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DYNAMICS AND BUCKLING OF SANDWICH PANELS WITH STEPPED FACINGS

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Sandwich panels have been developed to either produce lighter structures capable of carrying prescribed loads or increase the load-carrying capacity subject to limitations on weight. In these panels, facings carry bending and in-plane loads while the core functions similarly to the web of a beam, mostly resisting transverse shear. Improvements in the load-carrying capacity of sandwich panels can be achieved through modifications in their geometry, boundary conditions, and material distribution. One of the methods recently considered by the authors is based on using facings with a step-wise variable thickness that increases at the critical region of the structure.[1] It was illustrated that the strength of a sandwich panel can be considerably enhanced using such stepped facings, without a detrimental increase of the weight. The present paper expands the study of the feasibility of the stepped-facing sandwich panel concept concentrating on three structural problems, i.e. a possible improvement in stability, changes in the natural frequencies, and forced dynamic response to the explosive blast. It is illustrated that the stepped-facing design can improve stability of the panel and its response to blast loading. However, fundamental frequencies of stepped-facing panels decrease compared to those in their conventional equal-weight counterparts. Such decrease is detrimental in the majority of engineering applications representing a limitation of stepped-facing panels. Nevertheless, the usefulness of the stepped-facing design is proven in the problems of bending, stability, and blast loading. Numerous examples presented in the paper validate our suggestion that the combination of a relatively simple manufacturing process and an improved structural response of sandwich panels with stepped facings may present the designer with an attractive alternative to conventional sandwich structures.

Keywords: Sandwich panel; vibration; buckling; blast load; variable stiffness.

1. Introduction

Sandwich panels have to satisfy a number of requirements to their strength, stiffness, stability, and dynamic properties. These requirements can be met by enhancing the stiffness of the panel. This stiffness can be improved using one of the following methods:

(1) Stiffer facing material with higher strength;
(2) Functionally graded composite facings and core with variable in-plane or through the thickness properties (see for example, review by Birman and Byrd[2]); and
(3) Panels with a nonconventional geometry that includes continuously variable thickness of panels, stepped-facing thickness, or the use of internal ribs.

In particular, rib-reinforced facings were considered by Birman *et al.*[3] While this approach could present advantages in certain situations (e.g., if the location of a concentrated force is known in advance), in the case of a distributed static or dynamic loading such facings are of questionable value since the height of the ribs is limited by the thickness of the core (the ultimate expression of such design would be a web core). The performance of sandwich panels with a tapered core has been extensively studied by Gupta and Sharma,[4] Paydar and Libove,[5] Libove and Lu,[6] Lu and Libove,[7] Lu,[8] Peled and Frostig,[9] and Vel *et al.*[10,11] While tapered panels represent an interesting concept, its realization in engineering is problematic due to a variable thickness of the structure. On the other hand, designs using facings of stepped thickness and maintaining constant overall thickness of the panel can easily be realized in applications.

The panel with stepped facings, such as that shown in Fig. 1, can be manufactured using honeycomb or polymeric core sections of various depths. In the present paper, we concentrate on the "global" response of the panel. The analysis of local stresses at the junction between sections of unequal facing thickness that may affect local strength is outside the scope of the paper. Note that these local stresses may become essential in the case of polymeric cores, while the relevant effect for

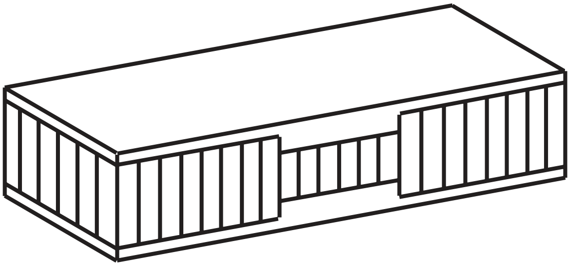


Fig. 1. Honeycomb sandwich panel with stepped facings. The central section of the panel has thicker facings providing a higher local stiffness.

honeycomb-core panels considered in the numerical analysis in this paper should be less prominent. In the case of a polymeric core local stresses at the junction between adjacent sections should be determined numerically since the analytical solution fails to detect the local stress concentration at the edge formed by the sections of the facing. In a honeycomb core manufactured from sections of different depths, local stresses are due to an abrupt change in the stiffness of adjacent sections of the facings and the associated discontinuity of the overall panel stiffness. The analysis yielding such stresses could be conducted modeling each section of the panel with a constant facing thickness by a first-order or higher-order shear deformation theory and applying the continuity conditions for deformations and stresses at the junction of the sections. The complexity of such analysis justifies using a three-dimensional (3D) finite element method.

The previous paper[1] illustrated that panels with stepped facings experience significantly smaller stresses compared to equal-weight counterparts with a constant thickness of facings. In addition, the applicability of the first-order shear deformation theory to the analysis of representative sandwich panels with commercially available honeycomb cores was considered. The comparison of analytical results obtained by the first-order theory with the 3D finite element analysis illustrated that the first-order theory remains sufficiently accurate at the width-to-thickness ratio equal to or exceeding 20. This conclusion is limited to the case of distributed loads since a 3D state of stress in the vicinity of concentrated forces precludes the use of the first-order theory in relevant problems.

The present paper is concerned with the analysis of potential advantages of stepped-facing sandwich panels in stability and dynamic problems. The latter problems include fundamental frequencies of the panel, i.e. free vibrations, and forced response to explosive blast. The desired outcome is an increase in the buckling load and fundamental frequency and a speedy reduction of blast-induced dynamic deflections and stresses compared to the equal-weight baseline structure. As is shown in the paper, while stability and the forced dynamic response of the panel can be improved by adopting stepped facings, the fundamental frequency invariably decreases reflecting on limitations of the stepped-facing design.

2. Problem Formulation and Numerical Analysis

The previous analysis[1] has already illustrated the potential of stepped-facing panels in static bending applications. In the present paper, the study is extended to eigenvalue problems, i.e. buckling and natural frequencies, as well as forced response of the panel to dynamic loading. Variables that can be changed to achieve the desired outcome include the thickness and size of the regions with increased thickness, their distribution over the planform of the panel and the lamination of these regions. The material of the panel is not varied, though it is possible to consider adding additional stiffer layers to the critical regions of the panel. The weight of the panel with stepped facings should be either equal to that of the conventional panel that is being replaced or it may differ from it by a small prescribed amount. Details of the finite element model employed in the stability and free vibration analyses are described in the previous paper.[1] The model of the panel developed using Nastran-2005 employed facings modeled by 2D shell elements, while the core consisted of 3D solid elements. In addition, a different finite element model was used in some examples concerned with free vibrations as well as for the blast response study to ensure that the conclusions were not influenced by modeling peculiarities. This model is described in the relevant section of the paper. Notably, both models yielded practically identical results in several representative examples.

In the present paper, the authors utilized the Monte Carlo method in buckling and vibration problems sampling various sizes of stepped-facing sections subject to the constraint on the total weight of the panel. The samples varied by the number of layers per facing and by the size and location of the sections of constant facing thickness. The facings considered in the numerical analysis were cross-ply laminated, i.e. we did not attempt to optimize the angle of lamination of individual layers. In all examples illustrated below, the dimensions of the section with thicker facings correspond to the most desired outcome for the objective function based on a number of trials varying dimensions and thickness of panel sections subject to the weight constraint.

In the stability problem, the objective function was the buckling load that was maximized subject to the constraints against wrinkling failure and weight of the panel. The core shear failure[12] did not occur in any of the samples considered in the study. In the free vibration problem, the goal was to increase the fundamental frequency. In numerous engineering applications, such increase effectively “shifts” the spectrum of natural frequencies above the range of driving frequencies of anticipated loads. In the problem of forced dynamic response, the goal was the reduction of deflections and stresses in the panel as well as shortening the duration of large-amplitude motion, while maintaining its weight close to that of the baseline conventional panel.

In addition to the numerical analysis, the analytical formulation for dynamic and stability problems for simply supported panels with stepped facings is presented in Appendix. The formulation utilizes the first-order shear deformation theory that has been proven accurate in bending problems for panels with the side-to-thickness ratio equal to or exceeding 20.[1] Buckling loads and fundamental frequencies generated in representative examples using the analytical solution were found in close agreement with finite element results.

2.1. *Buckling analysis*

Conventional and stepped-facing panels considered in the buckling study had cross-ply carbon/epoxy facings (T300/5208) and aluminum honeycomb hexagonal core with the foil thickness equal to 0.001 inch (see Table 1 for details). The equivalent properties of the core are available from the manufacturer (www.hexcel.com).

Table 1. Properties of carbon-epoxy employed in the facings and effective properties of hexagonal aluminum honeycomb core.

Material	E_1 (psi)	E_2 (psi)	E_3 (psi)	ν_{12}	G_{12} (psi)	G_{23} (psi)	G_{13} (psi)	Density (lb-s^2/in^4)
Carbon/epoxy (T300/5208)	2.63×10^7	1.49×10^6	1.49×10^6	0.28	1.04×10^6	5.28×10^5	1.04×10^6	1.50×10^{-4}
Aluminum honeycomb	3.77×10^4	4.52×10^4	1.85×10^5	0.33	28×10^3	28×10^3	70×10^3	6.75×10^{-6}

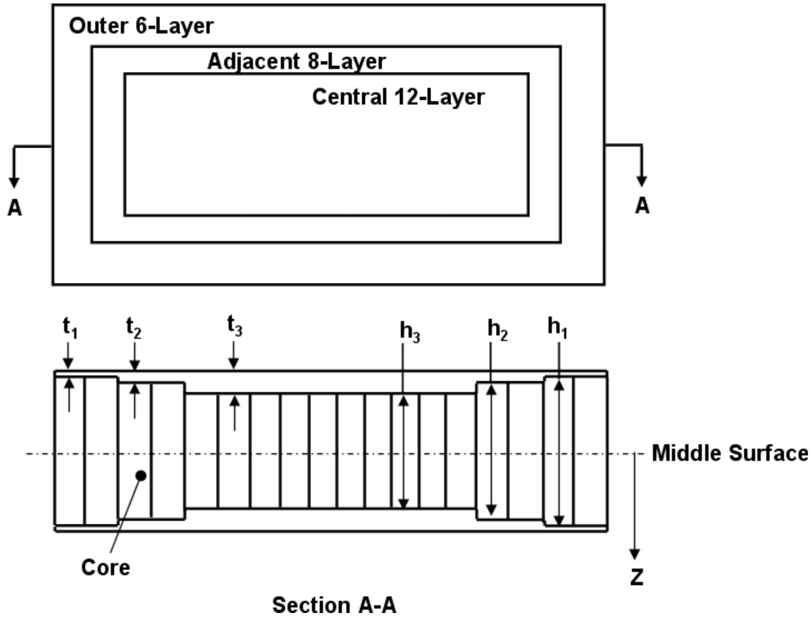


Fig. 2. Schematic illustration of a sandwich panel with three-stepped facings.

Rectangular panels considered in the study were 254 mm (10″) long and 127 mm (5″) wide. The total thickness of the panels was equal to 25.4 mm (1″), unless indicated otherwise. The facings of the baseline conventional panel consisted of eight layers, each of them 0.127 mm (0.005″) thick. Stepped-facing panels consisted of three sections as shown in Fig. 2. The facings of the "nearly optimum" (based on the Monte Carlo simulations) stepped-facing panel included the central 12-layer 7.5″ × 2.3″ section, the adjacent eight-layer section with the outer dimensions 8.5″ × 3.5″ and the outer six-layer section. While the stepped-facing panel was 5.7% heavier than the conventional counterpart, the effectiveness of the replacement was estimated through the comparison of the merit factor of two panels defined as

$$M = \frac{\left(\frac{N_{\mathrm{cr}}}{P}\right)_s}{\left(\frac{N_{\mathrm{cr}}}{P}\right)_c}, \tag{1}$$

where the subscripts refer to the stepped (s) and conventional (c) panels, respectively, N_{cr} is the critical force (or stress resultant) and P is the weight of the corresponding panel. The stepped-facing design is advantageous if $M > 1$.

The first example depicts the effect of stepped facings on the buckling load of a large aspect ratio panel simply supported along short edges and free along long edges that are parallel to the direction of the compressive load. As follows from Fig. 3, sandwich panels with the aspect ratio equal to 2 compressed along free long edges deform similarly to beams. The mode shape of buckling has one half-wave in the axial direction, while twisting and bending in the planes parallel to short edges are negligible. The buckling load was increased by 12.64% because of the use of stepped facings corresponding to the merit factor equal to 1.066.

The improvement in the buckling load of the same sandwich panel in the case where it is simply supported along all edges and subject to a uniform compression along the long edges was reported in the previous paper.[1] The stepped design resulted in an increase of the buckling load by 11.7%, the merit factor being equal to 1.057. The mode shapes of buckling of the stepped-facing and conventional panels were nearly identical.

In the present paper, we expand the spectrum of stability problems to examine whether the stepped-facing design is advantageous for different aspect ratios and boundary conditions. As an example, simply supported and clamped square sandwich panels are shown in Figs. 4 and 5, respectively. The length and width of the panels are equal to 254 mm (10″), while their thickness is only 0.2″ (thicker panels showed a tendency to wrinkling). Each facing of the conventional panel consisted of eight cross-ply layers. In the stepped design, the central section that had 12 layers per facing was 7.5″ long and 4.6″ wide, while the outer dimensions of the adjacent section with eight layers per facing were 8.5″ and 7.0″, respectively. The outer section adjacent to the boundaries had six layers per facing.

While the buckling load increased in both simply supported and clamped stepped-facing panels, the merit factor in the case of simple support (Fig. 4) was only 0.946

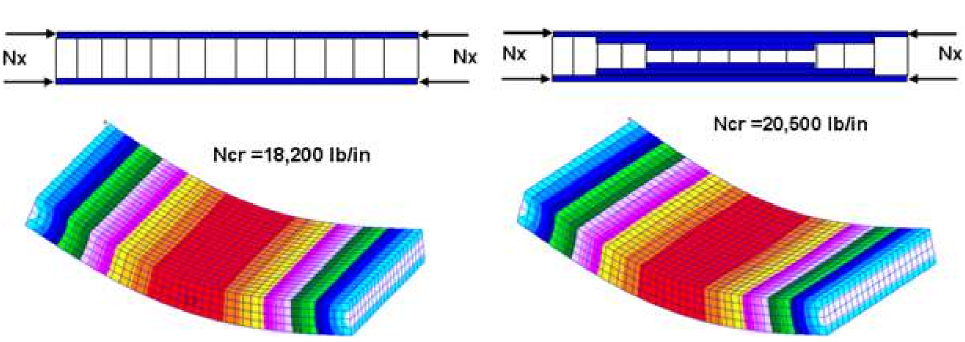


Fig. 3. Buckling of a rectangular panel with free long edges and simply supported short edges (left case: conventional panel and right case: stepped-facing panel). The buckling load is increased by 12.64% in the stepped-facing design.

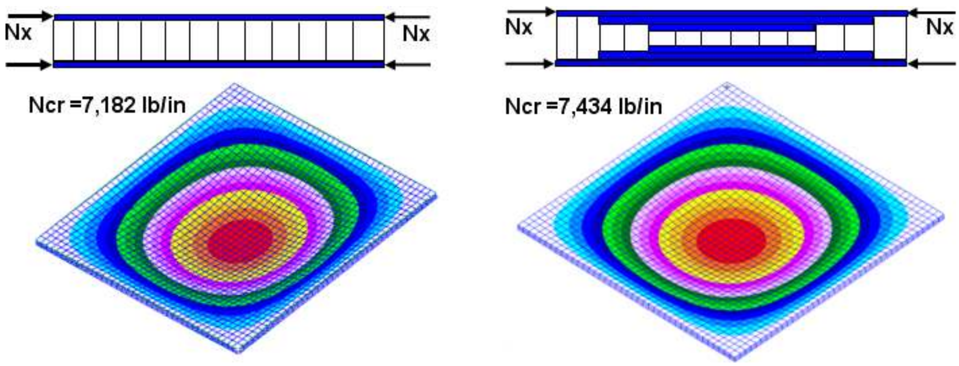


Fig. 4. Buckling of simply supported square conventional (left) and stepped (right) panels. While the buckling load is slightly increased, the added weight of the stepped panel makes it inefficient.

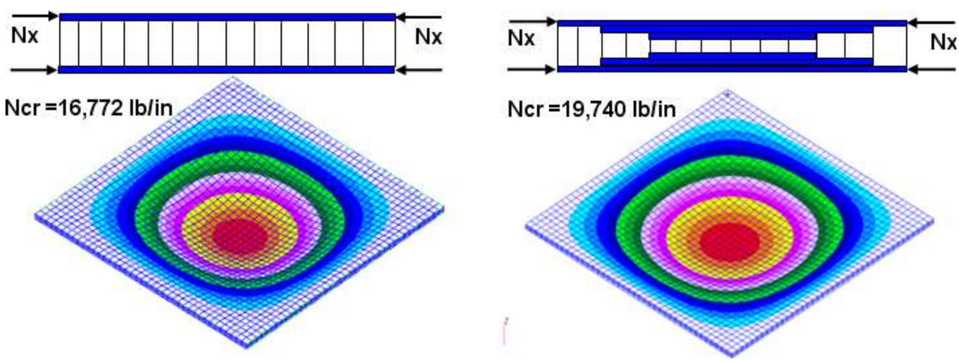


Fig. 5. Buckling of clamped square conventional (left) and stepped (right) panels. The merit factor (load-to-weight ratio) is improved justifying the stepped-facing design.

implying that the stepped design was inefficient. In the contrary, in clamped panels (Fig. 5) this factor reached the value of 1.075, indicating improved buckling capacity. The mode shapes of buckled conventional and stepped-facing panels were almost identical as is observed in Figs. 4 and 5.

In conclusion, the improvement in the buckling capacity depended on geometry and boundary conditions. Although buckling loads increased in all situations, the added weight made some of the stepped-facing designs inefficient. This conclusion is different from the results observed in the bending analysis that illustrated a significant improvement in the load-carrying capacity achieved in all stepped-facing panels compared to conventional counterparts.[1]

2.2. *Free vibration analysis*

The analysis was conducted comparing the same conventional and stepped geometries as in the buckling study. While a variety of panel thicknesses were considered, the following results are presented for the case where the total thickness was equal to 1″ (the exception is Fig. 8 below illustrating that the trend observed in the study is unaffected by the panel thickness). It appeared impossible to increase the natural frequencies, including the fundamental frequency, using stepped facings. For example, the comparison between simply supported conventional and stepped-square panels in Fig. 6 clearly illustrates that while the mode shape of motion corresponding to the fundamental frequency was practically unaltered in the stepped-facing design, the fundamental frequencies significantly decreased in the stepped-facing panel. The same conclusion follows from the analysis of rectangular panels with the aspect ratio equal to 2 depicted in Fig. 7. A significant decrease in the fundamental frequencies of stepped-facing panels compared to conventional panels is observed for both boundary conditions, i.e. simple support and clamping, in both rectangular and square panels. Numerous alternative stepped-facing designs illustrated the same tendency, i.e. a smaller fundamental frequency than that in baseline conventional panels.

The reason for such behavior is evident if we account for two conflicting effects of stepped facings on the natural frequencies. Increasing the thickness of the facings results in a higher stiffness of the central part of the panel that should increase the fundamental and higher frequencies. However, this increase in the stiffness is

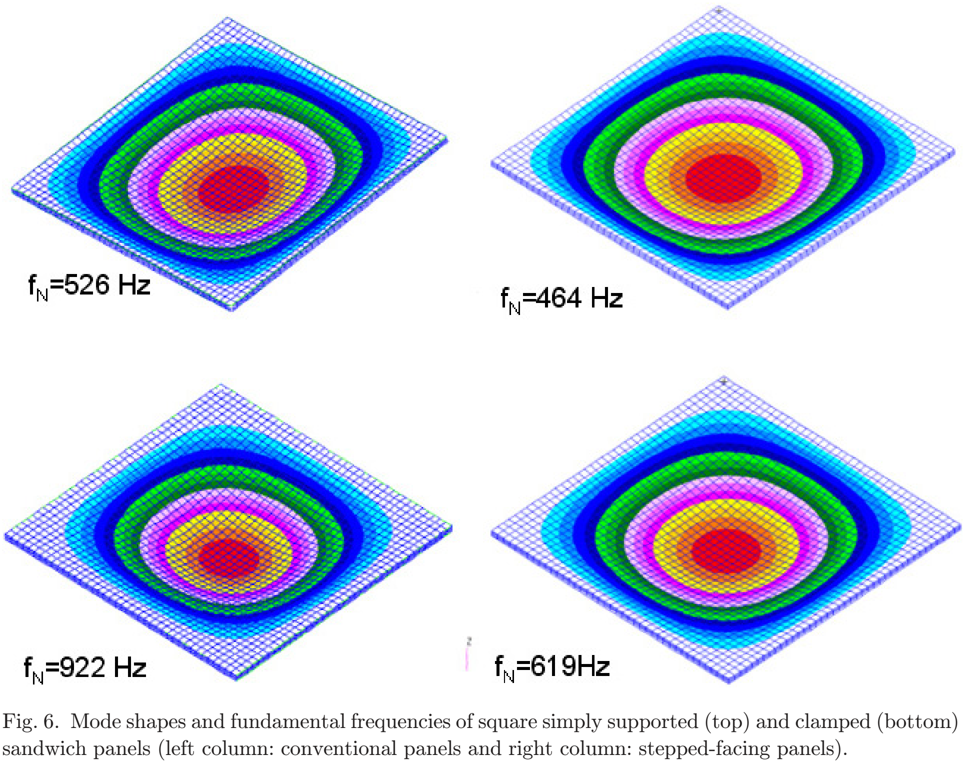


Fig. 6. Mode shapes and fundamental frequencies of square simply supported (top) and clamped (bottom) sandwich panels (left column: conventional panels and right column: stepped-facing panels).

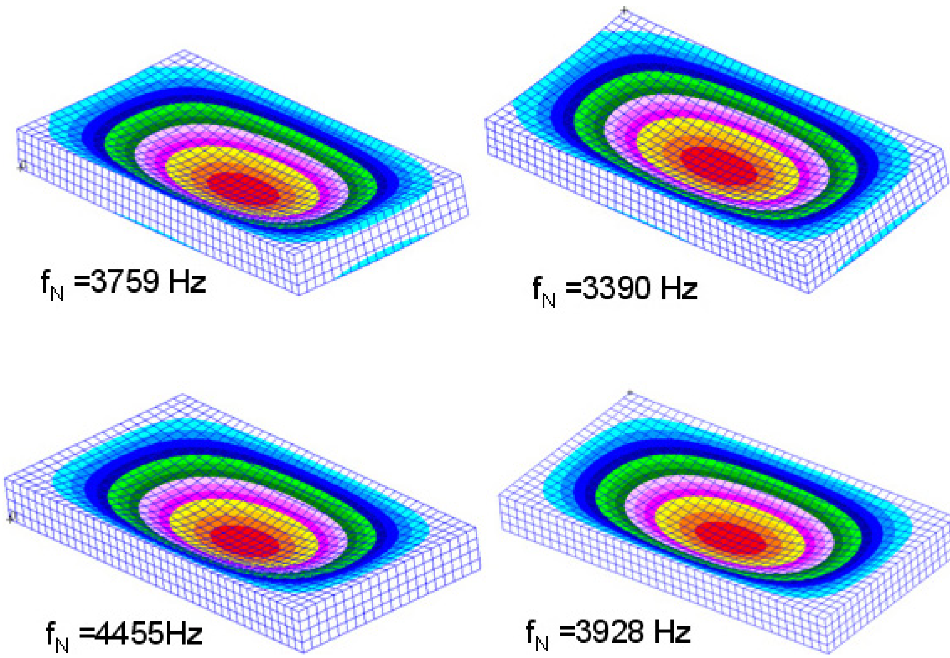


Fig. 7. Mode shapes and fundamental frequencies of rectangular simply supported (top) and clamped (bottom) sandwich panels (left column: conventional panels and right column: stepped-facing panels).

achieved at the cost of a larger mass concentration in the center. Apparently, the latter effect resulting in a lower natural frequency is dominant.

The effect of the total thickness of the sandwich panel on the fundamental frequency was similar for both conventional and stepped-facing panels of all aspect ratios as is reflected in Fig. 8. The geometry of the facings of these panels did not

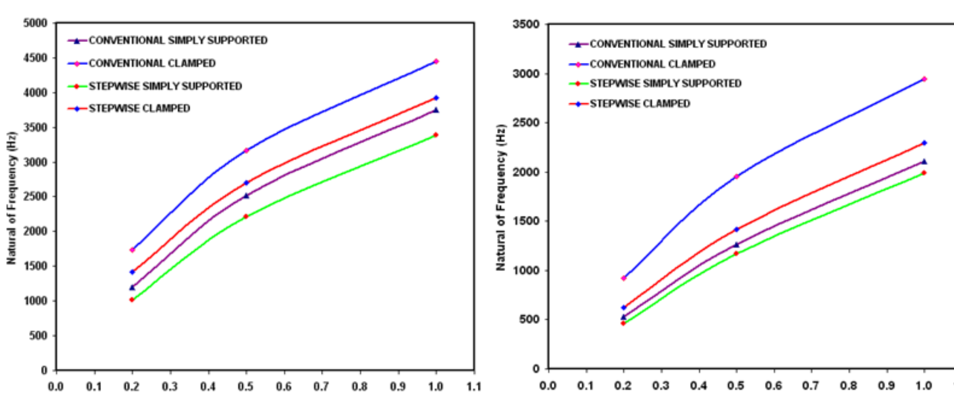


Fig. 8. Effect of sandwich panel thickness on its fundamental frequencies (left case: rectangular $10'' \times 5''$ panel and right case: square $10'' \times 10''$ panel).

vary (i.e., both conventional and stepped facings were identical to those described above). The variable thickness of the panels was achieved by varying the thickness of the core. The results shown in Fig. 8 are consistent, i.e. while a larger thickness results in a higher fundamental frequency, the use of stepped facings lowers the frequency of the panel compared to the conventional design of the same overall thickness.

Elevated temperature results in a reduction in the stiffness of the facing and core materials as well as in-plane thermally induced stresses. In the present study, the in-plane expansion of facings was not constrained, so that thermal stress resultants were absent. Accordingly, the effect of temperature reflected in Fig. 9 for simply supported rectangular panels was confined to reduced stiffness. As follows from Fig. 9, this effect is little affected by the design of the facings, i.e. the corresponding curves are almost parallel.

The effect of stepped facings on the first four natural frequencies and mode shapes of sandwich panels was considered for rectangular panels similar to those described above. The only difference was related to the number of layers in cross-ply sections of stepped facings modified to ensure a reduced weight of the stepped-facing panel compared to the conventional counterpart. Accordingly, while the central section had 12 layers, the adjacent section was constructed of nine layers, while the outer section consisted of five layers. Using such design the mass of the stepped panel was reduced by 3.2%. The finite element model utilizing ABAQUS 6.9 employed 3D brick element for both the facings and the core. Details of these elements are presented in Table 2.

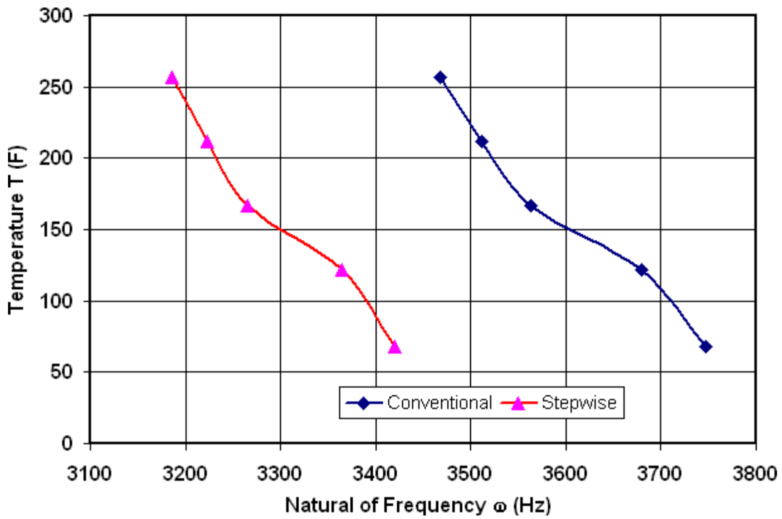


Fig. 9. Effect of temperature on the fundamental frequency of simply supported rectangular conventional and stepped-facing panels.

Table 2. Finite elements used in the modeling of facings and core.

Stepped and conventional sandwich component	Element type	No. of elements stepped panel/ conventional panel
Carbon/epoxy(T300/5208) Face plate	SC8R: An eight-node quadrilateral in-plane general-purpose continuum shell, reduced integration with hourglass control, finite membrane strains for free vibration. S4R: A four-node doubly curved thin or thick shell, reduced integration with hourglass control, finite membrane strains for blast loading.	7000/10000
Aluminum honeycomb	C3D8R: An eight-node linear brick, reduced integration, hourglass control	5000/6250

Table 3. The first four natural frequencies of conventional and stepped-facing rectangular panels.

	Simply supported		Clamped	
	Stepped	Conventional	Stepped	Conventional
Mode 1 (Hz)	3096.2	3341.6	4172.1	4647.3
Mode 2 (Hz)	4691.3	5083.1	6037.6	6561.0
Mode 3 (Hz)	6091.9	6291.1	7630.8	8157.3
Mode 4 (Hz)	6884.6	7322.7	8592.6	9231.1

The comparison of the first four frequencies is shown in Table 3. As is evident from this table, the trend of reduced frequencies in stepped-facing panels that was observed for fundamental frequencies is preserved for higher natural frequencies. While higher frequencies exhibit a large reduction in absolute terms, the relative reduction in the higher frequencies appears to be smaller compared to that for the fundamental frequency. The change from the conventional to stepped-facing design did not alter mode shapes corresponding to higher natural frequencies as is reflected in Fig. 10 (the change was already shown negligible for the fundamental mode shapes).

As a result of the analysis of numerous sample cases, some of them shown in this section, it was concluded that stepped facings cannot be employed to increase the fundamental and higher frequencies of sandwich panels. Obviously, if the goal is to modify the fundamental frequency by decreasing it compared to that of the conventional panel, stepped facings can be very effective. Such requirement may be entertained if there is a large gap between the fundamental and higher natural frequencies, i.e. the resonance with the latter frequencies in stepped-facing panels can be avoided, while all natural frequencies decrease. Then, a reduction in the fundamental frequency necessary to avoid the resonance does not trigger large vibrations due to the resonance with higher natural frequencies. This situation is plausible if the frequencies of applied loads are deterministic, contrary to the case where they can vary within a broad spectrum.

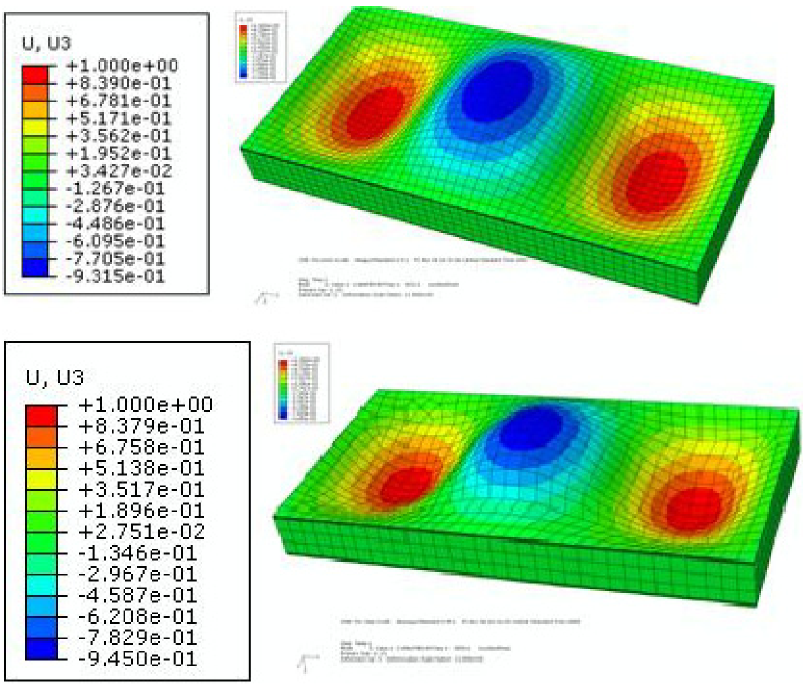


Fig. 10. Mode shapes of vibrations corresponding to the fourth natural frequency of clamped panels (top case: conventional panel and bottom case: stepped panel).

2.3. *Dynamic response: case of blast loading*

Based on encouraging results for the static bending response of sandwich panels with stepped facings that could withstand higher loads than conventional designs,[1] it was anticipated that these panels will also show a higher resistance against transverse dynamic loads. This was illustrated on the example of panels subject to an explosive blast overpressure uniformly distributed over the surface of the panel and varying with time according to the Friedlander equation[13–16]:

$$p(t) = p_0\left(1 - \frac{t}{t_\mathrm{p}}\right)\exp\left(-\frac{\Lambda t}{t_\mathrm{p}}\right), \tag{2}$$

where t is time, p_0 is the peak overpressure, t_p is a positive phase duration of the pulse, and Λ is an empirical decay parameter. Representative values for the parameters specified above are $t_\mathrm{p} = 0.1\,s$ and $\Lambda = 2$. The variation of the blast overpressure with time according to Eq. (2) is shown in Fig. 11.

Sandwich panels considered in the report were square ($10'' \times 10''$) and clamped around the perimeter (all displacements and rotations were prevented). The thickness of the panels was equal to $1''$. Each facing of the conventional panel consisted of seven cross-ply layers. The optimum design of the panel with stepped facings would be symmetric about the center; however, in the simulations, the panel consisted of the

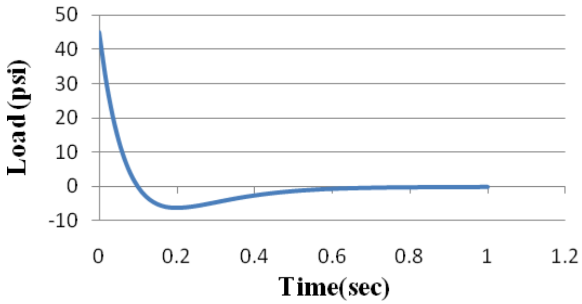


Fig. 11. Variation of the blast overpressure with time.

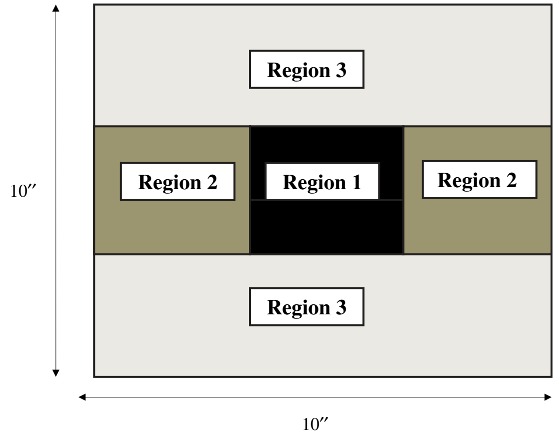


Fig. 12. Stepped-sandwich panel used in blast simulations.

central thicker section (Region 1 in Fig. 12) and four sections of unequal thickness (Regions 2 and 3 in Fig. 12). Using unequal thickness in Regions 2 and 3 was dictated by the objective of keeping the total weight of the stepped-facing panel as close to the weight of the conventional panel as possible. As followed from representative examples, such asymmetric design of the facings about the panel center resulted in negligible variations of maximum deflections and stresses from those in the symmetric design.

Four designs of stepped-facing panels considered in simulations are reflected in Table 4. The weight of the panels varied from that of the conventional baseline panel by less than 5%. All three cases of Type 1 panels shared the same size of the central region and varied in the number of layers allocated to each panel region (the comparison between these panels could be referred to as "thickness optimization"). Type 2 panel was considered to elucidate a possible beneficial effect of varying the size of the central Region 1 (the comparison between Type 1 and Type 2 panels could be referred to as "size optimization").

Variations of maximum deflections with time in response to the blast overpressure shown in Fig. 11 are illustrated in Fig. 13. As follows from this figure, forced

Table 4. Designs of stepped-facing panels used in blast simulations.

Type	Case	Dimensions, inch (Region 1)	Number of plies in Regions 1, 2, and 3 (see Fig. 12).
Type 1	1	4×4	12,8,6
	2	4×4	14,7,6
	3	4×4	18,7,5
Type 2	1	5×5	16,6,4

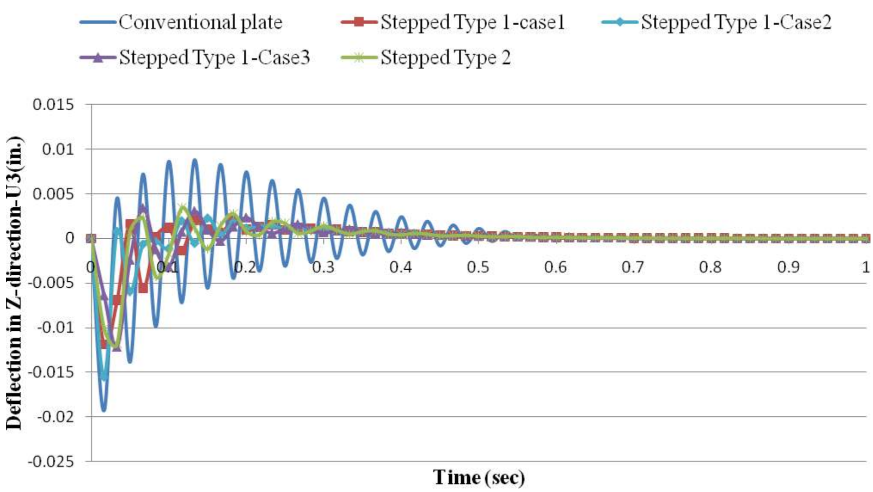


Fig. 13. Variations of maximum deflections of sandwich panels subject to blast loading with time.

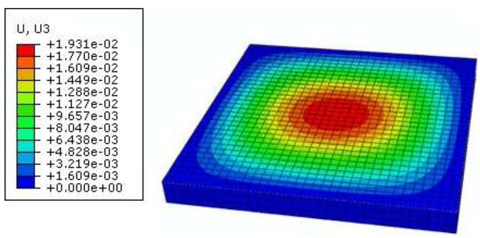
vibrations of all panels eventually decay to zero, but transient response varies significantly from panel to panel. In particular, the conventional panel has markedly higher deflections, while all stepped-facing designs result in smaller transient vibration amplitudes as well as a much shorter duration of transient motion. The latter observation may have important consequences in problems of sustained forced vibrations or periodic shocks reducing fatigue tendencies of the panel. In the present case of blast loading, the problem is dynamic failure, rather than fatigue, i.e. the reduction of amplitudes is more important. Among stepped-facing designs, Type 1 Case 1 and Type 2 panels exhibit the smallest amplitudes.

In addition to the analysis of deflections, the maximum stresses in the facings of panels subjected to blast overpressure are compared in Table 5. While Type 1 Case 1 panel has the smallest stresses among all panels, it is remarkable that all stepped-facing panel designs result in a drastic reduction of stresses compared to the conventional panel. Therefore, while stepped-facing design is counterproductive in the free vibration problem where the goal is an increase of the fundamental frequency, it is effective in the problem of blast loading.

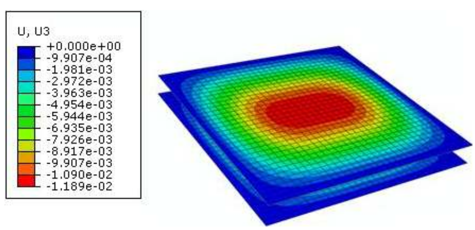
As follows from Fig. 14, the mode shapes of the deformation of conventional and stepped-facing panels experiencing blast loading are almost identical. The maps of

Table 5. Maximum stresses and deflections in conventional and stepped-facing panels subject to blast loading.

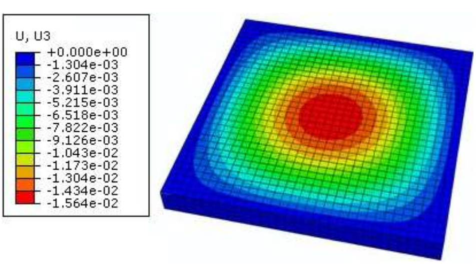
Type	Case	Weight (lb)	Number of plies (Regions-1, 2, 3)	Max. σ_x (psi)	Max. σ_y (psi)	Max. τ_{xy} (psi)	Max. W (in)
Conventional		0.6482	7,0,0	3.37E + 04	1.26E + 03	4.71E + 02	1.93E−02
Stepped	Type 1 Case 1	0.6725	12,8,6	1.24E + 04	4.39E + 02	3.12E + 02	1.19E−02
Stepped	Type 1 Case 2	0.677	14,7,6	2.65E + 04	1.11E + 03	4.00E + 02	1.56E−02
	Type 1 Case 3	0.6792	18,7,5	1.96E + 04	9.08E + 02	3.23E + 02	1.22E−02
Stepped	Type 2	0.6759	16,6,4	2.09E + 04	9.86E + 02	3.70E + 02	1.19E−02



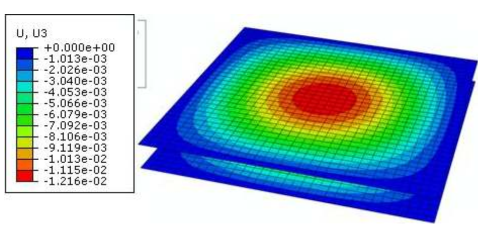
Conventional panel



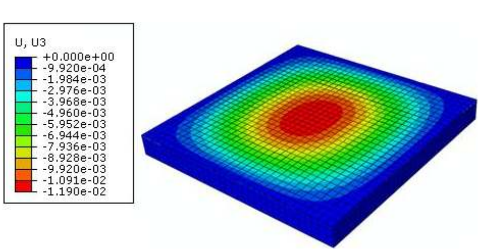
Stepped panel Type 1, case 1



Stepped panel, Type 1, case 2



Stepped panel, Type 1, case 3



Stepped panel, Type 2

Fig. 14. Mode shapes of deflections of conventional and stepped panels under blast loading (the shapes shown correspond to the maximum deflection).

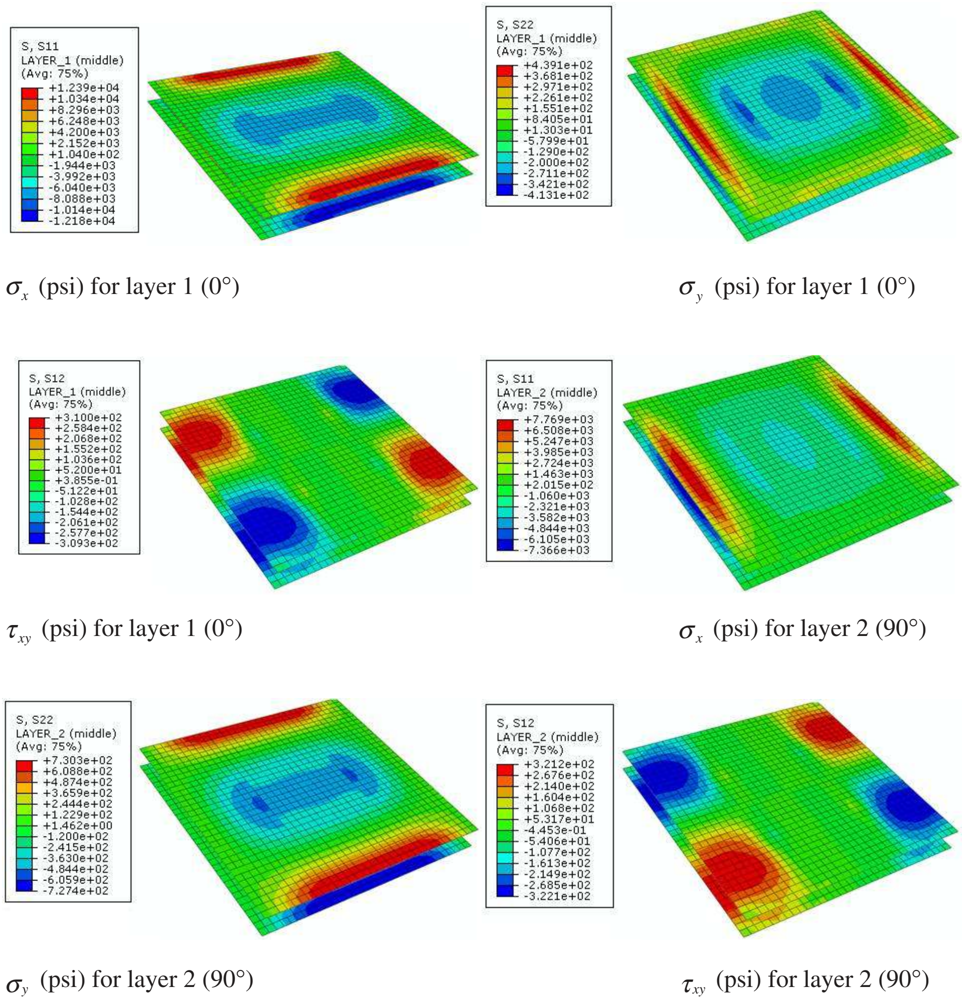


Fig. 15. Stresses in two outer layers of the compressed facing of Type 1 Case 1 panel subject to blast loading.

the representative distribution of maximum stresses in the compressed facing of Type 1 Case 1 panel are shown in Fig. 15. The maximum stresses are observed in the central section, although it has a larger thickness. This point to a possible design improvement by redistributing the material further reinforcing the center of the panel.

3. Conclusions

The paper illustrates the potential advantages and areas of the possible application of sandwich panels with stepped facings developed to enhance the performance with a minimum effect on the weight of the structure. Following the previously shown improved performance of such panels under static pressure, the problems analyzed here include buckling, free vibrations, and the response to blast loading.

The buckling load of stepped-facing panels could be improved over that of their conventional counterparts, dependent on the boundary conditions and aspect ratio. However, such improvement quantified by the use of the merit factor was relatively small compared to the previously observed improvement in the case of static bending.

Natural frequencies of stepped-facing panels were smaller than those of conventional panels. The reason was attributed to a larger mass concentrated close to the center of stepped-facing panels where the increase in the stiffness required thicker facings. Apparently, the effect of a larger mass in the central section of the panel on its natural frequencies overwhelmed the influence of a locally higher stiffness. Such decrease in the frequency is usually undesirable since in many practical situations the engineer faces the demand to increase the fundamental frequency. The trend that prevailed for all geometries considered in the paper leads to the conclusion that stepped facings cannot be used to increase the fundamental frequency.

The analysis of the response to blast loading was most encouraging, as stepped facings resulted in a quick decay of forced vibrations and much smaller amplitudes of motion and stresses. Such results (decreased deflections and stresses in stepped-facing panels) are not surprising since they are in agreement with the conclusion previously obtained for panels subject to static pressure. Therefore, stepped facings, while being of limited advantage in buckling problems and ineffective in typical free vibration problems, are promising in cases where the panel is subject to transverse static or dynamic pressure.

The mode shapes of deformation of stepped-facing and conventional panels were nearly identical in both static and dynamic problems. The maps of stresses in stepped-facing and conventional panels were also closely matched, differing only in their magnitude.

The additional cost and technological complications involved in the production of stepped-facing panels are limited. Therefore, such panels present an attractive option in the situations where the panel is subject to static or dynamic pressure.

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Appendix: Dynamic and Stability Analyses of Simply Supported Sandwich Panels with Stepped Facings by the First-Order Shear Deformation Theory

The potential energy of a symmetrically laminated panel with cross-ply or multiple-layer angle-ply facings subject to lateral dynamic pressure $p(x, y, t) = P(t)\bar{p}(x, y)$

and to in-plane stress resultants N_x and N_y is given by

$$\Pi = \frac{1}{2}\sum_{i,j}\int_{x_i}^{x_{i+1}}\int_{y_j}^{y_{j+1}}\left[\begin{aligned} & D_{11}^{(ij)}\left(\frac{\partial\psi_x}{\partial x}\right)^2 + 2D_{12}^{(ij)}\frac{\partial\psi_x}{\partial x}\frac{\partial\psi_y}{\partial y} + D_{22}^{(ij)}\left(\frac{\partial\psi_y}{\partial y}\right)^2 \\ & \quad + D_{66}^{(ij)}\left(\frac{\partial\psi_x}{\partial y} + \frac{\partial\psi_y}{\partial x}\right)^2 \\ & \quad + kA_{55}^{(ij)}\left(\frac{\partial w}{\partial x} + \psi_x\right)^2 + kA_{44}^{(ij)}\left(\frac{\partial w}{\partial y} + \psi_y\right)^2 \end{aligned}\right] dydx$$
$$- \int_0^a\int_0^b pwdydx - \frac{1}{2}\int_0^a\int_0^b \left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2\right]dydx, \tag{A.1}$$

where w are deflections and ψ_x and ψ_y are rotations of cross sections in the xz and yz planes, respectively. The limits of integration refer to the coordinates of the ijth section of the panel that has a constant facing thickness. Furthermore, $A_{gg}^{(ij)}$ and $D_{gs}^{(ij)}$ are extensional and bending stiffnesses of the ijth section of the panel and k is a shear correction factor introduced to compensate for a difference between the actual (warped) shape of a deformed cross section and the first-order idealization assuming that the cross section remains plane.

The kinetic energy of the panel is

$$K = \frac{1}{2}\sum_{i,j}\int_{x_i}^{x_{i+1}}\int_{y_j}^{y_{j+1}}\left[m_{ij}\left(\frac{\partial w}{\partial t}\right)^2 + I_{ij}\left(\left(\frac{\partial\psi_x}{\partial t}\right)^2 + \left(\frac{\partial\psi_y}{\partial t}\right)^2\right)\right]dydx, \tag{A.2}$$

where

$$\hat{m}_{ij} = \int_z \rho(x,y,z)dz \quad I_{ij} = \int_z \rho(x,\, y,\, z)z^2dz, \tag{A.3}$$

$\rho(x, y, z)$ being the mass density of the facings or core. The terms given by Eq. (A.3) represent the inertial coefficients in the ijth section of the panel.

The deflections and rotations of a simply supported panel can be represented in double Fourier series that satisfy the boundary conditions

$$\begin{aligned} w &= \sum_m\sum_n W_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \\ \psi_x &= \sum_m\sum_n F_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \\ \psi_y &= \sum_m\sum_n P_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \end{aligned} \tag{A.4}$$

where the amplitudes of harmonics W_{mn}, F_{mn}, and P_{mn} are functions of time in dynamic problems or constants in the buckling problem.

The Lagrange equations of motion adapted to the present formulation are

$$\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{\partial K}{\partial W_{mn}}\right)+\frac{\partial \Pi}{\partial W_{mn}}=0\\
&\frac{\partial}{\partial t}\left(\frac{\partial K}{\partial F_{mn}}\right)+\frac{\partial \Pi}{\partial F_{mn}}=0\\
&\frac{\partial}{\partial t}\left(\frac{\partial K}{\partial P_{mn}}\right)+\frac{\partial \Pi}{\partial P_{mn}}=0
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
&\sum_{i,j}\sum_{k,r}\left[\hat{m}_{ij}\,\ddot{W}_{kr}A^{(ij)}_{mnkr}+kA^{(ij)}_{55}\left(\frac{m\pi}{a}\frac{k\pi}{a}W_{kr}+\frac{m\pi}{a}F_{kr}\right)C^{(ij)}_{mnkr}\right.\\
&\qquad\left.+kA^{(ij)}_{44}\left(\frac{n\pi}{b}\frac{r\pi}{b}W_{kr}+\frac{n\pi}{b}P_{kr}\right)D^{(ij)}_{mnkr}\right]\\
&\qquad-P(t)\int_0^a\int_0^b p(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dydx\\
&\qquad-\frac{\pi^2}{4}\left[\frac{m^2 b}{a}N_x+\frac{n^2 a}{b}N_y\right]W_{mn}=0
\end{aligned}$$

$$\sum_{i,j}\sum_{k,r}\left[\begin{aligned}
&I_{ij}\ddot{F}_{kr}C^{(ij)}_{mnkr}+\frac{m\pi}{a}\left(D^{(ij)}_{11}\frac{k\pi}{a}F_{kr}+D^{(ij)}_{12}\frac{r\pi}{b}P_{kr}\right)A^{(ij)}_{mnkr}\\
&\quad+D^{(ij)}_{66}\frac{n\pi}{b}\left(\frac{r\pi}{b}F_{kr}+\frac{k\pi}{a}P_{kr}\right)B^{(ij)}_{mnkr}\\
&\quad+kA^{(ij)}_{55}\left(\frac{k\pi}{a}W_{kr}+F_{kr}\right)C^{(ij)}_{mnkr}
\end{aligned}\right]=0 \tag{A.6}$$

$$\sum_{i,j}\sum_{k,r}\left[\begin{aligned}
&I_{ij}\ddot{P}_{kr}D^{(ij)}_{mnkr}+\frac{n\pi}{b}\left(D^{(ij)}_{22}\frac{r\pi}{b}P_{kr}+D^{(ij)}_{12}\frac{k\pi}{a}F_{kr}\right)A^{(ij)}_{mnkr}\\
&\quad+D^{(ij)}_{66}\frac{m\pi}{a}\left(\frac{k\pi}{a}P_{kr}+\frac{r\pi}{b}F_{kr}\right)B^{(ij)}_{mnkr}\\
&\quad+kA^{(ij)}_{44}\left(\frac{r\pi}{b}W_{kr}+P_{kr}\right)D^{(ij)}_{mnkr}
\end{aligned}\right]=0.$$

The coefficients $A^{(ij)}_{mnkr}$, $B^{(ij)}_{mnkr}$, $C^{(ij)}_{mnkr}$, and $D^{(ij)}_{mnkr}$ are given in Ref.1.

Equation (A.6) can be employed to determine the response of the panel to dynamic pressure that is an arbitrary time function (including blast loading). In the general case, Eq. (A.6) represents a system of ordinary differential equations that can be numerically integrated by one of the initial value methods. In the particular case where the applied pressure is a harmonic time function, Eq. (A.6) yields a closed-form solution. Other particular cases, i.e. buckling and free vibrations, can also be solved using Eq. (A.6). Similar to the case of static pressure considered in

Ref.1, it was observed that the analytical approach yields the closed-form solution for buckling and free vibration problems that practically coincides with the solutions generated by the finite element method as long as the side-to-thickness ratio remains equal to or larger than 20.

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