Proposition 16.9	175
16.5. Comments and references	176
Index	177